ZDM (2020) 52:843–858
https://doi.org/10.1007/s11858-019-01116-x

ORIGINAL ARTICLE

Math MOOC UniTo: an Italian project on MOOCs for mathematics teacher education, and the development of a new theoretical framework

Eugenia Taranto[1,2] · **Ferdinando Arzarello**[1]

Accepted: 5 December 2019 / Published online: 10 December 2019

Abstract
This paper focuses on MOOCs (massive open online courses), a fairly recent paradigm in e-learning educational projects. Despite the high dropout rate, and the impossibility of benefiting from the opportunities that bring with it a face-to-face dialogue, several factors make MOOCs a good option for ongoing teacher professional learning. The MOOCs on which we focus address mathematics teacher education. In particular, we illustrate our experience based on four MOOCs that we organized in Italy in the last 4 years within the *Math MOOC UniTo* project, the aim of which is the professional development of mathematics in-service teachers. In the paper we articulate the conceptual framework that has guided the design, data collection, analysis and interpretation of findings for the project. It is mainly based on the hybridization of the meta-didactical transposition (MDT) model with the instrumentation/instrumentalization processes of the instrumental approach and the network of knowledge of Connectivism. We offer this new framework (called MOOC-MDT) to stimulate discussion with colleagues in the mathematics education research community about ways in which it might be refined and extended, towards building a shared understanding of the process of mathematics teacher education through MOOCs.

Keywords MOOCs · Mathematics teacher education · Meta-didactical transposition · Instrumental approach · Connectivism · MOOC-MDT

1 Introduction

MOOC is an acronym that stands for massive open online course. They are courses designed for distance learning involving a large number of users. Historically, the term MOOC was introduced in 2008, conceived by American universities and some scholars such as Siemens, who theorized Connectivism, a new learning theory that emphasizes the role of the learner in the dynamics of knowledge and its capacity of establishing connections and networks between knowledge, experiences and interactions (Siemens 2005).

✉ Eugenia Taranto
eugenia.taranto@unict.it

Ferdinando Arzarello
ferdinando.arzarello@unito.it

[1] Department of Mathematics 'G. Peano', University of Turin, Turin, Italy

[2] Department of Educational Sciences, University of Catania, Catania, Italy

Since 2012 (the so-called "year of the MOOCs" for the USA) some of the most widespread platforms for delivering MOOCs (Udacity, Coursera, edX) became active with a huge numbers of participants. In Italy, MOOCs have grown and multiplied since 2013, with platforms (e.g., POK, Eduopen, Federica.EU, …) that could manage a large number of members (mainly university students). In addition to traditional teaching materials (resources for deepening the topics, exercise sets, etc.), many MOOCs started to provide video-lectures and interactive forums to support community interactions between students, professors and tutors (Taranto et al. 2017a). In that sense, a MOOC can be considered as a digital resource with many other digital resources within it (Loisy et al. 2019). After some years, the first critical reflections on MOOCs are circulating (Zutshi et al. 2013; Onah et al. 2014), which we can summarize as follows:

– The drop-out rate is very high: if a MOOC is a formative opportunity easy to access and open to many, the experience shows that it is not easy for everyone to complete the path.

– As to the method, many MOOCs are nothing more than simple online courses, open to access by a large number of users.

To this we add also that, despite their success, the emergence and use of MOOCs for teachers' education (TE) is still rare, especially in mathematics, and this phenomenon is not sufficiently studied (Taranto in press).

Within this background about the state of the art in MOOC activities, a group of researchers (among them, the two authors of this paper) and expert teachers,[1] in 2015 decided to launch a MOOC program for TE. It was called *Math MOOC UniTo* and was designed for in-service mathematics teachers of all school levels, basing on the Moodle platform "DI.FI.MA." (https://difima.i-learn.unito.it/). This group of about ten people constituted the team who took care of the MOOCs in each of their aspects: from the design to their concrete implementation and delivery. Since then, every year there is a MOOC course, which is concentrated on one of the main topics of the Italian Mathematics Curriculum.

For us as researchers, these MOOCs were the opportunity for studying them in the specific case of TE courses: as we have mentioned above, these are not so common. All of us in the MOOC team had long experience with face-to-face (f2f) courses for TE and had elaborated suitable lenses for analysing them, some of which we consider below in Sect. 2.3: hence it was very natural that we tried to ask ourselves what lenses we needed to focus the new kind of courses, because of the differences we perceived between the two types of experiences. This curiosity in time generated the research question for our investigations: (i) What theoretical framework can be considered for analysing mathematics teacher education through a MOOC?

We also observed that, both in the f2f and online training, generally the teachers in training acquired new knowledge (Shulman 1986; Ball and Bass 2003) and compared it with what they already had, generating possible evolutions in their teaching practices. In an online environment, however, there is a multitude of resources from which teachers can acquire new knowledge, not only from the materials that trainers make available, but also from those that are created during peer interactions. We therefore wondered: (ii) How can such a diverse environment be described?

Moreover, because of this multitude of resources, it seemed that in a MOOC teachers are subject to a process that goes in two directions: on the one hand it is the MOOC that with its resources enriches the teachers who participate in it, on the other hand also teachers have the opportunity to contribute to the resources that the MOOC offers by sharing their experiences. This leads to the question: (iii) How could such a process be formalised?

In the following we explain how we have answered such questions. We start by focusing on the existing literature on MOOCs and on mathematics teacher education. Later, we move on to the theoretical aspects that help to answer the research questions (i) and (ii). Thanks to the new terminology that we then have introduced into the theoretical framework, we reformulate research question (iii). Then, we explain the research context of the *Math MOOC UniTo* project and what data we have collected to carry out the analyses that allow us to show how the new theoretical framework works in practice.

2 Literature review

2.1 MOOC types

Literature recognizes two main types of MOOCs: Connectivist massive open online courses (cMOOCs) and eXtended massive open online courses (xMOOCs). cMOOCs are focused on the learning community and connections between members of the community across the web, rather than on course content or the instructor (Rodriguez 2012). These courses are structured to provide a minimum of centralised control or content, and to develop participants' ability to contribute to, and learn from, the digital network. xMOOCs, offered through platforms such as edX (from which they get the name), are focused on giving many students access to an online course within the same platform. The style of learning is also different; xMOOCs tend to privilege individual studying, while cMOOCs focus on networked learning across several web tools and services (Rodriguez 2012). The cMOOC/xMOOC distinction is usefully descriptive of two different trajectories of development. However, recent literature is beginning to move away from what is increasingly seen as a simplistic categorisation. MOOC pedagogy is rapidly evolving (Boyatt et al. 2014) and, as some researchers are beginning to note, what goes on in any given MOOC is no longer clearly determined by its 'x' or 'c' status. Each MOOC is profoundly shaped by its designers, teachers, platform and participants.

2.2 Interaction and learning in MOOCs

MOOCs typically provide little or no instructional support beyond the prepared videos and course materials posted by instructors. Due to their scale, even MOOCs with active instructors make it impossible to provide the level of instructional feedback and support that would be expected

[1] By experts, we mean in-service teachers who had participated in a second level Master's for mathematics educators in the Department of Mathematics 'G. Peano' of Turin University.

in smaller f2f or conventional online course settings. This problem of scale, however, presents a unique opportunity for social networking and the development of peer support networks to fill this instructional void (Kellogg et al. 2014). Wenger et al. (2011) describe social networks as "a set of relations, interactions, and connections… with affordances for learning, such as information flows, helpful linkages, joint problem solving, and knowledge creation" (p. 9). Kellogg et al. (2014) underline that even with technology as basic as a discussion forum, MOOCs can be leveraged to foster these networks and facilitate peer-supported learning that results in the process of knowledge construction. McAuley et al. (2010) note that MOOCs have the potential to "model and build collaborative networks of unprecedented size that transcend time and space" (p. 35). From their side, Johnson et al. (2008) observe that in e-learning environment interaction must be intentionally designed into the learning context or it is unlikely to result spontaneously. Meaningful interaction is not just sharing opinions and information, but should stimulate the learners' intellectual curiosity (Pear and Crone-Todd 2002).

2.3 Looking at the existing theories on mathematics TE and more

The emergence and use of MOOCs for TE is still uncommon, especially in mathematics. In fact, although there is a wide choice of many different topics, when looking specifically for a MOOC aimed at mathematics TE the range is limited (Aldon et al. 2017; Borba et al. 2017). Nevertheless, there is growing interest in MOOCs involving mathematics teachers as participants (see http://www.icme13.org/files/tsg/TSG_44.pdf). However, the specific intersection of MOOCs and TE is poorly researched. Our initiative to provide MOOCs for mathematics TE has been the first example in Italy. Our MOOCs have been designed for TE and are studied as objects of research.

Arzarello and colleagues' long experience of f2f TE gave rise to the *Meta-didactical transposition* model (MDT: Arzarello et al. 2014), which was first used to analyse the first MOOC of the *Math MOOC UniTo* project. However, from the very first observations during the delivery of the online course, it became clear that the MDT has limits, when used in this new context (Loisy et al. 2019). The work of Taranto's doctoral thesis, supervised by Arzarello, brought to light a new theoretical framework, which we called MOOC-MDT. It was born thanks to the possibility of having taken part intensely both in the MOOCs activities design and in the monitoring of the MOOCs during their delivery. MOOC-MDT aims at understanding the complexities of the learning trajectories (Simon 1995) of the participants in a MOOC. The participants are in-service mathematics teachers (we call them MOOC-teachers) and the mathematics teacher educators (MTEs) who are involved in the MOOC design and delivery, are researchers and expert teachers. With *learning trajectory*, Taranto (2018) means how these participants interact online, both with the platform and with each other. In particular, the term addresses if and how these interactions change participants' knowledge and beliefs, and generate perception of change in their practices. Taranto (2018) revised the MDT and re-elaborated it from a framework apt to describe f2f TE to a new one, suitable for describing the TE dynamics within a MOOC environment. To do that, she integrated MDT through a *hybridization process*[2] (Arzarello 2016) with the *instrumental approach* (henceforth IA: Verillon and Rabardel 1995) and *Connectivism* (Siemens 2005), obtaining what she called MOOC-MDT. Before illustrating it, we briefly recall the key point considered in these three theoretical frameworks.

2.3.1 MDT

It focuses on the *community of the researchers* and the *community of the teachers* that participate in a f2f educational course. Every involved teacher has her own didactical praxeology (Chevallard 1999), namely tasks, techniques, and justifying discourses that the teacher, with her questions and actions, develops in her teaching practices with her students in the class. According to Arzarello et al. (2014), teachers' *meta-didactical praxeologies* are tasks, techniques, and justifying discourses that each teacher develops during a TE course, thanks to the interaction and the reflections with the community of researchers, who prompt teachers with suitable questions, examples, problems in order to promote the discussion and possibly the evolution of the didactical praxeologies of the teachers. In fact, the researchers have the aim of transposing to teachers some innovative knowledge, skills, and teaching practices, according to the institutions (e.g., national curricula, textbooks, national assessment, …) and recent research. Such researchers' meta-didactical praxeologies may produce teachers' didactical praxeologies in their classes, which may evolve from those present at the start, to new ones, hopefully thereby improving teachers' professionalism. From the other side, also the reactions of the teachers may determine a change in the meta-didactical praxeologies of the researchers. The two change in time because of these reciprocal influences. In the chapter by Arzarello et al. (2014), the authors discussed an example from a TE program described by Sullivan (2008), where the

[2] With the hybridization, one considers a particular component of a theory. This is 'implanted' in another theory that, for this reason, will be hybridized: the old theoretical framework is so enriched, and the language as well. Of course, the implanted component must satisfy coherence issues with respect to the theory into which it is embedded.

similarities and differences between didactical and meta-didactical praxeologies were carefully pointed out.

When we compare this situation, typical of f2f courses, with what happens in a MOOC, a big difference is apparent. While in a f2f course teachers and researchers can discuss the materials and content that are presented from time to time, so that they can reflect on these possible evolutions of the didactical praxeologies, in a MOOC this is unlikely to happen, for the following reasons. First, because the materials and contents must all be prepared and defined before the MOOC begins. Second, the MTEs do not have the possibility of discussion with the MOOC-teachers. Everything happens online, more in an asynchronous than a synchronous way. A community of MTEs and MOOC-teachers does not tend to be formed, because of the distance work. For those reasons, we need a fresh model to analyse the interactions in a MOOC and their impact on teachers: as we illustrate below we can accomplish this aim by modifying suitably the MDT model according to the specific affordances of a MOOC.

2.3.2 Instrumental approach (IA)

In this framework, an artifact is an object that presupposes a purpose and consequently a subject capable of creative activity that can incorporate some knowledge in this activity. An instrument is the artifact joined to the utilization schemes of a user on the basis of her culture and experience. The *instrumental genesis* (Verillon and Rabardel 1995) is the process that leads from an artifact to an instrument. According to the direction, it can be described as follows:

- *Instrumentation* (from the artifact to the subject): it leads to the development or appropriation of utilization schemes which progressively constitute techniques that allow humans to solve given tasks efficiently;
- *Instrumentalization* (from the subject to the artifact): it progressively transforms the artifact for specific uses with the related utilization schemes. So, it is the adaptation of the artifact to human cognitive structures.

Let us consider a MOOC. As we mentioned, a MOOC is inhabited first by the MTEs, who are the designers. Subsequently it is opened and inhabited by the MOOC-teachers. In general, since we are dealing with large numbers, it is not easy to observe the relationship artifact-subject referred to every single participant. Of course, it can be done if one is interested in the study of individuals; but our intent is to describe the phenomenon in its generality, without excluding or favouring anyone. And above all our intent is to describe it in its dynamic evolution.

We begin by observing that a MOOC is created to pursue specific purposes, in this case to accomplish a TE. In it the MTEs have inserted various digital resources that are supposed to communicate their training intentions at a distance. Hence the MOOC presupposes a purpose, and consequently, intelligence capable of creative activity that can grasp the knowledge that the MTEs want to transpose by incorporating it inside the MOOC. At this stage, we can consider the MOOC to be an artifact. Once it is opened to the MOOC-teachers (namely, when they start to explore it, select some resources, interact with others), it becomes an instrument for each of them. It is worth wondering how this evolution from artifact to instrument takes place. Everyone follows the MOOC in total autonomy. However, what the MOOC-teachers do on the platform has weight on how they interface and make use of the MOOC. Mathematics teachers' interaction with different resources has been theorized in various ways (e.g., Gueudet and Trouche 2009; Pepin et al. 2013). However, all of these studies show that such interaction is a participatory two-way process, of mutual adaptation, in which teachers are influenced by the resources, and at the same time the design and use of the resources is influenced by the teachers. To account for this two-way process in a MOOC, we need a theoretical concept that must take into account the interactions that individuals make individually with the MOOC. However, several individuals use the MOOC at the same time. Moreover, the individuals are not only influenced by the MOOC resources, but also by how many other individuals are interacting with these resources at the same time. The goal is to describe a process that is certainly a two-way process, but also iterated and intertwined in a very dynamic and complex way. The IA does not seem to be able to do it exhaustively. Therefore, there is the need to add another fragment from another theory.

2.3.3 Connectivism

In this framework, the notions of *personal knowledge* and *learning* are closely connected to the network theory. *Personal knowledge* is a particular kind of network, whose nodes are any entity that can be connected to another node (information, data, images, ideas, …), while an arc is a connection, a relationship between two nodes (Siemens 2005). *Learning* is a continuous process of building, developing, self-organizing knowledge—understood as a network (Siemens 2005). Therefore, learning not only adds new nodes, but also connects existing nodes and gives sense to these connections. The personal knowledge can be understood as an evolving network: the knowledge at a given moment corresponds to a timely conformation of the network, while the act of learning, of increasing knowledge, corresponds to the process by which the network expands.

It might seem that there is an 'abuse' of language with respect to the use of the term 'learning' for teachers. In fact, it is not the learning process that typically takes place in a classroom with students. Learning is understood in a

connectivist sense. It is not only a 'literal' learning of new things, rather it means to be able to see differently concepts that were already known: reflecting, thinking again, integrating them under a different perspective (Taranto 2018), and in this way, improving professionalism. Teachers' learning is therefore an expansion of their own network of knowledge, which is possible through the sharing of practices and didactical theories. This is in line with what was observed in the MDT. In fact, the expansion of the network of knowledge corresponds to the researchers and teachers' praxeologies that can evolve.

Protagonists are as follows: the MTEs, the MOOC-teachers and the MOOC, each with its own network of knowledge. According to Siemens (2005) "knowledge may reside in non-human appliances" (p. 5), so we can say that the nodes of the MOOC's network of knowledge are all the uploaded resources. The individuals' network of knowledge (MTEs and MOOC-teachers) evolves because of new connections within it that emerge when the individuals use the MOOC resources; and also, when they act in a specific context (the MOOC and their daily environment). Instead, the factors supporting the genesis of new connections within the MOOC's networks are the individuals' interactions within it. It is interesting, therefore, to focus on the study of the interactions that take place within a MOOC.

3 MOOC-MDT: a new theoretical framework

Considering the i*nstrumental genesis* from the IA and the *network of knowledge* from the Connectivism, Taranto (2018) made a hybridization (Arzarello 2016) of them with the MDT, obtaining the so-called MOOC-MDT, which we illustrate below, also using some figures.

In the design phase, the MOOC is a world inhabited by the MTEs. They want to transpose some ideal praxeologies in order to generate a (possible) evolution of the didactical praxeologies of the MOOC-teachers. So, the MTEs upload some resources in the MOOC and then wait for the enrolled people to start to access and live the results. At this stage, a MOOC can be considered as an *artifact,* namely a static set of materials. The *MOOC-artifact* has its own network of knowledge (the green one in Fig. 1): its nodes are the used content, ideas, images and videos; the connections are the links between their node pairs. The MOOC-teacher also has its own network of knowledge (which we imagine with orange nodes, Fig. 1).

When the MOOC-teachers enter in the MOOC, also for them it is a *MOOC-artifact* at the beginning. As soon as a MOOC-teacher begins even simply to think of considering

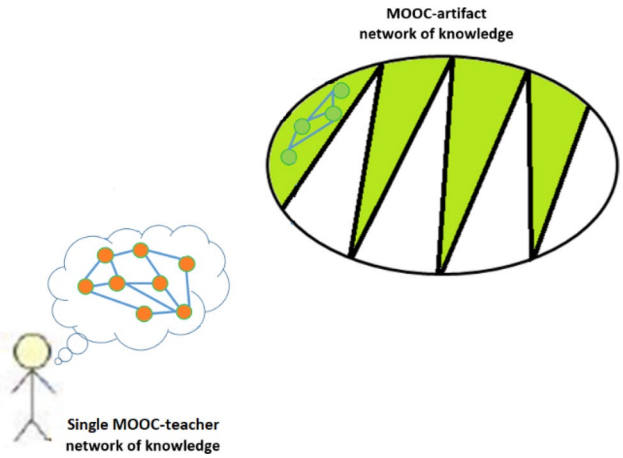


Fig. 1 Schematic idea of the MOOC-artifact network of knowledge

one resource rather than another, her network of knowledge and also the MOOC are subject to an evolution. Remember what happens in IA: if a subject applies proper utilization schemes, the artifact becomes an instrument for him. Here it is similar, but more complex, as we now describe.

When the MOOC is ready, it is opened in order to accommodate the entry of the MOOC-teachers. So, when a MOOC module is activated, it dynamically generates a complex structure that we call *ecosystem*, namely, "all the connections (exchange of materials, experiences and personal ideas/points of view) put in place by participants of an online community thanks to the technological tools through which they interact with each other, establishing connections within the given context" (Taranto et al. 2017b, p. 2481).

The network of knowledge of *MOOC-ecosystem* is composed not only of the MOOC-artifact network, but also of the contribution that each participant (remember that their number is massive) gives to this network (in fact it is green and orange, Fig. 2), exchanging and sharing material, ideas, thoughts, experiences. This network is not absolutely static, but develops as a dynamic structure, which is apparently chaotic, hence difficult to control in a timely manner.

When a MOOC-teacher *learns*, actually modifies her network of knowledge drawing on the ecosystem network (in fact her network of knowledge starts to be green and orange, Fig. 2), the MOOC-ecosystem becomes an instrument for her. The *MOOC-instrument* exists not only in the perspective of the use of the MOOC-artifact by the individual (when she explores the materials), but especially when the individual is part of the MOOC-ecosystem (when she takes advantage of

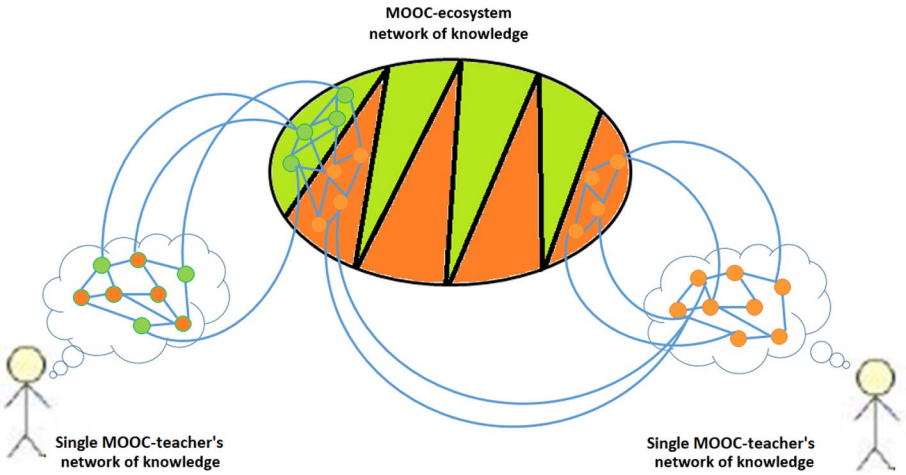


Fig. 2 Schematic idea of the MOOC-ecosystem network of knowledge

comments from other individuals or when she shares something of her own). Here lies the concept of self-organization of own network of knowledge.

Within the chaos[3] of the MOOC-ecosystem, each participant creates connections between her nodes and those of the MOOC network of knowledge. Each participant appropriates the resources made available in the MOOC, through self-organization processes. However, due to the massiveness that characterizes a MOOC, it is not possible to immediately identify the utilization schemes put in place by individuals. Everything happens in fractions of seconds with tens and tens of people at the same time. For this reason, it is more reasonable to consider the totality of the utilization schemes put in place, looking at what we call the MOOC-ecosystem.

The connections that trigger the ecosystem are linked to specific activities that the MOOC-teachers have to carry out within the MOOC, according to the instructions given by the MTEs. The MOOC-teacher has to solve tasks, through techniques, and properly justify the solution. In fact, she has to look at the proposed material, share her thoughts through sharing tools, and experience the MOOC activities. These tasks are not predetermined, depending on the time, approach and depth with which each single MOOC-teacher addresses them. The techniques are the ways in which the MOOC-teacher extends and modifies her network of knowledge, drawing on that of the ecosystem, and influencing it in turn (thus affecting all other MOOC-teachers). Therefore, the *MOOC-ecosystem*'s network of knowledge is dynamic: it evolves as the MOOC-artifact's network, thanks to the participants' contributions. Also, the individual's network of knowledge evolves as a personal self-organization (Siemens 2005, p. 4) of the ecosystem. The process of transformation from artifact to instrument (Verillon and Rabardel 1995) is here reinterpreted by the evolution from artifact to ecosystem/instrument and Taranto et al. (2017b) call it a "*double learning process*". It has the following components, intertwined and self-feeding each other (Fig. 3):

(i) *Instrumentation/self-organization* (from the ecosystem to the individual = E → I): process by which the MOOC-ecosystem network of knowledge expands the individual's network of knowledge. In particular, the instrumentation is the phase by which the chaos of the ecosystem network reaches the individual. The many novelties of views and experiences ensure that the individual compares herself with new utilization schemes. A phase of self-organization of the MOOC

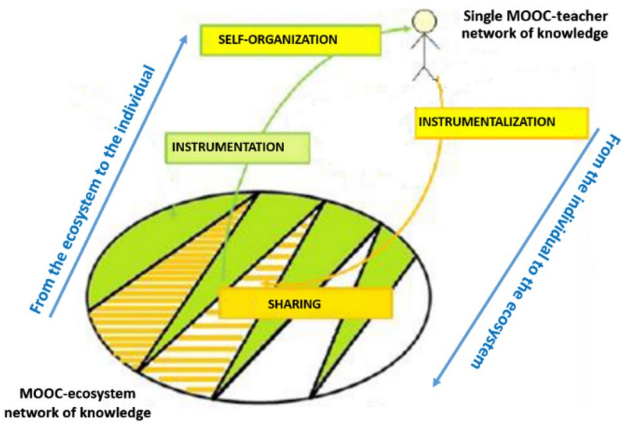


Fig. 3 The double learning process

[3] The complexity and multiplicity of connections can easily be perceived as *chaos*, information overload, in which it is difficult to find meaning or coherence in information. Siemens (2005) talks about chaos as "a cryptic form of order" (p. 4). Moreover, Siemens (2005, p. 4) states: "Unlike constructivism, which states that learners attempt to foster understanding by meaning making tasks, chaos states that the meaning exists—the learner's challenge is to recognize the patterns which appear to be hidden". Chaos becomes a new reality in the people's online learning process.

information follows: when the individual selects which utilization schemes proposed by the MOOC are valuable and which are not.

(ii) *Instrumentalization/sharing* (from the individual to the ecosystem = E ← I): process by which the individual's network of knowledge expands the MOOC-ecosystem network of knowledge. The instrumentalization is the phase by which the individual, with her renewed network of knowledge, independently builds new connections. The individual is stimulated by a task requested by the MOOC and she caters to the ecosystem to turn it according to her own (new) utilization schemes. The individual wants to integrate it with her own cognitive structures. Sharing is the phase by which the MOOC welcomes the contribution of the individual and makes it available to all: information goes towards (is available to) all members.

Since the MOOC participants are massive, the process is iterated: a phase of sharing is followed by a new instrumentation; a self-organization by an instrumentalization. It should be emphasized that the two processes are 'intertwined': there is no moment in which one ends and another begins.

The double learning process is the heart of the hybridization put in place between IA and Connectivism. It follows the principles of instrumental genesis, but under a new light, that is the complexity that the MOOC brings with it. In fact, the transformation from artifact to instrument (Verillon and Rabardel 1995) is here redefined as an evolution from artifact to ecosystem/instrument, creating a more complex system. It is important to note how the double learning process is linked to the two aforementioned theories also from a terminological point of view: the term *process* is taken from the IA; while the term *learning* is from Connectivism. In a sense, IA and Connectivism allow the incorporation, in the artifact and in its evolution as ecosystem/instrument, the role played in f2f courses by the MDT processes: the motor of these processes in the MOOC at the beginning is the inert starting content of the MOOC designed by the MTEs; then it is the dynamic process described by the connectivist model, through which the ecosystem-instrument develops.

We can start here to anticipate partial answers to our research questions. Considering (i), the theoretical framework described above, the MOOC-MDT, is the framework that we intend to consider in order to analyse mathematics TE through MOOC. With regard to (ii), the MOOC environment undergoes a dynamic evolution passing from artifact to ecosystem/instrument thanks to the entrance of the enrolled teachers. With respect to (iii), the answer concerns the double learning process, but to elaborate this issue properly, it is now necessary to formalize the research question with the terminology of the new theoretical framework. The question so becomes: (iiia) What is the degree of perception of the phases of the double learning process by the MOOC-teachers? And furthermore, given that teachers' interactions always and only take place through communication message boards, (iiib) To what extent do the MOOCs-teachers' posts show features of the double learning process?

The analyses below will allow us to answer all our questions.

4 Research context

The *Math MOOC UniTo* project started in spring 2015 at the Department of Mathematics 'G. Peano', University of Turin. It is focused on designing and delivering MOOCs for mathematics teachers, mainly from secondary schools. The aim of the project was to increase teachers' professional competencies and improve their classroom practices. Four MOOCs were designed, one for each of the main topics in the Italian mathematics curriculum: geometry, arithmetic and algebra, change and relations, uncertainty and data. The MOOCs were designed to be offered one per year, starting in 2015.[4] These MOOCs are open, free, and available online for teachers through the DI.FI.MA platform, as mentioned in the introduction. Each 10-week MOOC is subdivided into modules lasting one or 2 weeks.

The MTE team (a group of researches and expert teachers) is involved in each MOOC design, their delivery, and in monitoring their evolution in terms of interaction among participants. Digital resources replace the MTEs' voices and explanations that are usually done in f2f courses; therefore, MOOC-teachers interact with videos, images, interactive texts, software, etc. In this way, the MTEs are able to communicate their training intentions at a distance, so sharing research results, methodologies and teaching strategies that MOOC-teachers can then use in their own classes with students.

Each week, MOOC-teachers worked individually to become familiar with different approaches. In our MOOCs, these activities included the following: watching videos where an expert introduced the mathematical topic of the week, or reading about mathematical activities based on a laboratory methodology, and optionally experimenting with these in their classrooms. The MOOC-teachers were

[4] The fourth one has just been completed in April 2019 and its data are under investigation. The three that have been delivered are called respectively MOOC Geometria (on geometry contents, from October 2015 to January 2016), MOOC Numeri (on arithmetic and algebra contents, from November 2016 to February 2017) and MOOC Relazioni e Funzioni (on changes and relations concepts, from January 2018 to April 2018).

invited to share thoughts and comments about the activities and their contextualization within their personal experience, using specific communication message boards (CMBs). In the following we focus only on the forum because it is the only CMB where interventions are nested and it is easier to reconstruct the flow of an online discussion.

5 Methodology

Data came from two primary sources: MOOCs final questionnaires and MOOCs discussion forums, that we address more deeply in the following.

5.1 MOOCs final questionnaires

In each edition of our MOOCs we administered a final questionnaire to assess the degree of satisfaction and impact of this online educational experience on the MOOC-teachers. The questionnaire was produced using Google Modules, an open source application for online surveys, and uploaded on the Moodle course platform. It was divided into different sections and contained open-ended, semi-open, closed, and Likert scale questions. The analysis of responses was performed with Excel.

We have applied the theoretical framework in the design phases and in the methodological choices. The MOOC-teachers were not aware of our theoretical study; however, we included in the final questionnaire questions that were closely related to the double learning process. The intention was to understand how much the phases that we had observed during the monitoring and then described from the theoretical point of view, were also experienced in practice by the participants. For this reason, we take into consideration some data from the final questionnaire that allow us to make a quantitative analysis of the degree of perception of the phases of the double learning process by the MOOC-teachers. These are questions expressed on a Likert scale where a score of 1 means 'totally false' and a score of 5 'totally true'. The MOOC-teachers had therefore to express their degree of agreement with specific statements. We observe the following:

- E→I: includes the interactions that take place with new elements of the online world, from the platform in general to the resources in the modules in particular;
- E←I: concerns dynamics that are implemented when one is familiar with the online environment and its resources.

For each of these two categories, we identified five items to be submitted to the judgment of the MOOC-teachers. We show these in the analysis section. With respect to these analyses, we consider only the answers that come from the second and third edition of MOOCs. This is because the first edition was our first experience of TE with MOOCs and after it we refined the questionnaires (as well as the theoretical framework that benefited from our other MOOCs experiences to evolve to the current formulation).

5.2 MOOCs discussion forums

The forum keeps track of the date and time when a post is published. To make a qualitative analysis of MOOC-teachers' posts in the forum, in the light of the double learning process, we used the linguistic analysis of *lexical contrast* (Mohammad et al. 2013) as a basis for coding our data. Namely, in the sentences of MOOC-teachers' posts we identify couples of terms (specifically: verb tenses, adjectives) that have some degree of contrast in meaning with each other and mark the reference either E→I or E←I:

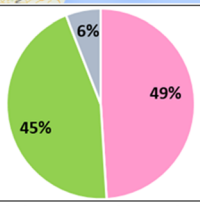
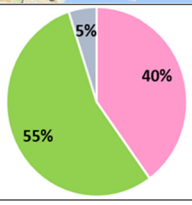
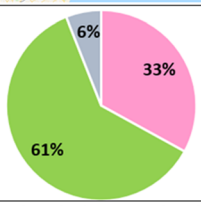
- For E→I: the verbs are in the future tense (e.g., I *will* do it, I *will* test it, I *will* use it) or the MOOC-teacher uses verbs or adjectives to express their own judgment (e.g. I *have* noticed, I *really* appreciated, *nice* idea).
- For E←I: the verbs refer to own self when one is creating new connection stimulated by the MOOC-ecosystem (e.g., I reflect, I know, I thought); while the verbs are in the present tense when a teacher shares her didactical praxeologies (e.g., I *do* this, I *use* that).

In the following, all the interventions are written in normal type, whereas bold type is inserted by the authors to accomplish the analysis.

On the one hand, choosing resources that support interactions (e.g., the CMBs) that increase the birth of new connections and/or nodes in the MOOC-teachers' network of knowledge, is a methodological choice that fosters the development of the instrumentation/self-organization phase. On the other hand, inserting specific stimulus questions or titles in the CMBs, or inviting MOOC-teachers to experiment with the activities with their own students is a methodological choice that promotes and increases the interactions among MOOC-teachers, hence the development of the instrumentalization/sharing phase. Moreover, the MTEs chose to limit their own interventions in the CMBs to a minimum in order to support the birth of a "MOOC-teachers only" online community. In fact, our MOOCs methodology aims to create collaborative contexts for teachers' work, where MOOC-teachers can learn from these kinds of practices.

During the analysis we also make some comparisons/contrasts between f2f and online education. This is because the peculiarities of MOOC-MDT emerged by observing how far away or similar it was from the MDT, which, as discussed

Table 1 MOOC-teachers of the *Math MOOC UniTo* project

	MOOC Geometria (October 2015 - January 2016)	MOOC Numeri (November 2016 - January 2017)	MOOC Relazioni e Funzioni (January 2018 - April 2018)
# teachers	424	278	358
Geographic origin			
Educational levels			
	Higher secondary school (9-13 grade)	Lower secondary school (6-8 grade)	Other (primary school, university, …)
Average age	44 (min 23 < 44 < max 63)	46 (min 27 < 46 < max 64)	48 (min 25 < 48 < max 72)
Completion rate	36%	42%	39%

above, is a lens suitable for analysing the processes of f2f or at most blended TE (Dello Iacono et al. 2019).

6 Data analysis

Table 1 shows data related to the MOOC-teachers enrolled in the first three editions of *Math MOOC UniTo* projects. The MOOC-teachers, for each edition, are a heterogeneous group. All of them are Italian in-service mathematics teachers, however, they have different geographical origins (they live in different regions of Italy). In each edition there is a very wide age range, and the teaching experience is equally varied because not only do the MOOC-teachers teach in different educational levels, but also some have been teaching for less than a year and some for more than thirty.[5]

We now focus on the quantitative analysis of data related to MOOC Numeri (Num) and MOOC Relazioni e Funzioni (R&F), from the final questionnaires, administered at the end of both these experiences. The data are shown in Fig. 5.

The first 5 statements of Fig. 5 refer to the E → I process. The MOOC-teachers are invited to reflect on their initial experiences in the online educational environment. Usually, in a f2f course we would not care to ask the participants what they think about the organization of the classroom in which they follow the course. With MOOCs, as these are virtual environments, it is worth asking. In both editions, we see how MOOC-teachers almost immediately understand how to move in the platform (51% in Num and 44% in R&F = 1 in the Likert scale). For them, therefore, the MOOC-artifact is a fairly intuitive yet virtual environment: they are not overwhelmed by the chaos of the network. In particular, it is always the majority that states that, after some access to the platform, they have become familiar with the structure of the MOOC (70% in Num and 64% in R&F = 4 + 5 together in the Likert scale). In a f2f course, the days and times of the meetings must be respected; typically, the way in which the lessons are followed is almost identical for all participants who receive handouts or in any case take notes; in addition, participants are called to deepen the materials they receive from the educators, in order to better interact in the succession of meetings. In the MOOC, instead, we see that everything is self-organized by the MOOC-teachers, with respect to the time in which to follow the course, the modalities with which to view the activities, and the level of deepening (third, fourth and fifth items of phase E → I in Fig. 5 that are 78%, 85%, 86% in Num and 80%, 83%, 83% in R&F respectively, if we consider 4 + 5 together in the Likert scale). In these moments of self-organization, the MOOC-teachers are considering whether the utilization schemes proposed by the MOOC are interesting and valid for them. In fact, the nodes inserted by the MTEs in the MOOC-artifact

[5] These data come from the analysis of the initial questionnaires submitted to the MOOC-teachers of each edition, but it goes beyond our purposes to show the exact percentages here. For more information, see Taranto (2018).

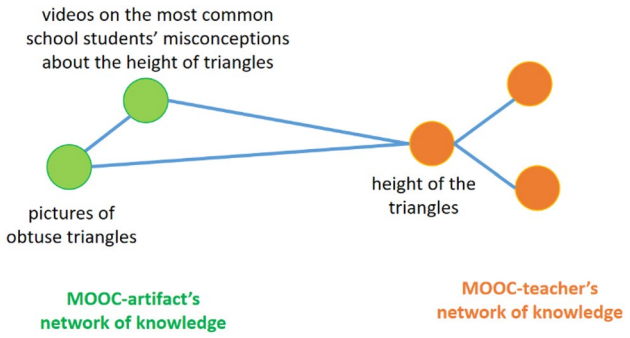


Fig. 4 Example of nodes and connections between nodes

network of knowledge are organized by thematic modules. For example, if in the first module the concept of the height of a triangle is addressed, possible nodes are for example "pictures of obtuse triangles" and "videos on the most common school students' misconceptions about the height of triangles" (Fig. 4). The MTEs generate a connection between these two nodes, specifying that the school students, working with obtuse triangles, often make the mistake of drawing the three heights all inside these triangles. For their part, the MOOC-teachers can benefit from connections in their network of knowledge by considering these nodes/connections proposed by the MTEs if, for example, they had not, before the MOOC, made a case for similar misconceptions about

		MOOC Num (#116)					MOOC R&F (#146)				
		1	2	3	4	5	1	2	3	4	5
(E→I)	When you first logged into MOOC, you knew how to move around the platform or at least it seemed intuitive to you	51%	17%	9%	14%	9%	44%	20%	17%	12%	7%
	After some access to the platform, you were familiar with the structure of the MOOC (weekly opening of the modules, use of resources, badge release, ...)	17%	7%	6%	24%	46%	16%	9%	11%	28%	36%
	I have self-organized my TIME in following the MOOC and its activities (I connected in my spare time, in the morning, in the evening, every day, only on Mondays, ...)	1%	7%	14%	13%	65%	3%	3%	14%	25%	55%
	I have self-organized the MODALITY with which to follow the MOOC and carry out its activities (I used to print, take notes, watch its videos, ...)	0%	4%	11%	18%	67%	1%	4%	12%	29%	54%
	I have self-organized the LEVEL OF DEEPENING (read the materials carefully, reviewed the videos several times, ...)	1%	3%	10%	22%	64%	2%	2%	13%	36%	47%
(E ← I)	I proposed to my classes some of the activities (even partially) seen in the MOOC	4%	9%	33%	26%	28%	15%	9%	21%	28%	27%
	I have shared with others my teaching practices	10%	22%	42%	21%	5%	19%	31%	25%	21%	4%
	I have shared with the other participants some of the materials I use in my classes	21%	29%	29%	16%	5%	26%	25%	29%	17%	3%
	I felt like I was part of a community that was growing collaboratively	4%	17%	29%	34%	16%	6%	15%	36%	32%	11%
	MOOC has contributed to my professional learning	1%	3%	15%	35%	46%	1%	1%	9%	33%	56%

Fig. 5 Phases of the double learning process perceived by MOOC-teachers (a score of 1 means 'totally false' and a score of 5 'totally true')

the "height of the triangles" node that is already present in their network of knowledge.

The last 5 lines of Fig. 5 refer to the E←I process. Having become familiar with the utilization schemes to be adopted in the online environment, the MOOC-teachers put in place specific dynamics. If they like the activities proposed by MOOC, these become part of their network of knowledge and can be proposed in their classes. This is not a mandatory practice, but optional in our MOOCs. However, we observe that more than half of the MOOC-teachers (54% in Num and 55% in R&F = 4+5) put them into practice. Sharing is not a practice of the majority (teaching practices: 26% in Num and 25% in R&F = 4 + 5; materials: 21% in Num and 20% in R&F = 4 + 5), but the following observations are worth considering. Sharing, in these MOOCs experiences, has been realized according to these two ways that we have included as items for the phase E←I: sharing of own teaching practices and sharing of own materials, that MOOC-teachers have since before attending the MOOC and that they use with their school students.

Sharing teaching practices was explicitly requested by the MTEs. In fact, in order to stimulate discussion on the CMBs, the MTEs asked the MOOC-teachers to share strategies or methodologies that they used in classroom to explain that certain topic that was being addressed in the MOOC. Sharing materials is a completely spontaneous fact that surprised us: the MOOC-teachers were never asked to do it, and yet since the first edition, this seemed to be a natural practice in the online environment.

Before continuing with the quantitative analysis, we give an example of each of these ways of sharing, through a qualitative analysis.

6.1 Sharing of own teaching practices

We consider an example of discussion taken from the forum of the first module of MOOC Geometria. We choose to present this example to underline that, although in the period of delivery of MOOC Geometria, the MOOC-MDT framework was still under development, even the dynamics that occurred in our first MOOC experience can be described in the light of the framework.

In the first module of MOOC Geometria the concept of perpendicular was addressed. The activities proposed were directed to avoid or overcome misconceptions related to this concept. We proposed activities of manipulation, discovery and conjecture. In general, school students often confuse the perpendicular with the vertical. For example, if the line with respect to which the distance of an external point is located is not horizontal, a student may be tempted to trace the vertical instead of the perpendicular. There can still be problems working with an obtuse triangle (as mentioned before), where it can happen that the height falls outside the triangle and thus outside the opposite side. The student who has not internalized the concept of straight-line distance could do everything to try to make this height fall inside the triangle. Working well on these concepts makes sure that the students internalize them well, avoiding the formation of misconceptions.

The forum of this module collected 24 discussions, each of them with from 0 to 62 responses, for 207 posts in total. In the forum, the MTEs have inserted a piece in order to stimulate the discussion among the MOOC-teachers: "Share your ideas and/or teaching experiences related to the conceptual nodes of the module". Let us consider the discussion in Fig. 6.

The MOOC-teachers were making considerations on the usefulness of the plumb line and exchanged reports of experiences of activities conducted in the classroom. MC entered into the information overload of the MOOC-artifact and experimented with the phase of instrumentation/self-organization.[6] She was rewarded by new activity proposals to use with her students to address the topic of distance, using round sheets and plumb lines. In self-organizing the information she had received, she activated the comparison with her utilization schemes. Precisely, based on her praxeologies, she self-organized this new information in her network. MC also put in place the instrumentalization/sharing. In fact, she hooked a node in the MOOC-ecosystem network (difficulty in drawing the heights in the triangles) with her personal experience in the classroom. Then she shared her didactical praxeologies related to the task "overcoming the misconceptions related to the height of the triangles". So, the strategies she shared are her utilization schemes, triggering the second process of the double learning process.

A few hours later, AR responded. Here, the first step of the process (E→I) is intended as implicit: AR had experienced the phase of instrumentation/self-organization, but responding to the colleague, she connected further with another node that had emerged, the plumb line. So, she shared her praxeologies for the purpose of showing she does not totally agree with the previous comment. In fact, she explained how, according to her, the plumb line can be used. An hour later, BP explained that she is still a beginner teacher. She entered the *chaos* of the MOOC and of the node that is shared by the other MOOC-teachers. In fact, notice how she said "your ideas are very exceptional", pointing out just how she before writing the post had self-organized the information of the module about her experiences (which by her own admission are few: "I teach recently"). Her network of knowledge seemed to be benefiting from

[6] Note that the comment by M.C. is not the first post in the forum. She joined a discussion started by other posts she had read before writing, in fact she started by saying "I noticed it too".

Discussion (started by X; on gg/mm/aa; at hh:mm)	Reply (by X; on gg/mm/aa; at hh:mm)	Intervention	Double learning process
MAINMAST… AND MORE (by M.C., 28/10/15; 16:04)		Hi, I'm M., I teach in a lower secondary school. **I noticed it** too: for many students the triangle has only one height .... and only one base! **I tried to make** it clear that it depends on the 'reference system', for example being standing or lying down ... [...] just think of a square put on a vertex, many say that it is a rhombus! Surely the [round] sheet without horizontal and vertical references helps to overcome misconceptions. The plumb line can be misleading [...] can confuse the concepts of perpendicular and vertical, better to go gradually [...]	E→I E←I
	A.R.; 28/10/15; 18:49	**I use** the plumb line in an initial phase ... that is when they do not understand that in a triangle we can consider each side as a base. In this step **I rotate** the triangle so that it rests on a different base each time ... in this way we see that the line marks the height. In a second step, **I test** the students by placing the base on an inclined plane ... at this point the plumb line is definitely deceptive ... here starts the reflection on the concept of perpendicularity.	E←I
	B.P.; 28/10/15; 19:29	The use of plumb line is actually **very useful** ... **I will try to use** it in the classroom too. Your ideas [*it is referred to the MOOC activities*] are **very exceptional**, I teach recently and I really need it :) thanks!	E→I
	M.P.U.; 28/10/15; 20:09	Hello, definitely plumb line and round sheets are **very useful**	E→I

Fig. 6 MOOC-teachers' interventions on the forum in module 1

her self-organization. She said "[the] plumb line is actually very useful", but she had never used it in person. She deduced it from the comments of the others. Siemens (2005) wrote, "we can no longer experience and acquire learning that we need to act. We derive our competence from forming connections" (pp. 3–4). Some time later, the last intervention in this comment was made by MPU. She was simply in agreement with what was written, so she limited herself to externalizing it. She did not add nodes either to her network, or to that of the MOOC-ecosystem. She simply connected, self-organizing the information overload.

6.1.1 Sharing of own material … and something more

We move to the MOOC R&F and consider the second module. The module focused on relational thinking. We wanted to invite the MOOC-teachers to reflect on the fact that mathematics is not made up of a series of rules to be memorized and applied. Rather, it should be understood as a context in which to pose significant problems and therefore to explore and perceive relationships and structures that are found in nature and in man's creations. In order to stimulate relational thinking, pupils can be encouraged to make use of representations or resort to analogies.

Let us focus on the forum of this module. It collected 20 discussions, each of them with from 0 to 58 responses, for 151 posts in total. The MTEs had inserted ain order to stimulate the discussion among the MOOC-teachers: "Highlight what is specific to the order of the school in which you teach. In these ways, can it be possible to reason and confront the vertical perspective of the educational path". Let us consider the discussion in Fig. 8.

In this examples MPB put in place the instrumentalization/sharing phase, sharing with the MOOC-ecosystem material that she used with her school students to explain the fractions (Fig. 7). However, something more emerged.

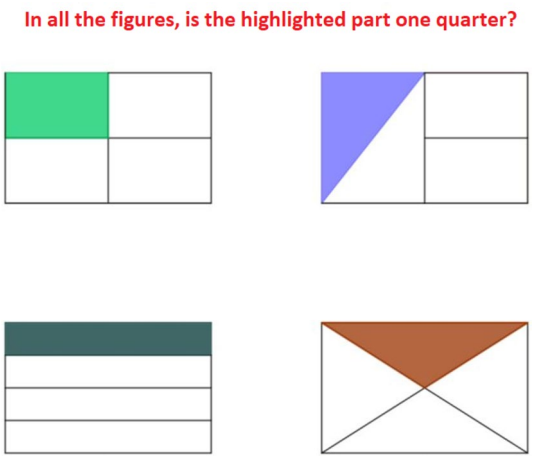


Fig. 7 Exercise of reflection on fraction proposed by MPB

Discussion (started by X; on gg/mm/aa; at hh:mm)	**Reply (by X; on gg/mm/aa; at hh:mm)**	**Intervention**	**Double learning process**
FRACTIONS (by F.A., 05/02/18; 10:35)	M.P.B.; 06/02/18; 13:00	[…] when I finished today's lessons, **I read** your posts and **felt the need** for an outburst/confrontation. AMAZING... it is true that some students are more motivated when you solicit mathematical thinking. However, APATHY prevails. **I often ask myself** how to stimulate critical thinking ... I teach in grade 10 too […]: at the beginning of the year, almost all of my first students (grade 9) said that for them math is to perform exercises. **I'm struggling a lot**, for their resistance, to scratch this belief […]. In these days I am explaining fractions and today **I have proposed** this image [Figure 6]. Most did not answer. Few, the most interested [students], expressed hypotheses on the second and fourth. **I asked** them to take a sheet and some scissors to verify their hypotheses. Only two students out of 29 considered this invitation and proposed their deductions. […]	E←I
	S.M.; 12/02/18; 19:03	Hi, I teach in a lower secondary school. **I really like** the example of M.P.B on the various representations of 1/4. It would be interesting to know how they evaluated the figure with the violet rectangle triangle. As far as I am concerned, **I am perfectly convinced** that the concept of a fraction is a fundamental obstacle. **I try to make** it very clear also as a ratio, talking about the concentration of colored solutions. The problem is that often the RELATIONAL concept is missing. I correct many exercises on the rectangles of the type the perimeter is 24 cm and the base is double the height, find the area. Many [students] manage to solve the problem, but when they draw the corresponding figure, the rectangles have random measurements. this thing **makes me angry**, **I think** their problem in the end is to try to get to a number as a result and not so much UNDERSTAND what is the relationship between the numbers. […]	E→I E←I
	M.P.B.; 16/02/18; 16:32	S.[M.], they said that both the purple and the brown part were not correct representations. **I invited** them to try to cut out the purple triangle so that it overlapped one of the two rectangles, then **I made** conjectures with the areas and then we generalized with the literal calculation. The same for the other representation.	E←I
	C.P.; 25/02/18; 11:23	**I find** the same problems too. Probably one struggles with the concept of division: to divide a whole, or more whole, into equal parts. My students, from the first lessons, in grade 6, as soon as they hear the word division, become quite agitated.	E→I
	G.C.; 25/02/18; 18:39	**I fully agree** with what you have said. **I also observe** a division between the "procedure" and its meaning.	E→I

Fig. 8 MOOC-teachers' interventions on the forum in module 2

The MOOC-teachers involved in the discussion of Fig. 8 were not just comparing teaching practices. They confided in each other the frustrations they experience in playing their teacher role, finding themselves with students who do not always have a motivation to learn. MPB began by writing "amazement". She was amazed at how her students showed no interest in the activity she proposed, both in the reflection phase and in the manipulative one. Only two students accepted the invitation to use the scissors, the other 27 are "apathetic", as she says. Reading what MPB wrote revived a connection of the SM's network of knowledge. In fact, SM found similarity in her profession as a teacher with the words of MPB She pointed out that according to her, students lack the concept of relationship. In particular, she underlined how she becomes angry when her students read the text of a problem and make representations that seem to have nothing to do with what they have read. They do not commit to understanding, they only perform calculations. With curiosity, she then asked MPB how her students answered on the figure with the triangle in violet (Fig. 7). MPB answered 4 days later. About 10 days later, also CP and GC intervened and agreed with the difficulty of understanding the distinction between meaning and procedure found in their students.

Although all interactions take place online and in a-synchronous mode, the MOOC also affects the beliefs and emotions of the participants. We do not go into these aspects in depth, but it is important to note how we can identify such facets in an online course. We also underline that similar interventions followed one another in the previous MOOCs. In these editions, several teachers highlighted how they felt comfortable with colleagues who shared this same formative occasion, with which to share experiences, with which to confront and even to confide.

Let us return to Fig. 5 and its last two lines that we had left outstanding. We had noticed that the sharing by MOOC-teachers of their own material had seemed surprising to us. We were even more surprised when they shared their emotions because it was between people who had never seen each other before. Moreover, this sharing takes place in, having as recipients of their own speech, the entire community of MOOC members and this does not seem to constitute a barrier to communication. This is in agreement with almost half the MOOC-teachers who claim to feel part of a community (50% in Num and 43% in R&F = 4 + 5). Finally, for most respondents to the final questionnaire, despite its strengths and weaknesses, MOOC is an environment that contributes to their professional development (81% in Num and 89% in R&F = 4 + 5).

7 Discussion and conclusion

Our paper has considered the three research questions listed in Sect. 1, with the third one (iii) further elaborated into (iiia) and (iiib) according to the theoretical framework we used, as described at the end of Sect. 3. Synthetically, the answer to (i) is the MOOC-MDT; to (ii) is the dynamic evolution of the ecosystem from artifact to instrument according to the IA approach within the connectivist model; to (iiia) and (iiib) consists in the results given by the analysis of the double learning processes through the lenses of the E$\leftarrow\rightarrow$I interactions and of the lexical contrast.

In the paper we have articulated the conceptual framework that has guided the design, data collection, analysis, and interpretation of findings for the *Math MOOC UniTo* project, the so called MOOC-MDT. It was created with the aim of explaining the dynamic phenomena that characterize the MOOC-ecosystem, suitably modifying the MDT model that we have elsewhere used to analyse f2f TE courses. We have discussed above how MDT is inadequate to capture all the facets of the virtual environment, but how it is in any case a good starting point for that. We have thus extended the MDT model, hybridizing it with other theoretical fragments: the IA and the Connectivism. This lens allows a dynamic description of what happens in MOOCs, which is in fact described as an evolving network of knowledge, which transforms the MOOC-artifact into an ecosystem/instrument through the double-learning process of instrumentation/self-organization and instrumentalization/sharing.

In particular, using this lens to analyse our three MOOCs, we have been able to pinpoint some important aspects that are features of a MOOC. In fact, the extension of the MTD model to the MOOC-MDT shows why and how a MOOC such as the one we described can work and produce a perception of some evolution in the didactical praxeologies of participants.

First, our model shows that in the dynamic structure of the MOOC, whose evolution happens according to the connectivist model, we find both the MTE's meta-didactical praxeologies as starters of the process, and the didactical praxeologies of some of the participants, who share them through the different tools of the platform. All of them evolve locally in different ways according to the interactions that happen within the MOOC networked structure and affordances. This marks a first big difference with respect to the f2f courses and is specifically explained by embedding the MDT within the connectivist model. It is a sort of spontaneous feeding and self-organisation of the process, which goes on and increases (dynamically) more and more because of the (inert) inputs given every week as a new fuel for the MOOC by the MTEs.

Second, the possible dynamic evolution of teachers' didactical praxeologies explained through this model, shows another aspect, which is not relevant in f2f courses: namely the role of the evolution of the ecosystem from artifact to instrument, insofar as it becomes the repository of a variety of more or less explicit meta-didactical praxeologies, which act as energizers for the possible evolution of (some) teachers' didactical praxeologies.

Third, suitable analysis tools have been elaborated to analyse the two primary sources of data, namely, the final questionnaire and the discussion forums. The two analysis tools described in Sect. 5 (the E$\leftarrow\rightarrow$I and the lexical contrast) allow researchers to pinpoint how the double learning process can happen concretely, as illustrated in Sect. 6. They allow us to see how the sharing (of materials and more) can produce the dynamic evolutions recalled above.

The points above allow us to draw some specific concrete consequences, which feature MOOCs courses, and allow us to compare them with f2f ones: they are summarized in the points below, which deepen further the answers given to our research questions.

(a) *The creation of a specific MOOC social environment, free from any space-time constraints*: It is well known that online courses are born to cover great distances (Borba and Villarreal 2006) and thus allow a wider audience of users to benefit from training and/or disseminating new knowledge on the territory. Italy is

not an 'immense' country but consists of islands and decentralized parts, which are not always able to benefit from adequate or frequent training offers: North is different from South and even more from Islands. With the MOOC a uniformizing socialization effect seems to be reached: the result is a richness in the interactions and exploration of the resources and practices that can be used in the classroom to overcome the difficulties teachers meet in their work (an example was showed in Fig. 8). A similar effect concerns the fact that in the MOOCs, because of the richness of interactions, teachers of different grade levels can become aware of resources before unknown to them, which can be used vertically (an example was showed in Fig. 6).

(b) *The birth of an online specific community, where practices and emotions are shared*: In every edition of our MOOCs, we have witnessed the birth of an online, free, voluntary community. A very specific feature of MOOCs is the remarkable sharing of their own ideas, experiences, practices and materials made spontaneously by the MOOC-teachers through CMBs. This is clearly a consequence both of the modality of interactions (made evident by the evolution processes pointed out through our model) and of the space uniformization effect discussed in (a). The MOOC-teachers benefit from professional development thanks to the materials made available by the MTEs, but also thanks to the comments and opinions of the community on the content that the MOOC offers. The ecosystem grows, favouring the professional growth of individuals. Moreover, in the MOOCs we sometimes see an emotional involvement of participants (as showed in Fig. 8): it seems to be the effect of the social network aspects of the new tool.

(c) *The emerging of aware practitioners within the online community*: We underlined that in each CMB the MTEs inserted a specific question to be answered or a title that served as a discussion point. The MTEs chose to limit their own interventions in these CMBs to a minimum. The aim was to support the creation of a community of 'aware practitioners', namely, teachers able to share practices, of whose didactical effects they became aware, after having discussed and shared some different approaches for teaching specific topics. For this the CMBs allowed by the platform were crucial: this aspect is a remarkable difference between a f2f course and our MOOCs. In fact, the interventions of the teachers in a CMB about a specific topic promoted by the MTEs generally produced a natural equilibrium between theoretical and practical issues: there is a 'natural' process of convergence towards it, which in a f2f course is not always possible to achieve. The asynchronous discussion through the CMB allows different examples and comments to be produced at different times: a-synchronicity allows a certain number of well-thought-out contributions, both from theoretical and practical standpoints. This is not generally possible in f2f courses, where at most one can have some minutes of discussion on the spot, and sometimes no discussion at all. In a MOOC each participant finds more space for discussing each topic of the course and for assimilating its content.

In conclusion, MOOC-MDT is a framework that can be used both to interpret the dynamics that characterize the TE through MOOC, and at a methodological level to support the design of the MOOC itself. We offer this framework to stimulate discussion with colleagues in the mathematics education research community about ways in which it might be refined and extended, and to contribute to building a shared understanding of the process of TE through MOOCs.

References

Aldon, G., Arzarello, F., Panero, M., Robutti, O., Taranto, E., & Trgalová, J. (2017). MOOC for mathematics teacher training: design principles and assessment. In G. Aldon & J. Trgalová (Ed.), *Proceedings of the 13th International conference on technology in mathematics teaching* (pp. 200–207), Lyon, France.

Arzarello, F. (2016). Le phénomène de l'hybridation dans les théories en didactique des mathématiques et ses conséquences méthodologiques. In *Conférence au Xème séminaire des jeunes chercheurs de l'ARDM*, Lyon, France.

Arzarello, F., Robutti, O., Sabena, C., Cusi, A., Garuti, R., Malara, N., et al. (2014). Meta-didactical transposition: a theoretical model for teacher education programmes. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era* (pp. 347–372). Dordrecht: Springer.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Ed.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3–14). Edmonton: CMESG/GDEDM.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2017). Digital technology in mathematics education: Research over the last decade. In *Proceedings of the 13th International Congress on Mathematical Education* (pp. 221–233). Cham: Springer.

Borba, M. C., & Villarreal, M. E. (2006). *Humans-with-media and the reorganization of mathematical thinking: information and communication technologies, modeling, visualization and experimentation*. New York: Springer.

Boyatt, R., Joy, M., Rocks, C., & Sinclair, J. (2014). What (Use) is a MOOC? In L. Uden, Y. H. Tao, H. C. Yang, & I. H. Ting (Eds.), *The 2nd International Workshop on Learning Technology for Education in Cloud* (pp. 133–145). Dordrecht: Springer.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en didactique des mathématiques*, *19*(2), 221–265.

Dello Iacono, U., Pierri, A., & Taranto, E. (2019). Peer review methodology in a blended course for mathematics teacher education. *The*

International Journal for Technology in Mathematics Education, 26(2), 49–54.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Johnson, R. D., Hornik, S., & Salas, E. (2008). An empirical examination of factors contributing to the creation of successful e-learning environments. *International Journal of Human-Computer Studies, 66*(5), 356–369.

Kellogg, S., Booth, S., & Oliver, K. (2014). A social network perspective on peer supported learning in MOOCs for educators. *The International Review of Research in Open and Distributed Learning, 15*(5), 263–289.

Loisy, C., Sabra, H., Courtney, S. A., & with the contributions of Rocha K., Glasnović Gracin, D., Aldon, G., Front, M., Gardes, M.-L., Taranto, E., Arzarello, F., & Robutti, O. (2019). Analyzing teachers' work with resources: Methodological issues. In L. Trouche, G. Gueudet, & B. Pepin (Eds.), *The 'resources' approach to mathematics education* (pp. 300–310). New York, NY: Springer.

McAuley, A., Stewart, B., Siemens, G., & Cormier, D. (2010). *The MOOC model for digital practice*. Retrieved from http://www.elearnspace.org/Articles/MOOC_Final.pdf. Accessed 13 Aug 2019.

Mohammad, S., Dorr, B. J., Hirst, G., & Turney, P. (2013). Computing lexical contrast. *Computational Linguistics, 39*(3), 555–590.

Onah, D. F., Sinclair, J., & Boyatt, R. (2014). Dropout rates of massive open online courses: behavioural patterns. In *Proceedings of the 6th international conference on education and new learning technologies* (pp. 5825–5834). Barcelona, Spain.

Pear, J. J., & Crone-Todd, D. E. (2002). A social constructivist approach to computer-mediated instruction. *Computers and Education, 38*(1–3), 221–231.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teacher work and interaction: new perspectives on resource design, use and teacher collaboration. *ZDM - The International Journal on Mathematics Education, 45*(7), 929–943.

Rodriguez, C. O. (2012). MOOCs and the AI-Stanford like courses: two successful and distinct course formats for massive open online courses. European Journal of Open, Distance and eLearning. http://www.eurodl.org/?article=516. Accessed 17 Apr 2019.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Siemens, G. (2005). Connectivism: a learning theory for the digital age. *International journal of instructional technology and distance learning, 2*(1), 3–10.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivistic perspective. *Journal for Research in Mathematics Education, 26,* 114–145.

Sullivan, P. (2008). Knowledge for teaching mathematics: An introduction. In P. Sullivan & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 1, pp. 1–12)., Knowledge and beliefs in mathematics teaching and teaching development Rotterdam: Sense Publisher.

Taranto, E. (2018). *MOOC's Zone theory: Creating a MOOC environment for professional learning in mathematics teaching education*. Doctoral thesis. Turin University.

Taranto, E. (in press). MOOCs for mathematics teacher education: new environments for professional development. In J. P. Howard & J. F. Beyers (Eds.), *Teaching and learning mathematics online*. Boca Raton: Chapman and Hall/CRC Press.

Taranto, E., Arzarello, F., & Robutti, O. (2017a). MOOC: repository di strategie e metodologie didattiche in matematica. *Annali online della Didattica e della Formazione Docente, 14,* 257–279.

Taranto, E., Arzarello, F., Robutti, O., Alberti, V., Labasin, S. & Gaido, S. (2017b). Analyzing MOOCs in terms of their potential for teacher collaboration: The Italian experience. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of European Society for Research in Mathematics Education (CERME10, February 1–5, 2017)* (pp. 2478–2485). Dublin, Ireland: DCU Institute of Education and ERME.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. *European Journal of Psychology of Education*, *10*(1), 77–101.

Wenger, E., Trayner, B., & De Laat, M. (2011). Promoting and assessing value creation in communities and networks: a conceptual framework. Retrieved from http://isites.harvard.edu/fs/docs/icb.topic1116077.files/AssessingLearningCommunities.pdf. Accessed 13 Aug 2019.

Zutshi, S., O'Hare, S., & Rodafinos, A. (2013). Experiences in MOOCS. *American Journal of Distance Education, 27*(4), 218–227.