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ORIGINAL ARTICLE

Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms

Vince Geiger[1]

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Abstract
In this article I describe an approach to task design and implementation that addresses the broad capabilities common to 21st Century Skills and the STEM education agenda. Principles of task design and implementation were co-constructed by ten teachers and a single researcher during a longitudinal study that took place over 3 years. A vignette that draws on observation data from a Year 1 class and a post-lesson teacher interview is used to illustrate the possibilities that exist for mathematics to support critical reasoning and enquiry in primary science. The article concludes with a reflection on the role of teachers in designing and implementing tasks aimed at promoting effective STEM teaching and learning.

Keywords STEM · 21st Century Skills · Mathematics education · Numeracy · Mathematical literacy · Science education · Critical reasoning

1 Introduction

Governments and educational policy makers are facing the challenge of identifying the broad capabilities needed by citizens to be successful in a future where change will be the default, that is, in a world characterized by escalating technological, economic and social transformation. International initiatives such as 21st Century Skills (Partnership for 21st Century Skills 2002) and The Definition and Selection of Key Competencies (OECD 2005) have attempted to respond to this challenge by outlining and describing the knowledge, skills, and processes needed to generate solutions to the complex problems society faces both now and into the foreseeable future. While some variation exists between the competency sets identified by these initiatives, problem solving, reasoning, decision-making, and the ability to communicate (e.g., Darling-Hammond 2007) are typically seen as vital.

At the same time, STEM education is receiving growing attention internationally (e.g., Bybee 2010; Charette 2013). Increasing interest in STEM education is in response to two perceived imperatives: (1) a need to upskill populations with capabilities seen as essential for the innovation needed to ensure future economic prosperity (e.g., ACOLA 2013; European Parliament 2015; Hopkins et al. 2014) and (2) an obligation to support STEM literacy development in order to promote an informed, participating and contributing citizenry (e.g., Bybee 2010; Charette 2013; Zollman 2012). Thus, both 21st Century Skills and STEM education agendas identify the need for young people and adults to develop the capacity to generate innovative, evidence-based responses to known and developing real world problems through the processes of problem solving, critical reasoning and inquiry. In the case of STEM education, there is also increasing acceptance that these processes involve the use of knowledge from two or more of the relevant disciplines (English 2016). While this is a laudable aspiration, it has been argued that teachers require significant professional learning support in order to develop the capability required to design tasks and/or learning environments that promote connection between different types of knowledge and foster critical enquiry in a meaningful way (Geiger et al. 2015a; Zaslavsky and Sullivan 2011). Further, Maass et al. (2013) have argued teachers must have access to models of teaching practice if they are to be effective in connecting different types of knowledge when addressing real world problems.

In this article, I report on the role of task design in developing and implementing classroom activities that encourage students' use of cross-disciplinary knowledge when addressing real world problems through critical reasoning and inquiry. Specifically, I report on the development of a

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✉ Vince Geiger
Vincent.Geiger@acu.edu.au

[1] Institute for Learning Sciences and Teacher Education, Australian Catholic University, Level 4, 229 Elizabeth Street, Brisbane CBD, Brisbane, QLD 4000, Australia

Table 1 A comparison of the frameworks of the Partnership for 21st Century Skills Project (2002) and the Assessment and Teaching of 21st Century Skills Project (2009)

Partnership for 21st Century Skills Project (2002)	Assessment and Teaching of 21st Century Skills Project (2009)
Learning and innovation skills: Critical thinking and problem solving; communication and collaboration; creativity and innovation	Ways of thinking: Creativity, critical thinking, problem-solving, decision-making and learning
Information, media and technology skills: Information literacy, media literacy, ICT (Information, Communications and Technology) literacy	Ways of working: Communication and collaboration
	Tools for working: Information and communications technology (ICT) and information literacy
Life and career skills: Flexibility and adaptability; initiative and self-direction; social and cross-cultural skills; productivity and accountability; leadership and responsibility	Skills for living in the world: Citizenship, life and career, and personal and social responsibility

framework aimed at supporting teachers in designing and implementing numeracy tasks across the curriculum. The framework was complemented by professional learning that focused on how mathematics could be utilised to enhance the teaching and learning of other subjects, particularly aspects of the curriculum in which evidence based critical thinking and reasoning are required. The use of the framework is illustrated via a vignette in which a teacher makes use of the framework to design and implement a task that made explicit use of mathematical knowledge and skills to support critical enquiry within a science lesson. This case is described and analysed in order to address the following research question:

Does a framework based on principles of design and implementation for numeracy tasks provide effective support for teachers when developing science lessons in which mathematics is used to as a foundation for reasoning and critical enquiry?

In the remainder of this article, I first provide a brief outline of current research and policy within both 21st century skill and STEM agendas. I then present a synthesis of current research into both task design and teachers as designers. Following attention to relevant literature, the methodological approach will be described in conjunction with the teacher/researcher collaboration that led to the development of the principles of design and implementation for numeracy tasks. The use of these principles by a teacher when designing and implementing a task in a Year 1 Science lesson will be illustrated via a classroom vignette. Finally, the implications for this research will be discussed in relation to 21st Century Skills and STEM learning in order to point to new possible directions in research.

2 Addressing the aspirations and challenges of 21st century skill and STEM education

In this section I first outline research related to the aims of 21st Century Skills and STEM education agendas and discuss associated challenges. Second, I present a synthesis of research into task design by way of posing a potential approach to addressing these aims and challenges.

2.1 21st Century Skills: promise and challenges for schooling

In response to rising concern about rapid societal, economic, technological and educational change, governments and educators have highlighted the need to identify the capabilities required to meet the demands of the 21st century (e.g., Partnership for 21st Century Skills Project 2002; Darling-Hammond 2007). These capabilities have typically been represented as frameworks in which the capacities needed to meet the current and future demands of personal, civic and work life are identified and described. While developed independently, the constituent capabilities of such frameworks typically coalesce around common key elements. To illustrate this point, the elements of two established frameworks, the Partnership for 21st Century Skills (2002) and Assessment and Teaching of 21st Century Skills (2009) are presented in Table 1.

In both schemes, creativity, problem solving and critical thinking capabilities with the capacity to use of digital tools for communication and access to information are identified as essential capacities. Social and cultural capabilities are also seen as vital for exercising responsibility as a citizen and for undertaking leadership roles.

While there is substantial agreement about foundational 21st Century Skills, in contrast, it remains unclear which approaches are most effective within the bounds of school curriculum implementation. Different approaches to this challenge have been trialled including: (1) the addition of a new stand-alone subject to the school curriculum; (2) incorporating 21st Century Skills into existing subjects; (3) establishing 21st Century Skills as the basis for the development of all subjects in the curriculum; and (4) using 21st Century Skills as the foundation for a radical new approach to curriculum (e.g., Voogt and Roblin 2012). As yet, however, none of these approaches have been demonstrated to be unproblematic in their implementation as each requires changes to both structural elements of curriculum and to current expectations of teachers' roles within the education of students. An additional confounding element is how to assess 21st Century Skills within any of these approaches (e.g., Jia et al. 2016).

The issue of changing teacher roles in the promotion of students' 21st Century Skills has been taken up via a collaboration between the American Association of Colleges of Teacher Education and the Partnership for 21st Century Skills (2010). The resulting report (American Association of Colleges of Teacher Education and the Partnership for 21st Century Skills 2010), sets out aspirational targets within teacher preparation including that: teachers and administrators will possess, teach and assess 21st century knowledge and skills; educator preparation programs will address 21st century knowledge and skills; and higher education leaders will work with leaders in P-12 and local communities to redesign educator preparation programs to meet the needs of the 21st century. While the report highlights the importance of 21st Century Skills, identifies core principles for educator preparation, and presents key questions that should be addressed in an educator preparation program, it stops short of providing the sort of definitive advice teachers require to make significant changes to their practice.

2.2 STEM education: an evolving and challenging agenda

Research into effective STEM education has received increased attention over the past decade due to the perception that it is essential for driving the innovation necessary for national prosperity (e.g., Honey et al. 2014). This interest parallels rising concern about current or future shortages in those qualified to take up STEM careers in many countries (English 2016). Often this has meant that education systems have been expected to implement initiatives in STEM that will encourage greater student participations in subjects related to science, technology, engineering and mathematics. For example, building capacity in STEM teaching and learning has been seen as the most effective way of addressing Australia's increasing STEM workforce demands (e.g., Commonwealth of Australia 2015). A focused approach to STEM education, however, has proved elusive for many reasons including that there is still no agreement on the nature of STEM itself. Hobbs et al. (2018), for example, have identified five models of STEM that have been implemented in schools. These models represent a continuum between the separate teaching of disciplines through to total integration.

While there are different ways in which STEM is implemented, there appears to be growing acceptance that effective teaching and learning practices in STEM require that students draw on more than one of the constituent disciplines when addressing real world problems. Balka, for example views STEM as

> … the ability to identify, apply, and integrate concepts from science, technology, engineering, and mathematics to understand complex problems and to innovate to solve them.

(Balka 2011. p. 7).

There are strong arguments for integrated or interdisciplinary approaches as a way of emphasizing the cohesive and coherent potential of STEM based approaches to problem solving (e.g., Maass et al. 2013; STEM Task Force Report 2014). Glancy and Moore (2013), for instance, argue that the divides between STEM disciplines that exist in schools are artificial as they are rarely present outside the classroom. In their view, STEM activity in authentic situations takes the form of practices, for example, design thinking in engineering or inquiry based reasoning in science, that draw on disciplines (e.g., mathematics, science) to develop responses to realistic problems. Accordingly, for students to make the types of connections they need to appropriate the modes of thinking and reasoning prevalent in STEM practices outside of school, they need to experience realistic problems that employ the use of multiple disciplines within school (Lesh and Zawojewski 2007).

There are, however, challenges associated with subject integration that schools often struggle to address (e.g., Venville et al. 2002). These challenges include: the current structure of school curricula; discipline-based teacher education; large scale assessment regimes; school infrastructure itself (e.g., science laboratories, technology centres); the high degree of organisational work required of teachers; and the limited number of resources available to support learning across the curriculum. Each of these factors serves to reinforce the status quo where disciplines are kept separate or integrated in a superficial way. The superficial integration of disciplines may also be related to a lack of confidence and/or the support needed to acquire new knowledge and practices, experienced by many teachers, in what has been identified by the Australian Council of Learned Academics (ACOLA) as the "capacity gap in STEM teaching" (2013, p. 17). Wong and Dillion (2019), in a study involving mathematics and science policy makers and teachers conclude that the lack of connection between science and mathematics teaching and learning is related to the asymmetric dependency of these disciplines on each other—science is dependent on mathematics but the reverse is not as strong. At the same time, they note the failure of science teachers to take advantage of opportunities to make use of mathematics in science in a way which promotes the learning of both discipline.

The issue of superficial integration has been flagged as a problem in the case of mathematics in particular, where often STEM is primarily associated with science, which diminishes the role of other disciplines (English 2016). As argued by Fitzallen (2015), many reports claim that STEM provides contexts for fostering mathematical competencies but not how "mathematics can influence and contribute to the understanding of the ideas and concepts of other STEM disciplines" (p. 241).

2.3 Task design

Burkhart and Swan (2013) argue that task design has a key role when the aim is to improve the teaching of mathematics. In support of this argument, research and development approaches to task design have been demonstrated to be effective in improving teaching practice through the success of long-term projects such as Connected Mathematics (Lappan and Phillips 2009). While the careful design of tasks has been shown to enhance student learning opportunities, the process is complex as factors such as intended purpose, alignment with local curriculum requirements and available resources must also be considered when developing a description of the planned activity (e.g., Brown 2009; Johnson et al. 2017; Jones and Pepin 2016; Kieran et al. 2013).

Task design also involves an anticipatory aspect in that it requires a designer to think forward about implementation, an issue taken up by Sullivan and Yang (2013) who found that embedding tasks in mathematics classrooms involved complex decision-making that often required in-the-moment choices. While some decision-making can be anticipated on the basis of teachers' prior experience, there are times when pedagogy and sometimes the task itself may need to be adapted in situ (Coles and Brown 2013), an advanced teaching capability. The implementation of tasks is also shaped by other factors including teachers' beliefs about the role of tasks within instruction (Haggarty and Pepin 2002).

Because design and implementation is a complex activity, teachers require significant professional learning support (Zaslavsky and Sullivan 2011) in order to develop effective practices. Some have argued, for example Pepin et al. (2013), that this professional learning should incorporate the involvement of teachers in the design of task they intend to implement. A theme also taken up by Brown (2009) in developing the notion of *pedagogical design capacity*—the ability to utilize existing curricular resources effectively to design instruction. His perspective is based on the position that "all teaching involves a process of design in which teachers use curriculum materials in unique ways as they craft instructional episodes" (p. 18). Brown's (2009) insight, however, is limited to how teachers select or adapt existing curriculum materials and does not address how teachers can generate tasks themselves or in partnership with others.

Jones and Pepin (2016), in agreement with de Araujo and Singletary (2011), comment that for successful implementation, teachers must understand the principles that underpin a task. They argue that an effective way of ensuring this level of understanding is through partnerships between teachers and others with expertise in task design.

3 Research design

3.1 Methodological approach

In this article I draw on data from a larger project that was conducted over a 3-year period in two Australian states. In Australia, education is a state responsibility, however, syllabus documents developed by educational jurisdictions must align with a national curriculum. Within this curriculum, numeracy is included as a General Capability, which means that teachers are expected to make use of mathematics, where it provides advantage to students' learning outcomes, in every school subject. The project described below, aimed to address this issue by developing principles for the effective design and implementation of numeracy tasks across the curriculum.

The research design was aligned with the framework devised by Loucks-Horsley et al. (2003) that situates effective professional learning within teachers' own school-based contexts. From this perspective, if professional learning is to be effective, teachers must be in a position to try out and validate new ideas and initiatives within their own classrooms. Consistent with this perspective, a research design-based approach was adopted as this is known to accommodate the complexity and contextual richness of teaching and learning in situ (Cobb et al. 2003). Accordingly, the methodology: involved iterative interventions; was initiated through specific theoretical intent; and developed and tested theory about how teaching practice and student learning might change, and how these changes can be identified as they emerge through the study (Cobb et al. 2003).

Within each year of the project, three teacher/researcher professional learning workshops and two rounds of school visits were conducted. School visits, in which the researcher documented the implementation of teacher-designed tasks, took place after the first and second workshops. Initial professional learning workshops were based on input from the researcher about the nature of numeracy (Goos et al. 2014), generic principles of task design synthesised from relevant research literature (e.g., Geiger 2016), and immersion activities, in the form of exemplar tasks drawn from previous numeracy projects (e.g., Goos et al. 2011). The aim of these initial workshops was to build teacher capacity to design numeracy tasks for implementation during school visits. Over time, professional learning workshops evolved into opportunities for teachers and the researcher to evaluate the success of implemented tasks that, in turn, fed into a cycle of development and revision that resulted in a framework based on principles of design and implementation for numeracy tasks (PDINT). Through this cycle, a revised PDINT then became the basis for the design of new tasks to be trialled during follow-up school visits, the evaluation of which informed the next revision of the PDINT. Thus, consistent with a design-based research approach, teacher professional learning and theory development, in the form of the PDINT, evolved in parallel throughout the project.

Data used in this article are drawn from field notes, video recordings of lessons, and a semi-structured post-lesson teacher interview gathered during a school visit towards the end of the second year of the project—after the teacher had been involved in five professional learning workshops and three previous school visits. Interview questions focused on the effectiveness of the trialled task and their alignment with elements of the emerging PDINT.

Video-recorded observations and audio-recorded interviews were transcribed and analysed via a process of constant comparison with the categories of the emerging PDINT. The analysis also provided insight into the category descriptions within the PDINT and contributed to its further development via feedback to teachers during workshops. While not all conversations could be categorised against the elements of these principles, all noteworthy episodes were documented. These aspects of analysis were then combined to present a holistic representation of classroom learning experiences, as defined by the PDINT, in the form of vignettes. The vignette presented in this article was selected because of its alignment with the elements of structuring and actualizing within the PDINT.

3.2 Participants

Recruitment for the project took place in two phases. Five teachers were recruited for Phase 1 of the project (year 1) with an additional five teachers, one from each of the Phase 1 schools, agreeing to participate in Phases 2 and 3 (years 2 and 3). Phase 1 teachers were purposively selected (Burns 2000), firstly for their capability with designing rich numeracy tasks, established through previous collaboration in numeracy based researchers projects (e.g., Goos et al. 2011), and secondly for representation across learning areas. Phase 2 teachers, who were less familiar with effective numeracy teaching and learning practices, were recruited in order to assess the effectiveness of the developing PDINT in supporting their task design and implementation efforts. Teachers represented a range of learning areas and sectors of schooling—Secondary English (1), History (1) Mathematics (1), Music (1), Science (1) Technology and Design (1), and Early Childhood/Primary teaching (4).

In this article the specific case of a primary teacher, Stef, is examined. Stef was recruited during Phase 1 of the project and had been involved in previous projects related to numeracy across the curriculum. She is an experienced teacher of primary and early childhood students and the beginning of the project marked the 2nd year she had been employed at her school. In previous school visits within the project, Stef had presented lessons on a variety of topics including developing the language of location within an information and communication technologies class involving robots and the scheduling of a trip on public transport by using a timetable. Through her engagement with the project, Stef had developed a particular focus on how to incorporate a critical question into her teaching. Her attempt to do so in a science lesson is the focus of the vignette presented later in this article.

4 Developing principles of numeracy task design

In this section, I outline the development of the PDINT via a process of teacher/researcher co-construction and provide a description of the framework.

4.1 Process of development

The process of co-construction began in the first workshop with the researcher presenting teachers with: (1) the model of numeracy for the 21st century; and (2) a research synthesis of characteristics associated with the design of effective numeracy tasks. These were introduced to inform teachers' understanding of task design, to provide structure for the development of tasks to be trialled in the first round of school visits and as a starting point for the development of the PDINT.

The Model of Numeracy for the 21st Century represents a synthesis of relevant research literature (see for example Goos et al. 2014) that has been validated through a series of research projects (e.g., Goos et al. 2011; Geiger et al. 2015b). The model incorporates four components: contexts, mathematical knowledge, tools, and dispositions, which are intertwined with an analytical and evaluative capability—a critical orientation. A description of these dimensions appears in Table 2 and is represented in Fig. 1. While initially conceived as a tool for planning and reflection in relation to teaching and learning practice in numeracy, the model has also been used as a scaffolding instrument for the design of numeracy tasks (e.g., Goos et al. 2013). Additional detail about this model can be found in Goos et al. 2014).

The synthesis of characteristics associated with the design of effective numeracy tasks consisted of five aspects: *fit to circumstance*, *challenge, accessibility*, *transparency, opportunities to make decisions and judgements*, and *complementary pedagogies*. A description of each of these aspects is presented in Table 3. Further detail about the development of these principles can be found in Geiger (2016).

Table 2 Descriptions of the dimensions of the model of numeracy for the 21st Century

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities.
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position

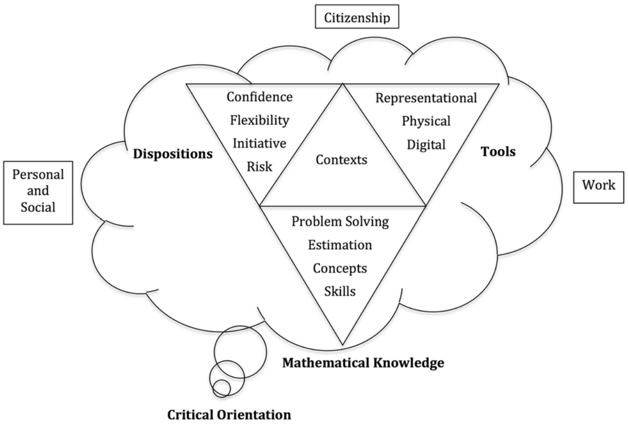

Fig. 1 A model for numeracy in the 21st century (Goos et al. 2014)

A draft PDINT was generated by the researcher from analysis of data collected during the first round of school visits and presented to teachers for comment at the second teachers' workshop. Teacher feedback led to a revised PDINT that was then used as a lens for the analysis of data collected during the next round of school visits. Through this iterative process of trialling and refinement, the PDINT was developed into a framework based on three broad types of activity: *identifying* an idea with the potential to form the basis of a numeracy task; *shaping* the initial idea into a classroom ready task; and *actualising* the task within a classroom. While descriptions of each of these categories of activity were discussed throughout the project their final form emerged at different junctures in the study. Perhaps because of the close connection to classroom practice, s*haping* an idea and *actualising* a task were completed first, towards the end of the second year of the project. I*dentifying* an idea requiring additional time, into the project's third year, as teachers took longer to develop the introspective abilities needed to bring to the level of consciousness, and then articulate, a process that had previously appeared to be innate. Thus, while the whole framework is presented here for completeness, only shaping and actualising are relevant to the vignette presented in Sect. 5 as this took place towards the end of Year 2 of the project.

4.2 Description of PDINT categories

Identifying an idea has three aspects *looking*, *noticing* and *seeing*. To begin the process of identifying a numeracy task, designers must develop a disposition to be always *looking*—a sensitivity or openness to ideas that could be brought into the classrooms in the form of numeracy tasks. Once *looking*, designers of numeracy tasks begin *noticing* events, phenomenon or experiences that that might form the basis for a task. *Seeing* is related to how an initial idea for a task aligns with curriculum documents or school-based teaching and learning programs. This process also includes how an initial idea will need to be adapted to

Table 3 A synthesis of characteristics associated with the design of effective numeracy tasks

Fit to circumstance	Accommodating curriculum requirements and other affordances or constraints within a school setting, for example, teaching materials available within a particular school.
Challenge	Extending students' thinking by including elements of challenge in tasks provides opportunity for reasoning, risk taking, and the justification of decisions
Accessibility	Tasks must feel achievable to all students regardless of their prior history of achievement
Transparency	In order for students to engage fully with tasks, activities must not only be accessible but also transparent in relation to expected outcomes—there is clarity around what is required of students to achieve success
Opportunity to make decisions and judgements	The opportunity to make decisions and judgements introduces a critical demand into a task and provides purpose for students to engage with an activity
Complementary pedagogies	The pedagogical approach must match the demands and instructional intention of the task

match curriculum objectives for both numeracy and the target subject area.

Shaping consists of two sub-activities: *structuring* and *fit to circumstance*. *Structuring* relates to the alignment of the task with the dimensions of the numeracy model—context, mathematical knowledge, dispositions, tools and critical orientation (discussed under Sect. 4.1). *Fit to circumstance* refers to how teachers select, adapt or create tasks that fit their personal teaching environments including:

> the specific learning needs of their students (e.g., non-dominant English language speakers)
>
> nuances of their school's interpretation of curriculum documents and education system requirements or expectations (e.g., development as an independent learner, importance of mental computation)
>
> teaching and learning resources within the school (e.g., commercial learning aids)
>
> the potential of school's built environment as a resource (e.g., swimming pool, school ovals)
>
> the potential of the local natural environment as a resource (e.g., nearby waterways, proximity to historical landmarks)
>
> how to manage the introduction of new activities when working with colleagues and school leaders (e.g., convincing colleagues to implement a task).

Teachers saw the effective *actualising* of tasks as dependent on what became known as a *pedagogical architecture*. This *pedagogical architecture* was based on features teachers identified as characteristic of lessons in which tasks were successfully implemented and included structuring elements as well as specific teacher capabilities considered necessary for the delivery of effective lessons including:

> an initial lesson setup focused on building students' understanding of the real world context in which the activity was situated and to make the task both transparent and accessible. The setup included a *critical question* that provided students with direction for the lesson.
>
> an initial selection of pedagogy(ies) appropriate for the task – usually initiated by teacher directed activity but with the body of the lesson providing opportunity for students to adopt an investigative approach.
>
> the capability to make adaptive use of a repertoire of pedagogies to accommodate both foreseen and unforeseen events during a lesson.
>
> the capability to adapt tasks in-the-moment in order to account for unanticipated student responses to a task.
>
> the use of a *measured responsiveness* when scaffolding student activity. This means providing just enough information/feedback for students to remain engaged with a task without diminishing the level of challenge.
>
> a conclusion in which students are brought together for review of their learning. Within the conclusion a summary of the lesson is orchestrated by the teacher with a particular focus on the original critical question. It is expected that perspectives, conjectures and opinions expressed by students during this phase are justified and supported by evidence drawn from their engagement with the task.

A summary of each of these features is presented in Table 4.

In the next section a classroom vignette will be presented in order to illustrate the shaping and actualising aspects of the PDINT.

5 Lesson Vignette

Stef began the lesson by revisiting progress across a week-long project with her Year 1 students. The project was concerned with the growth rate of bean sprouts when exposed to different levels of sunlight while holding other variables constant. In carrying out the experiment, Stef had also intended that students develop an understanding of specific *mathematical knowledge*—time, measurement, comparison (both graphically and numerically) and variance.

Each student was responsible for a bean that was contained in a zip-lock bag with a piece of wet cotton wool. The bags were then clipped, using a cloths peg, to a wire lattice in different positions near a window so that each bag received a different level of sunlight. Stef worked hard, at the beginning of the lesson, to ensure students were aware of the aim of their experiment—to investigate the factor(s) that resulted in different rates of bean growth.

Stef then began a discussion about how to determine the growth of their bean. Through this discussion she introduced students to the effective use of a ruler (a *physical tool*) and how to record their measurements, taken each day, in a table and as a graph in the form of a strip of paper with successive measurements marked on it (*representational tools*).

To highlight the purpose of the lesson, Stef asked students to read dot points from a white board that were followed by a *critical question*—What did we do/What could we have done to make the seeds grow better? (Figure 2). Stef then asked students to think about the possible factors that were making the difference to bean growth, for example, temperature, sunlight and amount of moisture.

At the end of this discussion, Stef handed each bag with a bean sprout to the appropriate student. Students laid the

Table 4 Principles for the design and implementation of numeracy tasks: shaping and actualising

Identifying	
Looking	A disposition to look for numeracy opportunities and to take advantage of demands.
Noticing	Identifying the source of potential numeracy tasks—(a) selection or adaptation of existing intra-school activities/resources, (b) creation of new activities based on extra-school experiences.
Seeing	Identifying how a proto-activity (preliminary task idea) might align with curriculum/curricula
Shaping	
Structuring	Numeracy model Context Mathematical knowledge Dispositions Tools Critical orientation
Fit to circumstance	Creating, selecting/adapting tasks to accommodate or take advantage of a teacher's/school's unique circumstances. Specific student needs School's interpretation of curriculum—the nuances a school places on aspects of curriculum Available teaching resources Aspects of the built environment Aspects of the natural environment Managing/convincing school leaders and/or colleagues
Actualising	
Pedagogical architecture	Initial setup—explaining and building understanding of context and task; asking a critical question(s) accessibility/transparency Selection of pedagogy(ies) (investigative, teacher directed etc.) Pedagogical repertoire and flexibility/adaptability Task adaptability—ability to change a task in-the-moment Measured responsiveness—providing just enough information/feedback for students to remain engaged in a task Bringing student learning together at the conclusion of a lesson to discuss the critical question—conjectures or opinions must be supported with evidence

bean sprout against their recording strip of paper made a new mark corresponding with the tip and recorded the length, measured with a ruler, in a table.

After every student had measured and recorded the length of their bean sprout, Stef called them together at the front of the room for a whole class discussion about the factors that influenced the growth of their bean. She began the conversation by reminding students of the *critical question* and then asked selected student to talk about how much their bean had grown and where it had been placed on the wire lattice. Throughout, Stef employed *measured responsiveness*, providing just enough information so she could progress the discussion with students. She was also insistent that any opinion or conjecture was supported by evidence gained from the experiment—a practice aimed at developing students' *critical* capability. In this case, students were expected to provide supporting evidence based on the lengths of bean sprouts against their position on the wire grid, that is, through mathematics based arguments.

Stef also took this opportunity to raise other issues associated with the experiment, for example, the issue of

Bean Seeds

- Measure
- See them grow
- See how much they grow
- Show mum and dad
- Different measurements

What did we do/What could we have done to make the seeds grow better?

Fig. 2 Lesson aims including a critical question measurement error, suggesting unusual observations should be checked—an important principle for data collection in science and an aspect of being *critical* when validating results.

6 Stef's perspective on the lesson

After the lesson, Stef was interviewed about the effectiveness of her task and its alignment with the structuring and actualising aspects of the PDINT. The discussion began with Stef describing the purpose of the lesson. Stef made it clear that she wanted her students to see that mathematics had an important role in learning science.

Stef … I wanted them to make that connection that it wasn't just maths, it was maths for a purpose when looking at science. We're looking at the environments that things grow in and the different ways environments can meet creatures' and plants' needs…so they could see there was a connection between the things plants need to grow and the maths components.

While students had made use of informal measurement techniques previously, for the purpose of this experiment, Stef wanted them to learn to use a ruler in order to determine bean lengths. She believed students would be more motived to acquire this new skill if there was a purpose. In this way she had *fit to circumstance* the intended learning outcomes for her students with their perceived need for a purpose.

Stef I really wanted to get them to using a ruler but I also wanted to give them a purpose.

Stef spoke of students' excitement when they observed changes to their bean sprouts as the experiment progressed—something she was pleased about as one of her aims for the lesson was to enhance students' *dispositions* towards science and mathematics learning.

Stef …they were excited about coming in each day and putting them on the windows. That was a focal point when they came in…it really got their interest

> In the beginning of the project, they'd grow like a centimetre, the roots would grow 2 cm and no sprout would come out, and then all of a sudden, roots got to a certain length and the sprout started to come out. So they were really excited about that.

Stef also noticed students' willingness to take risks and show initiative—an important aspect of *dispositions*.

Stef I was also looking at the disposition part…they also needed to take some risks because if they moved it into the wrong spot then they wouldn't get enough sunlight.

The experiment also provided students with the opportunity to develop an understanding of the connection between different types of mathematical representations of growth—numerical values and dates alongside graphical representations (*representational tools)*. Each a form of evidence that could be used to determine the growth of their bean.

Stef I wanted them to see the numbers were increasing but they also have the pictorial reference so they could see what that meant. They're not to scale, but you can see on some of them that as it's gotten bigger they've drawn the pictures bigger.

Stef prepared students to respond to the critical question on the basis of evidence by paying explicit attention to mathematical ideas within the lesson (a *fit to circumstance*). In this case, Stef was looking for students to develop conjectures about why beans in different positions grew more than others and to provide supporting evidence for their opinions. In this way, Stef was attempting to promote students *critical orientation*.

Stef I wanted them to do some problem solving. If they looked at their measurements and saw that their plant hadn't grown much I wanted them to use the data to think what they might have been able to do differently if they were to repeat it in the future. I wanted them to really show some initiative to think of how they might be able to affect the growth rate.

Stef explained further that she wanted to demonstrate the connection between scientific method and mathematics as an underpinning discipline that can provide supporting evidence for a relationship between two variables.

Stef I wanted them to try and draw some connections between sunlight and the position of the bags on the windows and whether there was anything we could do to enhance the growing so that the measurements would change.

Even though Stef was happy with the quality of the lesson, she offered ideas on how it might be improved. Her suggestions included what additional information students should record, for example, the "sunniness" of the day, and the position of a seed on the window grid. In this way, Stef was taking a critical orientation to her own lesson design by looking for how students might be encouraged to be more systematic in their approach to gathering evidence.

Stef	That's what I would do next time…I would have extra columns to see what the weather was like that day and a grid and they could colour in their spot where their bean seed was. Then we could draw better conclusions about whether that position had anything to do with it, or whether it was a super sunny day…yeah we didn't have enough data on the sheet.

Finally, Stef was asked if she was making progress as a "designer" of tasks that promote student learning. She indicated that the process of design had become more internalised as she participated in the project.

Researcher	That seemed to me like a really carefully designed lesson. Do you think you have developed as a task designer?
Stef	I think so, I've certainly become more relaxed about it. At the beginning of the project I would plan frantically and I would think "it's not good enough, is this what I need to do?" And now, I thought about this last night, I had it all planned in my head and I could see clearly where it matched up with the numeracy model…so while I didn't specifically sit down with the numeracy model in front of me, I knew it well enough to know what I wanted from my lesson

7 Discussion and conclusion

While the arguments of those who promote 21st Century Skills and STEM education are often compelling, the issue of how to meld, incorporate or integrate capabilities that enable students to respond to complex real world problems within the context of school classrooms is an area that requires further research (e.g., Honey et al. 2014; Jia et al. 2016). One way 21st Century Skills might be addressed within approaches to STEM education is to adopt a critical approach to enquiry that utilizes other discipline knowledge in a non-superficial way—for a purpose (e.g., Partnership for 21st Century Skills Project 2002; Assessment and Teaching of 21st Century Skills Project 2009). Yet, it appears that an approach that satisfies these expectations and in which knowledge from different STEM disciplines is utilized in a purposeful manner poses significant challenge for schools (e.g., Venville et al. 2002)—especially in the case of mathematics (e.g., English 2016; Fitzallen 2015; Wong and Dillion 2019). For teachers to implement approaches that bring together different STEM disciplines for the purpose of addressing 21st century problems, they must have access to models of teaching practice in which tasks are designed to connect types of knowledge in a cohesive manner (Maass et al. 2013).

I have argued that mathematics is an underutilized resource within STEM teaching and learning, especially in relation to critical aspects associated with evidence-based reasoning and judgement. At the same time, developing the capability to design tasks that utilize mathematics, as a means of enhancing the critical dimension of instruction, has been shown to be a challenge for teachers across the curriculum (Geiger et al. 2015a). While no attempt is made to generalize, the case presented here, however, demonstrates the potential of the PDINT, coupled with complementary professional learning opportunities, to support teachers in designing tasks in which mathematics is used to underpin reasoning and as the basis for evidence that supports propositions and conjectures within the process of scientific enquiry.

Brown (2009) and de Araujo and Singletary (2011) see teachers' involvement in the design of tasks as vital in the development of a deep understanding of the purpose of an activity and awareness of implications for implementation. Consistent with this position, the PDINT was co-constructed by teachers in the project and the researcher (Jones and Pepin 2016). Thus, teachers within the project had a stake in the development of the principles they used to design and implement tasks as well as the tasks themselves. Stef demonstrated her familiarity with the PDINT in the way in which she shaped and actualized her task.

Stef designed her task so that students were required to make use of *mathematical knowledge* such as measurement, comparison, variance and time in the *context* of a scientific enquiry about factors that influenced bean sprout growth. Integral to the gathering of this evidence was the use of *tools*—both physical (rulers) and representational (tables and graphs). She had made decisions about how to *fit* her intention for student to learn to use a ruler to the *circumstance* of instruction about scientific enquiry. In the post-lesson interview, Stef made it clear she had addressed the need to develop students' positive *disposition* toward mathematics and science learning, taking great pleasure in the enthusiastic manner in which students participated in the lesson. Throughout the lesson, Stef was insistent on students' attention to the *critical* question, introducing it to students at the beginning of the lesson and using it as a focus when bringing students together at the conclusion to review their learning. Throughout the process of lesson review, Stef made use of *measured responsiveness* to direct students' thinking towards her intended learning outcomes without trivializing the challenge embedded in the task. During the concluding discussion, Stef was insistent students provided evidence for their conjectures and opinions, directing students back to their data when they engaged in mere speculation. In this way, Stef demanded her students adopted *critical* reasoning practices.

As she had incorporated many of the principles of the PDINT in the lesson, Stef acknowledged she was "getting there". She further commented on her developing confidence as a designer as an outcome of her participation in the project. In particular, Stef noted she was more relaxed about designing tasks as she had internalized the tools she needed to be deliberate about her approach—a capability many teachers do not have opportunity to develop (Schoenfeld 2009). Stef's comments provide evidence for the type of meaningful professional learning that can take place when teachers are involved as partners in task design (e.g., Brown 2009; Pepin et al. 2013). That said, it should be noted that Stef had been involved in a previous project related to numeracy across the curriculum before achieving the success she had experienced in the lesson reported here. Thus, her experience with designing tasks across the curriculum was not restricted to the project alone. At the same time, Stef's long-term engagement with the ideas of numeracy task design is a reminder that significant professional learning is needed in order that teachers become effective designers (Zaslavsky and Sullivan 2011).

By extending Brown's (2009) notion of *pedagogical design capacity* by demonstrating the potential for teachers to create tasks, not just engage in the processes of selection and adaption, this study adds to current research in two ways. First, the PDINT represents the progression of research within the field of numeracy as it extends previous work based on the model for numeracy into the 21st century (e.g., Geiger et al. 2015b; Goos et al. 2014) by including explicit principles for the structuring and actualization of tasks. Second, Stef demonstrated that through the use of the PDINT she was able to design a science lesson in which mathematics had a significant role as the basis for supporting evidence within scientific enquiry. In this way, the approach developed though the project provides one possibility for addressing the underrepresentation of mathematics in STEM classroom activities (e.g., English 2016).

In considering further research that extends the findings of the study presented in this article, two issues come immediately to mind. First, this study was conducted in the context of a primary classroom where the integration of STEM knowledge is not limited by structural issues faced by teachers in secondary schools, for example, faculty boundaries (e.g., Venville et al. 2002). Thus, research is needed into the possibilities that exist within secondary school contexts for the use of mathematics in a purposeful way within other STEM subjects. Second, while the evidence presented here indicates the PDINT was an effective tool for shaping and actualising a task that brought together science and mathematics learning in a purposeful way, it did not account for how teachers identify ideas with the potential for development into effective tasks. Thus, further work is needed to investigate how teachers can be more deliberate about the use of processes that generate purposeful and effective tasks in STEM classrooms.

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