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ORIGINAL ARTICLE

Opportunity to learn problem solving in Dutch primary school mathematics textbooks

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Abstract

In the Netherlands, mathematics textbooks are a decisive influence on the enacted curriculum. About a decade ago, Dutch primary school mathematics textbooks provided hardly any opportunities to learn problem solving. In this study we investigated whether this provision has changed. In order to do so, we carried out a textbook analysis in which we established to what degree current textbooks provide non-routine problem-solving tasks for which students do not immediately have a particular solution strategy at their disposal. We also analyzed to what degree textbooks provide 'gray-area' tasks, which are not really non-routine problems, but are also not straightforwardly solvable. In addition, we inventoried other ways in which present textbooks facilitate the opportunity to learn problem solving. Finally, we researched how inclusive these textbooks are with respect to offering opportunities to learn problem solving for students with varying mathematical abilities. The results of our study show that the opportunities that the currently most widely used Dutch textbooks offer to learn problem solving are very limited, and these opportunities are mainly offered in materials meant for more able students. In this regard, Dutch mainstream textbooks have not changed compared to the situation a decade ago. A textbook that is the Dutch edition of a Singapore mathematics textbook stands out in offering the highest number of problem-solving tasks, and in offering these in the materials meant for all students. However, in the ways this textbook facilitates the opportunity to learn problem solving, sometimes a tension occurs concerning the creative character of genuine problem solving.

Keywords Non-routine problems · Heuristics · Learning facilitators · Opportunity to learn for all students · Textbook analysis · Comparing textbooks

1 Introduction

Mathematics is inextricably linked with problem solving. Problem solving is even considered the heart of mathematics (Halmos 1980; Schoenfeld 1992; Dossey 2017). However, despite the long-standing recognition of the importance of problem solving, there are still different interpretations of what is meant by it. The term is used in several ways, with different connotations (e.g., Schoenfeld 1992; Van Meriënboer 2013; Xenofontos 2010). Problem solving can refer to a skill, a process, an educational goal, and a teaching approach. Specifically, in the field of mathematics education, a distinction is made between teaching *of* mathematical problem solving and the teaching of mathematics *through* problem solving (e.g., Liljedahl et al. 2016). In the current study, the focus is on teaching *of* problem solving.

Several authors (e.g., Burkhardt 2014; Zhu and Fan 2006) have indicated that the term 'problem' itself can also be interpreted differently. In the meaning of a mathematical task on which students have to work, the term problem can refer to all types of tasks regardless of their cognitive demands, but it is also used for specific kinds of tasks, such as word problems in which previously learned mathematics has to be applied, or puzzle-like tasks which are new to the students and which they themselves have to figure out how to solve. The latter meaning is used in this study. By problems we mean non-routine mathematical tasks for which students do not immediately have a particular solution strategy at their disposal.

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In mathematics education, textbooks largely determine what teachers teach and consequently, what students learn (Stein and Smith 2010). In the Netherlands, this is very much the case (see Sect. 2.1). Generally speaking, if certain content is not included in the textbook, it will probably not be covered in the classroom (Stein et al. 2007). Thus, what is in the textbooks is of great importance for the learning opportunities students get, including the learning of problem solving. As a consequence, knowledge of the content of textbooks is very important. About a decade ago an analysis of Dutch primary school mathematics textbooks showed that non-routine problem-solving tasks were hardly included in the textbooks (Kolovou et al. 2009). The current study was meant as a follow up of this study and investigated to what degree non-routine problem-solving tasks are now included in Dutch textbook series. In addition to this purpose, it was also explored whether apart from offering these tasks, there exist other ways in which Dutch textbooks facilitate the opportunity to learn problem solving.

2 Background of the study and research questions

2.1 Textbooks and opportunity to learn in the Netherlands

The aforementioned research claim, that textbooks have a determining role in the enacted curriculum (Stein and Smith 2010), applies also to a great extent in the Netherlands. TIMSS research among Grade 4 teachers found that 94% of these teachers say that their textbook is the main source of their teaching (Meelissen et al. 2012). Other studies carried out in the Netherlands found that more than 80% of Grade 2 and 3 teachers indicate that they are following over 90% of the textbook content (Hop 2012). A minority of these teachers sometimes skip content of their textbook, but they still teach 60–90% of the content (ibid.). Another investigation revealed that only a minority of Grade 6 teachers use additional resources next to their textbooks, mainly materials for less able students and software for repetition (Scheltens et al. 2013). The results of these studies indicate that a vast majority of Dutch primary school teachers rely heavily in their teaching on the textbook series they use. This means that mathematics textbook series play a decisive role in Dutch daily teaching practice and therefore in the learning opportunities that students are offered. As a result, the textbook used has a significant effect on learning outcomes, as repeatedly shown in Dutch national evaluations of educational progress. For several learning topics, these studies have shown that students taught with different textbook series differ significantly in their mathematics achievement (e.g., Hop 2012; Kraemer et al. 2005; Scheltens et al. 2013).

2.2 Opportunity to learn non-routine mathematical problem solving

In his seminal work *How to Solve it* (1945), Pólya does not use the term 'non-routine', but he does define *routine* problems, namely as tasks that "can be solved either by substituting special data into a formerly solved general problem, or by following step by step, without any trace of originality, some well-worn conspicuous example" (p. 171). This implies that whether a task can be considered routine or not at least partly depends on factors other than the task itself, such as whether an example is given. Schoenfeld (1985, 2013) points out that difficulty alone does not define a task as a problem. Instead, it is not a property of the task itself which makes it a problem, but the "particular relationship between the individual and the task" (Schoenfeld 1985, p. 74). Thus, whether a task can be considered a problem can differ per person: a task that forms a genuine problem for one student may be a routine task for another student. Furthermore, whether a task is a problem may differ over time, after all, "the person who has worked on, and solved, a problem, is not the same person who began working on it. He or she approaches the next problem knowing more than before" (Schoenfeld 2013, p. 20). In other words, what is at first considered a problem can become a routine task.

The issue of this relative and personal character of problem solving has been raised already very often (e.g., Kantowski 1977; Lesh and Zawojewski 2007; Manouchehri et al. 2012; Lester 2013) and is also reflected in a recent OECD publication in which is stated that a mathematical problem "involves a situation, posed in either an abstract or contextual setting, where the individual wrestling with the situation does not immediately know how to proceed or of the existence of an algorithm that will immediately move toward a solution" (Dossey 2017, p. 61). Researchers may use different wording, but they generally agree that the feature that makes a mathematical task a problem is that the person who has to solve the problem does not directly have a solution procedure at his or her disposal. Otherwise this task is what Pólya (1945) calls a "routine problem" or what other authors call, with a more distinctive term, an "exercise" (e.g., Burkhardt 2014; Manouchehri et al. 2012; Schoenfeld 1985). Such tasks are solvable by straightforward calculation (Pretz et al. 2003), executing rules or procedures (Lesh and Zawojewski 2007), applying known algorithms or following worked out examples (Manouchehri et al. 2012), or by following a seen or taught solution pathway (Burkhardt 2014). In this study, we refer to such tasks as *straightforward*.

Different from straightforward tasks, non-routine problems require more than just executing the required calculations. Non-routine problems ask for more complex processes and set higher cognitive demands. Solving such tasks involves analyzing the problem at hand, relating procedures with their underlying mathematical concepts and making connections between different representations (Stein et al. 2000). Or, in the words of Lester (2013): “For non-routine tasks a different type of perspective is required, one that emphasizes the making of new meanings through construction of new representations” (p. 255). A related distinctive feature of a genuine problem is that it requires modeling (e.g., Lesh and Zawojewski 2007; English et al. 2008), which as Lesh and Zawojewski (2007) point out, in a way is *creating* mathematics. In other words, problem solving requires creative mathematical thinking, which is underlined in the recently published ICME ‘state-of-the-art’ report on problem solving (Liljedahl et al. 2016).

Another important perspective on problem solving that is also emphasized in this ICME publication is that of a heuristic approach (see Pólya 1945, 1962; Schoenfeld 1985, 1992, 2013; Mason et al. 2010). This approach involves the conscious use of a number of problem solving strategies that may help to find a solution, including acting the problem out with objects, drawing a diagram, guessing a seemingly reasonable answer and checking it, reasoning logically, making a systematic list or a table, restating the problem, simplifying the problem, solving part of the problem, thinking of a related problem, using a model or an equation, and working backwards (e.g., Fan and Zhu 2007; Lee et al. 2014).

According to Liljedahl et al. (2016), there can occur a certain tension between this approach and the non-routine, creative character of problem solving because the “problem solving heuristics that are based solely on the processes of logical and deductive reasoning distort the true nature of problem solving” (p. 19). Thus, for example, the heuristic “think of a related problem” may lead to the recalling of a known solution procedure for that particular type of problem, which means that in that situation no genuine problem solving occurs. Yet, the heuristic “make a systematic list or a table” can provoke divergent thinking, which is an aspect of genuine problem solving, and the heuristic “draw a diagram” can support the creative process of modeling a problem. However, drawing a diagram may also lead to the use of a well-known procedure, just as thinking of a related problem does not necessarily lead to recalling of a known procedure. Nevertheless, as Schoenfeld (1992) points out, when students are given intensive practice in certain heuristics, these become mere algorithms. Thus, just as problem solving and problems are relative in nature, heuristics can also be characterized as such—they either can or cannot contribute to the opportunity to learn problem solving, which, moreover, is partly due to how instruction takes form (see also English et al. 2008). For example, introducing heuristics in an isolated way could provoke students to see them as rules (Fan and Zhu 2007).

Although Lester (2013) claims that research does not tell enough yet about problem-solving instruction, he does list important principles that have emerged from research in the last decades. The two most important principles in his view are that students, in order to improve their problem-solving abilities, have to “work on problematic tasks on a regular basis over a prolonged period of time” (p. 272) and have to be “given opportunities to solve a variety of types of problematic tasks” (ibid.). In other words, the learning of problem solving is enhanced by the opportunity to actually work on genuine and varied non-routine problems. As a consequence, a clear way in which mathematics textbooks can contribute to the opportunity to learn problem solving is the inclusion on a regular basis of tasks that potentially can be genuine problems for students. In addition to including problem-solving tasks, Doorman et al. (2007) recommend that textbook series explicitly pay attention to heuristics. They also point out that genuine problem-solving is often believed to be only attainable by the best students (ibid.). However, Stein and Lane (1996) reason that all students, of varying abilities, may benefit from the opportunity to work on tasks with high level cognitive demands such as non-routine problem solving. More recently, Jonsson et al. (2014) found that cognitively less proficient students also profit from working on tasks that require creative mathematical reasoning. Based on their study, they argue that all students should be given the opportunity to be involved in problem solving (ibid.). That less able students indeed may have the ability to learn problem solving, given the opportunity, was for example demonstrated in a Dutch study with students that attend special education (Peltenburg et al. 2012). Thus, a final way in which textbooks can enhance the opportunity to learn problem solving is to include problem-solving tasks and heuristics not only in materials that are meant exclusively for more able students, but in materials meant for all students.

2.3 Research questions

About a decade ago, Kolovou et al. (2009) researched to what degree Dutch textbook series contained non-routine problem-solving tasks and so called “gray-area” tasks, which were defined as tasks that fall in between genuine problem-solving tasks and straightforward tasks (see Sect. 3.3. for a more precise description of these two types of tasks). All then available textbook series were investigated. The analyzed textbook materials were meant for the first half school-year of Grade 4. It was found that the proportion of non-routine problems was very low, varying from 0 to 2% of the total number of tasks. When taken together, the proportion of non-routine problem-solving tasks and gray-area tasks was still rather low—varying from 5 to 13% of all tasks. Furthermore, it was found that most non-routine and gray-area tasks were included in additional enrichment materials of the textbook series, meaning that the already limited opportunity to learn problem solving was even lower for students that were not given the chance to work with these materials.

Currently, all textbook series that were investigated then have been replaced by new editions or have been withdrawn from the market. New textbook series have also been published in the meantime. In order to find out whether these present mathematics textbooks have changed with respect to the opportunity to learn problem solving, we carried out a replication study, in which we investigated to what degree current mathematics textbooks offer non-routine problems. Further, bearing in mind the different perspectives on problem solving discussed in the previous section, we additionally researched whether textbooks facilitate the opportunity to learn problem solving in other ways, such as presenting heuristics. Finally, we analyzed to what degree learning opportunities are offered in materials meant for all students and in materials meant only for more able students. So, our research questions were as follows:

1. To what degree do current Dutch primary school textbooks contain mathematical problem-solving tasks?
2. In what other ways do these textbooks facilitate the opportunity to learn problem solving?
3. How inclusive are these textbooks with respect to offering opportunities to learn problem solving for students with varying mathematical abilities?

3 Method

To answer these research questions, we carried out a textbook analysis of primary school mathematics textbook series presently in use in the Netherlands.

3.1 Selection of textbooks and textbook materials

Nowadays in the Netherlands, there are eight different mathematics textbook series for primary school on the market. For selecting textbook series to be included in our study, we first looked at the textbooks' market share. We wanted to include textbook series that together are in use in a majority of schools because this would provide a sound basis for drawing conclusions regarding the Dutch situation in general. This led to the selection of three textbook series that together are used in approximately 90% of all schools.[1] These textbook series are *De Wereld in Getallen* (The World in Numbers) (Huitema et al. 2009–2014), *Pluspunt* (Plus Point) (Van Beusekom et al. 2009–2013) and *Alles Telt* (Everything Counts) (Van den Bosch-Ploegh et al. 2009–2013). Previous editions of these three textbook series were also included in the study of Kolovou et al. (2009). A fourth textbook we included in our textbook analysis was *Rekenwonders* (Wonder Calculators) (Projectgroep Rekenwonders Bazalt Groep 2011–2015), which is the Dutch version of the Singapore textbook series *My Pals Are Here! Maths* (Kheong et al. n.d.). As a result of the high performance of Singapore students as established in international research, a Dutch publisher took the initiative to translate and adapt this textbook series for the Netherlands. Compared to the other textbook series involved in our study, *Rekenwonders* has only a very small percentage of market share. This is not only because this textbook series is not that long on the market, but also because the content and teaching method are quite new for teachers and deviate somewhat from what is traditionally taught in Dutch primary schools. The reason that we nevertheless included this textbook in our study was that this textbook is purposely put in the market to enhance students' problem-solving skills. Thus, for us it is interesting to investigate what opportunities to learn problem solving this textbook series offers.

To make a comparison possible with the Dutch textbooks involved in the study of Kolovou et al. (2009) a decade ago, we included textbook materials for the same school period as was done in this earlier study, namely materials meant for the first half schoolyear of Grade 4. In order to determine possible differences within textbook series between materials meant for different grades, we also included materials for the first half schoolyear of Grade 6.

All four textbook series consist of the following materials: lesson books and work books for students, accompanying teacher guidelines, and additional materials such as work sheets and software. In the analysis we included all student materials that, as indicated in the teacher guidelines, belong to the daily lessons. Materials with no such link, such as software for repetition of basic knowledge and skills (e.g., the multiplication tables) were left out of our analysis. Because directions for instructional approaches, which are often included in teacher guidelines, are also of influence on the opportunity to learn (Remillard et al. 2014), we also included these guidelines in our analysis.

3.2 Unit of analysis

Although the four selected textbook series differ in their quantitative features such as number and size of student book pages, they all provide content for five daily mathematics classes per week, for 36 weeks per schoolyear. Also, in all four textbook series lessons are subdivided into numbered segments, mostly consisting of sets of tasks (see

[1] This estimation is based upon oral information from publishers.

Which number belongs to each letter?

a A + B = 7 A − B = 3 A = …… B = ……	**b** C + D = 10 C − D = 4 C = …… D = ……	**c** E + F = 14 E − F = 6 E = …… F = ……	**d** L + M = 11 L − M = 9 L = …… M = ……
e R + S = 18 R − S = 2 R = …… S = ……	**f** T × U = 12 T − U = 4 T = …… U = ……	**g** V × X = 12 V − X = 1 V = …… X = ……	**h** J + K = 14 J − K = 4 J = …… K = ……

Fig. 1 A set of tasks from *De Wereld in Getallen* meant for Grade 4, consisting of eight tasks. All examples of tasks have been translated by the authors of this article

Fig. 1). With the term 'task' we refer to the smallest unit that requires an answer from a student. In this study, we used the *set of tasks* as unit of analysis. This approach corresponds to the approach in the study by Kolovou et al. (2009). A difference between the two studies was that in the earlier study the teacher guidelines were left out of the analysis, while in our study we considered directions given in the teacher guidelines for a set of tasks as belonging to that set of tasks.

3.3 Analysis procedure

Because of the relative and personal character of problem solving, it is not easy to decide whether a task has to be classified as a problem or as a straightforward task. As Zhu and Fan (2006) reason, making such a judgement in textbook research is difficult, if not impossible, due to the fact that the features of the students solving the tasks are not known. Therefore, the methodological challenge of this study was to develop an analysis framework that indicates when a task should be classified as a genuine problem-solving task and when not.

We started the development of our analysis framework with several rounds of preliminary classifying tasks based on the theoretical insights as described in Sect. 2.2 and the analysis framework used by Kolovou et al. (2009). So, based on our judgement to what degree tasks require higher-order thinking skills such as analyzing or creative thinking, they were classified in three categories: straightforward tasks, non-routine problems, and gray-area tasks. If a set of tasks included tasks of more than one category, it was classified according to the highest category.

Figure 2 shows an example of a task that we classified as a non-routine problem. This task, meant for Grade 4, is a magic frame in the form of a triangle that has to be filled in with the numbers 1–9 in such a way that each side of the triangle adds up to 17. Students in Grade 4 will most likely have no known solution procedure at their disposal for this task and there are also no directions provided in the textbook on how to solve it. Therefore, we considered this a puzzle-like task that requires analyzing and creative thinking in combining numbers that add up to 17, while taking into account that the three numbers at the corners of the triangle

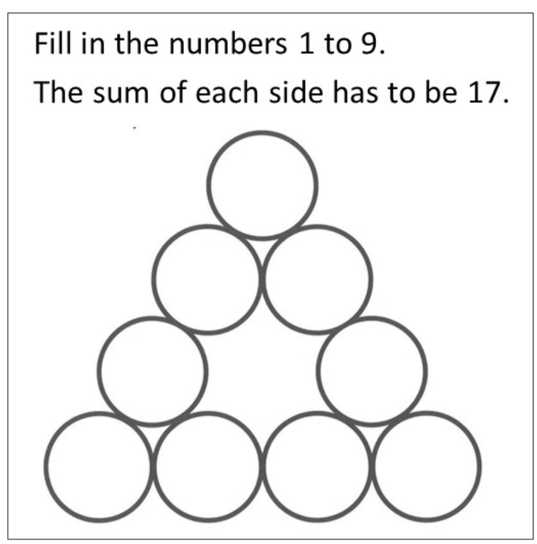

Fig. 2 A Grade 4 task from *Alles Telt* classified as a non-routine problem

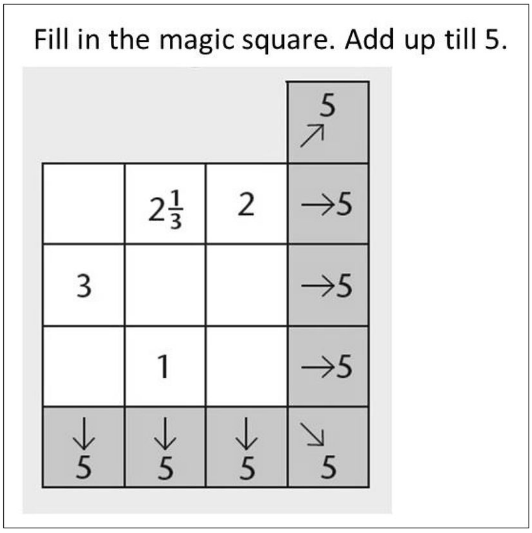

Fig. 3 A Grade 6 task from *Pluspunt* classified as gray-area

are used twice in a combination of numbers adding up to 17. So, placing higher cognitive demands, requiring creative mathematical thinking and being puzzle-like, classifies this task as non-routine.

The task in Fig. 3, meant for students in Grade 6, also concerns a magic frame. Again, the textbook does not provide directions on how to solve the task. However, because of the four already filled-in numbers, the cognitive demands of this task differ considerably from the task in Fig. 2. Based on the filled-in numbers in the upper row, it can be derived directly what has to be filled in in the upper left empty cell. Also, the number that has to be filled in in the middle cell can be derived directly from the two numbers already given in the middle column. In this way, the solution process can go on—every time an empty cell can be filled in from two given or earlier filled in numbers in the same row, column, or diagonal. Thus, the creative thinking needed and the cognitive demands are limited compared to those of the task in Fig. 2. Yet, its solution pathway cannot be qualified as completely straightforward, since solving this task requires finding a suitable starting point, as well as determining throughout the solution process what next step can be taken. Therefore, we classified this task as gray-area.

Based on the initial rounds of classification and a review of literature describing features of tasks, we developed further indicators to be used for the definitive classification of the tasks. These indicators concern features of tasks that are expected to provoke or require analyzing, modeling and creative thinking and therefore contribute to the opportunity to learn problem solving. Since all preliminary tasks labeled non-routine and gray-area were of the type that Pólya (1945, 1962) calls “problems to find”, we formulated indicators for each of the principal parts he distinguishes for these type of tasks, namely the data provided by the task, the unknown that has to be found, and the conditions that have to be fulfilled linking the unknown to the data. These principal parts are for the task shown in Fig. 3, for example, as follows: the unknown consists of five numbers that have to be filled in the empty cells; the data provided are the four already filled in numbers; and the condition is that the numbers in each row, column and diagonal have to add up to 5.

The number of relations between the provided data and the required conditions that have to be processed in parallel while solving a problem may influence the complexity of the problem (Jonassen and Hung 2008). This means that the more conditions that have to be fulfilled in a task, the more its complexity increases. Another way in which this occurs is when the data provided in the tasks are interdependent, as is the case in the task shown in Fig. 4, in which weights that have to be added are expressed in terms of each other. It also makes a difference whether or not data are provided in the same order as is needed for solving the problem (Goldin and McClintock 1979). Since increasing complexity gives more need for analyzing and modeling, we took these features—the number of conditions, interdependency of data and the order in which the data are provided—into account

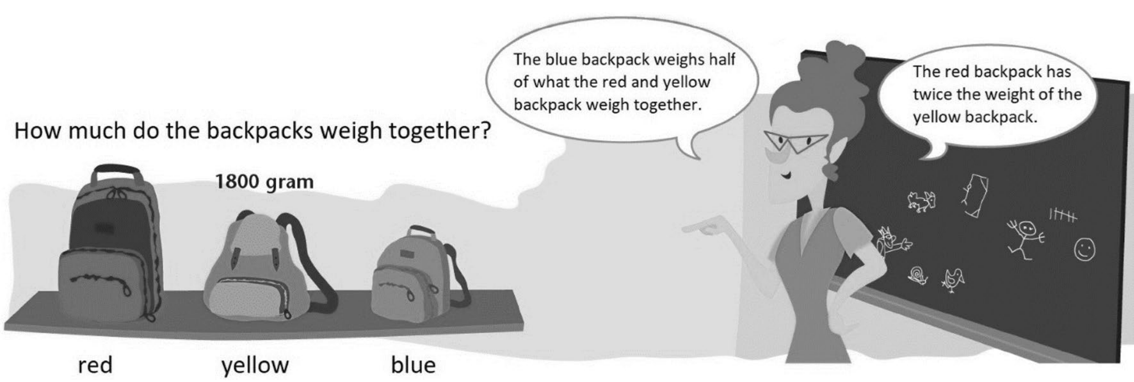

Fig. 4 A Grade 6 task from *Pluspunt* with interdependent data: the weights of the backpacks are expressed in terms of each other

Table 1 Analysis framework for classification of tasks

Category	Indicators and decision rules
Non-routine problems	The task meets *two or three* of the following features: The unknown has to meet three or more conditions The data provided are interdependent The data are provided in another order than needed for solving the task In case the task has multiple correct solutions: All possible correct solutions have to be given *or* The total number of all possible correct solutions has to be given
Gray-area tasks	The task meets *one* of the following features: The unknown has to meet three or more conditions The data provided are interdependent The data are provided in another order than needed for solving the task In case the task has multiple correct solutions: One possible correct solution has to be given *or* Some but not all possible correct solutions have to be given
Straightforward tasks	The task is solvable by straightforward calculation *or* The task is offered after an explanation or an example which demonstrates how it can be solved

as indicators for problem solving. Regarding the processing of data while solving a problem, the number of steps that have to be made can affect the nature of the task (e.g., Zhu and Fan 2006). A task that has no easily determinable starting point, such as the problem in Fig. 2, or that involves reasoning back and forwards, requires multiple steps in the solution process. However, we consider multiple steps not a distinctive feature as such—when multiple steps in solving a task involve nothing more than just straightforward calculation, we consider that task still to be a straightforward one.

Indicators regarding the unknown apply only to tasks that have multiple correct solutions. It makes a difference whether only one or some correct solutions have to be found, or that all correct solutions have to be found (e.g., Pólya 1962; Pretz et al. 2003). For example, in a combinatorics task, finding all correct solutions requires modeling and making a systematic analysis. This type of task can therefore be considered a non-routine problem (as long as a standard procedure for it is not yet known). The cognitive demands of a combinatorics task in which only a few correct solutions have to be given, is considerably lower. For solving such a task, a systematic analysis is not necessary, but providing a solution is still creative in nature. Therefore, we classified such a task as gray-area.

Altogether, we consider a larger number of conditions (below more on this), interdependency of data, and another order in the presentation of data than needed in the solution pathway as features of tasks that may lead to analyzing, modeling and creative thinking. Therefore, these features may serve as general indicators that a task may be a non-routine problem or a gray-area task. However, each of these indicators on its own may be applicable to non-routine problems as well as on gray-area tasks. It is the combination of multiple of these features which qualifies a task as non-routine problem and therefore we needed a quantitative decision rule (see also Goldin and McClintock 1979) to use for the definitive classification of tasks. Based upon the results of the preliminary classification rounds we decided to classify a task that meets two or all three of these features as a non-routine problem, and a task that meets one of these features (and that cannot be considered a straightforward task) as a gray-area task. Further, regarding the first feature of a larger number of conditions, we specified "larger" as three or more. This, again, is based upon the results of the initial analysis. Next to these general indicators, we added a specific indicator for tasks with multiple correct solutions, namely, when all possible correct solutions have to be given, the task is considered non-routine and when just one or some of the correct solutions is sufficient, the task concerning is classified as gray-area.

Table 1 shows our final analysis framework, including the decision rules. We illustrate how this framework was applied in the final analysis through the task shown in Fig. 5. The unknown that has to be found in this task is

> Linde is thinking of four whole numbers. When ordered from small to large, each number is half of the next number. All numbers are even. Together the numbers add up to 30.
> Which numbers is Linde thinking of?

Fig. 5 A Grade 6 task from *Alles Telt*

Table 2 Absolute and relative frequency of non-routine problems, gray-area tasks and straightforward tasks

	De Wereld in Getallen		Pluspunt		Alles Telt		Rekenwonders	
Grade 4 (first half school year)								
Non-routine problems	30	5%	3	0%	25	2%	43	5%
Gray-area tasks	24	4%	28	3%	23	2%	25	3%
Straightforward tasks	592	92%	926	97%	960	95%	832	92%
Total Grade 4	646	100%	957	100%	1008	100%	900	100%
Grade 6 (first half school year)								
Non-routine problems	16	2%	15	2%	40	4%	80	8%
Gray-area tasks	9	1%	35	4%	17	2%	8	1%
Straightforward tasks	666	96%	854	94%	1007	95%	920	91%
Total Grade 6	691	100%	904	100%	1064	100%	1008	100%

Due to rounding off, some percentages do not add up correctly

four whole numbers. These numbers have to meet all the given data, resulting in three conditions, namely (1) three of these numbers have to be half of another one of the numbers; (2) all numbers have to be even; and (3) the numbers have to add up to 30. The provided data are not interdependent (as opposed to the data of the task shown in Fig. 4). To combine the given data to find the unknown, they have to be processed in another order than they are given: the first piece of information that limits the possible correct answers—the numbers add up to 30—is given last. So, this task meets two of the features in our framework (there are three conditions; the provided data must be processed in another order), classifying it as non-routine.

Based on our framework, the final classification of tasks was done by the first author. An independent expert on mathematics education who was not involved in the development of the framework performed a reliability check of the classification. For this, we used a selection of tasks ($n = 100$) covering a quarter of the total tasks classified by the first author as non-routine or gray-area. In this selection, all found appearances of non-routine problems and gray-area tasks were included. Moreover, this selection also contained 'similar-looking' straightforward tasks ($n = 15$). The agreement between the classifications of the first author and the external rater was 87.8%. After discussing the differences between the two classifications the agreement was 96.5%.

For answering the second research question, a qualitative analysis was carried out in which we inventoried all directions for problem solving strategies included in the student books as well as in the teacher guidelines. For the latter, along with the final classification of sets of tasks, we systematically checked all the accompanying descriptions included in the directions for the daily lessons in these guidelines. In addition, we checked whether the general texts of the teacher guidelines include directions for the learning of problem solving.

4 Results

4.1 Mathematical problem-solving tasks in Dutch primary school textbooks

Our first research question considered the degree in which current textbooks contain mathematical problem-solving tasks. In all textbook series included in our analysis, the percentage of non-routine problems is low, varying from 0 to 5% in the materials meant for Grade 4 and varying from 2 to 8% for Grade 6 (Table 2). The percentage of gray-area tasks varies from 2 to 4% for Grade 4 and from 1 to 4% for Grade 6. In all textbook series the majority of tasks for both grades is of the straightforward category, varying from 91 to 97%. Compared to a decade ago, when Kolovou et al. (2009) found that the textbooks then in use had a proportion of straightforward tasks varying from 87 to 95%, this number has not changed much. All the textbook series are still mainly filled with straightforward tasks.

Within the low percentage of problem-solving tasks, the part of non-routine problems and gray-area tasks in the current textbooks has changed compared to a decade ago. The average over all textbook series of non-routine problems in the materials meant for Grade 4 was 1% and is now 3% (and 4% for Grades 4 and 6 together). The average of gray-area tasks for Grade 4 has shifted from 9 to 3% (and 2% for Grades 4 and 6 combined). So, the current textbooks include on average relatively more non-routine problems and less gray-area tasks (both relatively and absolutely). The combined average of non-routine problems and gray-area tasks however, dropped from 9 to 6%. This indicates that the current textbooks include even fewer problem-solving tasks than those investigated a decade ago.

For the textbook series *De Wereld in Getallen, Pluspunt* and *Alles Telt* a more precise comparison regarding Grade 4 can be made between the current editions analyzed in this study and the previous editions included in the study by

Table 3 Distribution of non-routine problems, gray-area tasks and straightforward tasks for Grade 4 in the current and former editions of *De Wereld in Getallen*, *Pluspunt* and *Alles Telt*

	De Wereld in Getallen				Pluspunt				Alles Telt			
	3rd edition		4th edition		2nd edition		3rd edition		1st edition		2nd edition	
Non-routine problems	10	2%	30	5%	0	0%	3	0%	6	2%	25	2%
Gray-area tasks	43	11%	24	4%	41	9%	28	3%	35	4%	23	2%
Non-routine and gray-area tasks together	53	13%	54	8%	41	9%	31	3%	41	5%	48	5%

Due to rounding off, some percentages do not add up correctly

Kolovou et al. (2009). This analysis shows that for each of these three textbook series on its own, the number of non-routine problems has increased and the number of gray-area tasks has decreased (Table 3). The combined percentage of non-routine problems and gray-area tasks is comparable in the two editions of *Alles Telt* but has decreased in the respective editions of *De Wereld in Getallen* and *Pluspunt*.

In all the textbook series, except *De Wereld in Getallen* the materials meant for Grade 6 provide more non-routine problems and gray-area tasks than the materials for Grade 4. Yet, the percentage of these tasks in Grade 6 is still low.

Out of all the textbook series, *Rekenwonders* provides the most problem-solving tasks in both grades. *Pluspunt* provides the least number of problem-solving tasks for Grade 4 and *De Wereld in Getallen* for Grade 6.

4.2 Other ways to facilitate the opportunity to learn problem solving

Our second research question concerned other ways, besides the offering of problem-solving tasks, in which Dutch textbooks facilitate the opportunity to learn problem solving. At this point, we found a striking difference between the textbook series *Rekenwonders* and the other three textbook series. Only in *Rekenwonders* did we come across regularly and systematically offered directions for both students and teachers that can be interpreted as facilitators for the learning of problem solving.

In the teacher guidelines of *Rekenwonders* problem solving heuristics are provided such as guess and check, making a systematic list and working backwards. Learning to use heuristics is explicitly mentioned as a goal. Furthermore, the teacher guidelines provide suggestions for questions to ask students to make them aware of the problem-solving process. These include asking students to sum up the data, conditions and unknowns of problems, and stimulating them to think about suitable representations of problems. However, it must be noted that these suggestions are also given for tasks that we classified as straightforward.

In the student books of *Rekenwonders*, two sorts of learning facilitators are provided. One involves the bar model, which is extensively used for different topics, in the way described by Kho et al. (2014). The bar model is also explicitly presented as a tool for solving non-routine word problems. This is done by providing partly worked out examples in which the steps for solving a specific word problem are already given and students have only to fill in the numbers (see Fig. 6 for an example). Thus, although the bar model is presented as a problem-solving tool, the way in which this is done remarkably requires little more than straightforward calculation.

The bar model is also present in the other three Dutch textbook series, but less so than in *Rekenwonders* and not as a tool for problem solving.

The second learning facilitator that *Rekenwonders* offers in the student books is presenting special text sections including summaries and reflections on particular learning content. For example, students are asked to think of other situations in which a particular way of solving a problem

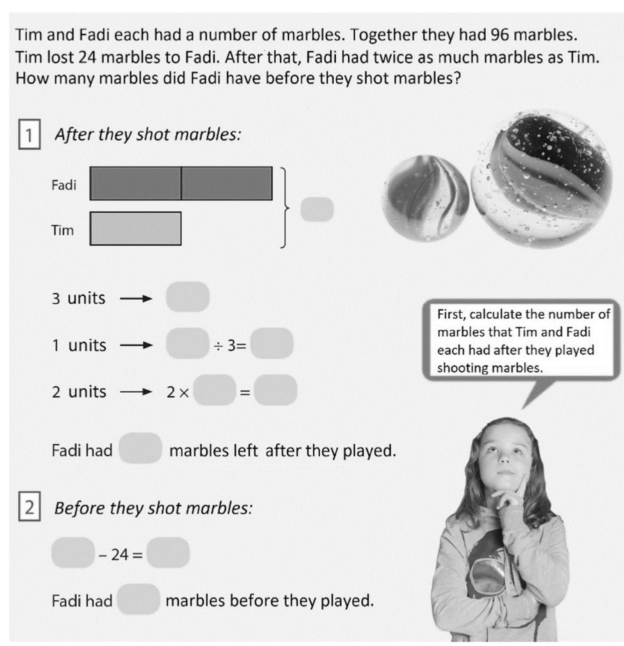

Fig. 6 A Grade 6 task from *Rekenwonders* in which is demonstrated how the bar model can be used for solving it

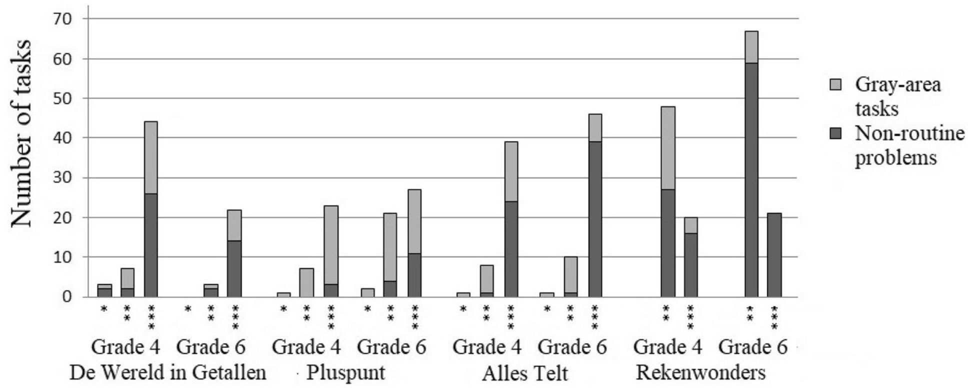

Fig. 7 Frequency of non-routine problems and gray-area tasks over materials meant for less able students (*), almost all students (**), and more able students (***)

also could be applicable. Similarly to the other facilitators already mentioned above, this one is not exclusively used for problem solving, but for all kinds of learning topics.

The other textbooks provide hardly any learning facilitators for problem solving comparable to those given by *Rekenwonders*. The teacher guidelines of these textbook series do provide suggestions for questions that can be asked of students, but not for the learning of problem solving. Only in a few cases *Alles Telt* provides the suggestion in the student book to draw a table. In *Pluspunt*, sometimes in the teacher guidelines it is emphasized that students should read a problem well and should work systematically. In *De Wereld in Getallen* we found no directions.

4.3 Opportunity to learn problem solving for students with varying mathematical abilities

Our final research question addressed the issue of how inclusive the current Dutch textbooks are with respect to offering opportunities to learn problem solving for students with varying mathematical abilities. All analyzed textbooks aim to a certain extent to be inclusive by having their materials organized in parts meant for different groups of students. In *De Wereld in Getallen, Pluspunt* and *Alles Telt* these parts contain differentiated tasks organized in three levels: tasks for almost all students, more cognitively demanding tasks for more able students, and easier tasks especially for less able students. *Rekenwonders* offers two levels of tasks: tasks for all students and more demanding tasks for more able students. Thus, in this textbook series the less able students also get the 'tasks for all', while in the other Dutch textbook series, these students get easier tasks. This means that *Rekenwonders* actually offers less able students more challenging tasks than the other Dutch textbooks do.

Figure 7 shows for the four textbook series the distribution of problem-solving tasks over the different levels. In *De Wereld in Getallen, Pluspunt* and *Alles Telt* most non-routine problems and gray-area tasks are included in the materials meant for the more able students. This was also the case in the former editions of *Pluspunt* and *Alles Telt*, as established by Kolovou et al. (2009). In the former edition of *De Wereld in Getallen* most non-routine and gray-area tasks provided were included in the materials meant for all students (ibid.), which in the current edition of this series is no longer the case. The situation in which most problem-solving tasks are meant for all students now applies only to *Rekenwonders*.

5 Conclusion and discussion

The importance of problem solving together with the finding from a decade ago that Dutch primary school mathematics textbooks hardly included problem-solving tasks at that time, led us to investigate the opportunity to learn problem solving provided by current Dutch textbooks. We found that in the textbook series *De Wereld in Getallen, Pluspunt* and *Alles Telt* this opportunity still turns out to be low. These textbooks provide only a small number of problem-solving tasks, incorporate hardly any other ways to facilitate the learning of problem solving, and the problem-solving tasks that *are* provided are mainly included in the parts that are meant for the more able students. The textbook *Rekenwonders* offers more opportunities to learn problem solving. This textbook provides the highest number of problem-solving tasks, systematically offers heuristics and other facilitators for learning problem solving, and moreover, includes most of the problem-solving tasks in the materials that are meant for all students.

All in all—also taking into account that *De Wereld in Getallen, Pluspunt* and *Alles Telt* together are in use in about 90% of Dutch schools and *Rekenwonders* only in a few schools—the opportunity to learn problem solving provided by current textbooks is for a vast majority of Dutch students very limited, just as was the case a decade ago.

Apart from bringing into view what mathematical content is offered to students directly, textbook analysis can also reveal implicit or hidden choices that are made in textbooks. Especially the comparison with textbooks that originate from different traditions in mathematics education may shine new light on content and teaching approaches that are taken for granted and can show that also other choices can be made. In this way our study can be of interest for a broader audience than only the Dutch mathematics education community. By not only doing an analysis on the content but also on the organizational structure of the textbooks, it was revealed that investigating the opportunity to learn offered in textbooks should also take into account what content is offered to whom. As a result of the organizational structure of *Rekenwonders*, in this textbook also less able students are offered genuine problem-solving tasks. This differs from the structure of the other three Dutch textbooks, in which the less able students obtain easier tasks which do not have a problem-solving character. This approach to problem solving as only an additional learning topic for the more able students is more or less in line with the official Dutch intended curriculum in which only limited attention is paid to problem solving (Van Zanten et al. 2018). Conversely, in Singapore problem solving plays a central role in the curriculum and is situated in the heart of the Mathematics Curriculum Framework (Ministry of Education of Singapore 2006).

Our study clearly shows how complex the concept of opportunity to learn is from the perspective of the textbook. Just exposure of the content does not tell the whole story. As we have described above, it is also necessary to bear in mind to which students the opportunity to learn applies. A further factor that determines whether an opportunity to learn really can be considered as such, is its quality. Therefore, in our study we did not look only at the exposure of problem-solving tasks but also at the offered learning facilitators and their quality. An example is the presenting of the bar model as is done in *Rekenwonders* as a tool for problem solving, in such a way that it requires little more than straightforward calculation. This use of the bar model is a clear illustration of the tension that can occur between the creative character of genuine problem solving and the use of certain problem-solving heuristics as rules to be followed. Another learning facilitator that also might not be so helpful for learning problem solving is what *Rekenwonders* offers for new types of problems, namely systematically partly worked out examples (such as shown in Fig. 6). These examples will not really trigger the creative problem-solving process of modeling, but this is rather a systematic exercise in using this particular model. Taking the quality of the opportunity to learn into account we have to put our initial conclusion that *Rekenwonders* offers more opportunities to learn problem solving into perspective. What in any case remains is that students in all four investigated textbooks are offered few opportunities to learn problem solving.

This brings us to our final thought. In our view, problem solving is an important learning topic for *all* students. After all, as Halmos (1980) puts it: “The major part of every meaningful life is the solution of problems” (p. 523). Or in the words of Freudenthal (1973): “How can mathematics be a discipline of the mind if people never experience mathematics as an activity of solving problems?” (p. 95). The chance of getting such experiences will be greatly enhanced if future Dutch mathematics textbooks—and this may apply for any mathematics textbooks—will provide more opportunities to learn problem solving—for all students.

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