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ORIGINAL ARTICLE

An interdisciplinary approach to designing online learning: fostering pre-service mathematics teachers' capabilities in mathematical modelling

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Abstract

In this article we describe and evaluate processes utilized to develop an online learning module on mathematical modelling for pre-service teachers. The module development process involved a range of professionals working within the STEM disciplines including mathematics and science educators, mathematicians, scientists, in-service and pre-service secondary mathematics teachers. Development of the module was underpinned by Bybee's five E's enquiry-based approach and Goos et al.'s twenty-first century numeracy model. Module evaluation data is examined in relation to the quality of pre-service teachers' learning experiences and interview data from the study is analysed through the lens of 'boundary crossing'. While the evaluation of the module was generally positive, aspects that required improvement were also identified including more meaningful inclusion of pre-service teachers and other stakeholders in the development process.

1 Introduction and background

Approaches to strengthening the personal knowledge of pre-service teachers within the Science Technology Engineering and Mathematics (STEM) disciplines is of international priority (e.g., English 2016). However, there is not yet a uniform understanding of what is meant by STEM or coherent or consistent development of STEM-oriented curricula (Fullan 2007). Similarly, there have been a number of attempts to articulate the connections between mathematics and science teaching and learning, for example, in the USA (Cady and Rearden 2007), Canada (Samson 2014), Korea (Kim and Bolger 2017) and Denmark (Blomhøj and Kjeldsen 2009). However, these studies have not provided the insight necessary to develop approaches that effectively promote pre-service teachers' understanding of the interrelationships between mathematics and science education (Song 2017).

Within the Australian context, the performance of school students on international comparative assessments such as the Programme for International Student Assessment (PISA) (Thomson et al. 2016) is a source of increasing concern for government, educational jurisdictions and the community at large. For example, Australian students' performance in PISA on mathematical literacy has steadily declined from 8th in 2006 to 20th in 2015. These results are paralleled by

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falling participation in mathematics, science and technology in Australia, raising serious questions about Australia's capacity to sustain a knowledge-based economy and society (ACOLA 2013).

The decline in Australian students' performance and interest in STEM has been linked to perceptions of these subjects as "largely about recipes or watching teachers following recipes" (Office of the Chief Scientist 2012, p. 9). In response, the Chief Scientist commissioned a report entitled, *Mathematics, Engineering and Science in the National Interest*, in which "inspiring teaching" was identified as "undoubtedly the key to the quality of our system and to raising student interest." (Office of the Chief Scientist 2012, p. 7). Further, current pedagogical approaches were not supporting the development of enquiry capabilities, "The issue is that science is not taught as it is actually practised: hypothesis, experimentation, observation, interpretation and debate" (Office of the Chief Scientist 2012).

To meet this challenge, the Australian government provided funding for *Enhancing the Training of Mathematics and Science Teachers* (ETMST) (2013–2017); a $12.6 m scheme targeting pre-service teachers (PSTs). Projects were required to develop courses of study aimed at promoting PST's personal discipline knowledge and effectiveness in utilizing enquiry approaches to learning science and mathematics. These programs were to be developed through collaboration between mathematicians, scientists, mathematics educators and science educators. Such collaboration was groundbreaking, in the Australian context, as there was little pre-existing culture of collaboration between these groups within PST education programs.

This article concerns one such project funded under the ETMST program, *Opening Real Science: Authentic Mathematics and Science Education for Australia* (ORS) that involved the development and evaluation of eight mathematics and 17 science online learning modules for primary and secondary initial teacher education programs. These modules were designed to utilize authentic contexts and enquiry-based pedagogical approaches within an online learning environment as a model for teaching and learning within school contexts.

We report on the development of an online learning module devoted to mathematical modelling for secondary mathematics PSTs, *Modelling the present: Predicting the future*, as an exemplar of the interdisciplinary work conducted within the ORS project. In doing so we will address the following research questions:

1. How can a module focused on mathematical modelling be designed for delivery to PSTs in an online learning environment?
2. What perspectives did the Module Development Team and stakeholders in mathematics, science and education bring to the design and evaluation of the online learning module?
3. What was the nature of the interdisciplinary collaboration between mathematicians, scientists, and mathematics and science educators in module development?

In responding to these questions, we will: (1) Describe and discuss the processes of module design and trialing with PSTs; (2) analyse survey and interview data from the Module Development Team and a range of stakeholders involved in module development; and (3) discuss implications for the ongoing program of strengthening Australian teachers' personal disciplinary knowledge and pedagogical processes in mathematics and science.

2 Instruction in mathematical modelling

Mathematical modelling is included as an instructional expectation within curriculum documents in a range of different countries (e.g., CCSM 2010) and has been integrated into teaching from primary to tertiary levels (e.g., Mousoulides and English 2011). From an international perspective, the significance of mathematical modelling as a key personal capability is evidenced through its inclusion as a core element within the *PISA 2012 Assessment and Analytical Framework* (OECD 2013). Within this document, the ability to mathematize, that is, transform a complex real world problem into a mathematical representation (a model), is identified as essential when using mathematics to solve real world problems. However, it is widely recognized that the ability to mathematize is insufficient, of itself, to develop effective solutions to authentic real-world problems, as a number of additional competencies (Maaß 2006) must be brought to bear. This suite of capabilities is most often represented in the form of a cyclic process.

The modelling cycle has been described in different ways by a number of authors (e.g. Galbraith and Stillman 2006; Blum and Leiß 2007), however, versions of these cycles tend to coalesce around a number of core phases: essential characteristics of a problem in the real-world are identified; the real problem is simplified in order to develop a workable model; justifiable assumptions are made to accommodate missing information; the real situation is translated into an idealized mathematical model (mathematizing); an initial solution is generated from the mathematical model; resulting solutions are brought back into relief against the initial real-world situation; the validity of a potential solution is considered; and the process is revisited until an acceptable solution is established. A typical representation of the cycle, as described by Blum and Leiß (2007), appears in Fig. 1.

It is important to note that while this analytical reconstruction of the modelling process is useful in describing the

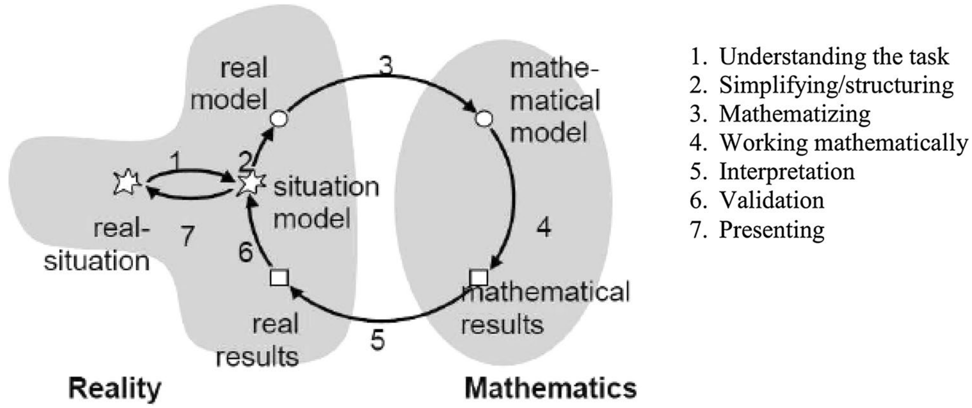

Fig. 1 A modelling cycle as described by Blum and Leiß (2007)

modelling process to novices, in practice, modelling does not proceed in a linear fashion as frequent switching between the different phases is necessary when moving towards a solution (e.g., Ärlebäck 2009) and is punctuated by *stumbling blocks* (Galbraith and Stillman 2006). Modelling is additionally demanding as *implemented anticipation* (Geiger et al. 2017; Niss 2010) is also required in order to mathematize. This means a modeller has to anticipate, before and during the mathematization process, the mathematical entities, such as knowledge and representations, that will prove useful in solving a problem, as well as how mathematical processes will be used to managed these entities. Consequently, carrying out mathematical modelling is a complex process that readily gives rise to learning difficulties (Jankvist and Niss 2015).

As an established field of research within mathematics education, there now exists a corpus of literature related to mathematical modelling, the foci of which include: the identification of modelling capabilities (e.g., Maaß 2010); the representation of individuals' modelling processes (e.g., Czocher 2016); the use of data in mathematical modelling (e.g., Doerr and English 2003), the presentation of scenarios and tasks suitable for mathematical modelling in school contexts (e.g., Lesh and Doerr 2003) and assessing modelling competencies (e.g., Mousoulides et al. 2008). Most studies, however, have sought to describe and understand the capabilities invoked when engaged in modelling, that is, how modelling is learned and practiced, or what inhibits learning, rather than those teaching practices which best promote the ability to engage in modelling.

It is important to distinguish between teaching students how to apply previously developed models and teaching within a modelling mode (Burghes and Huntley 1982). While both have value from a motivational and student interest perspective, the passive use of previously developed models is only a first step in moving away from conventional mathematics teaching towards a modelling approach.

According to Niss et al. (2007), quality instruction in mathematical modelling includes practices common to the teaching of pure mathematics as well as activities that are not a part of traditional mathematics classrooms. These activities are needed to accommodate the contextual and situated nature of modelling. Further, instruction in mathematical modelling varies according to a range of contextual factors such as: level of education, nationality, curriculum intention and expectation, the purpose of modelling, and availability of teaching resources. Different contexts have implications for the types of instructional approaches adopted to promote mathematical modelling capability as well as the tasks selected or created to support instruction (Blum and Niss 1991). Blum and Niss (1991) distinguish six different approaches to instruction related to mathematical modelling and applications:

- Separation—in which mathematics and modelling are separated in different courses;
- Two-compartment—with pure and applied elements within the same course;
- Islands—where small islands of applied mathematics can be found within the pure course;
- Mixing—in which newly developed mathematical concepts and methods are activated towards applications and modelling, although the necessary mathematics is identified from the outset;
- Mathematics curriculum integrated—here real-life problems are identified and the mathematics required to deal with them is accessed and developed subsequently;
- Interdisciplinary integrated—operates with a full integration between mathematics and extra-mathematical activities where mathematics is not organised as separate subject.

These approaches represent options for how teachers may realise modelling in their classrooms. Tasks can exist on a continuum between extended complex modelling problems in co-operative, self-directed learning environments (e.g., Blomhøj and Hoff-Kjeldsen 2006) through to more constrained versions of modelling tasks embedded within a traditional curriculum, such as the case in a recently revised national program in Singapore which included mathematical modelling for the first time (e.g., Chen 2013). Other factors, for example, available resources, will also impact on the way modelling tasks are designed (e.g., Geiger 2017). The employment of digital tools, for example, introduces an additional layer of complexity into the teaching of mathematical modelling. Previous research has reported that the use of digital technologies has the potential to make complex modelling problems more accessible to students and the successful implementation of technology "active" modelling tasks is largely dependent on teachers' expertise, confidence and beliefs about the nature of mathematics learning (Geiger 2011). While the use of digital tools in mediating the modelling process is receiving increasing attention (e.g., Greefrath et al. 2011), research has again tended to focus on how students learn to model within technology-rich environments rather than how to best structure approaches to teaching. This is especially the case for online learning environments where there appears to have been limited research related to effective instruction.

The influence of different teaching approaches on students' engagement with mathematical modelling has been the focus of a number of studies by Shukajlow and colleagues (e.g., Shukajlow et al. 2015). This work indicates that student-oriented modes of instruction have a stronger effect on students' enjoyment, valuing, interest and achievement in mathematical modelling activities than teacher directed approaches. However, these studies also found that students' enjoyment, interest and achievement in modelling are mediated by their self-perception of competence. A complementary study by Schukajlow and Krug (2014), however, using an inferential and path analyses method, found that approaches that encouraged students to find multiple solutions to problems had positive effects on student learning including higher levels of motivation and interest and feelings of competence and autonomy.

3 Theoretical foundations

In order to describe and analyse both the quality of the online learning module and the processes of interdisciplinary collaboration engaged in module development, it was necessary to utilize two different theoretical perspectives; instructional design and 'boundary crossing'. The instructional design approach was based on two models—the Biological Sciences Curriculum Study (BSCS) 5Es instructional model (Bybee 2009) and the twenty-first Century Numeracy Model (Goos et al. 2014); both consistent with a modelling approach to instruction. *Boundary crossing* is utilized to describe and analyse the nature of the interdisciplinary collaboration between mathematicians, scientists, and mathematics and science educators during the development of the module.

3.1 The 5Es instructional model

All ORS project modules utilised the 5Es instructional model (Bybee 2009) as a design framework. This model employs an enquiry-based approach to science education that consists of five phases: engagement, exploration, explanation, elaboration and evaluation. The 5Es model was chosen because it was consistent with the broad pedagogical intention of the project—to enable PST's to develop mathematical and scientific reasoning capabilities through examples or cases of authentic scientific exploration and findings. Phases within the model are cyclic and reflexive, with different components informing others as illustrated in Fig. 2. The context-based and exploratory approach promoted by the model provided a frame consistent with the teaching and learning demands of a module based on mathematical modelling, *Modelling the present: Predicting the future*.

A summary description of these phases is set out in Table 1.

3.2 A twenty-first century numeracy model

The *twenty-first Century Numeracy Model* (Goos et al. 2014) was chosen as the basis for structuring module case studies because its dimensions aligned with authentic modelling practices described by the Module Development Team (depicted in Fig. 3, summarised in Table 2).

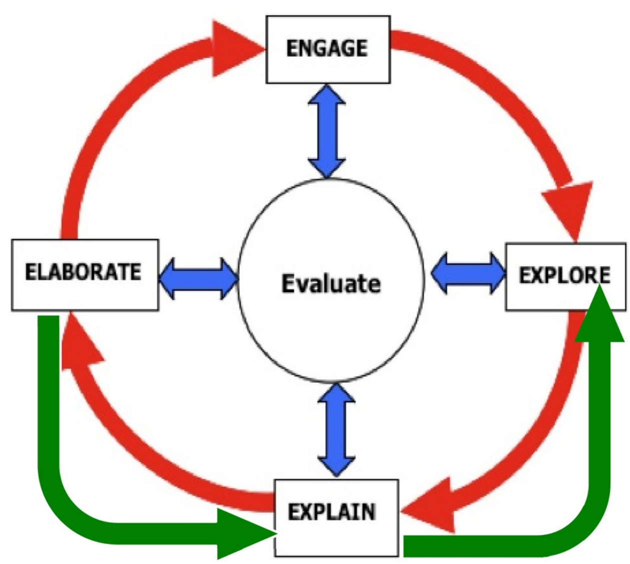

Fig. 2 Biological sciences curriculum study 5Es instructional model (Bybee 2009)

Table 1 Summary description of the 5Es instructional model (Bybee 2009, p. 8)

Engagement	Learners become engaged in a new concept. The activity should organize students' thinking towards specific learning outcomes
Exploration	Learners are provided with a common base of activities within which concepts, processes, and skills are identified and conceptual change is facilitated
Explanation	Opportunities for students to demonstrate conceptual understanding, process skills, or behaviors and for teachers to directly introduce a concept, process, or skill are provided
Elaboration	Teachers challenge students' conceptual understanding and skills and students progress their understanding of a concept by conducting additional activities
Evaluation	Students assess their own understanding and teachers evaluate student progress towards achieving the educational objectives

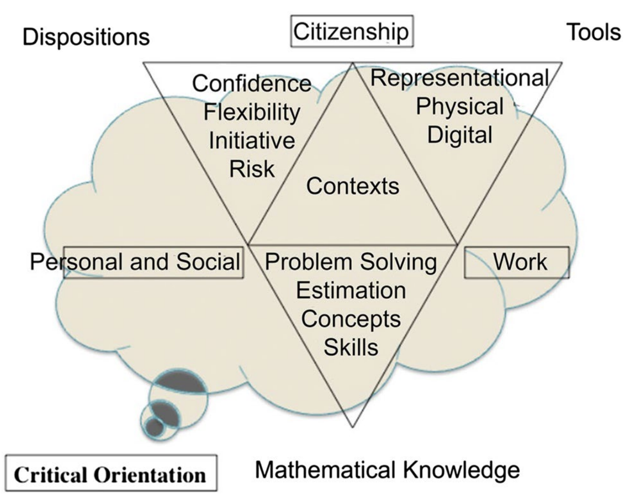

Fig. 3 A model for numeracy in the twenty-first century (Goos et al. 2014)

3.3 Communities of practice and boundary crossing

In his analysis of groups involved in shared practices within and across trades and professions, Wenger (1998) developed the notion of *communities of practice* in which group members contribute to each other's learning by engagement in a common activity. Wenger proposed three dimensions of collaborative pursuit within such communities: mutual engagement, joint enterprise and shared repertoire. He also described different ways of participating within communities of practice:

- Engagement: doing things together, talking, and producing artefacts.
- Imagination: constructing an image of ourselves, of our communities, and of the world, in order to orient ourselves, to reflect on our situation, and to explore possibilities.
- Alignment: a mutual process of coordinating perspectives, interpretations, and actions so they realise higher goals.

    (Wenger 1998).

Communities, by their existence, are defined by boundaries that separate groups of participants and non-participants. Such boundaries can both divide and connect communities (Akkerman and Bakker 2011) but where it is advantageous, members of different communities will seek out opportunities for boundary encounters (e.g., Sztajn et al. 2013). Such encounters represent points at which coordinated and coherent shared action and interaction can be established.

The concepts of *boundary crossing* and *boundary objects* are central to describing the ways in which different communities can engage with learning by sharing, coordinated action and gainful interaction (Akkerman and Bakker 2011). Boundary crossing refers to the transitions of individuals across communities and their interactions with new and different ideas and cultural norms. *Boundary objects* are those artefacts that act as bridging mechanisms by which a *crossing* is effected. The concepts of boundary crossing and boundary objects are of interest within educational contexts because of the potential for learning at intersections

Table 2 Descriptions of the elements and critical orientation of the numeracy model

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge
Tools	Use of material (e.g., measuring instruments), representational (graphs, maps) and digital (e.g., computers, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position

between communities who create and value different types of knowledge.

Suchman (1994) has argued that the term boundary crossing denotes the transition of an expert into an arena in which they are far less qualified. Such transitions have the potential for new learning and the development of new knowledge as boundary crossers bring together their expertise and the new knowledge and ways of reasoning that exist within the community to which they have transitioned.

Within mathematics and science education, the ideas of boundary crossing and boundary objects have been utilized to analyse one-way transitions such as school to work (e.g., Wake 2014). Bilateral exchanges have also been explored between groups including collaborations between mathematicians, educational researchers and in-service teachers (e.g., Goos 2013), mathematics teacher educators and teachers involved in teacher professional development (Sztajn et al. 2013).

4 Methodological approach

The method followed an adapted form of design study based on five iterations of activity where material was drafted, evaluated, enhanced and finally assembled into a coherent program of study. Five different groups of participants were involved at different levels of evaluation (described in Sect. 4.4).

4.1 Participants

Participants in the study included:

- the Module Development Team—eight academics with backgrounds in mathematics, science, and mathematics and science education, who volunteered their expertise in biological evolution, financial mathematics, astrophysics and environmental science as well as experience in the teaching and learning of mathematical modelling and instructional design
- an External Academic Expert Evaluator—a senior academic with expertise in, and experience with, the teaching and learning of mathematical modelling
- Teacher Education Professionals—six expert stakeholders and professionals in mathematics education (e.g., representatives of education systems)
- a Module Enhancement Workshop group—informed stakeholders including mathematics, science and education academic groups and in-service teachers
- Pre-Service Teachers (PSTs)—a cohort of 22 PSTs from a partner university who trialled module activities

4.2 Procedure for module development

The development phase of the model was coordinated by the Module Development Team leader and began with an introduction to the 5Es model as an overarching framework for all modules. This was followed by development of case studies/topics related to participants' expertise: epidemiology, financial mathematics, astrophysics and environmental chemistry. The intention was that each topic form the basis of a case study from which PSTs could develop an understanding of the use of mathematical models to describe natural phenomena and to make predictions. Consistent with the requirements of undergraduate teaching programs, the module was developed to deliver 36–40 h of academic study over 4–5 weeks. Given the constraint on time, the decision was taken to restrict instruction to the use of existing mathematical models (Burges and Huntley 1982). Module design was based around mandatory elements (Introduction, Case study 1, Reflection and capstone assessment) and options (one of case studies 2a, 2b, 2c). The module structure and its alignment with the 5Es model is represented in Fig. 4.

The Numeracy Model for the twenty-first century was used to develop a template for case study design as there was clear alignment between its dimensions and the initial ideas for module activities described by members of the Module Development Team. This alignment is presented below as a list which matches the common key elements of each initial idea for a case study with the dimensions of the numeracy model:

- a real-world scenario was central (context)
- mathematical knowledge was necessary to address case studies (mathematical knowledge)
- digital tools were needed to explore the case studies (tools)
- positive student dispositions toward the use of mathematics to solve real-world problems were targeted (dispositions)
- a decision or judgement was required in relation to each case study (critical orientation).

Thus, the case study template took the following form:

- Outline of a specific real-world context to which a mathematical model could be applied. The context should pique students' interest (context and dispositions).
- Introduction of a tool for exploration of an introductory problem. The problem should require the application of mathematical knowledge and be of interest to students (context, mathematical, knowledge, tools, dispositions).
- Presentation of an open-ended problem in which a scenario is explored using a mathematical model. Prediction(s) are required in response to questions (con-

Fig. 4 Organisation of module *Modelling the present: Predicting the future*

Introduction

Description of the modelling process; video examples of modelling real world phenomena

(Engagement)

↓

Case study 1

Epidemiology – Humanities greatest killers

Modelling infectious diseases to predict transmission rates. Assessment requiring the investigation of mortality rates in relation to influential factors in infectious disease spread (e.g. Ebola) based on the use of a prepared spreadsheet.

(Engagement, Exploration, Explanation)

↓

Case study 2a

Trading – Understanding investment and risk

Case study 2b

Environmental Chemistry – Impacts of catchments pollutants

Case study 2c

Astrophysics – Modelling the big, the strange, and the too far to see

Each case study included: a description of a real-world scenario and specified a related open-ended problem; support material for the use of a digital tool needed for investigating the problem; assessment requiring predictions about aspects of the scenario/problem.

(Engagement, Exploration, Explanation, Elaboration)

↓

Reflection and capstone assessment

Develop a teaching resource about mathematical modelling

(Engagement, Exploration, Explanation, Elaboration, Evaluation)

text, mathematical knowledge, tools, dispositions, critical orientation).

4.3 Module case study exemplar

An activity from the case study, *Environmental Chemistry—Impacts of Catchment Pollutants*, is presented in Figs. 5, 6 and Table 3 as a way of illustrating the type of modelling activity students were required to attempt.

The impacts of concrete on water chemistry were *outlined* in the introduction of the case study through a video in an attempt to pique student interest (dispositions) via connection with an environmental concern—the pollution of water ways (Activity 3.1, Fig. 5). Students were then introduced to a *tool* for the exploration of an initial problem (tools). The tool took the form of an Excel file, inclusive of relevant data including pH and electrical conductivity over time (Table 3). The expectation was that students would use the facilities of Excel to investigate the impact of concrete on water chemistry as measured by

Fig. 5 Activity 3.1: modelling the impacts of concrete on water chemistry

To examine data on the impacts of concrete on water chemistry, as measured by pH and electrical conductivity, open and use the MS Excel file, *Concrete and Tank Water* in the folder, *Resources: in-class experiment instructions and data files*. View the YouTube video, *How to make a line graph in Excel (Scientific data)*, to learn how to plot the data in a graph and then fit a line or curve to the data on the change in pH or electrical conductivity over time. Make separate graphs for pH and electrical conductivity (the latter is measured in micro siemens). Include your graphs in a report interpreting what they tell you about the impacts of concrete on water chemistry. Then submit your report (approx. 300 words) to the discussion forum.

Fig. 6 Activity 3.2: modelling solute concentrations from concrete abd PVC sources

In this second activity on the impacts of concrete on water chemistry, you will undertake comparisons of changes in pH and electrical conductivity of water from different sources, concrete and PVC (plastic) pipes. Use data in the MSExcel file, Concrete and Plastic pipe experiment, which is located in the folder, Resources: in-class experiment instructions and data files. Use MSExcel to plot graphs of the change in pH and electrical conductivity over time for concrete and for PVC pipe, i.e., plot four graphs and fit a trend line or curve to each. Include your graphs in a report (approx. 500 words):

* Document and compare the impact of concrete with PVC pipe on water chemistry and predict what the impact of this might be on storm water flowing from paved urban areas into natural streams

* Explain your result, i.e., your interpretation/understanding of the role that mathematical modelling plays in predicting water pollution from concrete

* Comment on some of the assumptions and limitations of the model. How do these impact the predictions it makes?

* Submit your report to the discussion forum and comment on at least one other participant's conclusions with reference to how they supported their conclusions with analysis of the evidence.

Table 3 Water chemistry data from the excel file used for Activities 3.1 and 3.2

Time (min)	Concrete pipe—metre 4		Plastic pipe—metre 1	
	pH	Conductivity (micro-siemens)	pH	Conductivity (micro-siemens)
0	4.92	26	4.79	26.1
5	5.88	26.3	5.11	25.2
10	6.35	26.7	5.24	24.9
15	6.71	28	5.39	24.7
20	6.98	29.4	5.54	24.7
25	7.23	31.8	5.67	24.7
30	7.33	32.9	5.76	24.7
35	7.41	34.2	5.81	24.8
40	7.59	36.2	5.89	24.9
45	7.56	37.1	5.97	24.9
50	7.57	38.2	6.00	25.00
55	7.61	40.4	6.07	25.2
60	7.65	41.7	6.11	25.4
65	7.7	43.6	6.17	25.4
70	7.77	45.8	6.21	25.5
75	7.84	47.6	6.29	26.1
80	7.85	49.2	6.32	26.3
85	7.86	50.6	6.36	26.5
90	7.91	51.8	6.38	26.6
95	7.93	54.9	6.41	26.8
100	7.94	56.2	6.45	27.1

pH and electrical conductivity by creating graphs from the data and looking for trends (context, dispositions, tools, mathematical knowledge and critical orientation). The purpose of this activity was two-fold: (1) to familiarise students with the role of mathematical models in addressing real world problems; and (2) to develop students' capability to use a digital tool as a means of exploring data related to the problem.

After an introduction to the general problem and providing opportunity for students to learn how to use a relevant digital tool, Activity 3.2 (Fig. 6) was presented in order to provide a more *open-ended* and critically demanding scenario for students to explore. The problem involved comparing the impact of PVC (context, dispositions, tools, mathematical knowledge and critical orientation) on water chemistry compared to concrete. In addition to describing the comparative effects of PVC and concrete on water chemistry, students were also required to state and justify any assumptions made in developing their conclusions and also discuss the limitations of the model as these related to their results (critical orientation).

4.4 Module evaluation components

There were four sources of module evaluation data.

An External Academic Expert Evaluator was engaged to provide critique on the quality of the module. Their brief was to provide a general impression of the module, indicating a score of between 1 and 10, and to comment on the quality of the learning activities aimed at promoting mathematical modelling capabilities.

Teacher Education Professionals were utilized for an independent review of the module using an online survey. The survey consisted of ten items with six open-ended questions focused on Teacher Education Professionals' views on the likely effectiveness of the module. Questions 1–3 were related to this group's background and general impressions of the module. Questions 3, 7 and 8, considered critical to the evaluation, were concerned with the quality of the learning process (See Sect. 5.2).

Module Enhancement Workshop evaluations were led by a member of the Development Team and provided advice on the quality of the module. Before the workshop, participants responded individually to four questions designed to document their impressions of the module using a six-point Likert scale (1 = 'not at all' to 6 = 'extremely well'). At the workshop, members discussed their individual responses in order to prepare a consensus report—an agreed Likert scale score and a qualifying comment.

Secondary mathematics PSTs' perspectives on the quality of the module were evaluated via a survey of confidence, understanding of content and mathematical scientific processes as well as functionality and usefulness. The survey included: an overall rating for the module; eight six-point Likert scale (indicating the range of agreement from 1 = 'strongly disagree' to 6 = 'strongly agree') items; three binary response items; and four open-ended questions. The evaluation was completed fully by 22 PSTs. Data from Question 12 are drawn upon for this article because of its alignment with the research questions.

4.5 Module development team interview protocol

As the focus of this study was both the quality of the online learning module and the development process, semi-structured interviews were conducted with each member of the interdisciplinary Module Development Team. The instructional designer, a member of the Module Development Team, conducted six interviews within 1 month of module completion with each being digitally recorded and later transcribed. The third section of the interview protocol, *Perspectives on interdisciplinary collaboration*, was relevant to our third research question and included the following prompts:

Describe, from your perspective, the experience of working in a interdisciplinary team to develop the module as a whole. For example

- What do you believe was the value in including contributions from different disciplines? Describe advantages/disadvantages.
- Are you satisfied/happy/impressed with the module as an outcome of the collaboration?
- Outline the opportunities/advantages for educators/mathematicians/scientists working together in promoting STEM education.
- Describe any limitation/constraints/barriers for educators/mathematicians/scientists working together in promoting STEM education.

4.6 Analysis of data

Data utilized in this article are drawn from participants' written responses to the evaluation components of the module construction process and audio-recordings of semi-structured interviews with Module Development Team members. The evaluation components of the study targeted the quality of PSTs' learning experiences through the module in relation to the development of their personal knowledge and understanding about the use of mathematical models. Interviews with team members captured their perceptions of the module development process as well as their views on the cross-disciplinary nature of its design.

Participants' responses to the evaluation components were first examined to identify specific references to mathematical modelling or the application of mathematics real-world phenomena. These selected responses were then subjected to a process of open coding (Strauss and Corbin 1990), seeking emergent themes within responses to each question or element within the question. The outcome of an initial round of coding included emergent themes such as quality of design, understanding of the modelling process and potential for enhancement. These themes were then used to structure a second examination of the data seeking to identify finer-grained categories of responses. This process resulted in sub-themes such as the potential for student engagement, the relationship between mathematical modelling and the STEM disciplines, variety of real-life contexts presented as case studies, the connection between mathematical modelling and scientific enquiry, and an understanding of mathematical modelling in specific scientific or mathematical contexts. While not all participants' responses were relevant to the overarching focus on mathematical modelling, all noteworthy comments were documented. Representative comments are presented as evidence of themes identified within each module evaluation component (set out in Sect. 5).

Module Development Team members audio-recorded interviews were transcribed through a process of constant comparison against a frame informed by Wenger's (1998) notion of community of practice and the concept of boundary crossing (Akkerman and Bakker 2011). Principle elements of this frame were drawn from the work of Akkerman and Bakker (2011) and Suchman (1994) and consisted of:

- How boundaries can connect communities.
- How boundaries can divide communities.
- The potential benefits of boundary encounters.
- The role of the mathematical modelling module as a boundary object.
- Perspectives on crossing a boundary into an arena in which an expert is less qualified.

Team members responses to interviews were examined through the lens of these principle elements and then synthesized into three broader themes (reported in Sect. 6):

1. Benefits of crossing boundaries.
2. Opportunities provided by collaboration across disciplines.
3. Limitations associated with interdisciplinary collaboration.

5 Module evaluation

A summary of evaluations at each level is provided below.

5.1 External academic expert evaluation

The External Academic Expert Evaluator scored the module an overall ranking of 7 out of 10 with detailed comments focused on both the quality of the modelling case studies and the effectiveness of the manner in which the modelling process was presented:

> The students should both learn how to use existing models, and for purposes of their future independence, understand the more general modelling process used to develop them. Given the nature of the module the second purpose can be built from experiences with the first.
> … the approach in this module is ideally suited to meeting objectives associated with engagement with STEM disciplines…Two desirable outcomes are enhanced knowledge around the substantive content areas of specific models, and appreciation of the modelling process sufficient to support future independent activity in other fields of application.

Further, the External Academic Expert Evaluator commented on the effectiveness and affordances of the module design as a collaborative process.

> The design of the module is a team project. This adds richness and depth by drawing on the strengths of individual contributors.

Recommendations for enhancement and review centred on aspects of time allocation and learning priorities and the need for an academic university tutor to provide guidance for effective and efficient delivery.

> There is a need for supplementary text material providing guidelines for students as to which resources to prioritize at times, and what they should expect to obtain from them.

5.2 Teacher education professionals

Surveys were completed by six Teacher Education Professionals (TEPs) from different universities and stakeholder groups. Responses to Questions 3, 7 and 8 from the survey are reported here because of their relevance to PST engagement and learning about the design process that underpins authentic mathematical modelling.

Question 3 of the survey sought each reviewer's "main impression". The six reviewers commented on some aspects of module design but were more concerned about how the mathematical modelling related to real-world "authentic" situations.

> TEP1: I felt this module was well designed and had a good scope of areas covered. The introduction provided interesting and relevant aspects of mathematical modelling and how it applied to science and real-world applications.

Five out of six Teacher Education Professionals commented on the strength of the potential engagement and interest of PSTs because of the use of modelling in a range of real world contexts. For example:

> TEP 2: The interdisciplinary case study examples include business, health sciences, environment and astrophysics. The diversity of subjects ensures that students will likely engage with the module.

For Question 7, *How effective do you think the module is in improving pre-service teachers' capability of designing authentic rich tasks in mathematics and/or science?*, there was unanimous agreement that the module could support the design of authentic tasks, however, two of six respondents believed some PSTs would not have sufficient mathematical content knowledge to effectively engage some aspects of mathematical modelling within the module.

> TEP 3: I think that some aspects of the module will do this really well. Some of the maths will kill them; some is high school maths (some may have seen it, many might not have or might have forgotten), some of the finance maths goes beyond that.

The alignment of the module content with the perceived requirements of the *Australian Curriculum: Mathematics* (ACM) was also questioned.

> TEP 4: It gives pre-service teachers an opportunity to look beyond the traditional methods of teaching and support the development of "rich-tasks" but there is no content that has a title of mathematical modelling. My concern would be that beginning teachers would find it difficult to implement such approaches into a normal mathematics classroom environment in the culture within New South Wales [an Australian state] schools at the moment.

Similarly, responses to Question 8, *How effective do you think the module is in improving pre-service teachers' capability in devising learning activities that reflect real scientific/mathematical knowledge with links to current "real-life" applications?* indicated that the module could help develop PSTs' capability to design real-life investigations but there was some reservation about how inexperienced teachers might implement such tasks.

TEP 5: The insight given by the module is worthwhile for all preservice students to experience mathematics as an interesting and relevant subject. The real life links are made obvious and powerful. The pre-service teacher would no doubt benefit from this knowledge, how that is utilised in the mathematics classroom as a beginning teacher would be the biggest concern.

Comments by Teacher Education Professionals 6 and 3 also raises the question of how PSTs' personal mathematical knowledge and skill, in addition to their mathematical modelling competencies, will be maintained and further enhanced once they transition to teaching in-service. This is especially pertinent in a curriculum context (e.g., the state of New South Wales) which does not explicitly describe modelling approaches to instruction.

5.3 Module enhancement workshop reviews

The consensus responses to questions from the Module Enhancement Workshop group are presented below.

For Question 1, *How well does the ORS module encourage pre-service teachers to ask and investigate questions about the world in which they live?*, the Module Enhancement Workshop group provided the following comment.

The module gave some very good examples of modelling and extended ideas and thoughts of modelling into the real-world with specific examples of where modelling is important. Modelling is often not regarded in mathematical learning in the normal mathematics classroom. This module gave PSTs very good practical examples and experiences of how important mathematics is in modelling in many experiences.

In response to Question 2, *How well does the ORS module introduce pre-service teachers to real scientists/mathematicians and their research?*, Module Enhancement Workshop reviewers responded:

All these examples are very interesting and real life. These are nice examples that give authentic examples. This is work that people are doing.
Inclusion of a social aspect of scientists' real-world modelling helps give a realistic perspective of mathematical modelling and how important social circumstances are.

The group was also asked how well the module presented the scientific process via Question 3, *How well does the ORS module articulate the concepts of scientific questioning, fair testing and data integrity?*

The module raises many questions and gives students an opportunity to look at the concepts from a scientific/mathematical perspective but essentially its real-world applicability.

Finally, Question 4, *Can authentic enquiry-based learning happen within this module? How and why?*, sought to ascertain the group's consensus view on the likelihood of promoting PSTs' understanding of mathematical modelling (as a form of enquiry-based learning) by engaging with the module.

I think this was a really good module, it was interesting with a variety of case studies and then so rich in its examples and varied. It opens up mathematical modelling and presents some exciting work and opens up enquiry learning to another level.

Thus, the Module Enhancement Workshop group saw the module as valuable in the way it promoted an understanding of mathematical modelling via rich and interesting examples drawn from authentic sources. The group also suggested that the focus on mathematical modelling within the module provided an opportunity to adopt enquiry-based approaches in mathemaitcs and science classroooms.

5.4 Pre-service teacher (PST) trial evaluation

The perspectives of PSTs in relation to the module trial are drawn from responses to Question 12 of the survey, *Describe any changes in your understanding*, as these were the most relevant to how well the module promoted an understanding of mathematical modelling.

PST's responses indicated engagement with the module had enhanced their understanding of the mathematical modelling process and its importance to mathematics and science in particular.

PST 5: This module has shown me how important modelling is to us. Not just scientists/mathematicians but also everyone else.
PST 4: I learnt about modelling and its application and importance in the scientific world.
PST 8: Basic fundamentals of modelling.

Responses to Question 12 also indicated that the use of models within the module had promoted their knowledge and understanding of real-world phenomena.

PST 5: I know about the stock market and how it works now.
PST 9: Better understanding of disease control, and financial options.
PST 6: I have become more knowledgeable about real-life events in the world which I knew nothing about before.
PST 10: I learnt some things I did not know and developed my understanding of the things I did.

Overall, PSTs indicated that they found the module content interesting and relevant. A limited number of PSTs indicated they had developed a broader understanding of mathematical modelling as a process but also deeper personal knowledge of the specific phenomena modelled within mathematics and science.

6 The nature of interdisciplinary collaboration

Themes related to opportunities and constraints that promote or inhibit interdisciplinary collaboration were emergent from analysis of Module Development Team interview transcripts. These themes are now discussed from the perspective of boundary crossing.

6.1 Benefits of crossing boundaries

Members of the Module Development Team (all identified using pseudonyms) were unanimous in their view on the module quality:

> Leonard: I'm quite happy with them [referring to case studies], part of me, the mathematician in me would like to take them both a little bit further mathematically but at the level they're aimed at that would not be appropriate, I think we stopped at the right level.
> Martin: I thought that the end product was fantastic… Whether you're naturally attracted to maths or not, and the big problems on this planet, I don't think we can solve without modelling...We have to model to foresee the future and we are all resource limited.

While most participants indicated they were pleased with the module, they also tended to view the product of the collaboration from their own disciplinary perspective—as in the case of Leonard, a mathematician, who explained he had to hold himself back from arguing for the inclusion of more sophisticated mathematics. An exception was Martin who could see the potential for a collaboration with a mathematician to strengthen the teaching of a first year biology course.

> Martin: You know I think if I do first year Biology, I will also need to bring in the mathematical expertise into it and it's not with me, it will be with someone that comes and helps me develop the maths behind it. But I know what the context is in which the maths is needed.

In summary, the module had acted as a boundary object that allowed team members from different disciplines to cross disciplinary boundaries. At the same time, Module Development Team members tended to judge the quality of the module from their own disciplinary side of the boundary. Thus, while boundaries were crossed during the collaboration, because of the potential for mutual benefit, most members crossed the bridge back to their own discipline when considering the quality of the final product. The effectiveness of the collaboration could be measured, however, by establishing how many of the Module Development Team, like Martin, were prepared to seek out opportunities to cross boundaries in the future, that is, they continue to seek boundary encounters because of the advantages they offer.

6.2 Opportunities provided by collaboration across disciplines?

All six interviewees expressed a belief that the modelling-based approach utilized in the module offered opportunities for PSTs' mathematics and science learning. However, most also raised a concern about the need to have a depth of personal discipline knowledge in order to develop effective and authentic real-world approaches in the future.

> John: The advantages…being able to use contexts that are really authentic and that they address real problems…[teacher] educators may not be quite okay with some of these cutting edge scientific problems such as the spread of disease if they haven't got an expert that can really help them inform how they should, or what datasets they should use and how they should be interpreting data.

This comment makes it clear that teacher educators could benefit through the input of discipline experts when designing learning tasks. Conversely, others commented on the insight provided by a teacher educator's perspective on teaching approaches within science or mathematics as disciplines.

> James: …we can do with a lot more learning support in academia [referring to science and mathematics disciplines] in general. I particularly liked that this module was collaboration, in the full sense, between scientists and educators.

Another interviewee looked at the issue more broadly.

> Leonard: There are certainly advantages for people to work together to promote STEM…I think we should take every opportunity to promote it. If people can work together, then perhaps we can create things that have more depth and breadth.

Module Development Team members could clearly see the benefits of the collaboration between discipline experts and teacher educators in terms of promoting the depth and breadth of student discipline knowledge. The comment by Leonard, however, identifies a broader purpose for crossing discipline boundaries—promoting STEM education.

6.3 Limitations associated with interdisciplinary collaboration?

Participants commented that the demands of prescriptive course/unit learning outcomes in academic programs of study was a potential limitation for cross-disciplinary collaboration.

> Martin: I think the limitations are if we think too specific and too small and if we go "we don't have room in our curriculum to link across because I need all my time to stuff it full of biology knowledge".

Another Module Development Team member raised the issue of how collaboration could take place within institutions that did not have mathematicians, scientists or mathematics and science educators on staff.

> John: Could this collaboration happen easily and effectively where there is no science or mathematics faculty attached to a university with a teacher education program?

Thus, there are institutional constraints that make collaboration between different communities of practice more difficult. Such restrictions need to be accommodated in new ways if the advantages of interdisciplinary collaboration are to be realised.

Taking a different perspective, one team member expressed concern about how PSTs would receive the explicit embedding of mathematics in her discipline of environmental science.

> Irene: When I first heard about this, I thought, mathematics? Environmental chemistry? Ah, from my experience with dealing with classes both at university, high school and primary school, my experience is generally that the idea of doing the maths would turn students off straight away.

Accordingly, it is important to note that not only is there a challenge associated with developing interdisciplinary collaboration between program and course/unit developers, but also in how the product of their collaboration is received by end users—their pre-service teachers. This is a reminder that the value of boundary crossing between two communities cannot be measured by reference to views of the collaborators alone as the outcome may influence and impact on other communities. Thus, potential for learning at intersections between communities who create and value different types of knowledge may need to be moderated by how new learning, practices and artefacts are received by others who did not participate in the collaboration.

7 Discussion and conclusion

The evaluation data reported here reflects the strength of the collaborative interdisciplinary approach adopted for module development as well as the effectiveness of strengthening PSTs' personal mathematics and science knowledge through attention to the use of mathematical models. Development was supported by a number of evaluation processes aimed at improving iteratively the quality of the module. The collaborative nature of the work between educators, mathematicians and scientists within the ORS project for the development of PST mathematics and science program units is unique in the Australian context. The online approach was found to be both effective and advantageous in terms of instruction in mathematical modelling. The findings of this study provide new insight into the development of online learning experiences related to mathematical modelling.

In relation our first research question, *How can a module focused on mathematical modelling be designed for delivery to PSTs in an online learning environment?*; the module development process brought together experts from a range of STEM disciplines and so an integrated interdisciplinary approach to instruction in mathematical modelling (Blum and Niss 1991) was adopted. Module development was underpinned by models of instructional design (Bybee 2009) and numeracy task development (Goos et al. 2014). As well as serving as structuring devices, both models acted as boundary objects that facilitated structured discussion about module development across different research communities (Akkerman and Bakker 2011), essential for the construction of a coherent learning experience for students. In recognition of the phases of *elaboration* and *evaluation* as part of the Bybee's 5Es model tasks at the complex, self-directed end of the modelling spectrum were included (Blomhøj and Hoff-Kjeldsen 2006). These aspects are essential to effective approaches to teaching and learning mathematical modelling which have been demonstrated as possible in online learning conditions through this study. This study also highlighted the need to accommodate factors such as institutional requirements and local curriculum demands (Geiger 2017).

Our second research question, *What perspectives did the module development team and stakeholders in mathematics, science and education bring to the design and evaluation of the online learning module?*; involved input from a range of stakeholders. The External Academic Expert Evaluator saw the module as a means of promoting integrated mathematical and scientific knowledge through the use of mathematical models in addressing authentic problems within relevant fields of research. The Module

Enhancement Workshop group also noted the module's potential to pique PSTs' interest and to promote their understanding of both modelling and scientific processes. In alignment with this view, Teacher Education Professionals indicated that the module was well designed and likely to promote PST engagement with, and an understanding of, the use of models within scientific and mathematical fields. At the same time, Teacher Education Professionals expressed concern that the mathematics underlying some of the case studies would be too challenging for PSTs and raised the issue of how instruction in mathematical modelling aligned with national curriculum requirements. One Teacher Education Professional suggested additional resources to support the implementation of the module in high school classrooms.

The views of the Teacher Education Professionals, expressed through the survey, were thus supportive of the approach adopted by the Module Development Team but they also appear to indicate a lack of full understanding about the purpose of the module—to enhance the personal mathematics and science knowledge of PSTs through a modelling approach and not specifically as a resource for implementation in secondary school mathematics classrooms. This discrepancy in understanding highlights a challenge related to the use of "independent" evaluators, in this case stakeholders from educator communities outside of those who developed the module; mainly secondary mathematics teachers. In boundary crossing terms, the Teacher Education Professionals represented willing parties in a collaboration with a broad common goal, enhancing mathematics and science teaching, but were really only allowed to put one foot over the boundary—through the evaluation. Thus, Teacher Education Professionals were not provided with the opportunity to make a full transition into the collaboration and, as a result, provided advice about an initiative they did not fully understand. As a consequence they conducted their evaluation from the perspective of their "home" community and so focused on student learning in school classrooms rather than their PSTs' enhanced personal mathematics and science knowledge that could be facilitated via a modelling approach.

PSTs indicated the module was helpful in developing their understanding of how mathematics can be applied to problems in the real world, however, they also remarked that the module was not as functionally accessible as they would like. For example, there was too much information on some pages and navigation was less than ideal. This observation is perhaps a consequence of having no published cases available where instruction in mathematical modelling has been conducted in an online environment. In such circumstances it may have been prudent to facilitate greater involvement of the stakeholder community—in this case by inviting PST's input earlier into the development process. This is an example of a community who were only provided with the opportunity for a partial crossing of a boundary, in this case that between developers and end users. This implies that the module could have included additional features that more effectively supported PST's understanding of and ability to use mathematical models if there had been greater inclusion of end users in the processes of development and evaluation.

Research *question three, What was the nature of the interdisciplinary collaboration between mathematicians, scientists, and mathematics and science educators in module development?*; was addressed through the analysis of Module Development Team members' interview data. Members of this group indicated their collaboration provided a rich boundary crossing experience. They were collectively pleased with the quality of the module, particularly in the way mathematical and science disciplinary knowledge was presented in authentic modelling contexts. Further opportunity for interdisciplinary collaboration where partners could benefit from the expertise held by one but not the other when working on real-world problems was also seen as desirable by some members. This perspective is consistent with Suchman's (1994) view that crossing denotes the transition of an expert into an arena in which they are far less qualified and that such transitions hold the possibility for the development of new knowledge and practices. It appeared that the boundary crossings that took place were bilateral, that is, educators benefitting from the input of discipline experts and vice versa with the modelling module acting as a boundary object. This reflects Sztajn et al.'s (2013) position that members of different communities will seek out opportunities for boundary encounters where there is perceived advantage in doing so. At the same time, this finding adds to new knowledge as it demonstrates the potential for mathematical modelling activities to act as boundary objects when bringing together individuals from different scientific disciplines to design learning modules in pre-service teacher education.

This study indicates that principles of design based on the work of Bybee (2009) and Goos et al. (2014) were effective underpinnings for an online learning module on mathematical modelling. Further, this research also suggests that there is great potential for productive collaboration between experts in education, science and mathematics for the purpose of designing PST education programs that target the strengthening of their personal knowledge through mathematical modelling. At the same time, careful consideration needs to be given to how to optimize the inclusion of different communities in boundary crossing collaborations to produce the best educational outcomes.

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