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ORIGINAL ARTICLE

# Video analyses for research and professional development: the teaching for robust understanding (TRU) framework

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**Abstract**
This paper provides an overview of the teaching for robust understanding (TRU) Framework, its origins, and its evolving use. The core assertion underlying the TRU Framework is that there are five dimensions of activities along which a classroom must do well, if students are to emerge from that classroom being knowledgeable and resourceful disciplinary thinkers and problem solvers. The main focus of TRU is not on what the teacher does, but on the opportunities the environment affords students for deep engagement with mathematical content. This paper's use of the TRU framework to highlight salient aspects of three classroom videos affords a compare-and-contrast with other analytic frameworks, highlighting the importance of both the focus on student experience and the mathematics-specific character of the analysis. This is also the first paper on the framework that introduces a family of TRU-related tools for purposes of professional development.

**Keywords** Analytic framework · Classroom analysis · Professional development · Robust understanding · Teaching tools · Video analysis

## 1 Introduction

Every framework for characterizing classroom practice represents a particular set of values (e.g., how highly prized is classroom management, or "teaching for understanding"?) and some degree of comprehensiveness (e.g., does it cover all aspects of classroom instruction, or focus on specific aspects of practice, such as classroom discourse?). Moreover, any such framework is likely to be of differential utility for research, administrative decision making, and professional development. Over time, the primary function of a framework and the tools associated with it may evolve. For example, early work in developing an approach to characterizing teaching may focus on research-related issues such as insuring the consequential validity and statistical reliability of the framework. Once the framework is found to be robust, it may be used primarily for any of the three functions just mentioned (research, evaluation, or professional development), or some combination of them.

For these reasons I begin with a general introduction to the teaching for robust understanding (TRU) framework. I then illustrate the main dimensions of TRU by discussing the three stimulus videos common to this volume. That discussion highlights both the mathematics-specific character of the version of TRU discussed here and its emphasis on students' experience of lessons as the fundamental shaper of student learning. I conclude with a discussion of some TRU tools for professional development.

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## 2 The teaching for robust understanding (TRU) framework

### 2.1 The origins and present state of TRU

The origins of the TRU framework are described in Schoenfeld (2013, 2014). Early on, we focused on the improvement of algebra instruction. As the work evolved, we saw that the issues of "robust learning environments" we were dealing with were relevant to all mathematics instruction. Thus TRU evolved into a framework for characterizing the quality of mathematics instruction in general, with an additional module specifying the kinds of content understandings that represent a robust understanding of algebra. The idea is that the general frame applies to all mathematics instruction, and additional modules can go more deeply into specific mathematical content. What I present here is the framework that evolved for all of mathematics.[1]

From the beginning, we intended to develop a framework that was comprehensive. Whether for purposes of evaluation or professional development, a framework is of limited value if it omits something of critical importance. That is: if one attends to everything in the framework, the result should be improvements in instructional practice that enhance student learning. At the same time, a framework needs to highlight a relatively small number of dimensions of practice. If there are too many aspects of instruction to attend to, professional development can easily become unfocused.

Before turning to the task of characterizing productive learning environments, the author had spent more than a decade building models of teaching, a project that resulted in the book *How We Think* (Schoenfeld, 2010). That research was grounded in the literature concerning mathematics teaching and learning, and teaching and learning more generally (e.g., as represented in the content of handbooks such as English 2008; Lester 2007; Richardson 2001; Sikula 1996; Wittrock 1986).

Rather than reinvent the wheel as we worked to conceptualize productive mathematics learning environments, we searched the literature for extant frameworks and their research support. For a variety of reasons elaborated below, none of the extant frameworks, among them the framework for teaching (Danielson 2011), the classroom assessment scoring system (Pianta et al. 2008), the protocol for language arts teaching observations (Institute for Research on Policy Education and Practice 2011), mathematical quality of instruction (University of Michigan 2006), the UTeach teacher observation protocol (Marder and Walkington 2012), the instructional quality assessment, (Junker et al. 2004), the performance assessment for California teachers (PACT Consortium 2012), and the systematic classroom analysis notation (Beeby et al. 1980), met our needs. We embarked on creating a new framework. Doing so entailed listing all of the aspects of powerful instruction suggested by an extensive literature review [including all of the handbooks and frameworks listed above, plus a collection of books and papers on powerful pedagogical practices (e.g., Engle 2011; Gresalfi and Cobb 2006; Stein, Engle, Smith, and Hughes 2008)].

As described in more detail in Schoenfeld (2013), our preliminary analyses identified hundreds upon hundreds of items that had the potential to affect student outcomes. The set of items listed was extensive—if was highlighted in the literature, it was on our list—but it was also unmanageable, by virtue of its size. To maintain the essence of the list but cut it down to manageable size, we created equivalence classes of those items. By virtue of the process used to create them, these categories were guaranteed to be comprehensive. The five main dimensions of classroom activity resulting from our analyses are given in Fig. 1.

The distillation of the literature into five coherent categories indicates that, in a metaphorical sense at least, classroom activities can be considered a five-dimensional space. The five dimensions of practice identified in Fig. 1 can be considered a set of basis vectors for the space of classroom activities. It is possible, of course, to choose alternative basis vectors; spanning sets are not unique.

Given the process that generated them, the five dimensions of TRU are comprehensive. Of course, with any description at this level of grain size, some issues of importance are not visible at the top level. Consider, for example, "classroom safety." It is essential for students to feel safe enough to venture ideas in the classroom. Yet, classroom safety is not highlighted or immediately visible in Fig. 1. However, it is impossible to have a classroom climate conducive to the development of agency, ownership, and identity (Dimension 4 of the TRU framework) if students do not feel safe setting forth their mathematical ideas. Thus classroom safety is deeply implicated in TRU, even if it does not appear at the top level. Similarly, the top level of TRU does not focus on classroom management. The need for it is there, however: in a poorly managed classroom there are few opportunities to do well along the dimensions featured in Fig. 1. In sum, everything that matters is captured somewhere in the TRU framework. The five dimensions can be considered necessary and sufficient for powerful learning environments. If one or more of the dimensions in Fig. 1 are not well represented in classroom activity, students will not emerge from that classroom as knowledgeable and flexible thinkers and problems solvers; if all are well represented, students will.

When we first distilled the collection of factors known to shape learning outcomes into the five dimensions in Fig. 1, the idea that those five dimensions sufficed for analysis and professional development was a preliminary hypothesis. Work needed to be done to operationalize what was meant by "doing well" in each of the five dimensions, and to see how "doing well" related to student outcomes. The TRU team developed a scoring rubric for assessing classroom activities. The summary rubric given in Fig. 2 describes

[1] In fact, the ideas generalize: modified versions of the TRU framework are now being used for instruction in science, English language arts, and history.

| The Five Dimensions of Powerful Classrooms | | | | |
|---|---|---|---|---|
| **The Content** | **Cognitive Demand** | **Equitable Access to Content** | **Agency, Ownership, and Identity** | **Formative Assessment** |
| *The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.* | *The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."* | *The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.* | *The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.* | *The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.* |

**Fig. 1** The five dimensions of robust classrooms—the teaching for robust understanding (TRU) framework

Whole Class Activities

| The Mathematics | Cognitive Demand | Equitable Access to the Mathematics | Agency, Ownership, and Identity | Formative Assessment |
|---|---|---|---|---|
| Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content (as specified in the Common Core Standards) | Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises. | There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue. | Students' speech turns are short (one sentence or less), in response to and constrained by external authority (teacher or text). | Student reasoning is not actively surfaced or pursued. Responses to student comments are limited to encouragement or corrective feedback. |
| Activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or engaging in mathematical practices. | Activities offer possibilities of conceptual richness or problem solving challenge, but interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle. | There is uneven access or participation, although there are some clear efforts by the teacher or in the materials to provide mathematical access to a wide range of students. | Some students present mathematical thinking, but they look to external authority (text or teacher) to judge correctness and guide conversations. Students ideas are not built on. | Ways to think about problems and/or common mistakes are mentioned, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic). |
| Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate), providing opportunities for engaging in reasoning, problem solving, and other key practices. | When needed, hints or scaffolds support students in productive struggle, providing opportunities to build understandings and engage meaningfully in mathematical practices. | Classroom activities support and to some degree achieve broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement. | Students explain their ideas and reasoning. Students may be ascribed ownership for their ideas, and/or, students respond to and build on each other's ideas. | Student thinking is solicited and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. |

**Fig. 2** Degrees of richness along each of the five TRU dimensions, for whole class activities

**Fig. 3** Observing a mathematics lesson from the student perspective

| Observe the lesson through a student's eyes | |
|---|---|
| The Mathematics | • What's the big idea in this lesson?<br>• How does it connect to what I already know? |
| Cognitive Demand | • How long am I given to think, and to make sense of things?<br>• What happens when I get stuck?<br>• Am I invited to explain things, or just give answers? |
| Equitable Access to Mathematics | • Do I get to participate in meaningful mathematical learning?<br>• Can I hide or be ignored? |
| Agency, Ownership, and Identity | • Do I get to explain, to present my ideas? Are they built on?<br>• Am I recognized as being capable and able to contribute in meaningful ways? |
| Formative Assessment | • Do classroom discussions include my thinking?<br>• Does instruction respond to my thinking and help me think more deeply? |

three levels of proficiency for each dimension (see Schoenfeld et al. 2014 for more extensive detail). A detailed scoring guide (Schoenfeld et al. 2015) is available for researchers. Our research team now uses a more fine-grained rubric for analytic purposes. For purposes of the discussion in this paper, where precision is not an issue, the levels of performance indicated in Fig. 2 will suffice. The main purpose of this discussion is to indicate what the framework highlights. In Sect. 3 of this paper I discuss each of the three stimulus tapes for this volume in turn.

The dimensions of the TRU framework comprise a "nearly decomposable" system. There is, of course, some overlap between dimensions: students cannot develop a sense of mathematical agency (Dimension 4), for example, if they do not get to participate meaningfully in classroom activities (Dimension 3). But the constructs can be defined so that the overlap is comparatively small. Each dimension coheres as an aspect of practice, and thus can be made the focus of professional development. Indeed, the main focus of TRU in its current state is on professional development (in the high stakes environments in the United States, assessments can have a significant impact on the careers of both students and teachers. We prefer that our tools not be used in punitive ways, but instead be used to enhance teaching and learning; hence our focus is on professional development rather than evaluation). We are constructing a set of tools that teachers, coaches, and professional learning communities can use for purposes of ongoing improvement. Section 4 of this paper introduces those tools and suggests their relevance for the three videos analyzed in Sect. 3.

Here by way of summary, we indicate five important aspects of the TRU framework.

1. *The five dimensions of TRU are necessary and sufficient to characterize the kinds of teaching that result in students being knowledgeable, flexible, and resourceful thinkers and problem solvers*. This point was elaborated above.
2. *TRU involves a fundamental shift in perspective, from teacher-centered to student-centered. T*he key question is *not*: "Do I like what the teacher is doing?" It *is*: "What does instruction feel like, from the point of view of the student?" The teacher's actions are critically important, of course—but what really matters are the ways in which the students have meaningful opportunities to make sense of the content. This perspective is represented in Fig. 3, drawn from the TRU observation guide (Schoenfeld, A. H. and The Teaching for Robust Understanding Project, 2016a, b, c).
3. *There are no* "thou shalts" *in TRU. TRU does not say how to teach, because there are many different ways to be an effective teacher*.

   TRU serves to problematize instruction. That is: asking, "how am I doing along this dimension; how can I improve?" can lead to richer instruction without imposing a particular style or norms on teachers.
4. *TRU is not a tool or set of tools. Rather, TRU is a perspective regarding what counts in instruction, and TRU provides a language for talking about instruction in powerful ways*.

   With this understanding, one can make use of any productive tools wisely. Of course, we have worked to create tools that are consistent with the values expressed in TRU; some of these are discussed below. But, from our perspective, any tool that is aligned with the values represented by TRU can be a good thing to use.
5. *At the* "meta level", *it must be understood that what is seen as good teaching is not an absolute, but an expression of a particular (often culturally embedded) set of values*.

Later in this paper I will discuss Ms. Young's lesson, in which some students are treated rather harshly. As discussed, this lesson is rather problematic with regard to Dimensions 3 and 4 of the TRU framework, "equitable access to the mathematics" and "agency, ownership, and identity." Quite possibly, most readers will agree with this perspective: the belief that all students should be supported in learning and have opportunities to develop their mathematical understandings as best they can is widespread. But, those values are not universal. For example, R. L. Moore developed a pedagogical approach known as the "Moore Method" that produced a large number of world class mathematicians (see Coppin, Mahavier, May, and Parker, 2009)—but which also rapidly separated mathematically "talented" students from others, and left the others in the dust. If one's goal is the production of world class mathematicians, maybe equity is not that important to you. Or, more broadly, there are societies in which there are strict limits, by examination scores, on who has access to certain jobs. Given that there is an abundance of candidates, the goal is selection: having *more* adequately prepared candidates is not a priority. So, equity in classrooms (or elsewhere) is not a universally prized value. Nor, for that matter, is the kind of agency, authority, and identity that I value and that is highlighted in Dimension 4. I recently asked a close colleague if I could have had the kind of career I have had if I were a citizen of his country. After some thought, he replied that it was unlikely—in arguing for the kinds of work I have thought important I have not been deferential to authority, and that would have harmed my career in his country. That is: the sense of agency that I have and that I try to promote could be counterproductive in some (mathematically successful!) societies. So, in response to the requests from reviewers of this paper that I give recommendations for the professional development of the three teachers we studied, I must stress that such recommendations come from a particular value system—the values represented in TRU, which are culturally specific. What is considered to be "good teaching" is not universal, as David Clarke's (2006–2014) series "The learner's perspective study" makes abundantly clear.

## 2.2 Key issues: methods, reliability, affordances and constraints

The TRU team has not conducted large scale validity studies, but it has been straightforward to achieve consistency in scoring. In our current scoring, TRU assigns scores of 1, 1.5, 2, 2.5, and 3 for each dimension. In our work on Schoenfeld, Floden, El Chidiac, Gillingham, Fink, Hu, Sayavedra, Weltman, Zarkh, and The Algebra Teaching Study and Mathematics Assessment Project (in preparation) (2018), as many as ten research group members, including coders who were new to the project, watched videos individually and scored independently. Tapes were segmented into episodes of up to 10 min of the same activity type—whole class discussion, small group work, seat work, and student presentations. Scorers then used the rubric to assign a score to each episode. Typically, the independent scores were within a range of 1—e.g., from 1.5 to 2.5. At the next research group meeting people explained their scores, justifying their decisions with respect to what they saw in the video and their understanding of the rubric. Between meetings individuals re-watched and re-scored the videos. Typically, the second set of scores for each dimension clustered on or between scores that were 0.5 apart.

To assign precise numerical scores, a formal weighted average score can be computed for each dimension. The score for each dimension is obtained by multiplying the score for each episode by the length of that episode, and then dividing the sum of those values by the sum of the lengths of the episodes. [That is, the dimension score is the weighted average, over time, of the episode scores for that dimension]. For many purposes, however, that degree of precision is unnecessary. In practice, trained raters can assign holistic scores for each dimension that correspond closely to the weighted averages obtained for each dimension. For this paper, the author rated each dimension holistically.

For reasons of cost and the fact that TRU was developed after the MET study (Measures of Effective Teaching, 2012) was initiated, we do not have large-scale data correlating performance on the TRU dimensions with student performance. Given the way it was derived, and its surface similarity to other frameworks, one would expect correlations as good as those found in the MET study. Schoenfeld, Floden, El Chidiac, Gillingham, Fink, Hu, Sayavedra, Weltman, Zarkh, and the Algebra Teaching Study and Mathematics Assessment Project (in preparation) (2018) provides data indicating parallels, and some differences in emphases, between TRU and two of the widely used frameworks, the framework for teaching (FfT) and MQI. Here is a brief summary of the similarities and differences. There are significant differences between TRU and FfT, which tends to place significant value on well-managed classrooms and, because of its content-independent nature, does not attend to disciplinary specifics. As a result, FfT rated some lessons well because the classrooms were well managed, while TRU gave them low ratings because the mathematical content of the lessons was superficial. Conversely, TRU gave high ratings to some "messy" lessons in which students were arguing in meaningful ways about content, while FfT gave them low ratings because they were not as well managed and the teacher did not seem in "control" in obvious ways. In contrast, TRU and MQI shared more in common, although, to give one example of mathematical differences, to earn high scores on the mathematics TRU demands that multiple representations be connected, contrasted, and used, whereas MQI gives high scores if different representations are discussed in the lesson. This reflects the point made in Sect. 2.1, that every framework embodies a particular set of values. We would also claim that the existence of five "necessary and sufficient" dimensions for analysis and professional development in TRU makes TRU easier to use and potentially more powerful than the other two frameworks.

I note two major constraints. First, by virtue of its scope, the TRU framework does not in itself go very deeply in detail into any of the five dimensions. Thus, for example, it does not in itself offer as detailed a characterization of classroom discourse as other frameworks might, even though discourse is an essential component of Dimension 4 (agency, ownership, and identity). Second, as noted, TRU foregrounds some aspects of learning environments and backgrounds others. Thus, for example, those who have an interest in classroom management will find little that addresses it directly, even though classroom management is an enabling condition for many of the dimensions.

## 3 The three videos

In what follows I discuss the three stimulus videos in sequence. For each video I work through the dimensions one at a time.

I should stress that discussing any lesson out of context is problematic. For example, a teacher may decide that her students need to review material for an upcoming unit, and a large part of that day's lesson may be below grade level—earning an automatic low score on the mathematics dimension. This degree of attention to below grade level content may be atypical of the teacher's overall practice. For that reason, we insist on there being at least five classroom observations before the framework is used for any evaluative purpose. The primary function of the TRU framework is to facilitate the improvement of teaching. Tools for that purpose are discussed in Sect. 4.

What follows is a discussion of the issues raised by the three classroom videos.

### 3.1 Mr. Smith's lesson

In many ways, this video seems typical of mainstream instructional practice in the United States. The class is well behaved. The teacher, known as "Mr. Smith" (a pseudonym) maintains calm control of the classroom, and the content is at grade level. A framework that prizes effective classroom management might well give this lesson high marks. Yet, there are issues.

#### 3.1.1 Dimension 1, the mathematics

The mathematics in this lesson is straightforward and, as far as one can tell, at grade level. It is clearly introductory. At the same time—presumably a function of a skills-oriented curriculum, which employed "SuccessMaker"—the content is superficial, offering almost no opportunities for student engagement with mathematics of consequence. The first part of the class is definitional: the class reviews the terms ray, vertex, acute, right, obtuse, and straight. There are reminders that a right angle must be indicated by the "little symbol" indicating 90°, and that angles should be measured and not judged by whether, for example, they "look like" they are right angles. Students are given mnemonic devices to remember that "*a cute* little" angle (that is, an acute angle) is small, while *ob*tuse angles share "*ob*" with "*ob*ese," which means that an obtuse angle is large. The students look at and classify angles projected on the screen in front of the class. They practice making right angles and 45° angles by folding paper. They practice using two forms of protractor, as projected on the smart board in front of the classroom. As the video ends, the students measure angles at their desks. Our characterization, as taken from the rubric in Fig. 2: *activities are at grade level but are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or engaging in mathematical practices*.

#### 3.1.2 Dimension 2, cognitive demand

Given that the day's work was focused on terminology and skills, the lesson provided very little opportunity for student engagement with challenging mathematical ideas. The most complex task faced by the students was to determine that the angle supplementary to a 47° angle had a measure of 133°. From the rubric: *classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises*.

#### 3.1.3 Dimension 3, equitable access to the mathematics

We stress that one lesson provides an inadequate sample from which to determine which students have opportunities to participate actively in the core mathematics of the class. That said, the teacher often invited choral responses from the class and much of the class seemed to respond. From the rubric: *there is uneven access or participation, although there are some clear efforts by the teacher or in the materials to provide mathematical access to a wide range of students*.

#### 3.1.4 Dimension 4, agency, ownership, and identity

For the most part, this lesson was a lesson in mastering other people's mathematics. The vast majority of talk was teacher talk, with a significant number of IRE sequences (teacher Initiates; student Responds; teacher Evaluates: see Cazden and Beck 2003; Mehan 1985) such as this exchange:

Mr. Smith: Are all my angles always gonna be facing the same direction?
Multiple students: No.
Mr. Smith: No. Some are gonna be facing different directions, so sometimes I'm gonna have to use the different side. Where do I always start counting from?
Multiple students: Zero.
Mr. Smith: From zero.

At one point [26:00] Mr. Smith asked a student to use the smartboard to create a 65° angle. The student overshot by a degree, and a number of students said "sixty-six." As the student at the whiteboard was adjusting the angle, a student said "It's the same thing." The following dialogue ensued:

Mr. Smith: No, actually, it's a degree different.
Student: But it's kind of the same thing.
Mr. Smith: No.
Student: [*Inaudible*]
Mr. Smith: If you decided to become an engineer 1 day and you get 66° and the answer's 65°, then it's not gonna be built well. If you decide you want to be an architect or civil engineer—should have the buildings be perpendicular. Shouldn't they form 90° angles? If you make your building 91° everybody's gonna be walking slanted a little bit. So, no, it's not the same.

In short, the students had to tow the mathematical line, as defined by others. From the rubric: *students' speech turns are short (one sentence or less), in response to and constrained by external authority (teacher or text).*

#### 3.1.5 Dimension 5, formative assessment

As I have noted, the lesson itself offered few opportunities for thinking; moreover, when students did venture opinions, they were judged. From the rubric: *student reasoning is not actively surfaced or pursued. Responses to student comments are limited to encouragement or corrective feedback.*

#### 3.1.6 Brief discussion

Again, I note that this is introductory material, and that one should never generalize from just one lesson. But, the character of the mathematics in this particular lesson, with a focus on mastering vocabulary and straightforward mechanical procedures, provided little by way of opportunity for students to become flexible and resourceful thinkers and problem solvers, or to come to see themselves as "math people" (Dimension 4).

#### 3.1.7 Notes on professional development

Professional development would focus on enriching the mathematics and finding more ways to support student participation in meaningful ways. My preference for such work is at the collective level, where a group of teachers working on the same content can discuss both the content and the ways in which students can be supported in engaging with it. Some key questions for conversations about the content would be, "What are the big ideas we want students to understand in this unit? Are there ways we can introduce them that involve active sense making on the part of the students? Are there tasks involving angle measurement that the students could see as interesting and challenging, that would provide opportunities for them to develop some ownership of the mathematics?" Given that there were issues related to Dimension 4 in all three of the tapes, I address professional development related to agency, ownership, and identity in more detail in Sect. 4.2.3.

### 3.2 Ms. Young's lesson

This is a complex and fascinating lesson. On the one hand, the level of mathematics is remarkably high, and a significant percentage of the students engage with the content in ways that are mathematically powerful. On the other hand, the students who do not engage in ways that are to the teacher's liking are dealt with harshly. We call the teacher Ms. Young.

#### 3.2.1 Dimension 1, the mathematics

The goal for the day is to investigate what happens to a product of two positive integers when one of those two whole numbers is doubled or halved—e.g., to understand and explain the relationship between $(16 \times 3)$ and $(16 \times 6)$. The students in this classroom are held to a very high standard of explanation. In an early exchange, for example, Ms. Young works with a student whose first explanation is somewhat incomplete until that student produces a lucid explanation that (in essence) $3+3=6$, so $(16 \times 3)+(16 \times 3)$ will equal $(16 \times 6)$. In the lesson, mathematical products are represented using boxes and arrays, and the products are then checked with arithmetic. Students are asked to produce alternative verbal explanations, and to generate stories that match the arithmetic problems they have just worked.

Mid-lesson, students are asked to find the relationship between the products $(15 \times 8)$ and $(30 \times 4)$. The students quickly note that the products are the same (they both equal 120), and the discussion turns to, in Ms. Young's words, "How can we justify that?".

Students work on the justification in multiple ways (again, employing both pictorial and verbal representations), in this case noting that in moving from the first product to the second, 15 is doubled but 8 is halved, so the product is unchanged. Moreover, Ms. Young consistently focuses on meta-level and strategic issues. For example:

> When you are given a story situation or a problem context or a problem that is working with numbers or with words, you try to see relationship between and among the numbers in the problem. Don't just rush and try solving it all from this question. Try to see if there is any way that the numbers are related and use that relationship that you identify to figure out the solution.

From the rubric: *classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate), providing opportunities for engaging in reasoning, problem solving, and other key practices*.

#### 3.2.2 Dimension 2, cognitive demand

In various ways, Ms. Young provides scaffolding that appears to allow those who participate to engage in productive ways. She shapes the responses of the first volunteer until his originally inchoate response is somewhat lucid, and then asks for another student to rephrase that explanation in his own words. The multiple representations employed (story problems, arrays, manipulatives, verbal explanations) allow for access to the content in a variety of ways. From the rubric: *when needed, hints or scaffolds support students in productive struggle, providing opportunities to build understandings and engage meaningfully in mathematical practices*.

#### 3.2.3 Dimension 3, equitable access to the mathematics

Here and in Dimension 4 Ms. Young's lesson presents significant challenges. Those who participate are rewarded; those who do not are punished. Ms. Young is a very strict disciplinarian. Early in the lesson, for example, a student who was not necessarily attending, but certainly not disruptive by most standards, was sent to the back of the classroom for the whole period. In the middle of an explanation Ms. Young directs her attention to a student and says "Student T, your eye is not here. Your mind is far away." Those who incur the teacher's displeasure pay a high price. As the lesson winds to a close Ms. Young turns to three students and says, "Thank you Student C, Student P, Student D for disrupting the lesson throughout the day."

From the rubric: *there is differential access to or participation in the mathematical content, and no apparent efforts to address this issue*.

#### 3.2.4 Dimension 4, agency, ownership, and identity

In this class, how much agency a student has or develops appears to depend on that student's status and participation. Ms. Young appears to call on those who raise their hands first. Once called upon, a student is helped to develop a lucid explanation and given credit for such; but most student turns are a phrase or a sentence long, as Ms. Young maintains strict control of the dialogue and moves the conversation in the mathematical directions that she deems productive. From the rubric: the form of classroom discourse lies somewhere between *Students' speech turns are short (one sentence or less), in response to and constrained by external authority (teacher or text)* and *Some students present mathematical thinking, but they look to external authority (text or teacher) to judge correctness and guide conversations. Students' ideas are not built on*.

#### 3.2.5 Dimension 5, formative assessment

Student thinking is solicited and refined, but used largely to the degree that it facilitates the teacher's agenda. Except for the few students who interact verbally with Ms. Young, it is impossible to gather what students are understanding as the lesson progresses. From the rubric: *but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic)*.

#### 3.2.6 Brief discussion

This lesson offers significant mathematical riches for those students who are capable of engaging, or who choose to engage. The activities themselves are powerful on multiple grounds: there is, for example, a focus on strategic thinking and the use of multiple representations. Moreover, there are enough mathematical handholds and enough scaffolding that it is possible for students with varied understandings to participate in meaningful ways. However, this is a strongly teacher-centered environment, and it does not seem very welcoming to student ideas; it is impossible to know what sense the majority of students are making of the lesson as it unfolds.

#### 3.2.7 Notes on professional development

Professional development would focus on ways to enfranchise more students and incorporate their mathematical voices more directly in the high quality mathematical work of the classroom (Dimension 3 and especially Dimension

4). See Sect. 4.2.3 for a discussion of related professional development.

## 3.3 Ms. Jones' lesson

One thing that should be stressed about TRU, and will be expanded below, is that four of the five TRU dimensions focus on the student's experience of the content—after all, it is that experience that results in learning. What the teacher says or does is important, of course; but the students' opportunities to make sense of the content are what truly matter. That framing becomes particularly salient in this video. The classroom project, to build posters demonstrating three different ways to multiply a whole number by a fraction, appears mathematically rich on the surface: conceptualizing operations on fractions in different ways is an important mathematical goal. However, the ways in which the students experience the construction of the poster make it difficult for students to make those connections. The teacher is "Ms. Jones."

### 3.3.1 Dimension 1, the mathematics

On the surface, the goal of the lesson is for students to produce a piece of mathematical work demonstrating mathematical connections. However, the lesson *as experienced by the students* verges on the incoherent. The directly mathematical part of the lesson is purely procedural, with students being guided step by step through a range of activities, including how to cut a circle into fourths. But there are extended parts of the lesson that this observer was unable to connect to the mathematics at all. For example, Ms. Jones has the students burp an imaginary baby five *times*, repeating the word "times" loudly [because the students are multiplying, and "times" signifies multiplication]. Ms. Jones tells a long story about a baby handing her blocks one at a *time*. From the rubric: *classroom activities are unfocused or skills-oriented, lacking opportunities for engagement with key grade level content*.

### 3.3.2 Dimension 2, cognitive demand

The students were led step by step through procedures that may or may not have made sense to them. Ms. Jones' explanations were hard for this observer to follow—and judging from the students' incorrect responses to questions Ms. Jones asked them, those explanations were hard for the students to follow as well. From the rubric: *classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises*.

### 3.3.3 Dimension 3, equitable access to the mathematics

As noted, access is very difficult to determine, especially in a whole class situation. Ms. Jones does appear to be attending to the whole of the class, and she circulates around the whole classroom as students work at their table. There is not enough information in this video to determine participation patterns.

### 3.3.4 Dimension 4, agency, ownership, and identity

The vast majority of classroom "air time" is taken up by Ms. Jones, with students typically limited to one- or two-word responses to specific prompts she provides. The following exchange is typical, as Ms. Jones is converting the "improper" fraction 15/4 to a mixed number:

> Ms. Jones: Very good. So I take 15 and I put it inside. It becomes my dividend. And 4 becomes—what is that word that we use for the number that's outside the box? Raise your hand. What is that word that we use, Student R?
> Student: The divisor.
> Ms. Jones: Divisor. So 15 becomes my dividend and 4 becomes my divisor, and I divide it out. Does 4 go into 1?
> Multiple students: No.
> Ms. Jones: No. So I put a zero. How many times does 4 go into 15? Remember, it's got to be something times 4 that will give me a number close to 15 without being too much. I know 4 times 2 is 8. I think I can get closer. So what do you think it is, Student J?
> Student: 3.
> Ms. Jones: You think it's 3. 4 times 3 is what?
> Student: 12.
> Ms. Jones: I'm gonna check one larger to make sure. What's 4 times 4?
> Multiple students: 16.
> Ms. Jones: It doesn't fit. I can't take 16 away from 15. So it's got to be 3. So 3 times 4 is 12. 15 minus 12 is what? 15 take away 12. What is 15 take away 12, Student D?

From the rubric: *students' speech turns are short (one sentence or less), in response to and constrained by external authority (teacher or text)*.

### 3.3.5 Dimension 5, formative assessment

Student thinking is not solicited in this lesson. Most questions from the teacher, as in the example given above, are IRE sequences (Cazden and Beck 2003; Mehan 1985) in which a known answer is expected by the teacher and is evaluated for correctness. Unexpected student thinking is not addressed, as in the following exchange where Ms. Jones is checking a group's answer to $5 \times \frac{2}{4}$:

Ms. Jones: How many whole circles do you think he's gonna have?
Multiple students: Two.
Ms. Jones: And how many fourth pieces do you think he's gonna have?
Student: 8.
Ms. Jones: You think 8 fourth pieces?
Student: 12.
Ms. Jones: You think 12 fourth pieces?
Student: Yeah.
Ms. Jones: No, no. He has two. How many fourths does he have left over?
Student: Two.
Ms. Jones: So he has two whole circles, right?
Student: Mm-hmm.
Ms. Jones: And two-fourths, two and two-fourths.
Student: Oh.
Ms. Jones: Weird.

From the rubric: *student reasoning is not actively surfaced or pursued. Responses to student comments are limited to encouragement or corrective feedback.*

#### 3.3.6 Brief discussion

There is little focus on student thinking in this lesson. The teacher lays out the mathematics as she understands it, but how that connects to student understandings is not clear.

#### 3.3.7 Notes on professional development

Professional development would focus on sorting out the mathematics in ways that supported clear presentation, and finding ways to open up classroom discourse for student contributions. There is a lot to be done on the mathematical side: the teacher needs support in understanding the mathematics, and being able to talk about it in clear ways. As noted above, the lesson as seen from the students' point of view verged on the incoherent. If the mathematics is not straightened out, little of value can follow. At the same time, there were significant issues related to Dimension 4: students had negligible opportunities for meaningful engagement with the content. Thus, opening things up is a key issue. See Sect. 4.2.3.

## 4 Concluding discussion

### 4.1 What TRU highlights in the focal lessons

Before turning to the issue of improvement in Sect. 4.2, I summarize the ways in which TRU illuminated key aspects of the student experience in the three classrooms. Again, I stress that three videos are *samples* of classroom practice; one should not assume that they are typical of those teachers' practices.

#### 4.1.1 Dimension 1, the mathematics

The quality of classroom discourse about the content establishes an upper bound on what most students are likely to learn. If there are few opportunities for mathematical sense making, for making connections, or for engaging in the mathematical practices, where will students develop these necessary understandings? In that sense, videos 1 and 3 were problematic. As noted, the mathematics in video 2 was quite rich. The question, then, becomes: who has access to it, in what ways? That question is addressed in Dimensions 2 through 5.

#### 4.1.2 Dimension 2, cognitive demand

If the mathematics in a lesson is trivial, students have no opportunities for "productive struggle," but if it is over their heads or does not connect to what they know, they cannot engage with the challenge. There are various ways to support meaningful engagement—through problems that have multiple access points, through discussions of multiple representations, and through scaffolding without giving answers away, for example. There was variation in the use and effectiveness of such techniques.

#### 4.1.3 Dimension 3, equitable access to the mathematics

Various teacher moves can support equitable access to the main content of a lesson—among them the use of "equity sticks" or other devices for distributing "air time," status interventions such as complex instruction (Cohen 1994; Cohen and Lotan 1997), and the use of various language prompts such as "sentence starters" for supporting safe and respectful classroom discussions. That they were not visible in these lessons is not necessarily problematic—but, such moves do offer pathways to progress toward more equitable instruction.

#### 4.1.4 Dimension 4, agency, ownership, and identity

The question is, what opportunities do students have to come to see themselves as "math people"? Consider the 4th row of Fig. 3. In lessons 1 and 3, IRE sequences dominated and students got to "fill in the blanks" in response to narrowly focused questions posed by the teacher. In lesson 2 there were opportunities to think and to explain, and students were supported in being more lucid as they explained. The question there is, which students? If some students come to see themselves as mathematically powerful and others see such thinking as being beyond their reach, there is still progress to be made.

#### 4.1.5 Dimension 5, formative assessment

A major function of making student thinking public is the sharing of ideas, which is good for access and for agency, ownership, and identity. But, at least as importantly, hearing what sense the students are making of the content is the way that teachers can adjust what they are doing so that students can engage with the content more fully. The conversations in these classrooms were mostly one-directional.

#### 4.1.6 Brief discussion

The discussion of the three lessons indicates the importance and interdependence of the five dimensions of the TRU framework. For example, the quality of the mathematics establishes an effective ceiling on student learning: it is unlikely that the students in Mr. Smith and Ms. Jones's classrooms will emerge with deeper understandings of angles and fractions, respectively, than were conveyed in the rather limited classroom discussions. However, the fact that the mathematics is rich, as in Ms. Young's classroom, does not mean that many students will emerge from the classroom with deep understandings or with the perception of themselves as mathematical sense makers. There are issues of equitable access: who gets to participate in ways that will produce learning? For those who spoke, there was feedback. But the majority of students did not speak, and what they knew was not apparent, so it is impossible to know whether those students who did try were able to engage in productive struggle. And, this was hardly a safe environment, conducive to the development of mathematical agency or identity. In sum, the preceding discussion has highlighted the importance of all five of the TRU dimensions as important factors in student learning.

See the following section for notes on professional development, specifically with regard to Dimension 4—a challenge in all three of the classrooms that were analyzed.

### 4.2 Tools for progress

The first challenge faced by the TRU team was to make sure that the TRU framework focused on the right things, in rigorous and reliable ways. With that accomplished, our primary motivation has been to make it useful for professional development—the idea being that *using the five dimensions of the TRU framework to problematize one's instruction and reflect on it* can be a mechanism for continuous improvement.

The TRU Community has been developing a range of tools for supporting professional learning communities. In what follows I describe some individual written tools, and then discuss how an ongoing professional development workshop makes use of them.

Given space limitations I discuss just one dimension of the framework, agency, ownership, and identity. (This was a key challenge in all three of the lessons discussed.) An introduction to the corpus of TRU tools can be found in Schoenfeld and the Teaching for Robust Understanding Project (2016a, b, c). Currently available TRU documents can be found at the websites (http://map.mathshell.org/trumath.php) and (http://edcollaboration.org/TRU-LS/trutools.html).

#### 4.2.1 The TRU math conversation guide

The first tool is *The TRU Math Conversation Guide* (Baldinger et al. 2016). This tool has been used by a wide range of professional learning communities (school districts, departments, teacher-coach pairs, and individual teachers) to problematize instruction. The idea is to try to enrich instruction along all five dimensions before the lesson takes place, and then to reflect on possible improvements. The page of the *Conversation Guide* that focuses on agency, ownership, and identity asks the community to reflect on the questions given in Fig. 4.

#### 4.2.2 The TRU math observation guide

The *TRU Math Observation Guide* (Schoenfeld and the Teaching for Robust Understanding Project 2016b) is based on development work done by the San Francisco Unified School District. This guide highlights "look fors"—the kinds of evidence that indicate that a classroom is doing well along each dimension. The "look fors" for agency, ownership, and identity in the observation guide are given in Fig. 5.

#### 4.2.3 A professional development workshop focusing on Dimension 4: agency, ownership, and identity

In all three of the videos discussed in this paper, there was comparatively little room for student "voice"— opportunities for students to present their thinking and have it refined in classroom discourse. This is a widespread issue. In our experience, it is also one recognized by teachers, and one that they are willing to work on. Here I describe the kind of professional development workshop we have created to address this aspect of teaching. We are in the process of developing a series of workshops to support ongoing learning related to all five of the TRU dimensions.

As noted above, my preference is to work with a teaching community in an ongoing way. No matter what the discipline, the development of expertise takes years (see, e.g., Ericsson and Smith 1991). TRU is aimed at continual improvement over time.

**Things to think about**

- Who generates the ideas that get discussed?
- What kinds of ideas do students have opportunities to generate and share (strategies, connections, partial understandings, prior knowledge, representations)?
- Who evaluates and/or responds to others' ideas?
- How deeply do students get to explain their ideas?
- How does (or how could) the teacher respond to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.)?
- How are norms about students' and teachers' roles in generating ideas developing?
- How are norms about what counts as mathematical activity (justifying, experimenting, connecting, practicing, memorizing, etc.) developing?
- Which students get to explain their own ideas? To respond to others' ideas in meaningful ways?
- Which students seem to see themselves as powerful mathematical thinkers right now?
- How might we create more opportunities for more students to see themselves and each other as powerful mathematical thinkers?

**Fig. 4** Agency, ownership and identity as supported in the conversation guide

| AGENCY, OWNERSHIP, AND IDENTITY<br>*The extent to which every student has opportunities to explore, conjecture, reason, explain, and build on emerging ideas, contributing to the development of agency (the willingness to engage academically) and ownership over the content, resulting in positive mathematical identities.* | |
|---|---|
| Each student… | Teachers… |
| • Takes ownership of the learning process in planning, monitoring, and reflecting on individual and/or collective work<br>• Asks questions and makes suggestions that support analyzing, evaluating, applying and synthesizing mathematical ideas<br>• Builds on the contributions of others and help others see or make connections<br>• Holds classmates and themselves accountable for justifying their positions, through the use of evidence and/or elaborating on their reasoning | • Provide time for students to develop and express mathematical ideas and reasoning<br>• Work to make sure all students have opportunities to have their voices heard<br>• Encourage student-to-student discussions and promote productive exchanges<br>• Assign tasks and pose questions that call for mathematical justification, and for students to explain their reasoning<br>• Employ a range of techniques that attribute ideas to students, to build student ownership and identity |

**Fig. 5** Agency, ownership and identity "look fors" in the observation guide

We begin our work with a department by conducting a workshop designed to illustrate the TRU framework and introduce teachers to the key constructs in it. The teachers watch three very different videos of mathematics instruction. They are encouraged to make any comments they wish about what they see. There is typically a very wide-ranging conversation, during which I (or the person moderating the discussion) write their comments on a whiteboard or a series of flip charts in the front of the room. When the discussion wraps up, the moderator organizes the comments and points out that all of them fall comfortably into the five categories of the TRU framework, which is then presented formally. Teachers are also given documents such as the introduction to TRU (Schoenfeld, A. H., and the Teaching for Robust Understanding Project 2016a), and the observation and conversation guides (Baldinger et al. 2016; Schoenfeld, A. H., and the Teaching for Robust Understanding Project 2016b).

In the next meeting the teachers discuss their goals for the year and how they are related to the TRU framework. With the recognition that it takes some time to build skills in any particular dimension, they often choose one dimension to focus on, possibly for a semester or for the year. Say they choose Dimension 4: agency, ownership, and identity (AOI).

At the first AOI meeting, the moderator provides some definitions:

> Agency: The disposition and capacity to act—"I can do mathematics, and I'm willing to jump in and give it my best."
> Ownership: Taking possession of knowledge by making it one's own—"I figured this out; it makes sense; it's not simply what 'they' told me is true."
> Identity: How you see yourself mathematically—"I am a math person. I like math; it makes sense; and I can figure things out."

and reminds the group of the various tools they have at their disposal, including the TRU conversation and observation guides.

The group then watches two pre-selected videos, each about 5 min long. In the first video the students are working an interesting problem, but their opportunities to contribute to the mathematical discussion are somewhat constrained. In the second video the same teacher has opened up classroom discourse substantially, and the students have many more opportunities to contribute their ideas and develop their collective understandings.

We work very hard to build productive norms for discussions, so the teachers are not judgmental ("I would not have done it that way") but, rather, focus on ways in which the current practice might be modified to open up more space for student contributions. Prior to watching each video, the teachers are "primed" with a set of questions:

- Who generates the mathematical ideas that get discussed?
- Who evaluates and/or responds to others' ideas? How does the teacher respond?
- Which students respond? How do they respond?
- How deeply do students get to explain their ideas?
- How are norms developing around students' and teachers' roles in generating mathematical ideas?

After each video, there is an open conversation, with some stimulus questions (in this case, for student presentations):

- How comfortable are the students taking control in front of the class?
  - Can they be further positioned and supported as leaders, with things to share?
- Whose math is it, theirs or external authority's?
  - Can the presenters be further supported in explaining their own thoughts and ideas?
- Who's the audience, the teacher or fellow students? Which students are invited into the conversation, in what ways?
  - How can presenter–student dialogues be supported, further opening up AOI to the whole class?

We repeat this kind of workshop until the norms for discourse are well established and the teachers feel comfortable with having these conversations. At that point we ask for a volunteer: would one of the teachers be willing to have us video his/her instruction, and use that tape for discussion? We work with the teacher to pick moments of interest in the video, and offer the teacher the role of co-facilitator.

From that point on, the workshops operate as follows. Each session begins with a description of a "problem of practice" the volunteer teacher wants to address, e.g., "When I lead a discussion there are lots of student contributions, but when students present, they look to me as the authority and the rest of the class does not contribute. You'll see that in the video. Can you help me think about how to get broader participation?" We watch the tape and discuss it, using the various tools (*Conversation Guide, Observation Guide*, AOI prompts, etc.) to scaffold the conversation. Once the group sees a volunteer take the risk of doing this, and profiting from the conversation, others volunteer. This structure becomes our modus operandi for the semester or the year. Toward the end of the allotted time, the group decides what its next focus will be.

#### 4.2.4 On target

Finally, we are developing a tool for examining individual classroom activities and enhancing them. That tool, known as *On Target* (Schoenfeld and the Teaching for Robust Understanding Project 2018), provides a visual metaphor for reflecting on the affordances of particular classroom activities and "zeroing in" on increasingly productive practices. One of the targets for Dimension 4 is given in Fig. 6.

### 4.3 A brief return to the three focal lessons

I am loath to say very much about the kind of professional development I would engage in with each of the three focal teachers for the following reasons: (1) opportunities for professional development depend very much on the local institutional context, (2) effective professional development depends on building productive working relationships with the teachers and their local community, and (3) it is dangerous to extrapolate from a sample of one lesson. There are, however, some general things I can say.

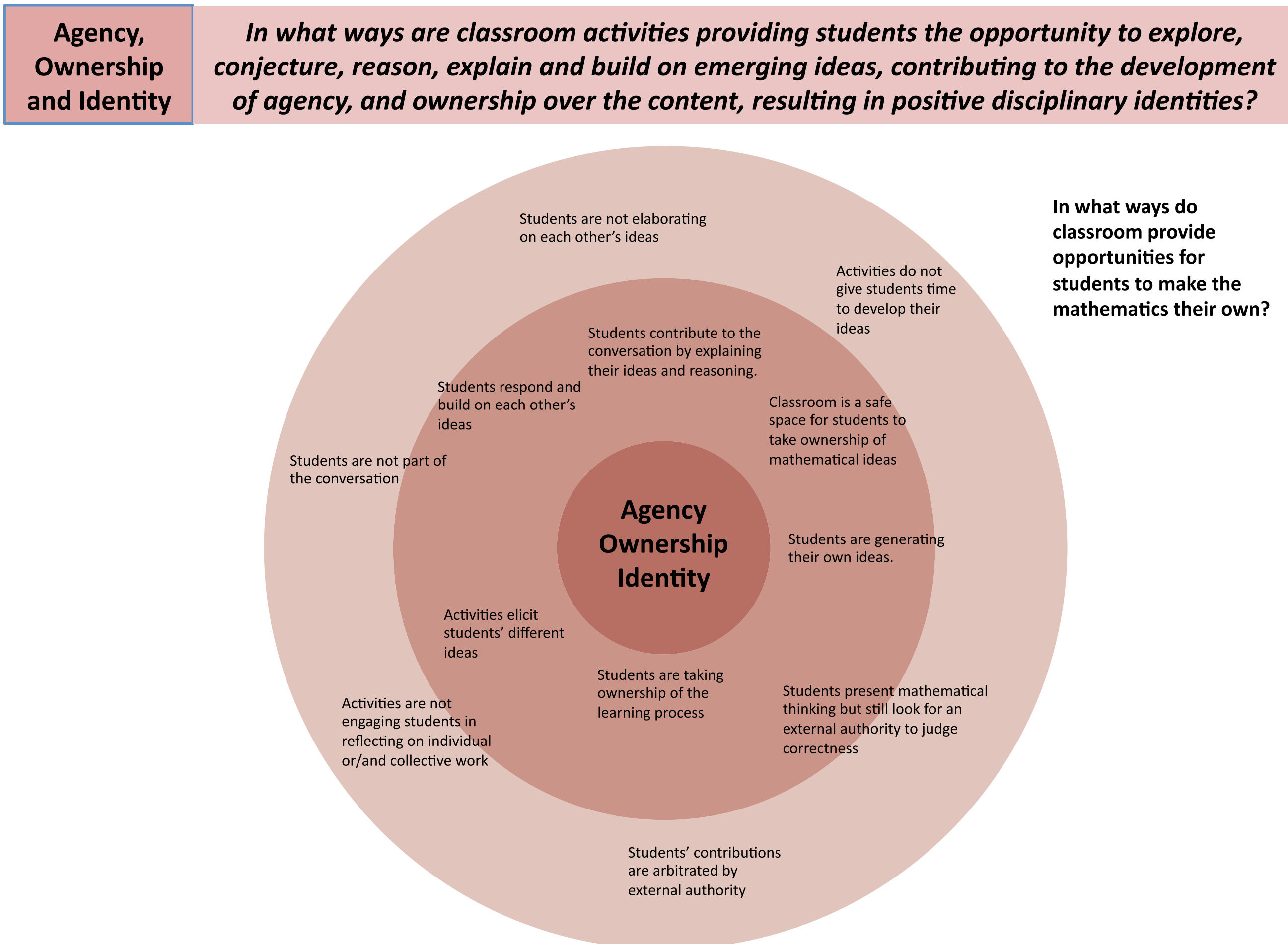


**Fig. 6** A "bull's-eye" representation of the richness of various activity structures

A major issue in the lessons taught by Mr. Smith and Ms. Jones was the richness and coherence of the mathematics being discussed. Some of the workshops we run are focused on the question "How can we make the mathematics at the core of this lesson both richer and more accessible?" The parts of the TRU Conversation Guide (Baldinger et al. 2016) and TRU Observation Guide (Schoenfeld and the Teaching for Robust Understanding Project 2016b) that are devoted to Dimension 1 (the mathematics) provide some resources for this conversation: we look specifically at ways to view the mathematics from different perspectives, employ multiple representations, etc. (When we engage in collective lesson design, as in TRU-lesson study, we look into various textbooks and the research literature for ideas, and we role play the lessons ourselves.) This provides a way to "problematize" the way the content is conceived and experienced by the students.

Along the lines of "student experience of the mathematics" in both the cognitive demand and agency/ownership/identity dimensions, we also discuss ways of de-centering the teacher's role and giving students more voice. For example, there is a technique known as "think/pair/share," in which the teacher (1) poses a problem to the students and gives them a chance to think it over, (2) has them discuss it with their neighbors, and then (3) takes suggestions and discusses them with the whole class. When teachers make good use of this technique, the students have more "think time" and more of an opportunity to work through their own and each other's ideas.

This technique might well be of use to Mr. Smith and Ms. Jones. Interestingly, this technique might also be a non-threatening way to ease into a discussion with Ms. Young. Think/pair/share can be introduced as a way to bring more students into the mathematical conversation. If Ms. Young is part of a teacher learning community that agrees to take up the challenge of increasing student "voice," someone else might volunteer (or be induced to volunteer) to try the technique. If that teacher's attempt is videotaped and discussed, it might open the door to Ms. Young trying it as a positive way of building on her already strong mathematical practice.

Finally, I cycle back to the strengths and limitations of TRU as a frame for analysis and professional development. There is every reason to be confident that TRU identifies "what counts" in a lesson; the three focal lessons studied in this volume are cases in point. Of course, as with any non-algorithmic observation system, the acuity of the observations depends on the skills of the observer. But the framework is in place. The TRU community now provides a beginning set of tools that coaches, administrators, and teacher learning communities can use in the service of improving instruction. Similarly, the effectiveness of these tools depends on the skills of those using them.

Our mission, in sum, has been to understand what really matters in classrooms, and to provide tools built on those understandings that will enhance instruction. This is, of course, an ongoing process.

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