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SURVEY PAPER

Applying cognitive psychology based instructional design principles in mathematics teaching and learning: introduction

Lieven Verschaffel[1] · **W. Van Dooren**[1] · **J. Star**[2]

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Abstract This special issue comprises contributions that address the breadth of current lines of recent research from cognitive psychology that appear promising for positively impacting students' learning of mathematics. More specifically, we included contributions (a) that refer to cognitive psychology based principles and techniques, such as explanatory questioning, worked examples, metacognitive training, exemplification, refutational texts, multiple external representations, which are considered as well-established principles or techniques for instructional design in general, and (b) that explore, illustrate and critically discuss the available research evidence for their relevance and efficacy in the specific curricular domain of mathematics teaching and learning. The special issue ends with an interview with Paul Kirchner about the relationships between cognitive psychology, instructional design and mathematics education.

Keywords Instructional design · Cognitive science · Domain-general theories · Domain-specific theories · Conceptual change · Cognitive load · Metacognition · Embodied cognition

✉ Lieven Verschaffel
lieven.verschaffel@kuleuven.be

1 Center for Instructional Psychology and Technology, Van den Heuvel Instituut, KU Leuven, Dekenstraat 2, Post Box 3773, 3000 Leuven, Belgium

2 Harvard School of Education, 13 Appian Way, Cambridge, MA 02138, USA

1 Aims and scope of the special issue

For several decades, the research on mathematical thinking and learning mainly occurred from a cognitive point of view, but at least since the 1990s, the field of mathematics education has been enriched by perspectives from many other disciplines, including philosophy, sociology, economics, anthropology, and neuroscience (De Corte, Greer, & Verschaffel, 1996). Yet, because cognitive psychology is centrally concerned with learning processes such as thinking, remembering, and transfer, our attempts to understand and improve the learning and teaching of mathematics continue to be heavily influenced by cognitive psychology. At the same time, cognitive psychologists have long claimed that their research is relevant for teaching and learning in school, particularly in major subject-matter domains such as language, science and mathematics (Star and Verschaffel 2017).

Historically, the relationship between cognitive psychology and (mathematics) education is a complex one. As argued by Star and Verschaffel (2017), whereas the decade of the 1980s represented the apex of cognitive psychology's impact on (mathematics) education, some would argue that, today, its influence appears to be less substantial—either because it shares the stage with other disciplines such as sociology and/or because many in the field no longer feel that (cognitive) psychological studies of learning are relevant to school settings. The goal here is to summarize some of the instructional design principles or techniques that have been derived directly and explicitly from recent research in cognitive psychology and that have been claimed to be informative for the learning and teaching of mathematics.

In some sense, this special issue relates to a series of recent publications from cognitive psychologists bringing psychological findings to education in general. These publications are similar in structure, in that they claim to synthesize research in order to identify a set of learning or instructional principles that have strong support from cognitive psychology, with the hope that educators will adopt these principles. As a first example, we provide the ten "cornerstone findings" about learning identified by Schneider and Stern (2010): (1) It is the learner who learns. (2) Optimal learning builds on prior knowledge. (3) Learning requires the integration of knowledge structures. (4) Optimal learning is about learning concepts, skills and metacognitive competence in a balanced way. (5) Optimal learning builds complex knowledge structures through the hierarchical organization of more basic pieces of knowledge. (6) Optimally, learning uses structures in the external world to organize knowledge structures in the mind. (7) Learning is constrained by capacity limitations of the human information-processing architecture and capacity. (8) Learning results from the dynamic interplay of emotion, motivation, and cognition. (9) Optimal learning builds up transferable knowledge structures. (10) Learning requires time and effort. Whereas this list is strongly focused on learning, and only indirectly addresses the issue of instruction, other lists are more explicitly conceived and phrased in terms of principles for instructional design. Take, for instance, Pashler et al. (2007) list of seven recommendations for improving student learning: (1) Space learning over time, (2) Interweave worked examples solutions and problem solving exercises; (3) Combine graphics with verbal descriptions; (4) Connect and integrate abstract and concrete representations of concepts; (5) Use quizzing to promote learning; (6) Help students to allocate study time efficiently; (7) Help students to build explanations by asking and answering deep questions. Recently, several other examples of such general lists of principles of learning and/or instructional design have been produced: Graesser and colleagues (2008) identify their 25 "learning principles", Roediger and Pyc (2012) discuss three "inexpensive techniques" from cognitive psychology that can inform educational practice, Koedinger, Booth, and Klahr (2013) write about five techniques for improving learning, while Dunlosky and colleagues (2013) describe their own five instructional techniques.

This special issue does not aim to duplicate these efforts to produce lists of recommendations from cognitive psychology for improving learning, but rather wishes to extend the discussion in two important ways.

First, we consider these principles or techniques derived from recent cognitive psychological research using a disciplinary lens. So, rather than providing, as the various syntheses cited above, a review that is content general, identifying recommendations that are presumed to improve learning generally rather than in the specific case of mathematics, we look critically at these principles and techniques from the perspective of mathematics education. For many years, leading scholars of the field of mathematics education have pleaded strongly in favor of such a domain-specific approach (Verschaffel and Greer 2013). For example, Freudenthal (1991, p. 149) was highly critical of the work of cognitive and educational psychologists relating to mathematics "as long as, for the researcher, mathematics is no more than an easily available and easily handled subject matter, chosen to test and apply general ideas and methods, with no regard for the specific nature of mathematics and mathematics instruction". Likewise, Fischbein (1990, p. 10) stated that "(m)athematics education raises its own problems, which a professional psychologist would never encounter in his own area and that the methodology should also be adapted to the specificity of the domain". Wittmann (1995, p. 356) also strongly pleaded for mathematics education to be validated as a scientific field in its own right, one that cannot be developed by simply combining the insights from other fields like mathematics, general didactics, pedagogy, and psychology; "rather, it presupposes a specific didactic approach that integrates different aspects into a coherent and comprehensive picture of mathematics teaching and learning, and then transposing it to practical use in a constructive way". A recent illustrative case of a discussion between the community of cognitive scientists and mathematics educators relates to the use of concrete or abstract examples. Kaminski, Sloutsky, and Heckler (2008) published in *Science* a study on "The advantage of abstract examples in learning math". Their main claim, which was supported by their data, was that students benefit more from learning mathematics through an abstract symbolic representation than from concrete examples. However, the paper elicited many critical comments by mathematicians and mathematics educators (e.g. Jones 2009a, b; Podolefsky and Finkelstein 2009). Based on a combination of theoretical arguments and empirical results of a carefully designed replication study, De Bock, Deprez, Van Dooren, Roelens, and Verschaffel (2011) raised serious concerns about the interpretation by Kaminski et al. (2008) on the specific mathematical concepts that students actually acquired in the study and on the educational recommendations these authors derived from their study. So, in this special issue, we seek recommendations for improving mathematics learning from cognitive psychology that are tested in and supported by research in mathematics education, preferably by studies that representatives of the mathematics education community would find most valuable.

Second, the various contributions to this special issue not only review and discuss these instructional design principles in relation to mathematics education in general terms; in addition, each contribution also includes a report of a specific study or a set of studies that concretely illustrates how that specific principle or technique is actually used in the design of a mathematics learning environment and/or how it is subjected to an empirical test of its effectiveness. Furthermore, using that exemplarily study or set of studies as an illustrative case, each author (team) has been asked to reflect not only on the strengths, but also on the pitfalls of their instructional design principle or technique for improving mathematics education and to do suggestions for further research.

In the present special issue, a dozen of such instructional design principles or techniques that have been derived from cognitive psychology and that appear promising for impacting students' learning of mathematics, are reviewed, illustrated, and discussed. These principles are listed briefly below. But before doing so, we briefly explain how we arrived at this list. Actually, following up the approach applied by Star and Verschaffel (2017), we combined two complementary approaches. On the one hand, we looked at the recent research syntheses mentioned above, where cognitive scientists had conducted extensive literature reviews to identify research-supported recommendations for improving student learning in general (e.g., Dunlosky et al. 2013; Koedinger et al. 2013; Pashler et al. 2007; Schneider and Stern 2010). By looking across these syntheses, we identified several distinct principles for improving student learning that had been identified as having strong research support from cognitive psychology, and we considered whether we were aware of manifest and explicit applications of these principles in the domain of mathematics education. On the other hand, we looked in the research community of mathematics education for (teams of) scholars who explicitly and systematically refer to principles or techniques of instructional design in their attempts to enhance students' mathematical learning, with a preference for evidence that emerged from studies in "real" mathematics classrooms (where it existed). Using these two sources of data, we finally used our professional judgment and experience as mathematics educators to select a list of principles and techniques that are addressed in the various contributions to the present special issue. We recognize that this process was not entirely objective, but rather included some personal bias about what we considered to be particularly illustrative and useful for the topic of this special issue. Furthermore, it should be clear that some of the chosen principles and techniques are sometimes termed differently in other, similar overviews and/or are grouped into a broader instructional design principle or technique (e.g., Booth, McGinn, Barbieri, et al., 2017; Star and Verschaffel 2017).

Each invited author (team) was asked to write his/her chapter according to a fixed structure, consisting of the following elements: (1) Explanation of the instructional design principle and its theoretical underpinnings; (2) Short review of the relevant empirical research and its main results and conclusions in general and in the domain of mathematics education in particular; (3) A summary of an illustrative study (or a line of research) by the author's own research team; (4) Discussion of theoretical and methodological issues for future research; (5) (Provisional) recommendations for mathematics educators.

The initiative for this special issue grew out of a workshop entitled "Providing Support for Student Learning: Cornerstone findings, implications and recommendations from Cognitive Psychology for the Teaching of STEM (Science, Technology, Engineering and Mathematics)", which was organized in October 2015 in Leuven. However, the specific themes addressed in the special issue and the authors involved coincide only partially with those involved in that workshop.

2 The different contributions

The first principle, addressed by Vamvakoussi, is *instructional analogies*. The problem of adverse effects of prior knowledge in mathematics learning has been amply documented and theorized by mathematics educators as well as cognitive (developmental) psychologists. This problem emerges when students' prior knowledge about a mathematical notion comes in contrast with new information coming from instruction, giving rise to systematic errors. Conceptual change perspectives on mathematics learning argue that in such cases reorganization of students' prior knowledge is necessary. Analogical reasoning, in particular cross-domain mapping, is considered an important mechanism for conceptual restructuring.

Relying on the same conceptual change framework, Lem, Onghena, Verschaffel, and Van Dooren (2017) discuss an instructional design technique called *refutational text*. The aim of refutational text is to transform misconceptions into conceptions that are in line with current scientific concepts. This is done by explicitly stating a misconception, refuting it, and providing a correct conception. This technique has been applied to various content domains, particularly in science teaching but, more recently, also in the domain of mathematics, and is being argued to be effective in inducing cognitive conflicts in students and remediating misconceptions.

Walkington (2017) reviews *personalization* as an instructional design principle that involves presenting mathematics tasks to students in the context of their interests in areas like sports, music, or video games. Personalization may allow for students' understanding of domain principles to become grounded in their concrete and familiar experiences. It is argued that by making connections to students' prior knowledge, personalization may reduce extraneous cognitive load, freeing up cognitive resources for the acquisition of new ideas.

Another intensively investigated instructional design principle is that of using *multiple representations*. To make complex mathematics concepts accessible to students, teachers often rely on visual representations. But because no single representation can depict all aspects of a mathematics concept, instruction typically uses multiple representations. Much research suggests that multiple representations can have immense benefits for students' learning, particularly in the domain of mathematics—a domain that is fundamentally characterized by the intrinsic inaccessibility of its (thinking) objects (Duval 2002). However, some research also cautions that multiple representations may fail to enhance students' learning if they are not used in the "right" way. In their contribution to the special issue, Rau and Matthews (2017) review research-based principles for how to use multiple representations effectively so that they enhance student mathematics learning.

An additional instructional design principle with particular potential in early mathematics education is *embodiment*. According to Dackermann, Moeller, Fischer Cress, and Nuerk (2017), there is accumulating evidence that the acquisition of basic numerical representations (e.g., magnitude understanding) can be corroborated by embodied trainings allowing children to move their body in space. Following a brief summary of recent embodied training studies, the authors try to integrate the respective results into a unified model framework which elucidates the working mechanisms of embodied trainings, allowing for the identification of the age group and/or numerical content for which embodied trainings should be most beneficial.

Kullberg, Runesson Kempe, and Marton (2017) present and discuss *variation* as a guiding principle of pedagogical design in the domain of mathematics education. The variation theory of learning, which underlies this principle, emphasizes variation as a necessary condition for learners being able to discern new aspects of an object of learning. In a substantial number of studies these authors have used the theory to analyze and design the teaching and learning of mathematics.

Probably one of the best-known and empirically most tried-and-tested instructional design principles resulting from cognitive research, reviewed and discussed by Renkl (2017), is learning from *worked examples*. Such examples consist of a problem formulation, the final solution, and typically also the steps leading to the final solution. When applied properly, learners studying such examples are assumed to engage in self-explanations that justify not only the demonstrated solution steps but also their underlying principles.

Underlying Durkin, Rittle-Johnson and Star's (2017) article is the idea that *comparison* is a fundamental cognitive process that has been shown to support learning in a variety of domains, including mathematics. The authors report results of their own classroom-based research on using comparison to help students learn mathematics, indicating that comparing different solution methods for solving the same problem is effective for supporting procedural flexibility across students and for supporting conceptual and procedural knowledge among certain groups of students.

Rittle-Johnson, Loehr and Durkin (2017) address *self-explanation* (i.e., generating explanations for oneself in an attempt to make sense of new information) as another powerful instructional technique that has been shown to improve learning in a range of domains. Theoretically, self-explanation is thought to guide attention to structural features over surface features of the to-be-learned content and in doing so promote knowledge generalization and integration. The authors report a meta-analysis of the self-explanation literature for mathematics learning that provides empirical support for the benefits of prompting for self-explanation for improving comprehension and transfer in mathematics.

Another recommendation from cognitive psychology that has a quite strong evidentiary base in mathematics classrooms is that *metacognition* plays a significant role in mathematical problem solving and that it therefore is productive to train students in the use of metacognitive strategies. After having reviewed the research on the assessment and stimulation of metacognition in general, Baten, Praet, and Desoete (2017) report a study highlighting that instructional designs where metacognition is stimulated to enhance children's mathematical proficiency at a young age.

The starting point of the contribution by Lehtinen, Hannula-Sormunen, McMullen and Gruber (2017) are theories of expertise development that highlight the crucial role of *deliberate practice* in the development of high level performance. Deliberate practice is practice which intentionally aims at improving one's skills and competencies. It is not a mechanical and repetitive process of making performance more fluid. Instead, it involves a great deal of thinking, problem solving, and reflection for analyzing, conceptualizing, and cultivating a developing performance. Expertise studies in music and sport that have described early forms of deliberate practice among children have inspired the authors to analyze and design various forms of practice in early and elementary school mathematics.

The special issue ends with an interview with Paul Kirschner, reflecting upon the theme of the special issue in general and the contribution of the different articles to the theme.

3 Some preliminary comments

For the special issue, we have selected a dozen principles and techniques derived from cognitive psychology that are considered by cognitive scientists as well as mathematics educators as particularly promising for the field of mathematics education.

Surely, this is not the first attempt to come up with such a list. In this respect, we refer to two recent publication, namely Booth et al. (2017) discussing evidence for a somewhat different subset of principles from cognitive psychology (including principles such as abstract and concrete representations, analogical comparison, feedback, error reflection, scaffolding, distributed practice interleaved practice, and worked examples), and Star and Verschaffel (2017) discussing a much smaller selection of three such principles (i.e., explanatory questioning, worked examples, and metacognitive training).

As argued by the authors of these similar reviews, a common feature of all these principles reviewed is that they are grounded in well-articulated general theories of human cognition and learning and that they have been applied to and tested in mathematics education settings, and, thus, have the potential to enhance students' mathematics learning processes and outcomes. But at the same time, these principles differ from each other in various ways (see also Booth et al. 2017; Star and Verschaffel 2017).

First, they differ in terms of the underlying theoretical notions and models. For instance, while some principles are grounded in conceptual change theories, others are derived from cognitive load theory and still others from theories such as variation theory or embodied cognition.

Second, these principles differ in terms of the available empirical evidence in their support in the field of mathematics education. For instance, whereas the principles of explanatory questioning, worked examples, and metacognitive training have already been amply tested in the field of mathematics education (see Star and Verschaffel 2017), others, such as refutational text, rely on much less evidence from within this field. Also, some principles have been well-tested in laboratory studies, while others were mainly subjected to classroom research.

Third, the principles also differ with respect to their application field. Some principles seem primarily applicable and/or have been primarily tested for simpler versus more complex mathematics and/or for younger rather than older students (Booth et al. 2017). Given what is known about differences between distinct subdomains and developmental levels or levels of expertise, it would be dangerous to simply transfer results and recommendations from one mathematical domain, type of problem, and/or target group to another.

Fourth, these principles also differ largely in terms of how easy or difficult it is to implement and enact them in the real mathematics classroom, at least if one wants to use them not merely occasionally but on a rather systematic basis (Booth et al. 2017; Star and Verschaffel 2017). For instance, whereas a technique like instructional analogy or refutational text can be quite easily incorporated into existent mathematics textbooks or lessons, others, such as multiple representations or personalization may be difficult to implement systematically without the help of educational technology.

Notwithstanding these differences into theoretical background, empirical support, field of application and difficulty of implementation, the contributions in this special issue provide an interesting overview of the main instructional design principles and techniques that have derived from general cognitive theories of thinking and learning and applied to the mathematics education—either alone or in combination with other principles or techniques—with a view to enhance students' learning processes and outcomes in this particular curricular domain. In the interview that closes this special issue, Paul Kirschner will reflect upon the promises and pitfalls of such an attempt.

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