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ORIGINAL ARTICLE

The relevance and efficacy of metacognition for instructional design in the domain of mathematics

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Abstract The efficacy of metacognition as theory-based instructional principle or technique in general, and particularly in mathematics, is explored. Starting with an overview of different definitions, conceptualizations, assessment and training models originating from cognitive information processing theory, the role of metacognition in teaching and learning is critically discussed. An illustrative training program in kindergarten demonstrates that explicit and embedded metacognitive training can have an effect on mathematics learning even in very young children. Theoretical and methodological issues for future research, and recommendations for mathematics educators, are analyzed within the framework of the Opportunity-Propensity and Universal instructional design framework, demonstrating the relevance of metacognition in the domain of mathematics teaching, impacting children's learning of mathematics and their active involvement in their learning process.

Keywords Metacognition · Mathematics achievement · Instructional design · Opportunity propensity model

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1 Metacognition as instructional design principle: its underpinnings in mathematics

Mathematics achievement is among the strongest predictors of later academic success (Duncan et al. 2007; Wang et al. 2013). Mathematics is also central to get a mortgage, buy a car, sort out household bills, or just understand the vast amount of information thrown at us (Budd 2015). The Opportunity-Propensity (O-P) model suggests that children are more likely to realize their potential for learning mathematics if they are provided Opportunities (O) to learn that content at school and in other contexts and have the motivation and capability or propensity (P) to benefit from the opportunities provided to them (Wang et al. 2013). Within this model, metacognitive skills can be considered as P-factor, whereas powerful learning designs can be seen as O-factor, both positively impacting the learning of mathematics.

Metacognition refers to 'cognition about cognition' (Furnes and Norman 2015) or to self-referential confidence, check and balance (Fleming et al. 2012; Nelson 1996).

According to Nelson (1996), metacognition has a role in forming a representation of cognition based on monitoring processes as well as in exerting control over cognition based on the representation of cognition (Nelson 1996). In this paper we explore, illustrate, and discuss the definitions, conceptualization, assessment and stimulation of 'metacognition' and its impact on learning mathematics. We first explain the theoretical underpinning of metacognition and the instructional principle derived from it.

Metacognition originates within instructional design from cognitive information processing theory, with roots traced back to Piaget (Robson 2016; Schneider and Lockl 2002). The construct itself was introduced by Ann Brown and John Flavell as the knowledge concerning one's own cognitive processes and products and anything related to them (for a review, see Schneider and Artelt 2010). The first studies were developmental in nature (Brown et al. 1983; Flavell 1976; Flavell et al. 2002; Schneider and Artelt 2010), focusing on meta-memory or children's knowledge about information storage and retrieval. The concept was very successful, stimulating numerous studies but also leading to various definitions and methodologies, confusion and, sometimes, contradictions (Focant et al. 2006; Veenman 2013; Vermeer et al. 2000). Once metacognition gained popularity, most researchers agreed to differentiate a reflective component (or metacognitive knowledge) and an executive component (metacognitive skills; Brown 1987; Georghiades 2007; Pintrich 2004; Schraw et al. 2006). Metacognition became an important predictor of learning performance (Marulis et al. 2016; van der Stel and Veenman 2014) and effective teaching (Donker et al. 2014).

Metacognitive knowledge refers to the awareness of and reflection on cognitive strengths and weaknesses, the application of resources and strategies, and their situational appropriateness. It consists of one's 'correct' and 'false' beliefs about the subject and nature of mathematics (Schneider and Artelt 2010). Nunes et al. (1993) described this as "mature and immature task knowledge", pointing to the fact that "a lack of metacognitive knowlege" in children is not a static and fixed propensity. Metacognitive knowledge is sometimes referred to as the 'declarative knowledge' at one's disposal, without the guarantee for using this knowledge whenever it is needed (Van der Stel and Veenman 2014). Metacognitive knowledge does not develop automatically in all children, thus teachers play an essential part in the enlargement of metacognition by enhancing reflection on learning experiences and giving feedback on the planning of further learning tasks (de Jager et al. 2005).

Metacognitive skills encompass the 'active' control of engagement in learning, adapting to situational learning demands and optimising learning processes or outcomes (Azevedo 2009; Pintrich 2004). Metacognitive skills depend on procedural knowledge or the actual regulation of and control over one's learning activities (Van der Stel and Veenman 2014). Task analysis and goal setting or orientation, planning, monitoring, and evaluating or elaboration have been described as basic metacognitive skills (Brown 1987; Desoete and Roeyers 2002; Lucangeli et al. 1998; Pintrich 2004; van der; Stel and Veenman 2014; Wall et al. 2016).

A related conceptualization distinguished declarative, procedural and conditional (or strategic) metacognitive knowledge (Brown 1987; Schraw 1998). Declarative metacognitive knowledge can be defined as the 'what' knowledge or the knowledge of the strengths and weaknesses of one's own processing ability as a learner and the knowledge about cognitive strategies (Brown 1987; Georghiades 2007). Procedural metacognitive knowledge can be described as knowing 'how' to successfully employ particular cognitive strategies in order to achieve learning objectives (Perfect and Schwartz 2002; Schraw 1998). Conditional metacognitive knowledge can be used for 'when' and 'why' knowledge, referring to knowledge of the appropriateness of particular cognitive strategies when taking into account external learning conditions, including awareness of the underlying reasons for cognitive strategies' effectiveness (Zimmerman and Schunk 2011).

From a developmental perspective, elementary forms of contextualized metacognitive knowledge were detected already in 3- to 5-year olds (Marulis et al. 2016; Whitebread et al. 2007). In addition, Larkin (2009) found a relation between metacognitive knowledge and strategy use in 5- to 6- year olds. There was also some evidence for metacognitive knowledge as a necessary precursor to metacognitive skills (van der Stel and Veenman 2014).

Also metacognitive skills were found to develop, with studies revealing qualitative changes in the type of processes between 5 and 7 years of age (Bryce and Whitebread 2012). Spiess and colleagues (2016) demonstrated that monitoring skills were relatively well developed by the age of 8 (Spiess et al. 2016). Veenman and colleagues (Van der Stel and Veenman 2014; Veenman 2006) described an increase in quantity (frequency) and quality (depth) of metacognitive skills between 9 and 22 years with different components of metacognitive skills not developing at the same pace nor continuously. This result is in line with the study of Focant and colleagues (2006), revealing that goal setting, planning and control skills were acquired at different ages. Planning was found to be not acquired in most grade 5 children, although these children had no problems with the goal setting and succeeded in some tasks in which they had to use control skills. Furthermore, it was observed that planning was more closely related to school performance than control, and among the substrategies of control, control of operations was the only one positively influencing problem solving.

Although the metacognitive concept is more than 40 years old, researchers keep using different concepts for phenomena overlapping with or including metacognition (Tarricone 2011). There is empirical research of shared characteristics between metacognition and selfregulation (DiDonato 2013; Volet et al. 2013; Winne 2011; Zimmerman and Schunk 2011), temperament (Dragan and Dragan 2013), and executive functions (Ardila 2013; Roebers and Feurer 2016; Spiess et al. 2016). Efklides (2006, 2008) added the importance of the affective component of metacognition, with the construct of metacognitive experiences, making the person aware of his or her cognition and triggering control processes that serve the pursued goal of the self-regulation process (Efklides 2008; Koriat 2007).

Moreover, there is growing attention to social metacognition (De Backer 2015). Collaborative learning and reciprocal tutoring are assumed not only to encourage children in the processes of adopting and refining their personal metacognitive skills, but are also assumed to engage them in social forms of regulation skills as well (De Backer 2015; Iiskala et al. 2011; Järvelä et al. 2013; Rogat and Adams-Wiggins 2014). In addition, studies to understand the neurocognitive underpinnings of metacognition have been set up (e.g., Desender et al. 2016).

The assessment of metacognition remains an issue of discussion (Azevedo 2009; Cavanaugh and Perlmutter 1982; Veenman 2013). Metacognitive 'knowledge' is often assessed with questionnaires that are administered either prospectively or retrospectively to performance on a learning or problem solving task. Such questionnaires have the advantage that they are both easy to administer, especially in large samples, and easy to analyse (De Jager et al. 2005). Teacher ratings in grades 3 and 4 were found to account for about 22% of mathematics performance, pointing to the value of an experienced teacher and questionnaires as appropriate measures of metacognitive knowledge. There also appeared to be convergent validity for prospective and retrospective child ratings, but no significant relationship with the other metacognitive measures (Desoete 2008). However, questionnaires do not have only pros. There are also certain cons, such as the fact that they might measure the perception of metacognition rather than their actual metacognitive knowledge. In addition young children can find it hard to reflect on learning behaviour in a questionnaire and older children might have the tendency to answer in a way that they consider to be social desirable (De Jager et al. 2005), reducing the validity of questionnaire outcomes (Schneider and Artelt 2010). In young children, therefore, nonverbal assessment procedures such as concurrent videotaped observation of cognitive strategies have been used. Moreover, reciprocal peer tutoring has been used with older children (grades 3–6), asking them to teach a memory strategy, such as sorting items into semantic categories, to younger children. Tutors' instructions were taped and scored on the extent to which the instructions included appropriate strategy instructions (Schneider and Artelt 2010). In addition, metacognitive knowledge can also be assessed by evaluating and ranking the quality and usefulness of different strategies. The correspondence between the ranking of children and the optimal ranking is used as an indicator of metacognitive knowledge. Finally, an obvious problem with the retrospective assessment by means of questionnaires is the risk of memory distortions due to the time lag between the actual performance of problem solving and the verbal reports afterwards (Schneider and Artelt 2010). A method of overcoming this memory-failure problem is the 'stimulated recall'. The stimulated-recall technique requires participants to review a videotape of their performance on a specific task and to reproduce what they thought while performing the task. Similar issues remain for the assessment of 'metacognitive skills'. The most studied type of concurrent assessment of skills is the evaluation of how well one is doing while thinking or learning, also referred to as ease-of-learning (EOL), judgment of learning (JOL) and the feeling-of-knowing (FOK) judgements. Moreover, monitoring involves knowing when to terminate a study (recall readiness) and allocation of study time. A problem with these paradigms is that they not only tap metacognitive skills, but also address motivational variables (Schneider and Artelt 2010). Secondly, think-aloud protocols and observations have been used to assess metacognitive skills, instructing children to verbalize their thoughts during mathematics task performance (Desoete 2008; Veenman 2011). A third measure of metacognitive skills is the concurrent and systematic observation by judges who are present during task performance (participant observation) or the use of information from log-files (Hurme et al. 2006; Järvelä et al. 2013) and other on-line registrations, such as registration of answering time or eye movements (Azevedo et al. 2010; van Gog and Jarodzka 2013). In the next section, a literature overview of research on metacognition in the domain of learning, and in particular mathematics learning, is given.

2 Metacognition in mathematics education

The importance of metacognition for learning has been well established. Adequate metacognitive skills were found to advance the depth of learning and to correlate with more active cognitive processing, and better understanding, as well as improved performance (Azevedo et al. 2010; Winne 2011; Zimmerman 2002). Metacognition appeared especially effective for mathematics performance. Although metamemory has not always been found to be strongly related to cognitive performance (Cavanaugh and Perlmutter 1982), Schoenfeld (1992) provided a theoretically well-elaborated overview of problem solving, metacognition, and sense making in mathematics. In his conceptualization of mathematical thinking, metacognition, beliefs, and mathematical practices play a crucial role (Schneider and Artelt 2010). Children need to learn to be aware of and verbalize their knowledge and skills. Learning mathematics is therefore, in part, developing self-understanding and expressive language to join a community that values (or should value) discussion, argument and proof (Ginsburg et al. 2015). Vo and colleagues revealed that numerical metacognition in 5 year olds predicted school-based mathematics knowledge (Vo et al. 2014). Metacognitive beliefs and monitoring processes were revealed to be deeply involved in mathematical problem solving according to Cornoldi et al. (2015). Erickson and Heit (2015) demonstrated a greater overconfidence (or children predicting to have higher scores then they actually get) in predicting mathematics performance compared to academic subjects such as biology and literature, with overconfidence and anxiety adversely affecting metacognition, leading to mathematics avoidance.

Several studies demonstrated correlational evidence for a relationship between metacognition and mathematics performance. Carr et al. (1994) found in second graders significant relations between metacognitive knowledge and motivation (effort attribution) with both concepts contributing to increase in mathematics performance. Metacognitive knowledge was correlated with mathematics performance (correct decomposition strategy use). A replication showed that metacognitive knowledge significantly influenced young elementary children's developing strategy use (Carr and Jessup 1995). Schneider and Artelt (2010) also found that the impact of declarative metacognition on mathematics performance was substantial, sharing about 15–20% of common variance in fifth grade (9-10-year-old children). Özsoy (2011) found an even stronger relationship in fifth grade children, with 42% of the total variance of mathematics achievement explainable by metacognitive knowledge and skills. Moreover, findings from the PISA study demonstrated the importance of metacognition in 15-year-olds, with roughly 18% of the variance in mathematics performance explainable by the metacognition indicator (Schneider and Artelt 2010). In addition Veenman (2006) demonstrated that both intelligence and metacognitive skills influenced mathematics performance, but metacognition outweighed intelligence as a predictor of mathematics learning performance in secondary school. To conclude, most studies reveal that metacognition plays an important role in mathematics performance (Blair and Razza 2007; Morosanova et al. 2016; Özsoy 2011; Schneider and Artelt 2010), especially in new and effortful tasks (Carr et al. 1994; Vermeer et al. 2000; Verschaffel 1999), and that metacognitive knowledge instruction is valuable in primary and secondary education (Donker et al. 2014). In the following section an overview is given of the research literature in the domain of metacognitive training.

3 Metacognitive training

Fostering metacognition has been an important educational objective of several studies. Hartman and Sternberg (1993) categorized the available studies into four main approaches to improving metacognition. In the first approach 'general awareness' was promoted, with teachers modelling metacognitive skills and stimulating with a kind of reflective discourse the self-reflection exercises of children (e.g., Olsen and Singer 1993). In the second approach, teachers focused on improving 'metacognitive knowledge' by handing out overviews of successful approaches/strategies and clarifying how, when, and why to use specific strategies. Children for example learned to slow down on more difficult tasks (e.g., Desoete et al. 2003), activate prior knowledge, make a mental integration, and build up diagrams. The third approach aimed to improve 'metacognitive skills'. This approach included presenting a variety of heuristics that were intended to support reflective activities focusing on planning, monitoring and evaluation (e.g., Schraw 2001). The heuristically and metacognitively oriented intervention study of Verschaffel (1999) can be situated within this approach. In addition, Osman and Hannafin (1992) differentiated metacognitive skills that may be embedded or integrated within a criterion lesson and skills, which may be taught separately–detached–from academic subjects. Schunk (2004) pointed to the importance of teaching metacognitive skills in conjunction with more than one task, to make children see that the skills are applicable to more than the tasks highly similar to those they learned. If metacognitive skills and conditional metacognitive knowledge are trained, Moore (2005) would define this as 'direct instruction'. Within this perspective, Clarke and colleagues (1993) fostered reflective journal writing in mathematics (Clarke et al. 1993; Schneider and Artelt 2010). In addition to these approaches, Mevarech and Kramarski's IMPROVE is another example of instruction that emphasizes reflective discourse by providing the opportunity to be involved in mathematical reasoning. Teachers are trained to use metacognitive (comprehension, connecting, strategic and reflection) questions about the nature of the problem, the use of appropriate strategies for solving the problem and the relationships with previous knowledge (Kramarski and Hirsch 2003). In this way, the IMPROVE method guided the childrens' thoughts and actions on what, when, how (prediction, planning) and why (monitoring, evaluation). Study outcomes demonstrated that systematic reflection support was effective for developing mathematics pedagogical content knowledge and strengthening metacognitive knowledge of mathematics teachers (Kramarski 2009). This approach can be defined as teaching 'with' and 'for' metacognition. In the fourth type of training researchers focused on a 'powerful' teaching environment. These teaching environments fostered self-reflection and improvement, and helped children to attribute their success to the use of adequate strategies and self-regulation within a growth mindset. Tanner and Jones (1995) found that the dynamic scaffolders and the reflective scaffolders were the two most successful teaching styles to enhance mathematical development. Moore (2005) described this as an 'infusion' approach in which direct instruction was combined with exercises in several situations.

Veenman (2013) pointed to the fact that to enhance metacognition, informed, prolonged and embedded training is needed. This is in line with the study of Desoete et al. (2003), who revealed that that we cannot expect metacognitive skills to develop spontaneously in all children as they grow older and have more experience with mathematics. Explicit exposure, training and instructional programs are needed to enhance metacognition. Pennequin et al. (2010) and Cornoldi and colleagues (2015) revealed that especially low performers benefitted from such metacognitive instruction. However, Donker and colleagues (2014) did not find differential effects for different ability levels, and Schneider and Artelt (2010) stated that numerous intervention studies demonstrated that 'normal' learners as well as those with especially low mathematics performance did benefit substantially from metacognitive instruction procedures.

Most metacognitive training focuses on older children (Desoete et al. 2003; Donker et al. 2014; Schneider and Artelt 2010; Verschaffel 1999), although there is evidence of metacognition in younger children (Vo et al. 2014). In addition, studies on the differential effect of abilities remains inconclusive. In what follows, an illustration of training in kindergarten, of children with different ability levels, is given.

4 An illustration of metacognitive training in kindergarten

The possibilities of metacognitive exposure in young children is illustrated in the following study on 167 children (90 boys) in the last year of kindergarten, randomly assigned to the following: counting ($n=43$), comparison ($n=39$), counting and comparison (($n=18$), metacognition and counting and comparison ($n=19$) conditions, or to a gaming control condition ($n=49$). All groups included low performers ($n=55$) having problems with early mathematics skills (or a Z-score on an early mathematics test of $<.0.5$) and at least average performing peers ($n=112$). Multiple treatments were performed at each school.

The interventions took place in all groups in nine Computer Assisted Interventions (CAI) in the classroom, for 25 min each time. Visual feedback was given by a happy or a sad smiley face. Auditory feedback was given by a sob when they made a mistake, or by applause when they succeeded. Children in the *counting CAI* (see Fig. 1) learned to count without mistakes.

They were asked :"How many penguins are there?" and had to count and register by tapping the number on the keyboard (see Fig. 1). Children in the *comparison CAI* (see Fig. 2) learned to compare organized and non-organized objects, by pointing the mouse to the group that had the maximum number of elements, making abstraction of the size of animals and dots.

Fig. 1 Screenshot of a counting condition

Fig. 2 Screenshot comparison of quantity groups

For more information about the counting and comparison CAI, we refer to the work of Desoete and colleagues (2016), in which similar training was given. Children in the *combined counting and comparison CAI* learned to count and to compare. Half of the exercises were counting exercises (see Fig. 1). The other half were comparison exercises (see Fig. 2). Children in the '*metacognitive CAI*' had to remember sequences of Arabic numbers and quantities (see Fig. 3). They had to learn from their mistakes to improve their performance.

Thus, the CAI focused on memory monitoring and self-regulation. They learned 'how' to remember information, in line with what Brown and colleagues (1983) defined as task specific procedural metamemory. After 10 min of meta-memory training, children completed exercises similar to the counting or comparison CAI for 15 min.

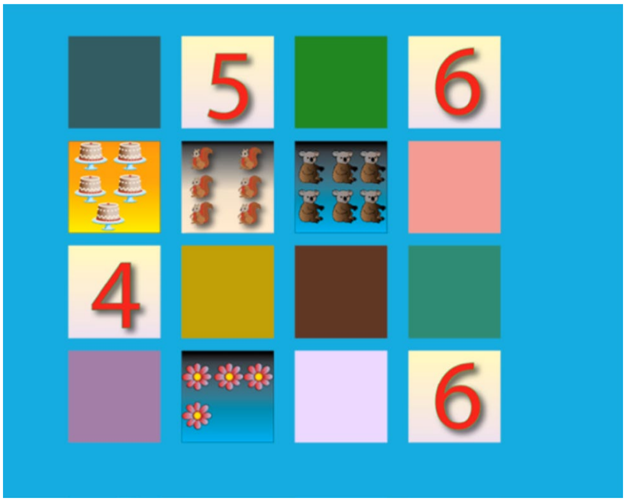

Fig. 3 Screenshot meta-memory groups

Control subjects (reference group) received as active control CAI, nine gaming sessions in regular kindergarten activities (intervention as usual). They had the opportunity to play reading games on the computer.

All groups had comparable intelligence ($p = .581$), measured with the WIPPSI-NL (Wechsler et al. 2002). They also had comparable pretest performance ($p = .779$) on exercises such as "Here you see two red balloons and three blue balloons. How many balloons are there together?") of the TEDI-MATH (Grégoire et al. 2004). Cronbach's alpha was 0.84.

To evaluate the value of metacognitive training in kindergarten, an ANCOVA was conducted with intelligence as covariate and the calculation results (on the TEDI-MATH as posttest in kindergarten) as dependent variable. The ANCOVA revealed significant differences between the groups in the posttest ($F(4,158) = 20.89$; $p < .001$, $\eta^2 = 0.35$) and a significant effect for intelligence ($p < .001$, $\eta^2 = 0.15$). The metacognitive CAI and the Counting and comparison CAI outperformed the comparison CAI and the active control condition where children made exercises on reading. The comparison CAI was less effective than the metacognitive CAI.

To study the differential effect on low or average performers, a 2×5 ANCOVA was conducted with child characteristics (low performer, average performer) and type of intervention (counting CAI, comparison CAI, counting and CAI, metacognition, counting and comparison CAI, control CAI) as independent variables, intelligence as covariate and posttest results in kindergarten as outcome variable. The ANCOVA revealed a significant effect for the covariate ($p = .010$; $\eta^2 = 0.04$), the type of intervention ($p < .001$; $\eta^2 = 0.37$) and the ability level ($p < .001$; $\eta^2 = 0.09$) but no significant interaction effect ($p = .699$; $\eta^2 = 0.01$), meaning that all children benefitted from the training. All intervention groups did better than the active control group in the posttest. However, the metacognition CAI, the combined counting and comparison CAI and the isolated counting CAI were more effective than the isolated comparison CAI in improving early mathematics skills in kindergarten for children at risk. The gain scores of the low performing young children were higher than those of their average performing peers.

To conclude, this study revealed that metacognitive training was efficient to enhance early mathematic skills in kindergarten. This is in line with a previous study involving older children (Desoete et al. 2003), and with Cornoldi and colleagues (2015) and Veenman (2013), who pointed out the fact that metacognition can be trained with a follow-up effect on mathematics problem-solving knowledge. In addition, especially poor performers benefited from the intervention. This is also in line with Pennequin et al. (2010) and Cornoldi et al. (2015) claiming that children with low mathematics propensities gained from the training, and especially benefitted from the metacognition training, making them reflect on how they memorized objects. This study, however, demonstrated that children with average or good mathematics performance as well as peers with especially low mathematics performance benefitted from the metacognitive opportunities provided by the training in kindergarten.

There are certainly some limitations on this study. The first limitation is the ceiling effect as an alternative for the lower gains in the average achieving groups. Thus, it seems 'natural' and 'logical' that the gains that low performers can demonstrate are larger than those of high performing children. Future research with tasks without ceiling effects are needed to study the impact of metacognitive training on children with different abilities. Moreover, there was the lack of a follow-up measure in grade 1, and the inclusion of a group of children attending only metacognitive training would have made it possible to examine the separate effects of cognitive and metacognitive instruction. This was unfortunately impossible due to time constraints, but it is important to bear this aspect in mind when planning future research. In addition, we assessed only a small group of low performers. The need for research with larger groups of low performers is indicated. Finally, propensities might be a variable concept, varying both within the subject of mathematics, as well as across years. Motivation as well as context variables such as home and school environment should be included in order to obtain a complete overview of the development of these children.

Nevertheless, this study highlighted that early numeral skills are highly susceptible to preventive training and opportunities. In addition, mathematics learning can also be enhanced in children with limited propensities or in low performers. In conclusion, the present results make a contribution to the existing literature on the role of metacognition in kindergarten. We knew that metacognition can be trained in elementary and secondary school (e.g., Cornoldi et al. 2015; Desoete et al. 2003; Schneider and Artelt 2010), but these data reveal that metacognition is already relevant and teachable in kindergarten. In addition, the study illustrated that instructional design and training where metacognition and/or counting is stimulated were efficient as 'opportunity', independent of the ability or 'propensity' levels of children.

5 Theoretical and methodological implications

From a theoretical point of view, the Opportunity–Propensity (O-P) Framework might be useful to understand the relevance of metacognition for mathematics learning and teaching. This O-P framework suggests that children are more likely to realize their potential for learning mathematics if they are provided Opportunities (O) to learn and have the Propensities (P) to benefit from the opportunities provided to them (Wang et al. 2013). Previous studies revealed that metacognitive skills, as P-factors, are strong predictors of later achievement in a domain of mathematics (e.g., Schneider and Artelt 2010). In addition, empirical studies demonstrated that metacognitive instruction is efficient (Donker et al. 2014), and relevant since metacognition was found not to develop automatically in all children (e.g., De Jager et al. 2005), pointing to the role of mathematics teachers in the opportunities (O) for children to learn metacognitive skills (e.g., Donker et al. 2014), suggesting the need for teacher-initiated metacognitive trainings in an enriched environment.

There is research evidence that metacognitive training is worthwhile for children with low mathematics performance (e.g., Cornoldi et al. 2015) as well as for average and high performing children (e.g., Schneider and Artelt 2010), pointing to the fact that metacognitive training as principle or technique for instructional design can be an effective component of 'Universal Design for Learning' (UDL). The UDL framework aims at creating powerful learning environments and adopting teaching materials and practices that allow for participation by all children, regardless of individual learning differences (Hanna 2005; Hitchcock et al. 2002). As such, UDL principles lend themselves to implementing metacognitive training as inclusionary practice in general educational settings, because they consist of flexible approaches that can be customized and adjusted for individual needs (Hitchcock et al. 2002). Thus, enriched metacognitive instructional designs can be effectively integrated within a preventive UDL perspective. Future research using the O-P model and UDL principles might be relevant to understand the nature and modifiability of metacognition impacting the propensity of learning and the exposure to metacognitive opportunities during mathematics teaching.

Although metacognition appears to be a powerful predictor of mathematics learning (Cornoldi et al. 2015; Verschaffel 1999), there remains a lack of consensus on several topics. The variety of measures being used to assess metacognition (questionnaires, observations, interviews, stimulated tutoring, judgments of performance and progress, recall readiness and allocation of children's study time, thinking aloud protocols, …) make study outcomes difficult to compare. Moreover the choice of assessment matters, since there is evidence that how you test is what you get (Desoete 2008; Desoete and Roeyers 2002, 2006). Therefore we suggest that researchers who are interested in metacognition use multiple-method designs, including think-aloud protocols in combination with teacher questionnaires and computerized tests (Veenman et al. 2006). Additional research comparing assessment techniques in all age groups seems indicated.

Not only 'how' you test, but also 'what' you test and how you label it, is what you get. There remain many issues in the conceptualization of metacognition and the relationship with self-regulation, temperament, and executive functions. Future research seems indicated to study whether the Opportunity–Propensity (O-P) Framework (Wang et al. 2013) can combine these measures and approaches as components predicting and enhancing mathematics learning.

6 Recommendations for mathematics educators

This literature study revealed empirical research on the relevance and efficacy of metacognitive training as a theory-based instructional design principle in mathematics teaching and learning. Several studies have educational implications resulting in recommendations for mathematics educators.

Firstly, there is evidence that metacognition is important in new and effortful tasks (Carr et al. 1994) and during the initial and final stage of mathematics learning (Verschaffel 1999), pointing to the recommendation that mathematics educators should asses the representation of the problem, analysis of the task demands and the reflection upon prior.

content knowledge and personal learning goals in mathematics. Mathematics educators should observe planning (Focant et al. 2006) and checking of calculation outcomes and highlight the importance of evaluation of the process and product of problem solving.

Secondly, there appears to be no 'golden standard' or agreement on the assessment of metacognition. All techniques have shortcomings. If teachers want to measure metacognition in a verbally skilled group of children, child questionnaires together with teacher ratings seem worthwhile. However awareness of the shortcomings of questionnaires, such as 'socially desirable response tendencies' have to be taken into account (Schneider and Artelt 2010). In addition teachers and mathematics educators should be aware of the possibilities of assessing metacognition while observing children, especially during reciprocal peer tutoring. Much information on metacognition can also be collected by being attentive to the judgment of performance and allocation of children's study time.

Thirdly, because metacognition, as propensity, does not develop automatically in all children (De Jager et al. 2005), teachers play an essential part in its development, by providing opportunities of exposure in a metacognitive enriched learning environment. School teachers and mathematics educators should explicitly instruct metacognitive knowledge and model and teach metacognitive skills to their children about mathematics learning. Moreover, mathematics teaching should focus on the development of 'mature task knowledge' (Nunes et al. 1993), within the awareness that 'a lack of metacognition' in children is not a static and fixed propensity, but an indication of the need of growth with informed, prolonged and embedded metacognitive training (Veenman 2013). Metacognitive knowledge and skills can be enhanced by modelling (Hartman and Sternberg 1993), scaffolding (Tanner and Jones 1995), reflective discourse (Kramarski 2009), adaptive feedback and prompts ((Zimmerman and Kitsantas 2005) as well as by explicit exercises on mental representation (Cornoldi et al. 2015; Desoete et al. 2003; Verschaffel 1999) and monitoring (Cornoldi et al. 2015) in elementary school children. In addition, teachers need to understand that explicit metacognitive training with direct instruction and cognitive apprenticeship (de Jager et al. 2005), time and modeling of reflective journal writing in mathematics (Clarke et al. 1993; Schneider and Artelt 2010), and training using a reflective discourse with comprehension questions, connecting questions, strategic questions and reflection questions (Mevarech and Fridkin 2006) are efficient as metacognitive training in secondary school children. It is clear that teachers need to be trained and supervised to implement these instructional models of metacognitive training in their classrooms successfully.

Fourthly, given that metacognitive knowledge was detected already in kindergarteners (Marulis et al. 2016) and the modifiability of metacognition in kindergarten (see illustrative study), it is worthwhile to observe the exposure of young children to metacognitive opportunities. Specific activities and training focusing on metacognition in kindergarten might perhaps help us to reduce the gap between children getting or not getting academic or metacognitive stimulation at home. As such, children with low mathematics performance can benefit from the adjusted enhancement and adequate preventive support of metacognitive knowledge and skills embedded in kindergarten education.

Finally, there were also substantial benefits and positive outcomes for children with regular mathematics performance (e.g., Desoete et al. 2003; Donker et al. 2014; Mevarech and Kramarski 2003; Schneider and Artelt 2010). Thus, we recommend that attention to metacognition be included in each pedagogical-didactical model of 'good' learning and teaching of mathematics and as part of a Universal Design for Learning. In a metacognitive and cognitively enriched design, all children are exposed to 'opportunities' to focus explicitly and reflect on what, how and when they learn. This can be done by modelling, verbalization, scaffolding, feedback on planning and allocation of study time, daily relooping prior knowledge and skills and by explicit focusing on awareness building within an 'infusion approach' (Moore 2005). Reciprocal peer tutoring activities and self evaluation tasks offer opportunities to think before, during and after mathematics learning. Children learn to explore and verbalize task demands, activate prior knowledge and analyze and structure task instructions. They learn to plan in advance and make a time-schedule, select strategies and notice comprehension or lack of comprehension, and learn to monitor progress and learning by reflecting on strategy-use, correctness, completeness and effectiveness of the solution as well as on personal efficiency, task-difficulty and self-efficacy.

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