

# Interest development during the first year at university: do mathematical beliefs predict interest in mathematics?

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**Abstract** We examined the development of interest in first-year university students in a lower secondary school teachers' program as well as connections between learners' belief systems and interest. Students' mathematics-related belief systems include their personal understanding of the nature of mathematics as a scientific domain (in the present study: aspects of process, application, schema, and formalism). Data (N=92) on beliefs and mathematical interest were collected at the beginning of the first (T1) and second terms (T3). In addition, students' interest was assessed at the ends of both terms (T2 and T4). Results showed that (1) students' interest in mathematics remained stable during the first academic year, (2) application beliefs showed positive correlations with interest in the first but not the second term, and (3) application beliefs at the beginning of the term predicted students' interest at the end of term in the second but not the first term; moreover, process, schema, and formalism beliefs did not predict interest in the first or in the second term. We discuss these results with respect to the influence of belief systems on interest but also with respect to possible effects that are based on differences between school mathematics and university mathematics.

**Keywords** Interest development · Mathematical beliefs · Teacher education · Secondary-tertiary transition

## 1 Introduction

In mathematics, the secondary-tertiary transition brings many changes (e.g., in thinking modes, mathematical communication, and the didactical contract; Guedet 2008). One aspect that changes is how the topic of mathematics is characterized, such that more emphasis is placed on formal aspects (Tall 2008). Definition, rigid logic, and proof are new to most of the students, whereas calculations are much less important than in school (Engelbrecht 2010). Students often struggle and are under-prepared for this new type of mathematics (Hourigan and O'Donoghue 2007), an issue that is viewed as especially problematic in teacher education (Goulding et al. 2003). In qualitative studies, students have reported a substantial decrease in their motivation as a result (Daskalogianni and Simpson 2002; Liebendörfer 2014; Liebendörfer and Hochmuth 2013). One part of students' under-preparedness may be a result of the inappropriate beliefs about the nature of mathematics that they carry over from school (Daskalogianni and Simpson 2001). For the present study, we chose students' interest as a motivational construct because interest is closely related to achievement (Schiefele et al. 1992) in the domain of mathematics education in particular (Köller et al. 2001; Lazarides and Ittel 2013; Schukajlow and Krug 2014a). In this paper we investigate the development of interest of lower secondary school preservice teachers during their first academic year at university and clarify the role that beliefs about the nature of mathematics play in determining their future interest in mathematics.

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## 1.1 Interest

Interest is defined as a motivational person-object relationship. It is specific to a person, but, unlike other motivational concepts (Eccles and Wigfield 2002), it is also specific to a (mental) object, which may be mathematics, for example. Interest has a cognitive component that refers to high personal value, and an emotional component that is related to positive affect (Krapp 2005, 2007; Hidi 2006). We focus on individual interest, which is defined as a rather stable disposition in contrast to situational interest.

Individual interest has gained importance due to empirical evidence that it is connected to students' use of metacognitive and deep learning strategies as well as effort, as has been shown across various disciplines and settings (Schiefele and Schreyer 1994). Furthermore, interest is related to good learning outcomes (Schiefele and Schreyer 1994; Schiefele et al. 1992), a finding that holds in particular in mathematics in school (Heinze et al. 2005; Köller et al. 2001; Singh et al. 2002) and in mathematics teacher education (Schwippert et al. 2013). Interest is also connected to students' enjoyment (Schukajlow 2015). Teachers' interest in the subject tends to increase in importance in teacher education as it is positively linked to students' motivation and learning (Long and Hoy 2006), and their interest is also related to other desirable outcomes such as job satisfaction and low levels of burnout (Kunter et al. 2011) as well as enjoyment and flow (Schiefele et al. 2013).

Interest development (Krapp 2005) is based on the satisfaction of three basic psychological needs: the needs for perceived competence, autonomy, and social relatedness postulated in the framework of self-determination theory (Deci and Ryan 1985, 2002). Empirical studies on interest in mathematics have confirmed the positive influence of factors such as students' experience of competence (Schukajlow and Krug 2014b) as well as competence support (Rakoczy et al. 2013) and autonomy support (Valås and Sjøvik 1994). Other studies have highlighted the importance of teaching methods (Bikner-Ahsbahs and Halverscheid 2014; Prendergast and O'Donoghue 2014; Schukajlow et al. 2011; Sonnert et al. 2015) for interest development, all of which are more or less explicitly related to basic need satisfaction.

Case studies have indicated that interest in mathematics may decrease after the transition to studying mathematics (Daskalogianni and Simpson 2002; Liebendörfer 2014), but some students may recover their interest toward the end of the first year. One reason for this decline is the formal nature of mathematics presented at university (Tall 2008). For example, students need to dismiss methods of argumentation such as giving examples in favor of offering rigorous proofs. Thus, although they may hold a clear image of the mathematical content in mind, they might not see how to

produce correct proofs, which may lead to a frustration of their basic needs for competence (Liebendörfer and Hochmuth 2016) and autonomy (Liebendörfer and Hochmuth 2015). In German mathematics teacher education, one longitudinal study provided evidence that higher secondary school preservice teachers who chose to study mathematics showed a decline in interest during the first term (Rach and Heinze 2013). Similarly, in a sample of future primary school teachers who had compulsory mathematics lectures, Kolter et al. (2016) found a decline in the first term but no significant change in the second term. To the best of our knowledge, no studies have tested this hypothesis in a sample of future lower secondary school teachers. On the basis of the literature review, we expected that future lower secondary school teachers' interest would also decline in the first term at university (Hypothesis 1a) and would not change significantly in the second term (Hypothesis 1b).

## 1.2 Beliefs

In mathematics education, the term "beliefs" became widespread after the influential work of McLeod (1992), who characterized beliefs as stable and low-intensity affective measures. In our study, we held beliefs to be "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp 2007, p. 259). In their nature, beliefs are more cognitive than emotions and, unlike knowledge, they may be held with varying degrees of conviction and are not consensual.

Beliefs do not stand in isolation but are typically organized in clusters and are more or less central, which means strongly held. Thus, it is helpful to consider the manner in which beliefs are organized into a belief system (Törner and Pehkonen 1996; Philipp 2007). Belief systems were found to be important because they influence teachers' and students' actions as well as their perceptions (Philipp 2007). In the domain of mathematics, beliefs can be differentiated into beliefs about mathematics, the participants' self, mathematics teaching, or the social context (McLeod 1992). In our study, we focus on beliefs about the nature of mathematics, sometimes also called mathematical world views or conceptions of mathematics. In teacher education, they have been the subject of research with respect to their impact on teaching practices; for example, beliefs were found to shape teachers' intentions (Bräunling and Eichler 2015; Liljedahl 2008). In addition, beliefs about the nature of mathematics are connected to how university students learn mathematics (Crawford et al. 1994) and to their motivation-related measures (Daskalogianni and Simpson 2002) such as interest.

Grigutsch et al. (1998a) developed a classification of belief systems consisting of the four aspects of process, application, schema, and formalism. The process aspect

describes mathematics as a vivid field of ideas, trial, and discovery as well as the process of understanding. The application aspect highlights the practical use of mathematics in everyday tasks and problems, for jobs, and for society. The schema aspect focuses on procedures such as algorithms and tools such as formulas. The formalism aspect characterizes mathematics as a rigid system with precise language and thoughts and determined by logic. For example, in the context of the derivative of a function, process aspects can be found in the discovery of connections between a function  $f$  and its derivative  $f'$  by using examples. Application aspects can be found by using differential calculus to model a real-life problem to find an optimal solution. Schema aspects are given by rules such as  $(f + g)' = f' + g'$  and their application in standard exercises. Finally, formalism aspects include the definition of the derivative as a limit of difference quotients. This belief system has an additional structure: The first two aspects are rather dynamic, whereas the last two aspects are rather static in their nature. This theoretical connection is reflected in positive correlations between process and application beliefs as well as between schema and formalism beliefs in cohorts of students in different grades in school (Grigutsch 1996) and teacher education programs (Felbrich et al. 2008), school teachers (Grigutsch et al. 1998b), and university teachers (Grigutsch and Törner 1998).

### 1.3 Relations between beliefs and interest and effects of beliefs on interest

The belief system interacts with other affective factors (Martino and Zan 2011). As Goldin et al. (2009) pointed out: “mathematical beliefs [...] are *embedded* in complex structures that are important to understand students’ motivations and behavioral patterns” (p. 13). They state that we possibly “need to consider how changes in belief may contribute—positively or negatively—to the affective structures that govern student engagement with mathematical ideas” (p. 13). Similarly, Kloosterman (2002) postulated that motivation is the result of beliefs about different areas, including mathematics as a discipline. Pepin and Roesken-Winter (2015) concluded that one of the core issues of research on students’ beliefs is the question of whether “students’ beliefs influence their interest in and motivation to learn mathematics” (p. 9).

In more detail, Schoenfeld (1985) described that mathematical beliefs influence how and how long students work on a given problem. Similarly, Daskalogianni and Simpson (2001, 2002) described in their qualitative study on the secondary-tertiary transition how beliefs shape students’ expectations and perceptions and thus their study behavior, experience, and motivation. Students who hold schema beliefs, for example, might not be used to and might not

expect to have to explore mathematics, which is often necessary for developing a proof. Instead, they might expect a clear schema and might consequently try to find it, an approach that will often fail. Similarly, students who hold application beliefs might lack a readiness for abstraction. Consequently, “students’ developing loss of interest in mathematics [is] due to a combination of cognitive and affective factors with a focus on the persistence of their mathematical beliefs” (Daskalogianni and Simpson 2002, p. 104), by which they mean beliefs that do not correspond with the manner in which mathematics is learned at university. We would therefore expect the fit between students’ beliefs and the mathematics they expect and encounter to be crucial for their interest and its development.

Process aspects of mathematics including problem solving and proving regularly appear in both school and university mathematics; consequently, process beliefs should be positively linked to interest. In addition, students encounter new processes of problem solving and the development of proofs at university, so process beliefs should help them to manage such new demands. The literature provides some evidence for this hypothesis. In an analysis of cross-sectional data from middle and high school students, Kaya (2007) reported positive correlations between “sophisticated” beliefs (including process aspects) and the interest-related measures of intrinsic goal orientation. Furthermore, Kolter et al. (2016) found positive correlations between process beliefs and interest in preservice primary school teachers’ first year of study. We thus expected that process beliefs would be positively correlated with interest (Hypothesis 2a). In two cross-sectional studies, “constructivism” beliefs, which described mathematics as a game, had a positive effect on the interest of German upper secondary school students (Köller 2001), and “sophisticated” beliefs, which highlighted multiple ways of solving mathematical problems, positively affected the motivation of middle and high school students (Kaya 2007). Games as a human activity and multiple ways of solving problems are closely related to process aspects. We thus expected that process beliefs would positively predict interest (Hypothesis 3a), despite the fact that students’ interest at the end of the first or second term was not predicted by process beliefs assessed at the beginning of the respective term in Kolter et al. (2016) study.

For the role of application beliefs, we should consider not only the fit between beliefs and courses but also that mathematics is sometimes presented as a self-contained theory, which might appear meaningless to students and might thus disappoint them if they see no applications, in particular in university (Brown and Macrae 2005). Application beliefs might protect students from such disappointment and might thus establish a positive link to interest and its development. Empirical results support

this perspective. Application beliefs showed positive correlations with interest in school (Baumert et al. 2000); and in a university sample that covered different disciplines, Kaldo and Hannula (2012) found a strong correlation between application beliefs and personal value, which is connected to the cognitive component of interest. Application beliefs were also positively correlated with interest in Kolter et al. (2016) study. These results provide evidence in support of our expectation that application beliefs should be positively correlated with interest (Hypothesis 2b). But in predicting interest development, the results appear to be contradictory. Köller (2001) reported positive effects of application beliefs on interest; Kolter et al. (2016) reported no effect of application beliefs on interest; and after conducting their case studies, Daskalogianni and Simpson (2001) hypothesized that application beliefs would negatively affect students' motivation. However, such contradictory observations may have resulted from differences in how mathematics was characterized in the courses in the different studies. Köller worked with students in upper secondary school, where applications are addressed regularly. Kolter et al. investigated courses that focused primarily on mathematical theory but included connections to its teaching. Daskalogianni and Simpson's study focused on undergraduate mathematics courses, which did not include applications in the first term. In addition, Daskalogianni and Simpson did not analyze the aspect of the meaningfulness of mathematics in their case studies, which can result in a positive effect of application beliefs on motivation. Since the preservice teacher program we investigated also highlighted applications of mathematics, we expected that application beliefs would be a positive predictor of interest (Hypothesis 3b).

Schematic aspects such as routine skills form the central aspect of mathematical beliefs in upper secondary school, yet they show low negative correlations with interest in school (Baumert et al. 2000) and are ascribed a negative impact on interest. Since schematic aspects are less central in many university courses, students holding such beliefs should encounter more problems such as searching for a schema to solve a task that does not ask for routine skills. Thus, there should be a negative link between schematic beliefs and interest at university. Using a university sample, Liston and O'Donoghue (2009) showed that the interest-related variables of enjoyment and value were negatively related to fragmented conceptions of mathematics, which are closely connected to schema beliefs. We therefore expected that schematic beliefs would be negatively correlated with interest (Hypothesis 2c). Schema beliefs were furthermore ascribed a negative impact on interest in the studies by Köller (2001) and Daskalogianni and Simpson (2001) but not Kolter et al. (2016), who found no

significant effect. On the basis of theoretical and empirical results, we expected that schema beliefs would negatively predict interest (Hypothesis 3c).

Formalism is very prominent in university mathematics (Tall 2008), and consequently, formalism believers should be better prepared for the new type of mathematics, so formalism beliefs should be positively linked to interest and its development. We found only one study that investigated a correlation between formalism beliefs and interest, and this study reported a nonsignificant correlation (Kolter et al. 2016). In line with theoretical considerations, we hypothesized that we would find a positive correlation between formal beliefs and interest (Hypothesis 2d). In the same previous study, formalism beliefs served as a predictor of positive interest development (Kolter et al. 2016), confirming our theoretically derived expectation that formalism beliefs would positively predict interest (Hypothesis 3d).

In the current study, we sought to extend the findings on interest development (Hypotheses 1a–1b) and the relation between beliefs and interest in the sample of lower secondary school students (Hypotheses 2a–2d). Further, we aimed to verify Hypotheses 3a–3d with longitudinal data, because these hypotheses were drawn mainly from qualitative and cross-sectional studies.

#### 1.4 Research focus and hypotheses

On the basis of theoretical considerations and previous empirical findings, we tested the following hypotheses in this study:

##### Hypothesis 1

- Lower secondary school teachers' interest in mathematics will decrease during the first term.
- Lower secondary school teachers' interest will not change during the second term.

Hypothesis 2 Interest in mathematics and beliefs will have the following correlations:

- There will be a positive correlation between process beliefs and interest.
- There will be a positive correlation between application beliefs and interest.
- There will be a negative correlation between schema beliefs and interest.
- There will be a positive correlation between formalism beliefs and interest.

Hypothesis 3 Students' belief systems will predict their future interest. More specifically, in addition to students' present interests:

- a. Process beliefs will positively predict interest.
- b. Application beliefs will positively predict interest.
- c. Schema beliefs will negatively predict interest.
- d. Formalism beliefs will positively predict interest.

## 2 Method

### 2.1 Design and participants

To test the hypotheses, we conducted a longitudinal study on students in a program for a lower secondary school teachers' degree (Lehramt an Haupt-, Real- und Gesamtschulen) at a large German University. The students had opted for mathematics as one of their two subjects. Four consecutive surveys were administered during lectures (T1–T4), each one at the beginning or end of the courses on arithmetic (first term) and geometry (second term), which form a compulsory module and the only mathematics module scheduled for the first year. Interest was measured in each survey, whereas belief systems were measured at the beginning of each term (T1 and T3). Students were not rewarded for their participation, and they were granted anonymity.

The lectures were all given by the same lecturer, addressed students' content knowledge and pedagogical content knowledge, and demonstrated a connection between university and school mathematics. Topics covered the positional notation of natural numbers, figurate numbers, divisibility, geometrical objects, theorems such as Pythagoras' theorem, symmetry, congruence, similarity, and preserving transformations. The lectures addressed process aspects by inviting students to discover mathematical relations and highlighting alternative approaches to solving problems. Application aspects were generally addressed by parts of pedagogical content knowledge such as educational goals. More specifically, in the second term, topics from geometry were motivated by everyday phenomena such as symmetries and tessellations. Formalism was emphasized more than in school but less than in other mathematics courses as objects were not given by an axiomatic theory; however, axiomatic systems were also mentioned in the lectures. The modes of argumentation included both formal and pre-formal proofs. Since the few routine tasks for students were not ultimate goals but taken as the basis for further reflections, toolbox aspects hardly appeared in the lectures.

Our analysis included 92 students who participated in at least one survey. However, the participation rates in the four surveys were lower: 77 (T1), 51 (T2), 57 (T3), and 54 (T4) students answered the questionnaires; thus, missing data for all variables ranged from 16 to 45% across the time points. Since participation in the lectures was not compulsory,

missing data may have resulted for various reasons such as illness but also a lack of motivation to attend the lectures. In order to avoid losing information in our data set, we used multiple imputation, which has proven to be effective also with samples as small as  $N=50$  and missing rates as high as 50% (Schafer and Graham 2002). Since recommendations for the number of data sets range from 3 to 100 (Manly and Wells 2015), we created 100 imputed data sets by applying the MCMC method (Graham 2012) as implemented in SPSS 23 with 10 iterations, including the variables of interest and beliefs of all four time points as well as age, term of study, and sex at T1. We analyzed the pooled data set according to Rubin's (1987) rules and Harel's (2009) rules for adjusted  $R^2$ . We checked whether the subsample with complete data differed from the sample with missing data using pairwise comparisons of the two groups on the 15 variables of age, sex, and study term at T1, interest at T1–T4, and the four belief aspects at T1 and T3. For the single tests, the significance level was chosen as  $\alpha=0.01$  in order to avoid alpha errors. The analysis showed that the two groups did not differ significantly on mean value or standard deviation of any variable.

Biographical data were collected from 77 students at T1: Their ages ranged from 18 to 30 years (mean = 20.79,  $SD=2.29$ ); the majority of 73 students (95%) were in their first term; and 47 students (61%) were female.

### 2.2 Instruments

To assess both interest and beliefs, we used paper-based questionnaires with Likert scales for beliefs and interest, which took the students approximately 5 min to answer. The response format ranged from one (absolutely not true) to six (absolutely true). To measure interest, we used Wendland and Rheinberg (2004) well-evaluated scale, which has also been applied in a number of other studies (e.g., Fechner 2009). We modified this scale for the learning of mathematics at university by replacing "school" with "university" and selected items that were appropriate for university students. The scale consisted of six items. A sample item is as follows: "I am not interested in mathematics" (reverse-scored). The reliability coefficients (Cronbach's alpha) had the acceptable values of 0.69, 0.82, 0.75, and 0.82 on the four surveys, respectively.

We measured students' belief systems with shortened versions of the scales developed by Grigutsch et al. (1998a; see Grigutsch and Törner 1998 for an English translation of the items), consisting of the four factors of *process*, *application*, *schema*, and *formalism*. These scales have formed the basis for many studies (e.g., Baumert et al. 2000; Felbrich et al. 2008), usually with the reduced scales that consisted of five to seven items, which showed sufficient reliability and validity (e.g., Kolter et al. 2016; Mischo and

Maaß 2013). We used six items from the process, application, and schema scales and seven items from the formalism scale. Sample items are as follows:

Process aspect: “Mathematics requires new and sudden ideas.”

Application aspect: “Mathematics helps to solve daily tasks and problems.”

Schema aspect: “Mathematics is the memorizing and application of definitions and formulas, mathematical facts and procedures.”

Formalism aspect: “Very essential aspects of mathematics are its logical strictness and precision, i.e., ‘objective thinking.’”

We checked the factor structure of the belief systems by applying an exploratory factor analysis with the number of factors given as four. The factor structure was generally reproduced on both surveys; however, four items showed cross-loadings as high as the main loading or even higher, two items from the formalism scale and one item from each of the process and schema scales. Reviewing these items showed that three items related to aesthetics of formalism, everyone’s ability to reinvent mathematics or the amount of practicing needed to work with mathematical rules. These items address the students’ beliefs on humans doing mathematics rather than the mathematics itself. The fourth item addresses the need for a precise mathematical language as part of its formalism. In a course with integrated didactical elements, both aspects of the learners and the language to convey mathematics might be seen as different from mathematics itself. As we focus on mathematical worldviews, we deleted these four items. The remaining items were distributed across the four factors as expected, with primary loadings above 0.4 and cross-loadings below 0.4 and at least 0.1 smaller than the primary loading on both surveys. The Cronbach’s alpha values of the resulting belief scales ranged from acceptable to good, see Table 1.

For the statistical analyses, we used one-tailed tests to verify Hypothesis 3, which specified directional effects, and two-tailed tests elsewhere.

### 3 Results

#### 3.1 Preliminary analysis

Table 2 displays the correlations for interest and beliefs across the different surveys as well as their means and standard deviations.

The interest values showed high correlations across the surveys. The belief aspects of application and schema were rather stable across time, whereas the aspects of process and formalism were less stable. The correlational patterns for the belief systems at the beginning of the two terms only

**Table 1** Reliabilities (Cronbach’s alphas) for the belief scales

	Number of items	T1	T3
Process beliefs	5	0.69	0.68
Application beliefs	6	0.76	0.83
Schema beliefs	5	0.77	0.80
Formalism beliefs	5	0.65	0.68

partly reproduced the findings from the literature because in both surveys, the two static aspects (i.e., schema and formalism) were positively correlated, but the two dynamic aspects (i.e., process and application) were not.

#### 3.2 Interest development

Hypotheses 1a and 1b predicted that the interest values would decrease during the first term and would not change during the second term. However, the mean values for interest did not change significantly during the first term (between the T1 and T2 surveys), and they also did not change during the second term (T3 and T4). The *t* tests clearly did not support Hypothesis 1a, but they supported Hypothesis 1b, see Table 3.

#### 3.3 Relations between interest and beliefs

Hypothesis 2a predicted that the process beliefs would be positively correlated with interest. This would mean that students who see mathematics as a vivid field, including exploration and creative activities, would report higher interest in mathematics. This hypothesis was confirmed only in the first term (T1) but not in the second term (T3). Hypothesis 2b predicted that application beliefs would be positively correlated with interest. Students who see mathematics as being of high practical use were expected to report higher interest. This hypothesis was also confirmed in the first term but not in the second term. Hypothesis 2c predicted that schema beliefs would be negatively correlated with interest, which means that students who see mathematics as a bundle of rules and formulae would report lower interest. This hypothesis was not confirmed because, in both terms, the correlation was not significantly negative. Hypothesis 2d predicted that formalism beliefs and interest would be positively correlated, such that students’ views of mathematics as a logical system would be connected to their interest in mathematics. This hypothesis was rejected in both the first and second term.

#### 3.4 Beliefs as predictors of interest

To predict interest with the beliefs (Hypothesis 3), we computed a linear regression with interest and the beliefs at the

**Table 2** Correlations, means, and standard deviations of all variables in the study

Variable	1	2	3	4	5	6	7	8	9	10	11	M	SD
1. Interest T1	1											3.62	0.72
2. Interest T2	0.658***	1										3.68	0.80
3. Interest T3	0.620***	0.687***	1									3.63	0.86
4. Interest T4	0.498***	0.767***	0.668***	1								3.76	0.87
5. Process beliefs T1	0.335**	0.167	0.112	0.178	1							4.36	0.77
6. Application beliefs T1	0.279*	0.222	0.247	0.384**	0.146	1						4.12	0.83
7. Schema beliefs T1	0.054	-0.052	0.040	-0.022	-0.186	0.067	1					4.21	0.84
8. Formalism beliefs T1	0.172	0.145	0.164	0.031	-0.238	0.183	0.379**	1				4.39	0.65
9. Process beliefs T3	0.031	0.176	0.006	0.181	0.242	0.267	-0.111	0.083	1			4.40	0.81
10. Application beliefs T3	0.046	0.329*	0.191	0.403**	0.184	0.454**	-0.156	-0.069	0.272	1		4.31	0.95
11. Schema beliefs T3	-0.115	-0.109	-0.149	-0.240	-0.184	-0.063	0.590***	0.305*	-0.287	-0.221	1	4.14	0.90
12. Formalism beliefs T3	-0.065	0.048	-0.040	-0.003	-0.322*	0.027	0.268	0.269	-0.110	-0.048	0.431*	4.37	0.77

\*p < .05, \*\*p < .01, \*\*\*p < .001

beginning of a term as independent variables and interest at the end of a term as the dependent variable. Table 4 shows the regression coefficients and levels of significance in both the first and second terms.

The analysis revealed that in the first term, none of the belief-system aspects was a significant predictor of interest. The only significant predictor was prior interest. Thus, Hypothesis 3 was not supported. In addition, the value of the adjusted R<sup>2</sup> was 0.45 and thus did not substantially exceed the squared correlation of 0.43 between interest at T1 and interest at T2, indicating that adding the belief-system variables as predictors in the regression model offered no substantial gain in explaining variance in interest. In the second term, there was a clear positive contribution of application beliefs but not process, schema, or formalism beliefs. The adjusted R<sup>2</sup> was 0.56 and thus exceeded the squared correlation between interest at T3 and interest at T4 of 0.45, indicating that including the belief-system variables as independent variables increased the predictive power of the model. Thus, Hypothesis 3b was confirmed, whereas Hypotheses 3a, c, d could not be confirmed. We want to note that excluding application beliefs from the regression model did not result in any other belief becoming a significant predictor of interest in the second term.

### 4 Discussion

In our study, we measured beliefs and interest in a cohort of preservice secondary school teachers with a longitudinal design. In accordance with theoretical considerations (Krapp 2007), interest was stable in terms of rather high correlations over time. Since a decline in its mean values during the first term (Hypothesis 1a) was not observed, we conclude that there is no decline in the development of the interest of preservice lower secondary school teachers, unlike students in upper secondary school (Köller et al. 2001) or vocational education (Krapp and Lewalter 2001). In particular, our results differed from findings by Kolter et al. (2016) and Rach and Heinze (2013), who found a decrease in interest in the first term of German teacher education. In the second term, the data confirmed Hypothesis 1b, which predicted that the mean interest values would not change, so our main finding is that the development of interest for students in the lower secondary teachers' program differs from students in other study programs in the first term. What is the reason for our findings? In contrast to the students we observed, the preservice primary school teachers in Kolter et al. (2016) study were required to take courses in mathematics, which most likely was not conducive to maintaining their interest. A major difference between the lower secondary school teachers' courses we investigated and higher secondary school teachers'

**Table 3** *t* Tests comparing the mean differences in interest

	Pretest <i>M</i> ( <i>SD</i> )	Posttest <i>M</i> ( <i>SD</i> )	<i>t</i> ( <i>df</i> )	<i>p</i>	Cohen's <i>d</i>
T1 → T2	3.62 (0.72)	3.68 (0.80)	−0.601 (505)	.548	0.07
T3 → T4	3.63 (0.86)	3.76 (0.87)	−1.241 (386)	.215	0.15

courses—along with the corresponding motivational problems of the latter (Rach and Heinze 2013)—is that the higher secondary school teacher courses in mathematics are taught “at a much more deeper and more theoretical level” (Krauss et al. 2008, p. 878). The lower secondary school preservice teachers thus face less difficult mathematics courses and might not share the negative impacts on their competence (Liebendörfer and Hochmuth 2016) and autonomy (Liebendörfer and Hochmuth 2015) experienced by higher secondary school preservice teachers. Another reason may be the conception of the mathematics courses that we chose to use in the current study for our sample of future lower secondary school teachers. These courses addressed not only content knowledge but also pedagogical content knowledge and were strongly connected to school mathematics, which is the object of interest for students at the beginning of their university courses. The hypothesis that preservice teachers' interest can benefit from such combined mathematics courses should be examined in future studies.

In the first term, our hypotheses about the correlations between beliefs and interest were confirmed only for process and application beliefs. The expected negative correlation between interest and the schema belief aspect was not confirmed. This might be a special characteristic of the group of preservice teachers who chose to teach mathematics in lower secondary school because, in Germany, the lower secondary school curriculum tends to focus more strongly on calculations and schematic procedures than the primary or higher secondary school curricula do (Drüke-Noe 2014; Henn and Kaiser 2001). Similarly, the expected negative correlation between interest and formalism beliefs was also not found. Again, we stress that the group of students who opt for mathematics as a core subject in their studies might show different patterns than students in

engineering and science (Liston and O'Donoghue 2009) or students in secondary school (Baumert et al. 2000). In the second term, interest showed no correlations with any beliefs. This result indicates that patterns of correlation may change during students' studies, presumably depending on the students' courses. For example, students might hold strong process beliefs but might not have much interest in mathematics since they struggle with the processes of problem solving and writing proofs, processes that turn out to be new and difficult for many students (Furinghetti et al. 2012; Moore 1994; Selden 2012). Similarly, application beliefs might strengthen perceptions of utility value, which has been suggested to prevent motivational problems in formal mathematics (Brown and Macrae 2005) and has generally been found to be closely related to students' interest (Harackiewicz and Hulleman 2010). However, application beliefs may also hinder students' learning (Daskalogianni and Simpson 2001), so these opposing effects may interact. In particular, the role of formalism beliefs should be investigated more intensely. The formal perspective on mathematics is one of the key changes in the secondary-tertiary transition (Tall 2008), and formalism beliefs should thus have a positive relation to a successful transition and in particular to interest. Another open question is how far the belief patterns in different teacher education programs and other forms of mathematics studies at university compare because studies based on a mixture of study programs (Kaldo and Hannula 2012) or that focused on primary teacher education (Kolter et al. 2016) found positive correlations between interest and application beliefs.

The third hypothesis referred to beliefs as predictors of interest and was confirmed in part. In the first term, the analysis revealed no effect of any belief aspect on interest in contrast to the positive correlations of process, application, and formalism beliefs with interest in the first term. In the second term, only application beliefs but not process, schema, or formalism beliefs predicted interest beyond what was predicted by previous interest, and the increase in the adjusted  $R^2$  reflected a more suitable model. This finding underscores the idea that beliefs are an important aspect of interest development, at least in the later parts of students' university careers. The roles of the different beliefs might result from the different nature of the mathematics

**Table 4** Standardized regression coefficients for interests and beliefs in the first and second terms (one-sided)

	First term (T1 → T2)			Second term (T3 → T4)		
	$\beta$	<i>t</i>	<i>p</i>	$\beta$	<i>t</i>	<i>p</i>
Interest	0.666	5.018	<.001	0.617	4.700	<.001
Process beliefs	−0.075	−0.516	.697	0.086	0.680	.249
Application beliefs	0.048	0.403	.343	0.250	2.006	.023
Schema beliefs	−0.124	−0.967	.353	−0.109	−0.758	.275
Formalism beliefs	0.052	0.376	.167	0.088	0.598	.225



presented in the courses. The fit of beliefs with learning situations and demands (Cobb 1985) might form an important factor for adequate perceptions and learning behavior (Daskalogianni and Simpson 2001; Schoenfeld 1985) and the subsequent experiences of competence (Liebendörfer and Hochmuth 2016) and autonomy (Liebendörfer and Hochmuth 2015), both of which are major factors in interest development (Krapp 2005). For the modeling of interest development, it would thus be interesting to see whether the belief system has indirect effects on interest with factors such as the experience of competence or enjoyment as intervening variables. These variables have been found to be important for the development of interest in the domain of mathematics (Schukajlow and Krug 2014b; Schukajlow and Rakoczy 2016).

Methodologically, the lack of predictive power found for beliefs despite corresponding correlations in our longitudinal study, in particular in the first term, reminds us that the proposed effects that were based on cross-sectional data in other studies (Kaya 2007; Köller 2001) should be interpreted with caution.

#### 4.1 Strengths and limitations of the study

The present study was based on theoretical models for the development of interest in mathematics and the mathematics-related belief system. Further, well-established instruments were used to measure interest and components of the belief system, and the results reproduced the factor structure of the belief system found in previous studies. However, future studies could include more belief aspects such as Grigutsch and Törner (1998) Platonism aspect, which could also be important for interest development. Another goal might involve investigating how students' adaptation of beliefs interacts with their interest development. In contrast to many other studies on beliefs, the longitudinal data set we collected is another clear strength of the study. Although causal implications cannot be derived directly from such a correlational data set, such data can provide evidence for the potential effects of beliefs on interest. Experimental studies are needed in this research area. In order to increase the generalization of our findings over time, we assessed interest at four occasions and beliefs at two. However, one limitation involves the generalization of the results, as the data came from only one university and might thus reflect local specifics. In addition, our handling of missing data was based on the assumption that the data were missing at random, a condition that cannot be checked. In particular, students' motivation might be a reason for missing data. Further, the study should be replicated with a larger sample. From a pedagogical point of view, we would like to note that interest is not only about

quantity but also about quality (Frenzel et al. 2012), which cannot be measured easily with quantitative instruments.

## 5 Conclusion

For academic research, we confirmed the theoretically derived hypothesis that the mathematical belief system—more precisely application beliefs—can predict the development of interest in the first year at university. In addition, we found that in the first academic year of teacher education, interest does not necessarily decline. For the teaching of mathematics at university, we furthermore conclude that apart from improving support for students' autonomy and competence (Krapp 2005; Liebendörfer and Hochmuth 2015, 2016; Schukajlow and Krug 2014b; Schukajlow et al. 2015), not only are university students' application beliefs important as an outcome in teacher education (Maaß 2006), but such beliefs should also be addressed to support students' interest.

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