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SURVEY PAPER

Teachers' perception, interpretation, and decision-making: a systematic review of empirical mathematics education research

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Abstract Research in mathematics education has investigated teachers' professional knowledge in depth, comprising two different approaches: a cognitive and a situated perspective. Linking these two perspectives leads to addressing situation-specific skills such as perception, interpretation and decision-making, indicative of revealing a teacher's knowledge while in the act of teaching. The aim of this study is to systematically review empirical research into mathematics teachers' situation-specific skills. From the databases Eric, PsycINFO and MathEduc a total of 60 articles were included in the review, based on specific criteria. The studies were categorized with respect to theoretical frameworks used, designs and methods applied as well as the main findings of each study. Teachers' noticing or teachers' professional vision, and teachers' (situated) professional knowledge were found to be the most frequent frameworks. Designs ranged from comprehensive case studies with a variety of methods to confirmatory studies testing a large sample with standardized instruments. The main findings suggest: (1) Teachers' expertise and experience positively influence noticing and teachers' noticing can be successfully fostered by (video-based) professional development programs. (2) Pre-service teachers struggle with perceiving and interpreting students' work. Thereby, their mathematical knowledge plays an important role. (3) Teachers' in-the-moment decision-making is influenced by their knowledge, beliefs and goals. (4) Teachers' knowledge and belief facets predict their situation specific-skills which in turn correlate with aspects close to instructional practice. (5) Teachers have difficulties interpreting tasks and identifying their educational potential. Methods and implication of this systematic review are thoroughly discussed.

Keywords Teacher professional knowledge · Teacher cognition · Situation-specific skills · Perception · Interpretation · Decision-making

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1 Introduction

Teachers' subject-specific professional knowledge is a strong predictor of students' achievement (Hattie 2009; Helmke 2009; Kunter et al. 2013; Sowder 2007). In the last decade, many studies investigated teachers' professional knowledge, affective-motivational beliefs, instructional practice, and in-the-moment performances in the classroom (Baumert et al. 2010; Blömeke et al. 2011; Kaiser et al. 2014; Kunter et al. 2013; Schoenfeld 1998).

Based on Shulman (1986), different frameworks of mathematics teachers' professional knowledge emerged contributing analytically distinguishable knowledge facets (Ball 2000; Ball and Bass 2000; Baumert and Kunter 2006; Kaiser et al. 2014). These approaches have pursued a *cognitive* perspective and emphasized the significance of teachers' profound subject-specific knowledge base for the quality of instruction.

Other research contributions in mathematics education have rather drawn on a *situated* perspective on teachers' professional knowledge. These studies adapt frameworks and methods from expertise research (Berliner 1992; Borko et al. 1992; Carter et al. 1988). Comparing novice and expert teachers' perception and interpretations of teaching situations is characteristic of such research alignments. In contrast to purely cognitive approaches, these studies use proximal measures of teachers' abilities such as classroom videos, video vignettes or exemplary student work (e.g. Jacobs et al. 2010; Kersting 2008; van Es and Sherin 2002).

Linking these two perspectives on teachers' professional knowledge, can contribute to a more comprehensive understanding (Depaepe et al. 2013; Kaiser et al. 2014; Santagata and Yeh 2016). In this regard, Blömeke et al. (2015a) stated that "processes such as the perception and interpretation of a specific job situation together with decision-making may mediate between disposition and performance" (p. 7). The aim of this article is to present a systematic review of mathematics teachers' situation-specific skills: perception, interpretation, and decision-making. The review reports the different conceptualizations and methodological approaches used in mathematics education empirical research and lists the main findings. The guiding questions are: To what theoretical frameworks does empirical research on mathematics teachers' situation-specific skills refer? What designs and methods are used to assess perception, interpretation and decision-making of prospective and practicing mathematics teachers? What results do the studies on situation-specific skills offer, and can these findings contribute to a comprehensive understanding of the link between teachers' dispositions and their performances?

2 Teachers' situation-specific skills in mathematics education research

In the last 30 years, many efforts have been made to explore the connection between mathematics teachers' professional knowledge and their instructional practices, with respect to their students' achievement. Rowland and Ruthven (2011) raised the question "whether mathematical knowledge in teaching is located 'in the head' of the individual teacher, or is somehow a social asset, meaningful only in the context of its application" (p. 3). Current discussions in the field label these two perspectives on mathematics teachers' professional knowledge as *cognitive* and *situated* (Depaepe et al. 2013). The aim of this section is first to outline essential contributions to both perspectives. Second, the role of situation-specific skills as mediating *what teachers know* and *how they act* is explored.

2.1 Perspectives on mathematics teachers' professional knowledge

Large-scale assessments like cognitive activation in the classroom: the orchestration of learning opportunities for the enhancement of insightful learning in mathematics (COACTIV; Kunter et al. 2013; Bruckmaier et al. this issue), teacher education and development study in mathematics (TEDS-M; Blömeke et al. 2010) and the follow-up study TEDS-FU (Kaiser et al. 2014; Hoth et al. this issue) have contributed substantially to conceptualizing and measuring mathematics teachers' professional knowledge. Based on Shulman's (1986) seminal work, these studies analytically distinguished mathematics teachers' knowledge and belief facets and explored diverse relations. The COACTIV study revealed positive effects of mathematics teachers' pedagogical content knowledge (PCK), enthusiasm for teaching, and self-regulatory skills on their instructional quality and students' outcomes (Kunter et al. 2013). Based on the notion of competence (Weinert 2001), the TEDS-M study investigated the professional knowledge as well as affective-motivational characteristics of (prospective) mathematics teachers. Summarizing the international state-of-the-art, Blömeke and Delaney (2014) emphasized that in advance of TEDS-M there has been limited *systematic* research on teachers' professional knowledge.

Other research traditions have placed emphasis on revealing conditions of effective teaching practice close to real classroom situations. While also considering teaching as a "knowledge-intensive domain with different knowledge and affective-motivational resources" (Kaiser et al. 2015 p. 370), these research approaches focus on aspects of teachers' professional knowledge in use. Schoenfeld (1998) has contributed a theory of Teaching-in-Context and modeled teaching as a function of a teacher's knowledge, goals, and beliefs. Later, he extended his approach to a theory of goal-oriented decision-making and replaced the concepts of knowledge and beliefs by resources and orientations. Particularly, he pointed out that "the notion orientation/resource/goal clusters is a lens through which teacher activity can be examined—and studies of coherence and change along these dimensions could be very interesting and useful" (Schoenfeld 2010 p. 194). In their "provisional framework for proficiency in teaching mathematics", Schoenfeld and Kilpatrick (2008) highlighted mathematics as a knowledge-intensive domain. For effective teaching, they considered as equally important knowledge about students' learning, managing adequate learning environments as well as substantially supporting classroom discourses. Ball et al. (2008) also demanded a practice-based theory of mathematical knowledge for teaching "to unearth the ways in which mathematics is involved in contending with the regular day-to-day, moment-to-moment demands of teaching" (p. 395).

Another line of research, drawing on expertise research (Berliner 2001), elaborates on mathematics teachers' professional vision to describe and analyze their teaching practice (Jacobs et al. 2007, 2010; Sherin and van Es 2005; van Es and Sherin 2008). Although the definitions and conceptualizations used partly differ, teachers' abilities to analyze teaching are in the focus.

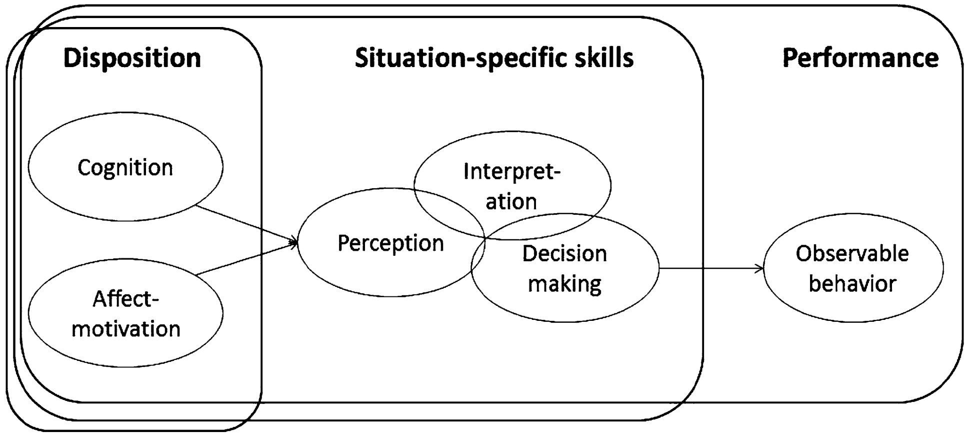

Fig. 1 Competence modeled as a continuum (Blömeke et al. 2015a, p. 5)

As presented above, mathematics teachers' professional knowledge has been investigated differently. In their systematic review on pedagogical content knowledge, Depaepe et al. (2013) provided evidence for distinguishing a cognitive and a situated perspective:

> Adherents of a cognitive perspective, in which PCK is conceived as a category of teacher's knowledge base, typically define–in line with Shulman–a limited number of components to be part of PCK and distinguish PCK from other categories of teachers' knowledge base, such as content knowledge and general pedagogical knowledge. By contrast, proponents of a situated perspective on PCK as knowing-to-act within a particular classroom context, typically acknowledge that the act of teaching is multi-dimensional in nature and that teachers' choices simultaneously reflect mathematical and pedagogical deliberations. (p. 22).

Based on their findings, Depaepe et al. (2013) demand a more integrated view on conceptualizing and assessing teachers' professional knowledge. Rowland and Ruthven (2011) already criticized that many research studies treat "mathematical knowledge for teaching as residing solely in the classroom teacher" (p. 2). Thus, the next section elaborates on the processes that link mathematics teachers' knowing and acting.

2.2 Relevance of situation-specific skills

Depaepe et al. (2013) revealed several shortcomings for the two perspectives discussed above: Within the cognitive perspective, research on teachers' professional knowledge is disconnected from real classroom situations. Neither the socio-historical context nor the way different accounts of teacher knowledge interact were considered. Within the situated perspective, the sample sizes are often small and the findings have only limited validity. Also, teachers' choices during teaching and their justifications are not accessible by classroom observations only.

Blömeke et al. (2015a) emphasized the connection between teachers' cognition and affective-motivational beliefs (dispositions) and their teaching behavior (performance). For integrating a cognitive and a situated perspective, Blömeke et al. (2015a) suggested considering competence as a continuum (cf. Fig. 1).

The frame work considers competence as a multidimensional construct, and resolves the dichotomy of "disposition versus performance" as follows: "[…] our notion of competence includes 'criterion behavior' as well as the knowledge, cognitive skills and affective-motivational dispositions that underlie that behavior" (Blömeke et al. 2015a, p. 3). Following this understanding, a key role is assigned to *situation-specific skills*. That is, perception, interpretation and decision-making are linking teachers' professional knowledge to observable behavior.

So far, only a few studies have combined the two perspectives on teachers' professional knowledge. One prominent example is TEDS-FU; the study enriches the rather cognitive alignment of TEDS-M by assessing teachers' performances proximal to their classroom behavior (Kaiser et al. 2014). In TEDS-FU "professional experience, deliberate practice and the ability of perceiving essential details in class are included as well as aspects of performance like dealing with heterogeneity in a flexible manner" (Kaiser et al. 2015, p. 373). Drawing on the framework proposed by Blömeke et al. (2015a), Kaiser et al. (2015) elaborated on situation-specific skills relevant for teaching mathematics in their so-called *PID-model*: (P) Perceiving particular events in an instructional setting, (I) Interpreting the perceived activities in the classroom and (D) Decision-making, either as anticipating a response to students' activities or as proposing alternative instructional strategies (p. 374). The PID-model can be applied to reveal specific aspects as, for instance, teachers' diagnostic competence (Hoth et al. this issue).

Lindmeier et al. (2013) also integrated a cognitive and a situated perspective on teachers' professional knowledge.

Their aim was “to capture facets of teacher cognition that go “beyond” knowledge in the sense that the scales depend on professional knowledge but mirror further abilities to use knowledge in typical teaching tasks” (p. 439). Particularly, teachers’ abilities to address students’ cognition, to cope with student’s individual strategies and misconceptions, and to handle representations and explanations during instruction were analyzed. Although Lindmeier et al. (2013) did not refer explicitly to situation-specific skills, the aforementioned facets imply such aspects.

A situated perspective on teachers’ knowledge emphasizes teachers’ professional experiences, deliberate practice and ability to perceive and to attend to essential classroom situations (Putnam and Borko 2000). Focusing on teachers’ situation-specific skills draws attention to seminal research on teacher expertise (Berliner 1992; Chi 2011; Li and Kaiser 2011). Research on situated skills such as *perception accuracy* (Carter et al. 1988) reveals how expert and novice teachers differ fundamentally in what and how they perceive classroom incidents. The concept of *noticing* addresses diverse facets of teacher expertise relevant for acting in the classroom (König et al. 2014; van Es and Sherin 2006). Teacher noticing builds on the notion of *professional vision* defined by Goodwin (1994) as “socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group” (p. 606). Sherin et al. (2011b) have “focused on noticing as professional vision in which teachers selectively attend to events that take place and then draw on their existing knowledge to interpret these noticed events” (p. 80). The components of “attending to particular events in an instructional setting” and “making sense of an event in an instructional setting” are commonly shared among researchers dealing with noticing (Sherin et al. 2011a). However, Sherin et al. (2011a) emphasized that research purposes vary as studies address either the diversity of what teachers notice or teachers’ subject-specific expertise in depth. “Making sense” includes teachers’ interpretations of classroom events such as classroom discussions or students’ work. However, Sherin et al. (2011a) emphasized that “it is not helpful to think of teacher noticing as simply another category of teacher knowledge. […] The word *noticing* names a process rather than a static category of knowledge” (p. 5). In their framework, van Es and Sherin (2002) define the concept of noticing as follows:

> (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions. (p. 573).

Although the framework of van Es and Sherin (2002) is often referred to, integrating and applying the construct vary substantially. First, although many researchers conceptualize noticing as “attending to” and “making sense” of particular events in the classrooms, there is no consensus on what making sense means. Second, there is a debate on the scope of the notion. For instance, Star and Strickland (2008) considered as teachers’ noticing “what catches their attention and what they miss […] when they view a classroom lesson” (p. 111). Jacobs et al. (2010) took a broader view on professional noticing as not only including teachers’ attention to and interpretation of classroom situations, but also teachers’ intended responding. Thus, the use of the concept of teachers’ noticing ranges from including perception solely, connecting perception with interpretation to also comprising decision-making.

2.3 Purpose of this study

Teachers’ situation-specific skills are processes linking their professional knowledge and performance. Systematically reviewing research on mathematics teachers’ situation-specific skills is the aim of the study. The following research questions are pursued:

1. What situation-specific skills are investigated in empirical research in mathematics education?
2. To what theoretical frameworks does empirical research on mathematics teachers’ situation-specific skills refer?
3. What designs and methods are used to assess perception, interpretation and decision-making of mathematics teachers?
4. What results do the studies on situation-specific skills offer?

3 Method

A systematic review (Petticrew and Roberts 2008) of the research literature was conducted in the three databases ERIC, PsycINFO and MathEduc. Since searching for the comprehensive but rather specific term “situation-specific-skills” (typeset between quotation marks to ensure that the entire term is included in the searching process) had not led to any significant results, the term was decomposed into corresponding concepts. Thus, the *processes* perception, interpretation and decision-making were addressed by searching[1] for “perception*”, “attending”, “interpret*”, “decision*”,

[1] By using truncation characters at the end of terms (*) it is specified that the search algorithms of ERIC, PsycINFO, and MathEduc includes all possible word endings, particularly plural forms or gerund (e.g. teacher, teachers or teaching).

"notic*, "professional vision", "situated", and "video-based". In addition, overarching *concepts* were included by referring to the search strings "competenc*", "knowledge", "skill*", "education", and "cognition". Since the systematic review is restricted to (prospective) teachers in the *domain* of mathematics, the mandatory search terms "math*" and "teach*" were additionally considered. In addition to the mandatory terms math* and teach*, one term concerning *processes* and one term concerning *concepts* was obligatory.[2] The search was carried out across the titles, keywords, and abstracts included in the databases. The search was restricted to peer-reviewed journal articles (written in English) published between January 1st, 1995 and January 31st, 2016. Applying these initial search criteria ensures a broad spectrum of high quality international research.

In total, the search algorithm yielded 1418 results (1001 in ERIC, 437 in PsycINFO, 549 in MathEduc; among them 569 duplicates retrieved from two or all three databases). The contributions in this special issue—if matching the criteria–were also included. The articles matching the search terms were then checked for six exclusion criteria (EC):

(EC1) The article reports empirical data. Editorials, (narrative) literature reviews, discussion papers, theoretical articles or commentaries were excluded as they do not provide information on their database (in sum 304 articles).

(EC2) The focus of the study is on teacher variables. Thus, articles focusing on student achievement, student motivation or emotion, students' gender or ethnicity, parents' involvement, parents' views, or parents' socio-economic status were excluded (in sum 293 articles).

(EC3) The article's context is teaching mathematics in pre- to secondary school or in tertiary education. Thus, studies focusing on other subjects (e.g. science, engineering, arts or social studies), on special education or mathematic education for other professions (e.g. medicine) were excluded (in sum 173 articles).

(EC4) The article investigates teachers' cognition or practice embedded in mathematics. Therefore, articles on curriculum (reform), on policy as well as articles evaluating software, specific materials or specific lesson designs were excluded (in sum 332 articles).

(EC5) The study investigates aspects of teachers' cognition or practice that are specific for teaching mathematics. Hence, studies dealing with general pedagogical topics such as classroom management or technical skills (even if conducted in mathematics lessons) were excluded (in sum 60 articles).

(EC6) The article is concerned with investigating situation-specific skills. Thus, articles using "perception" in the sense of an attitude or opinion, contributions that dealt with decisions on a higher level (e.g. curriculum decisions) as well as studies assessing teachers' knowledge from a cognitive perspective only were excluded (in sum 205 articles).

Applying these criteria resulted in a final database of 60 research articles.[3] Nine of these studies are published in this special issue. Each study was read and analyzed by two authors of this systematic review. The articles were reviewed and *summarized* with respect to the theoretical framework, research questions; sample size and characteristics of the participants; research design and methods, and main results. With respect to investigating situation-specific skills and their relation to disposition and performance, a *coding scheme* was applied: Research on *perception*, *interpretation* and/or *decision-making* (a dichotomous coding for each aspect was applied with 0 = not investigated; 1 = investigated) and focus on *research on skills per se*, *in relation to dispositions* and/or *performance* (a dichotomous coding for each aspect was applied with 0 = not investigated; 1 = investigated). The percentages of agreement ranged between 75 % for decision-making and 86 % for interpretation with an agreement of 78 % for perception. For "focus of research" the coding showed substantial agreement: 78 % for situation-specific skills, 85 % for disposition and 82 % for performance. Disagreements were thoroughly discussed among the three authors until consensus was reached.

4 Results

In this section, the research questions of this systematic review are answered successively. First, *situation-specific skills* investigated in the studies are summarized (Sect. 4.1). Then, the *theoretical frameworks* referred to are analyzed (Sect. 4.2). Subsequently, the *designs and methods* used to assess perception, interpretation and decision-making are reported (Sect. 4.3). Finally, the main focus lies on analyzing the *results* the studies on situation-specific skills report (Sect. 4.4).

4.1 What situation-specific skills are investigated in empirical research in mathematics education?

Most of the studies investigated interpretation (78.3 %), the majority of studies perception (63.3 %) and about half of the articles researched decision-making (53.3 %). Research

[2] Combination of Search Fields in detail (for ERIC): TI, AB, IF (teach* AND (competenc* OR knowledge OR skill* OR education OR cognition) AND (perception* OR attending OR interpret* OR decision* OR noticing OR notice OR "professional vision" OR situated OR "video-based") AND math*).

[3] The articles are marked with an * in the reference list.

on teachers'[4] perception or interpretation varied from identifying the potential of mathematical tasks (Klymchuk and Thomas 2011) and elaborating on student errors (e.g. Pankow et al. this issue) to recognizing instructional features in a classroom video (e.g. Star and Strickland 2008). Decision-making was primarily accessed by asking teachers to respond to a classroom situation (e.g. Jacobs and Empson this issue) or by analyzing teachers' planning and enactment of instructional decisions (e.g. Escudero and Sánchez 2007). Studies often examined teachers' perception *and* interpretation (19 of 60 studies) or *all* three situation-specific skills (15 of 60 studies). Table 1 gives an overview of the teachers' situation-specific skills investigated and the material used.

4.2 To what theoretical frameworks does empirical research on mathematics teachers' situation-specific skills refer?

The studies referred to a variety of concepts or constructs in their theoretical frameworks. Teachers' noticing or teachers' professional vision, and teachers' (situated) professional knowledge were the most frequent frameworks. About half of the studies (31 studies) were related to teachers' professional knowledge, and used a relevant framework. Several studies focused on PCK with respect to a specific mathematical theme such as fractions or proportional reasoning (e.g. Houssart 2000; Jakobsen et al. 2014; Son 2013; Son and Sinclair 2010). Other studies investigated multiple facets of teachers' professional knowledge, including teachers' situation-specific skills (Blömeke et al. 2015b; Bruckmaier et al. this issue; Dunekacke et al. 2015; Dunekacke et al. this issue; Knievel et al. 2015).

Numerous studies (26 studies) referred to teachers' noticing or professional vision in their framework. Drawing on the noticing framework by van Es and Sherin (2002), most of these studies included perception and interpretation. Other studies considered noticing as merely being perception (Star and Strickland 2008) or as additionally including decision-making (Jacobs et al. 2010; Schack et al. 2013). Three studies took a theme-specific perspective and investigated teachers' noticing of multiple representations (Dreher and Kuntze 2015), mathematics problem solving (Fernández et al. 2013) or the derivative (Sánchez-Matamoros et al. 2014). Further concepts or constructs referred to were lesson analysis (Amador and Weiland 2015; Santagata 2009; Santagata et al. 2007) and teachers' resources, goals and orientations (Paterson et al. 2011; Thomas and Yoon 2014; Zimmerman 2015).

[4] The term "teachers" is used for pre- and in-service teachers in this section, if not further specified.

4.3 What designs and methods are used to assess perception, interpretation and decision-making of mathematics teachers?

This section reports on the samples included as well as the research designs and methods used to access teachers' situation-specific skills. Additionally, the studies are checked for assessing teachers' dispositions or performances in relation to situation-specific skills.

4.3.1 Sample size and characteristics of the participating teachers

Table 2 reports sample sizes (i.e. the number of participants included in the data analysis), characteristics of the participants (pre-service teachers, in-service teachers and teacher trainers/lecturers) as well as school level (defined by the grades that the participating teachers taught or for which they were certified).

The studies analyzed very different sample sizes with $N = 1$ being the minimum (case studies) and N = 676 being the maximum (Kersting et al. this issue). The mean sample size is 56.35 participants ($SD = 106.50$) and the median is 19.50 participants. About half of the studies focused on pre- *or* in-service teachers' situation-specific skills (28 and 26 studies). Only five studies included pre- *and* in-service teachers. Of these five studies two analyzed differences between pre- and in-service teachers' situation-specific skills (Dreher and Kuntze 2015; Jacobs et al. 2010). One study concentrated on the development of primary teachers and thus reported data ranging from the last year of teacher education to 4 years of teaching experience (Blömeke et al. 2015b). With regard to school level, about half the studies assessed elementary, middle or secondary school teachers, respectively. Only a few studies investigated pre-school teachers (Dunekacke et al. 2015, this issue) or higher education teachers or lecturers' situation-specific skills (e.g. Paterson et al. 2011; Thomas and Yoon 2014).

4.3.2 Research design and methods

The studies differ with respect to their research design and the methods used to investigate situation-specific skills. Studies that included only one or a few teachers and reported results case-wise were categorized as case studies.

Studies investigating the effects of some form of intervention (e.g. a professional development course) were categorized as intervention studies. Studies conducted to confirm hypotheses or presumptions were categorized as confirmatory studies. The research methods used to assess

Table 1 Investigated aspects of teachers' situation-specific skills

References	P	I	D	Material used to assess teachers' situation-specific skills
Alsawaie and Alghazo (2010)	X	X		Video clips of mathematics lessons
Amador and Weiland (2015)	X	X		Student thinking in a mathematics lesson
Blömeke et al. (2015b)	X	X		Video material of classroom situations
			X	Responding to classroom situations
Bruckmaier et al. (2016)			X	Responding to classroom situations on video
Colestock and Sherin (2009)	X	X		Video clips of teacher and students discussing mathematical ideas
Cooper (2009)	X	X		Errors and misconceptions in children's work
			X	Instructional strategies
Derry et al. (2007)		X		Students' solutions of multiple representations in algebraic tasks
Dreher and Kuntze (2015)		X		Written vignettes on multiple representations in of classroom situations
Dunekacke et al. (2015)	X			Video clips of mathematics-related situations
			X	Planning actions to foster mathematical learning
Dunekacke et al. (2016)	X			Video material of mathematics-related situations
			X	Planning actions to foster mathematical learning
Dyer and Sherin (2016)		X	X	Teaching a mathematics lesson
Escudero and Sánchez (2007)			X	Planning lessons and instructional adaptions in the classroom
Fernández et al. (2013)	X	X		Students' problem solving in written answers
Gal (2011)	X	X		Students' difficulties during instruction
			X	Coping with difficulties during instruction
Galant (2013)		X		Mathematical content of multiplication tasks
			X	Sequencing of tasks for teaching
Hines and McMahon (2005)		X		Students' proportional reasoning strategies in written answers
Ho and Tan (2013)		X		Classroom practices
Hoth et al. (2016)	X	X		Video clips of classroom situations and written student solutions
			X	Responding to classroom situations and students
Houssart (2000)	X	X		Mathematical tasks (partly) on pattern
Huang and Li (2012)	X	X		Video material of two mathematics lessons (prize-winning vs. traditional)
Ingram (2014)	X	X		Video material of teaching sequences with mathematical/pedagogical focus and four reactions to each sequence
Jacobs and Empson (2016)			X	Teaching mathematics lessons
Jacobs et al. (2010)	X	X		Student thinking in video and written work
			X	Problem to be posed next
Jakobsen et al. (2014)		X		Students' work on a task
Kersting (2008)	X	X		Video clips of classroom episodes of teacher helping behavior/student mistakes
			X	Alternative teaching strategies
Kersting et al. (2016)	X	X		Video clips of classroom episodes of teacher helping behavior/student mistakes
			X	Alternative teaching strategies
Klymchuk and Thomas (2011)	X			Mathematical tasks
Knievel et al. (2015)	X	X		Video material of classroom situations and written student solutions
			X	Responding to classroom situation or student solution
Lande and Mesa (2016)			X	Animations of community college classroom situations
Lee and Kim (2005)	X	X		Mathematical problems
Magiera et al. (2013)	X	X		Algebraic tasks and students thinking in written solutions
Nickerson and Masarik (2010)		X		Tasks and student responses
			X	Pedagogical moves
Norton et al. (2011)		X		Student thinking in video
Osmanoglu et al. (2015)	X	X		Quality of instruction, activities and student thinking in lesson video
Pankow et al. (2016)	X	X		Student error(s) in written solutions
Paterson et al. (2011)			X	Teaching a mathematics lecture

Table 1 continued

References	P	I	D	Material used to assess teachers' situation-specific skills
Roth McDuffie et al. (2014)	X	X		Students' mathematical knowledge bases shown in a lesson video
Sánchez-Matamoros et al. (2014)		X		Students' understanding revealed by written solutions
Santagata (2009)	X	X		Student thinking and understanding in lesson video
			X	Alternative teaching strategies
Santagata and Yeh (2016)	X	X		Video material of classroom episodes
			X	Responding to classroom episodes
Santagata et al. (2007)	X	X		Video clips of mathematics lessons
			X	Responding to classroom episodes
Schack et al. (2013)	X	X		Children's mathematical thinking in a video clip
			X	Problem to be posed next
Sherin and van Es (2005)	X	X		Video material of mathematics lessons
Sherin and van Es (2009)	X	X		Video material of mathematics lessons
Sherin et al. (2008)	X	X		Video material of mathematics lessons
			X	Selection of noteworthy clips
Sleep (2012)	X	X		Teacher's own lesson on video
			X	Planning and teaching of a mathematics lesson
Son (2013)		X		Students' error(s) in written teaching situations
			X	Responding to student errors
Son and Kim (2015)			X	Mathematical tasks from textbook and their enactment in teaching
Son and Sinclair (2010)		X		Students' error(s) in written teaching situations
			X	Responding to student errors
Star and Strickland (2008)	X			Instructional features of a classroom video
Stockero (2008)	X	X		Video clips of students solving tasks
Stockero and Van Zoest (2013)			X	Pivotal teaching moments
Thomas and Yoon (2014)			X	Teaching a mathematics lesson
van Es and Sherin (2002)	X	X		Video material of mathematics lessons
van Es and Sherin (2006)	X	X		Video material of mathematics lessons
van Es and Sherin (2008)	X	X		Video material of mathematics lessons
Wager (2014)	X	X		Children's participation in lesson on video
			X	Responding to children's participation
Weiland et al. (2014)	X	X		Students' thinking in formative assessment interviews
Zahner et al. (2012)		X		Students' conceptual understanding and error(s) in a lesson
			X	Responding to student contributions and error(s) in a lesson
Zimmerman (2015)			X	Teaching a mathematics lesson

P perception, *I* interpretation, *D* decision-making

situation-specific skill were tests,[5] questionnaires,[6] interviews, lesson observations, other observations (e.g. observation of discussions), and the analysis of documents (reflection papers, lesson plans or homework assignments). Table 3 gives an overview of the research designs and methods.

[5] Instruments were categorized as tests, when they were (partly) derived from already validated instruments or provided information on reliability and validity of the instrument applied. Furthermore assessments composed of mathematical tasks teachers had to solve were categorized as tests.

[6] Video-based assessments with open-end format as well as interviews that were conducted in written format were categorized as questionnaires.

4.3.3 What is the specific situation?

The studies drew on rather different *situations* ranging from interpreting mathematical tasks (e.g. Galant 2013) to deciding upon teaching moves during instruction (Jacobs and Empson this issue). Studies investigating teachers' situated PCK primarily used written documents of students' work (e.g. Hines and McMahon 2005; Son 2013). Some studies applied videos of students solving tasks (e.g. Knievel et al.

Table 2 Sample and school level of participants

References	N[a]	Pre-service teachers	In-service teachers	Teacher trainer/ lecturer	School level (certification)				
					Pre	Elementary[b]	Middle[c]	Secondary[d]	Higher education
Alsawaie and Alghazo (2010)	26 (13)	X					X	X	
Amador and Weiland (2015)	32	X	X	X		X			
Blömeke et al. (2015b)[e]	231	X	X			X			
Bruckmaier et al. (2016)	284		X				X	X	
Colestock and Sherin (2009)	15		X				X	X	
Cooper (2009)	86	X				X	X		
Derry et al. (2007)	20/10		X				X		
Dreher and Kuntze (2015)	144	X	X				X	X	
Dunekacke et al. (2015)	354[f]	X			X				
Dunekacke et al. (2016)	354[f]	X			X				
Dyer and Sherin (2016)	2		X					X	
Escudero and Sánchez (2007)	2		X					X	
Fernández et al. (2013)	36	X				X			
Gal (2011)	1	X					X		
Galant (2013)	46		X			X			
Hines and McMahon (2005)	11	X					X	X	
Ho and Tan (2013)	2		X	X		X			
Hoth et al. (2016)	133		X			X			
Houssart (2000)	26		X			X			
Huang and Li (2012)	20		X					X	
Ingram (2014)	19	X				X	X	X	
Jacobs and Empson (2016)	1		X			X	X		
Jacobs et al. (2010)	131	X	X			X			
Jakobsen et al. (2014)	49	X				X			
Kersting (2008)	62	X	X			X	X	X	
Kersting et al. (2016)	676		X			X	X		
Klymchuk and Thomas (2011)	203		X					X	X

Table 2 continued

References	*N*[a]	Pre-service teachers	In-service teachers	Teacher trainer/ lecturer	School level (certification)				
					Pre	Elementary[b]	Middle[c]	Secondary[d]	Higher education
Knievel et al. (2015)	85		X			X			
Lande and Mesa (2016)	20		X						X
Lee and Kim (2005)	22	X				X			
Magiera et al. (2013)	18	X				X	X		
Nickerson and Masarik (2010)	4		X				X		
Norton et al. (2011)	42 (19)	X				X			
Osmanoglu et al. (2015)	15	X				X			
Pankow et al. (2016)	137		X				X		
Paterson et al. (2011)	8		X						X
Roth McDuffie et al. (2014)	73	X				X	X		
Sánchez-Matamoros et al. (2014)	8	X						X	
Santagata (2009)	33		X				X		
Santagata and Yeh (2016)	3	X				X			
Santagata et al. (2007)	35/30	X					X	X	
Schack et al. (2013)	94	X				X			
Sherin and van Es (2005)	4/12[g] (6)	X	X				X	X	
Sherin and van Es (2009)	4/7[g]		X			X	X		
Sherin et al. (2008)	1		X					X	
Sleep (2012)	17	X				X			
Son (2013)	57	X				X	X	X	
Son and Kim (2015)	3		X			X	X		
Son and Sinclair (2010)	54	X				X			
Star and Strickland (2008)	28	X					X	X	
Stockero (2008)	21	X					X	X	
Stockero and Van Zoest (2013)	6		X				X	X	
Thomas and Yoon (2014)	1		X					X	
van Es and Sherin (2002)	12[g]	X					X	X	

Table 2 continued

References	N[a]	Pre-service teachers	In-service teachers	Teacher trainer/ lecturer	School level (certification)				
					Pre	Elementary[b]	Middle[c]	Secondary[d]	Higher education
van Es and Sherin (2006)	7[g]/6		X			X			
van Es and Sherin (2008)	11 (4)		X			X			
Wager (2014)	13		X			X			
Weiland et al. (2014)	2	X				X			
Zahner et al. (2012)	3		X				X		
Zimmerman (2015)	6	X					X	X	

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

[b] Studies involving teachers from grade 1 to 4 were categorized as 'elementary school'

[c] Studies involving teachers from grade 5 to 8 were categorized as 'middle school'

[d] Studies involving teachers from grade 9 to 13 were categorized as 'secondary school'

[e] Blömeke et al. (2015b) report longitudinal data of primary teachers from their last year of teacher education to 3 years in the profession

[f] Studies report on the same sample, but conducted different analysis

[g] Studies report on the same sub-sample

2015; Stockero 2008) or participating in an assessment interview (Weiland et al. 2014).

Studies analyzing teachers' noticing mostly used video of classroom situations (e.g. Colestock and Sherin 2009; Huang and Li 2012; Sherin and van Es 2009; Star and Strickland 2008; van Es and Sherin 2008). Exceptions were those studies that investigated theme-specific noticing by written documents of students' work (Dreher and Kuntze 2015; Fernández et al. 2013; Sánchez-Matamoros et al. 2014). A few studies used a combination of both written documents of students' work and video of classroom situations (Hoth et al. this issue; Jacobs et al. 2010; Knievel et al. 2015). Some studies took different approaches such as animations (Lande and Mesa this issue) or lessons to be observed live or taught (e.g. Amador and Weiland 2015; Jacobs and Empson this issue; Santagata and Yeh this issue). Table 1 reports the situations used to investigate teachers' situation-specific skills.

4.3.4 Are teachers' situation-specific-skills investigated in relation to their dispositions or teaching performance?

A study that included cognitive or affective-motivational aspects (e.g. content knowledge or beliefs) in the data analysis was considered to investigate dispositions.

For coding the studies as including performance data, a rather strict criterion was applied. Only if data of actual teaching and instruction practice had been reported, the study would have been coded accordingly. About one-third of the studies analyzed teachers' dispositions (i.e., their knowledge or beliefs). Twelve studies were concerned with teachers' performance. Of these studies two reported data on teachers' dispositions and their teaching practice (Bruckmaier et al. this issue; Son and Kim 2015). Table 3 indicates whether studies included aspects of dispositions or performance in their data analysis.

4.4 What results do the studies on situation-specific skills offer?

The studies report on a variety of results due to the different aspects of situation-specific skills investigated. Thus, in order to maintain clarity and comprehensibility, the results of the studies are summarized with respect to similar constructs or concepts and aims. The findings are presented along the following research lines:

(1) teachers' skill to notice classroom situations, (2) teachers' skill to perceive, interpret and respond to students' mathematical thinking, (3) teachers' situation-specific skills embedded in practice, (4) teachers' situation-specific skills in relation to their knowledge (or other dispositions) and (5) and teachers' skill to perceive and interpret mathematical tasks and their educational potential. Studies reporting different aspects of situation-specific skills were allocated to several foci. The emphasis is on studies reporting quantitative results. Effect sizes are presented if reported in the studies or, if possible, were

Table 3 Research design and methods used

References	Research design	Research methods						D/P
		Test	Question-naire	Interview	Lesson observation	Other observation	Analysis of documents	
Alsawaie and Alghazo (2010)	I						X	
Amador and Weiland (2015)	I					X		
Blömeke et al. (2015b)	CO	K; B; X						D
Bruckmaier et al. (2016)	CO	K; B; X						D, P
Colestock and Sherin (2009)	CO			X				
Cooper (2009)	CO						X	
Derry et al. (2007)	I	K				X	X	D
Dreher and Kuntze (2015)	CO	K	X					D
Dunekacke et al. (2015)	CO	K, X						D
Dunekacke et al. (2016)	CO	K; B; X						D
Dyer and Sherin (2016)	CA			X	X			P
Escudero and Sánchez (2007)	CA			X	X			D
Fernández et al. (2013)	CO		X					
Gal (2011)	CA		B	X	X		X	P
Galant (2013)	CO			X				D
Hines and McMahon (2005)	CO			X			X	
Ho and Tan (2013)	CA				X			
Hoth et al. (2016)	CO		X[a]					D
Houssart (2000)	CO			X				
Huang and Li (2012)	CO		X					
Ingram (2014)	CO					X		
Jacobs and Empson (2016)	CA				X	X		P
Jacobs et al. (2010)	CO		X					D
Jakobsen et al. (2014)	CO		X					D
Kersting (2008)	CO	X						D
Kersting et al. (2016)	CO	X						D
Klymchuk and Thomas (2011)	CO		X					
Knievel et al. (2015)	CO	K; X						D
Lande and Mesa (2016)	CO					X		
Lee and Kim (2005)	I		X	X			X	
Magiera et al. (2013)	CO	K		X		X	X	D

Table 3 continued

References	Research design	Research methods						D/P
		Test	Question-naire	Interview	Lesson observation	Other observation	Analysis of documents	
Nickerson and Masarik (2010)	I			X				
Norton et al. (2011)	I	X						D
Osmanoglu et al. (2015)	I			X			X	
Pankow et al. (2016)	CO	X						
Paterson et al. (2011)	CA			X	X		X	
Roth McDuffie et al. (2014)	I					X	X	
Sánchez-Matamoros et al. (2014)	I		X					
Santagata (2009)	I	K	X				X	D
Santagata and Yeh (2016)	CA	X		X	X			P
Santagata et al. (2007)	I		X				X	
Schack et al. (2013)	I		X					
Sherin and van Es (2005)	I					X	X	
Sherin and van Es (2009)	I			X	X	X		P
Sherin et al. (2008)	CA			X	X			
Sleep (2012)	CO			X	X			
Son (2013)	CO	K	X					D
Son and Kim (2015)	CA		X	X	X			D; P
Son and Sinclair (2010)	CO	K	X					D
Star and Strickland (2008)	I		X					
Stockero (2008)	I					X	X	
Stockero and Van Zoest (2013)	CO				X			P
Thomas and Yoon (2014)	CA			X	X		X	P
van Es and Sherin (2002)	I						X	
van Es and Sherin (2006)	I			X				
van Es and Sherin (2008)	I			X		X		
Wager (2014)	I						X	D
Weiland et al. (2014)	CA					X		P
Zahner et al. (2012)	CO				X			P
Zimmerman (2015)	CO			X	X			

CO confirmatory study, *CA* case study, *I* intervention study, *D* disposition, *P* performance, *K* method to capture knowledge from a cognitive perspective, thus not situated, *B* method to capture beliefs

[a] Hoth et al. (2016) used items of TEDS-FU, but conducted a qualitative analysis of open-ended answers

calculated based on the data reported.[7] Case studies or qualitative data are briefly summarized as well.

4.4.1 Teachers' skill to notice classroom situations

A large part of the studies included in this systematic review address mathematics teachers' noticing or teachers' professional vision. Some of these studies investigated teachers' noticing with a focus on students' mathematical thinking, whereas other studies took a broader perspective on noticing. The findings indicate *what and how teachers notice* and *how teachers' noticing can be improved* (Table 4).

Studies allocated to this research line took a rather situated approach. Several studies revealed that *what* teachers notice in a classroom as well as *how* teachers notice classroom events is related to their expertise and teaching experience. Experienced or expert teachers tended to show higher levels of noticing or noticed more events (Dreher and Kuntze 2015; Fernández et al. 2013; Huang and Li 2012; Jacobs et al. 2010). Ho and Tan (2013) found a researcher's and a teacher's professional vision of the same lessons to differ. Studies considering decision-making as a component of pre-service teachers' noticing showed this skill to be the least developed (Jacobs et al. 2010; Schack et al. 2013). Ingram (2014) described how teachers notice differently when discussing teaching videos on mathematical or pedagogical situations, whereas Colestock and Sherin (2009) provided evidence that different teachers used rather similar sense-making strategies when viewing video of classroom situations.

Many studies reported on successful interventions to foster teachers' noticing: a majority of these studies provided evidence for improving pre-service and in-service teachers' noticing skills by video-based training tools (Alsawaie and Alghazo 2010; Osmanoglu et al. 2015; Roth McDuffie et al. 2014; Schack et al. 2013; Sherin and van Es 2005, 2009; Star and Strickland 2008; van Es and Sherin 2002, 2006, 2008; Wager 2014). Other contributions provided evidence for different formats of professional development (Amador and Weiland 2015; Sánchez-Matamoros et al. 2014).

4.4.2 Teachers' skill to perceive, interpret and respond to students' thinking

Twelve studies examined mathematics teachers' perception and interpretation of students' thinking or products of students' thinking and their responding to students' work. The results give insight into teachers' ability to identify errors and to interpret students' solutions. They provide also information on how to improve teachers' ability to analyze students thinking. Table 5 provides an overview of the results.

There was evidence in the included studies that pre-service teachers had difficulties in perceiving and interpreting students' errors and solutions. This applied especially for common misconceptions or student errors (Hines and McMahon 2005; Jakobsen et al. 2014; Pankow et al. this issue; Son 2013; Son and Sinclair 2010). Some studies indicated that teachers' skills to perceive and interpret students' solutions and mathematical thinking were related to their professional knowledge. Teachers' own difficulties with mathematics tasks influenced their perception and interpretation (Hoth et al. this issue; Jakobsen et al. 2014; Magiera et al. 2013). Teachers' proposed instructional strategies for dealing with students' misconceptions or errors seemed to rather focus on "reteaching" (Cooper 2009) or showing students' how to do it correctly (Son 2013).

Other studies reported on promising formats to improve teachers' situation-specific skills with regard to student thinking, among them video-based approaches or contrasting case activities (Derry et al. 2007; Nickerson and Masarik 2010; Norton et al. 2011; Stockero 2008; Weiland et al. 2014).

4.4.3 Teachers' situation-specific skills embedded in practice

Most of the studies reporting on teachers' in-the-moment decision-making were case studies or reported mainly qualitative data. The same applies to those studies that investigated teachers' situation-specific skills close to practice. Three studies in this review explored the effects of lesson study on teachers' situation-specific skills. An overview on the studies that explored teachers' situation-specific skills embedded in practice is provided in Table 6.

Studies allocated to this research line took a clear situated approach. Several studies revealed that teachers' professional knowledge accounts for situation-specific skills embedded in practice (Gal 2011; Escudero and Sánchez 2007; Paterson et al. 2011; Santagata and Yeh this issue; Thomas and Yoon 2014). Another part of studies provided evidence for the complexity of in-the-moment decision-making and how multiple intentions have to be considered by teachers during instruction (Lande and Mesa 2016; Sleep 2012; Stockero and Van Zoest 2013; Zahner et al. 2012; Zimmerman 2015). Sherin et al. (2008) described teachers' professional vision in action using in the moment videotaping, whereas responsive teaching and teaching moves were categorized by Dyer and Sherin (this issue)

[7] Effect sizes (Cohens' d or r) are reported, if given in the studies or if they could be calculated from presented data. When structural equation models were undertaken in the studies, standardized coefficients (βs) are reported. When latent class analysis was conducted, odds are reported. Information about significance is provided, when presented in the studies.

Table 4 Results of the studies examining teachers' skill to notice classroom situations

References	Design[a] (N)	Focus	Findings
What and how do teachers notice?			
Huang and Li (2012)	CO (20)	Noticing of mathematics classroom events	Expert teachers paid more attention (than novice teachers) to Developing mathematical thinking and ability ($r = 0.73^{***}$) Developing knowledge coherently ($r = 0.55^{**}$) Teachers' enthusiasm and passion ($r = 0.51^{**}$) Developing higher-order thinking ($r = 0.41^{*}$) Students' participation ($r = 0.37^{*}$) Novice teachers paid more attention (than expert teachers) to teachers' effective guidance ($r = -0.37^{*}$)
Jacobs et al. (2010)	CO (131)	Professional noticing of children's mathematical thinking	Four groups (with growing level of expertise) with significant monotonic trend in Attending to children's strategies ($d = 0.58 - d = 0.66$) Interpreting children's understanding ($d = 0.49 - d = 1.06$) Deciding how to respond on the basis of children's understanding ($d = 0.88 - d = 0.99$)
Dreher and Kuntze (2015)	CO (144)	Theme-specific noticing (theme: multiple representations)	ISTs and PSTs showed low frequency of theme-specific noticing ISTs showed still higher theme-specific noticing than PSTs ($d = 0.72^{**}$) *Dispositions* and theme-specific noticing The view that changing between representations is necessary for understanding showed a positive relation to theme-specific noticing for ISTs ($r = 0.32^{**}$) Specific CK and theme-specific noticing showed a positive relation for PSTs ($r = 0.25^{*}$)
Fernández et al. (2013)	CO (36)	Noticing of students' mathematical thinking	Level of PSTs' noticing of students' mathematical thinking (proportional and un-proportional reasoning) Most teachers were on level 1: No discrimination of proportional and additive problems (25 of 39 PSTs or 64 %) Only few PSTs were on higher levels (level 2, 3 and 4)
Colestock and Sherin (2009)	CO (15)	Sense-making strategies/noticing	Substantial overlap in what different ISTs notice Substantial overlap in strategies used to make sense of classroom instruction
Ho and Tan (2013)	CA (2)	Professional vision	The researcher developed the following categories to capture professional vision: heuristics-instruction, teaching of concepts and skills, going over assigned work, allocating class time for student activities The teacher did not characterize his teaching the way the researcher did
Ingram (2014)	CO (19)	Professional vision	When events in the video shifted to mathematics, PSTs focus changed from themselves as teachers to the learners (conversely for video on classroom management issues) After PSTs watched video of possible reactions to a classroom situation, comments became more evaluative and interpretative

Table 4 continued

References	Design[a] (N)	Focus	Findings
Fostering teachers' noticing by using video			
Roth McDuffie et al. (2014)	I (73)	Noticing of students' multiple mathematical knowledge bases	Video-case activity improved PSTs depth of noticing and moved their foci from attending primarily to teacher moves to becoming aware of significant interactions
Osmanoglu et al. (2015)	I (15)	Noticing of teacher actions	PSTs' noticing of teacher actions that reflect specific domains of teacher knowledge (CK, PCK, GPK) increased over time (by online discussions and video case-based activities)
Schack et al. (2013)	I (94)	Noticing of children's early numeracy	Five-session-module with video-excerpts of diagnostic interviews improved PSTs noticing significantly from pre- to post-assessment: Attending: $d = 0.79$*** Interpreting: $d = 0.82$*** Deciding: $d = 1.29$***
Sherin and van Es (2005)	I (4/12 (6))	Noticing	Study 1: Video club participation shifted ISTs attention from pedagogy to students mathematical thinking Study 2[b]: PSTs working with VAST (video analysis support tool) developed more interpreting stance and showed more evidence-based comments
Sherin and van Es (2009)	I (4/7)	Professional vision	Participation in one of the two video clubs (Nile and Mapleton) influenced ISTs noticing as exhibited in video club meetings, interviews and during instructional *practice*
Star and Strickland (2008)	I (28)	Noticing	PSTs participating in a methods course using video (among other activities) showed Significant general improvement of noticing ability Significant improvement in four of five categories of noticing: classroom environment; tasks; mathematical content; communication
van Es and Sherin (2006)	I (7/6)	Noticing	Results for Mapleton club cf. Sherin and van Es (2009) Wells Park club ISTs started with a narrow range of noticing and developed a range of perspectives for discussing the video segments.
van Es and Sherin (2008)	I (11 (4))	Noticing	There were three paths identified along which ISTs learned to notice (all reaching a narrow vision): Direct, cyclical and incremental
Alsawaie and Alghazo (2010)	I (26 (13))	Analyzing mathematics teaching	PSTs participating in course (including case-methods and video analysis) learned Paying attention to student learning Interpreting classroom events (not merely describing or evaluating) Making connections between classroom events and the NTCM vision of teaching and learning
Wager (2014)	I (13)	Noticing children's participation	Two groups of ISTs identified based on the number of comments: Frequent noticers (FN) and emergent noticers (EN): Groups showed significant differences in the components of noticing (attending, interpreting, responding)

Table 4 continued

References	Design[a] (N)	Focus	Findings
Fostering teachers' noticing by other interventions			
Amador and Weiland (2015)	I (32)	Professional noticing	PSTs' development while participating in the lesson study Initially, PSTs focused on student thinking in 33 % of their comments Focus on student thinking dropped to 18.8 % in the fifth lesson Concerning the last lesson, about 33 % of the comments again addressed student thinking
Sánchez-Matamoros et al. (2014)	I (8)	Noticing of students understanding of the derivative concept	PSTs' levels of noticing of students' understanding increased after participating in the teaching module on students' understanding of the derivative concept

CO confirmatory study, *I* intervention study, *CA* case study, *PST* pre-service teacher, *IST* in-service teacher

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

[b] The same results are reported in van Es and Sherin (2002) and will not be presented repeatedly

and Jacobs and Empson (this issue). Finally, three studies reported on successfully fostering pre-service and in-service teachers' situation specific skills by lesson analysis (Amador and Weiland 2015; Santagata 2009; Santagata et al. 2007).

4.4.4 Teachers' situation-specific skills in relation to their knowledge (or other dispositions)

Some studies assessed teachers' perception, interpretation and decision-making by validated, standardized tests and in relation to dispositions. Most studies revealed evidence for the impact of CK, PCK or beliefs on teachers' situation-specific skills. Table 7 gives an overview of these findings.

The studies allocated to this research line took a rather cognitive approach but included situated measures of teachers' situation-specific skills. The six studies provided evidence for linking teachers' dispositions and situations-specific skills. Dunekacke et al. (2015, this issue) showed that MCK and MPCK are predictors of pre-school teachers' perception of classroom situations and (mediated by perception) of their planning of actions. Similarly, Bruckmeier et al. (this issue) reported correlations between situated reaction-competency and CK, PCK and beliefs. In addition, this study reported a significant relationship between a sub-facet of situated reaction-competency and aspects of teachers' instructional quality (Bruckmaier et al. this issue). Blömeke et al. (2015b) provided evidence for the impact of knowledge, beliefs and a school climate of trust on beginning mathematics teachers' perception, interpretation and decision-making skills. Two studies revealed a strong interrelation of teachers' knowledge facets and situation-specific skills (Kersting et al. this issue; Norton et al. 2011). Kersting (2008) and Knievel et al. (2015) reported evidence on the reliability and validity of their developed instruments and found teachers' knowledge related to their situation-specific skills.

4.4.5 Teachers' skill to perceive and interpret mathematical tasks and their educational potential

Some studies focused on the material used during instruction. Studies on mathematical tasks found teachers' perceptions, interpretations and decision-making to differ partly from curriculum guidelines or research recommendations. Table 8 gives an overview of the results.

Three studies indicated that pre-service as well as in-service teachers struggled with differentiating routine from non-routine mathematics task and choosing adequate formats for fostering their students' learning (Galant 2013; Klymchuk and Thomas 2011; Lee and Kim 2005). In addition, teachers' interpretation of task-related features (Houssart 2000; Magiera et al. 2013) and their decision-making corresponded with their professional knowledge and beliefs about student thinking (Son and Kim 2015).

5 Conclusion and discussion

This systematic review reports on 60 empirical research studies on teachers' situation-specific skills. These studies, published in English-speaking peer-reviewed journals, were selected based on a systematic search in the databases ERIC, PsycINFO and MathEduc as well as in this Special

Table 5 Results of the studies examining teachers' skill to perceive, interpret and respond to students' thinking

References	Design[a] (N)	Students' work	Findings
Teachers' perception, interpretation and responding to student thinking			
Hines and McMahon (2005)	CO (11)	Proportional reasoning strategies	PSTs considered students' solutions as developmentally advanced, if equations were used or a routine problem solving procedure was consistently applied
Magiera et al. (2013)	CO (18)	Solutions of algebraic tasks	PSTs' own algebraic thinking was related to their ability to recognize students' overall ability during one-to-one interviews, but were not related to their ability to analyze students' overall ability based on written solutions
Jakobsen et al. (2014)	CO (49)	Elementary students' work on fraction task	PSTs had difficulties in solving fraction tasks PSTs revealed difficulties in making sense of solutions different from their own solution
Son and Sinclair (2010)	CO (54)	Elementary students' errors in geometric tasks	30 of 54 PSTs (56 %) identified students' errors to be based on conceptual aspects of reflection rather than on procedural aspects About the same number or PSTs coped with these errors by invoking procedural knowledge (22 of 54 PSTs or 41 %) or conceptual knowledge (25 of 54 PSTs or 46 %), respectively
Son (2013)	CO (57)	Errors (ratio and proportion in similar rectangles)	Over half of PSTs identified students' errors as being linked to procedural aspects of similarity, although the errors were linked to conceptual aspects of similarity These PSTs proposed interventions focused on procedure-based instruction Pedagogical strategies were majorly showing or telling how. Using the student's error was proposed by less than half of the PSTs
Cooper (2009)	CO (86)	Computational errors	All PSTs could identify the error pattern 67 of 86 of PSTs (80 %) proposed a reasonable rationale 54 of 86 of PSTs (67 %) proposed some form of "reteach" as an instructional strategy (e.g. focusing on procedures or simplifying the problem)
Pankow et al. (2016)[b]	CO (137)	Student's error	Non-complex tasks ISTs that correctly and ISTs that falsely identified the error, showed a short anticipation time. Complex tasks ISTs that correctly identified the error showed a longer anticipation time (i.e. are slower in identification) Significant differences in anticipation time between correct and false answers for the three most complex tasks: $d = -0.7^{***}$; $d = -0.4^{*}$; $d = -0.8^{*}$
Hoth et al. (2016)[b]	CO (133)	Diagnostic competence	ISTs coped with diagnostic tasks during teaching differently Different perspectives in perceiving and solving diagnostic situations could be reconstructed: a content-related mathematical perspective and a student-related, more pedagogical perspective *Dispositions*: ISTs taking a content-related perspective tended to have higher mathematics content knowledge and general pedagogical knowledge

Table 5 continued

References	Design[a] (N)	Students' work	Findings
How can teachers' perception, interpretation and responding to student thinking be improved?			
Derry et al. (2007)	I (20/10)	Algebraic thinking	Contrasting case activities improves ISTs' analysis of student work in terms of Sophistication of description of representation/solution ($d = 0.87$*/$d = 1.5$**) Inferences about students' ability and understanding ($d = 0.95$**/$d = 1.39$**) Pedagogically useful inferences about students' mathematical trajectory ($d = 1.17$**/$d = 1.41$**). No improvement of teachers' metacognitive reflections
Nickerson and Masarik (2010)	I (4)	Middle-school students' work	The professional development program improved PSTs' interpretive power. PSTs showed shifts in their ability to anticipate students' responses
Norton et al. (2011)	I (42 (19))	Video of students' solving tasks	PSTs participating in iterative model building" course (IMB) vs. control group Video-based prediction assessment rubric for CK, model (of students' thinking), prediction (accuracy and detail) and use of model Components of assessment rubric (apart from CK) correlated with participation in IMB course Components interactions were stronger after participation in IMB course
Stockero (2008)	I (21)	Linear functions	PSTs participated in video-based curriculum for "Learning and Teaching Linear Functions" (LTLF) PTSs started to analyze teaching in terms of how it affects students' thinking, to consider multiple interpretations of student thinking and to develop a more tentative stance of inquiry PSTs' reflective stance improved during the video curriculum and transferred to their course field experience
Weiland et al. (2014)	CA (2)	Formative assessment	PSTs developed their questioning practice within the context of a face-to-face interaction with students PSTs showed two areas of questioning practice needing improvement: asking leading questions and missing opportunities to probe students' thinking

CO confirmatory study, *I* intervention study, *CA* case study, *PST* pre-service teacher, *IST* in-service teacher

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

[b] The study is to be published in this special issue

Issue. The systematic review was guided by the following research questions: *What situation-specific skills are investigated in empirical research in mathematics education? To what theoretical frameworks does empirical research on mathematics teachers' situation-specific skills refer? What designs and methods are used to assess perception, interpretation and decision-making of mathematics teachers? What results do the studies on situation-specific skills offer?*

Regarding the first research question, most studies investigated *interpretation* (47 studies), followed by *perception* (38 studies) and *decision-making* (32 studies). One-third of the studies explored perception *and* interpretation. One quarter of the studies analyzed *all* three situation-specific skills.

Table 6 Results of the studies examining situation-specific skills embedded in practice

References	Design[a] (N)	Focus	Findings
Teachers' situation–specific skills embedded in practice (and their development)			
Gal (2011)	CA (1)	Development after course	One IST expanded and deepened her understanding of students' ways of thinking She increased her awareness of her students' processes of thinking to identify their difficulties She enhanced her ability to retrieve and utilize knowledge while making instructional decisions
Santagata and Yeh (2016)[b]	CA (3)	Development of beginning teachers' competence	While at each moment in time teachers' own understanding of mathematical ideas and their beliefs about children's mathematics learning informed their sense making, interviews also highlighted how teachers sometimes made decisions based on particular instructional approaches recommended by their colleagues or required by their school leadership
Escudero and Sánchez (2007)	CA (2)	Knowledge integration in decisions	ISTs had similar backgrounds and experiences but showed differences in the domains of knowledge they integrated into their planning decisions as well as their decisions during instruction
Paterson et al. (2011)	CA (8)	Lecturer decisions	Schoenfeld's framework of resources, goals and orientations tended to be useful for explaining lecturers' decisions
Thomas and Yoon (2014)	CA (1)	Lesson on graphical antiderivatives	Presents details of one teacher's resources, orientations, and goals and how this was related to resolutions of the conflict between his competing goals and the decisions he made.
Zahner et al. (2012)	CO (3)	Lesson on interpreting graphs of motion	The more successful ISTs allowed time for students to use the curriculum and software and discuss it with peers. They used formal mathematical discourse along with less formal language, and they responded to student errors using higher-level moves
Lande and Mesa (2016)[b]	CO (20)	Lesson on trigonometry	Both groups of faculty members (full-time and part-time) justified their decisions in similar ways; the way in which they talked differed Part-time faculty members' language was more tentative, which hints at their tenuous status in their institutions
Sleep (2012)	CO (17)	Steering instruction towards the mathematical point	Steering instruction towards the mathematical point involves several tasks, e.g. Attending to and managing multiple purposes Developing and maintaining a mathematical storyline Keeping focus on meaning
Stockero and Van Zoest (2013)	CO (6)	Pivotal teaching moments (PTMs)	Study developed a preliminary framework for helping teachers to learn to identify and respond to PTMs that occur during instruction Results highlight the importance of preparing teachers to understand the mathematical terrain their students are traversing, to notice high-leverage student mathematical thinking and to act productively on that thinking

Table 6 continued

References	Design[a] (N)	Focus	Findings
Zimmerman (2015)	CO (6)	Practical intentions	Different practical intentions often occurred simultaneously Four prominent intentions: the desire to maintain lesson momentum; the desire to cover content; the desire to support student needs; and the desire to foster independent student thinking
Sherin et al. (2008)	CA (1)	Professional vision in action	Study investigated a new technology to study professional vision in action (small head-camera) that allowed the teacher to capture clips of events he considered as noteworthy The collected clips varied from whole class discussions, small group work, and student presentations to teacher talk Reasons for selecting these clips were student thinking, discourse, teacher moves, teacher strategies and student engagement
Dyer and Sherin (2016)[b]	CA (2)	Responsive teaching	Three types of instructional reasoning about interpretations of student thinking used by the ISTs: making connections between multiple specific moments of student thinking, considering the relation between the mathematics of student thinking and the structure of a mathematical task, and developing tests of student thinking
Jacobs and Empson (2016)[b]	CA (1)	Teaching moves	The study developed a framework with four major categories of teaching moves Ensuring the child is making sense of the story problem, exploring details of the child's existing strategy, encouraging the child to consider other strategies, connecting the child's thinking to symbolic notation
Fostering teachers' situation-specific skills with lesson study			
Amador and Weiland (2015)	I (32)	Lesson study	PSTs participating in the lesson study showed higher level of noticing than classroom teachers or university facilitators
Santagata (2009)	I (33)	Lesson study	ISTs encountered difficulties with questions on the basic understanding of target mathematics topics, knowledge of their students understanding, and the analysis of students' work and reasoning beyond classification into right and wrong answers
Santagata et al. (2007)	I (35/30)	Lesson study	PSTs' ability to analyze lessons improved significantly on all five criteria: elaboration, mathematics content, student learning, critical approach, and alternative strategies

CO confirmatory study, *I* intervention study, *CA* case study, *PST* pre-service teacher, *IST* in-service teacher

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

[b] The study is to be published in this special issue

With concern to the second research question, the studies referred to two main theoretical frameworks that are teachers' noticing and teachers' (situated) professional knowledge. Only a few studies combined both frameworks.

Articles included in this review were case studies, intervention studies or confirmatory studies. These studies used a variety of methods to investigate pre-service and in-service teachers' situation-specific skills, ranging from standardized tests to observing teachers during instruction. Only a few studies combined diverse methods or compared pre- and in-service teachers' situation-specific skills.

Table 7 Results of the studies examining teachers' situation-specific skills in relation to dispositions

References	Design[a] (N)	Findings
Dunekacke et al. (2015)	CO (354)	*Dispositions*: Pre-school PSTs' mathematical content knowledge (MCK) Was direct predictor of PSTs' perceptions of pre-school situations ($\beta = 0.55^*$) Was an indirect predictor of PSTs' planning of actions and mediated by the perception of the situation ($\beta = 0.43^*$) PSTs' perception of a situation was a predictor of their planning of actions ($\beta = 0.95^*$)
Dunekacke et al. (2016)[b]	CO (354)	*Dispositions* Pre-school PSTs' MCK could predict their mathematical pedagogical content knowledge (MPCK) ($\beta = 0.45^*$) MPCK and an application orientation (epistemological belief) could predict perception ($\beta = 0.60^*$ and $\beta = 0.29^*$) MPCK was an indirect predictor of PSTs' planning of actions and was mediated by the perception of the situation ($\beta = 0.51^*$) PSTs perception of a situation was a predictor of their planning of actions ($\beta = 0.94^*$)
Blömeke et al. (2015b)	CO (231)	*Dispositions* Development of beginning primary teachers' knowledge/beliefs: significant increase of general pedagogical knowledge (GPK) and dynamic belief After 3 years of profession three profiles identifiable (unfavorable; regular and optimal profile) based on knowledge and beliefs Climate of trust in school reduced odds of having an unfavorable versus a regular profile (0.34*) and the odds of having an unfavorable versu a favorable profile (0.20**) Perception, interpretation and decision-making skills in mathematics teaching and in classroom management were significantly higher in the optimal profile than in the unfavorable profile
Bruckmaier et al. (2016)[b]	CO (284)	*Dispositions of ISTs* Situated reaction-competency (SCR) correlated with CK ($r = 0.28^{**}$), PCK ($r = 0.33^{**}$) and beliefs (constructivist belief: $r = 0.26^{**}$ and transmissive belief: $r = -0.32^{**}$) SCR differed with school type (academic track > other school types: $d = 0.51$ and $d = 1.34$) *Performance* Subject-specific sub-competency showed significant relation with aspect of instructional quality (cognitive activation: $\beta = 0.22$)
Kersting (2008)	CO (62)	Development of classroom video analysis survey (CVA) to measure knowledge of teaching mathematics in concrete teaching situations Indications for reliability and validity of CVA Four rubrics: mathematics in the clip; student thinking; suggestions of improvement; overall interpretation depth and coherence Moderate correlation of CVA-Score with mathematical knowledge for teaching (paper-pen-test; $r = 0.53^{**}$)
Kersting et al. (2016)[b]	CO (676)	Results of a different approach to the CVA instrument were reported. They are consistent with the view that usable teacher knowledge requires individual knowledge components as well as an overarching ability to access and apply those components that are most relevant in a teaching situation
Knievel et al. (2015)	CO (85)	Development of an instrument to measure teachers' subject-specific competences in and for teaching mathematics with threefold structure: basic knowledge (BK), action-related competence (AC) and reflective competence (RC) Indications for the reliability and validity of the instrument Moderate difference for RC and BK between elementary ISTs without and with certification for teaching mathematics ($d = 0.63^{***}$ an $d = 0.77^{***}$)
Norton et al. (2011)	I (42 (19))	Development of a video-based prediction assessment instrument as a measure of PSTs' ability to model students' mathematical thinking Indications for reliability and validity of instrument Prediction assessment rubric for CK, model (of students' thinking), prediction (accuracy and detail) and use of model Components of assessment rubric correlated with participation in "iterative model building" course Components interactions are stronger (and all significant) after participation in IMB course

CO confirmatory study, *I* intervention study, *PST* pre-service teacher, *IST* in-service teacher

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

[b] The study is to be published in this special issue

Table 8 Results of studies examining teachers' skills to perceive and interpret mathematical tasks and their educational potential

References	Design (N)[a]	Tasks	Findings
Magiera et al. (2013)	CO (18)	Algebraic tasks	PSTs recognized some features of tasks (to engage students in algebraic thinking) more often than other features Predicting pattern > chunking information ($d = -1.38$) Predicting pattern > different representation ($d = -1.64$)
Klymchuk and Thomas (2011)	CO (203)	Calculus tasks (advanced)	Most secondary ISTs and nearly all lecturers did not identify non-routine problems and found them suitable for year 13 students
Houssart (2000)	CO (26)	Tasks (partly) on pattern	The word "pattern" was used frequently Some ISTs had a more sophisticated view on pattern than others
Galant (2013)	CO (46)	Multiplication tasks	Eight of 46 ISTs (17 %) chose the "advanced" tasks as the first to be done
Son and Kim (2015)	CA (3)	Tasks in textbooks	Analysis revealed four particular aspects that are related to teachers' decisions on selecting and enacting textbook problems Match between beliefs and goals and these of the textbooks, views on the textbooks, interpretation of state curriculum framework and assessment, and knowledge or orientation toward student thinking
Lee and Kim (2005)	I (22)	"Good" problems	Majority of PSTs rated routine problems as good After input/discussion most PSTs would have changed ratings, but expected difficulties in utilizing non-routine problems

CO confirmatory study, *I* intervention study, *CA* case study, *PST* pre-service teacher, *IST* in-service teacher

[a] *N* refers to the number of participants analyzed. When more than one study is reported the samples are marked by a/; When participants were partly assigned to a control group the number is given in ()

The last research question addressed the results obtained by the studies. The results revealed evidence for the significance of expertise or experience on teachers' noticing. Pre-service teachers tend to have difficulties in perceiving or interpreting students' work. These skills seemed to be influenced by their level of mathematical knowledge. A noteworthy finding is that video-based professional development programs can foster teachers' noticing successfully.

Decision-making appeared to be most challenging for pre-service teachers. Teachers' showed deficits in terms of proposing instructional strategies to foster students' understanding that go beyond "showing how to do it right". Case studies revealed the complexity of teachers' in-the-moment decision-making. Based on these studies, factors hypothesized to influence teachers' decisions were ranging from teachers' knowledge, beliefs to goals. Multiple intentions influenced teachers' decisions.

These hypotheses were confirmed in studies assessing the relations between teachers' knowledge, beliefs and situation-specific skills by using standardized tests and large samples. The studies provided evidence for MCK, MPCK and beliefs being predictors of situation-specific skills, which in turn correlate with aspect of instructional quality.

Based on selection and restriction criteria, this review systematically searched for and included empirical studies. Due to the specific selection and restriction criteria applied, this review might be biased. First, the limitation to English-speaking empirical journal articles may have caused a possible bias. Excluding all non-English articles could have resulted in overlooking substantial research published in other languages. Second, the search terms (individually or combined) as well as the inclusion or exclusion criteria might have impacted on the sensitivity and specificity of the search. Due to the diverse terms and concepts used in mathematics education research, the search strategy–especially combining the different terms–might have led to a specific subset of studies. On the one hand, studies that analyze situation-specific skills, but use terms other than the chosen search terms, could have been missed. On the other hand, the criteria for including papers into the systematic review were rather soft. That is, articles were included that investigated situation-specific skills but did not explicitly refer to perception, interpretation, and decision-making.

The article is appearing in the *ZDM Mathematics Education* special issue on "Perception, interpretation and decision-making: understanding the missing link between competence and performance". The studies of this special issue report on important and diverse topics. In case they met the review criteria, the articles were included in the review. Several studies taking a rather cognitive approach (i.e. measuring teachers' professional knowledge) and a strongly situated approach (i.e. observing teachers' practice) were considered. The three commentary papers by Mason, Schoenfeld and Schreiner (this issue) discuss the contributions of the special issue thoroughly and emphasize a huge variance on the two levels of theoretical considerations and methodological choices. This systematic review has a broader frame, as the last 20 years of empirical research in mathematics education were analyzed. Some striking observations were made in terms of conceptual clarity: across the studies different terms were used for the same aspect as well as the same terms were used for different aspects. The same lack of clarity can be observed in the theoretical frameworks used. For instance, the definitions of noticing and what situation-specific skills constitute noticing vary strongly. As Jacobs et al. (2010) stated, "researchers define noticing in a multitude of ways, but the connecting thread is making sense of how individuals process complex situations" (p. 171).

The different terminologies and conceptualizations of situation-specific skills also impact on how perception, interpretation and decision-making are studied empirically. In their systematic review on PCK, Depaepe et al. (2013) pointed out that measurements can be distinguished along the cognitive and situated perspective, that is:

> Advocates of a cognitive perspective on PCK believe it can be measured independently from the classroom context in which it is used, most often through a test. […] Adherents of a situated perspective on PCK, on the contrary, typically assume that investigating PCK only makes sense within the context in which it is enacted. (p. 22).

In this systematic review, the distinction between a cognitive and a situated perspective on teachers' professional knowledge is even more challenging when it comes to methodological approaches. There is a growing body of research developed from a cognitive perspective that reflects upon situation-specific skills as *knowledge-based* skills which are applied in contexts approximating classroom situations (Blömeke et al. 2015a; Kaiser et al. 2014). These approaches develop standardized video-based instruments displaying classroom situations closer to practice but not embedded in practice Other studies investigate teachers' behavior near or in the classroom and consider teacher knowledge as integral part of teaching. Studies that investigate the long-term development of teachers' situation-specific skills *and* include both perspectives are scarce: so far only Blömeke et al. (2015b) have conducted a longitudinal study.

This review shows that a considerable body of research contributions dealing with perception, interpretation and decision-making from either a cognitive or a situated perspective already exists. Comprehensive and integrative approaches that connect teachers' situations-specific skills to teachers' competence in terms of professional knowledge and performance are scarce yet. Research would highly benefit from combining both a cognitive and a situated perspective not only theoretically but methodologically as well. In this respect, Kersting (this issue) aptly emphasizes:

> Understanding what mathematics teachers need to know, and what it takes to be able to apply that knowledge in the classroom, is critical for helping teachers improve their practice and their students' learning. For years, progress toward this goal was hampered by imprecise and inconsistent use of terminology, a lack of well-developed theories, and a paucity of measures. (p. 1).

The initial aim of this systematic review was to explore teachers' situation-specific skills, i.e., perception, interpretation and decision-making. These skills display the missing link between mathematics teachers' dispositions (professional knowledge, affective motivational features) and their performance (observable behavior) (Blömeke et al. 2015a). Approaching teachers' situation-specific skills from a rather cognitive or situated perspective led to substantial research findings. These two approaches could be brought closer by acknowledging the respective advantages and findings. Existing frameworks and methods might be used to develop integrative research designs that allow for dealing more effectively with the complexity of teaching.

In this systematic review, research on situation-specific skills in mathematics education research was thoroughly analyzed. Following Petticrew's (2015) reflections, systematically reviewing research contributions concerning teachers' situation-specific skills does not provide a comprehensive overview on "what works", but rather describes "what happens" in this field. Having mapped this landscape, researchers can now proceed to direct research in this area on to solid ground where reliable findings can be gained essentially to advance teaching practice.

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