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SURVEY PAPER

A rich interpretation of numeracy for the 21st century: a survey of the state of the field

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Abstract This article is a state-of-the-art synthesis of literature concerned with the concept of Numeracy (also known internationally by other terms such as mathematical literacy), and the teaching, learning and assessment practices associated with this construct. Numeracy is a concept used to identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life, and to participate in society as informed, reflective, and contributing citizens. While there is an increasing focus on numeracy internationally, there is not yet a widely accepted definition for this construct or of how to best promote the development of numeracy capabilities. In this article, we first outline the development of the concept of numeracy internationally. Second, research on numeracy practice is presented through a number of distinct facets: a critical view; the workplace; the role of technology; and statistical and financial literacy. Third, studies that explore the teaching and learning of numeracy are examined. Fourth, we scrutinise the role played by national and international assessment regimes in providing information about the numeracy capabilities of a nation's citizenry and the consequences of making such data public. Finally, we reflect on the future directions of numeracy research across the spectrum of contexts to which it is relevant.

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1 Introduction

Numeracy is a term used to identify the knowledge and capabilities required to accommodate the mathematical demands of private and public life and to participate in society as informed, reflective, and contributing citizens. The term numeracy has existed since the time of the Crowther Report, *15–18*, and was originally defined as the mirror image of literacy, but involving quantitative thinking (Ministry of Education 1959). Since this time, other terms and associated definitions have emerged internationally (e.g., Cockcroft 1982; Steen 1999; OECD 2010). While it is more common to use the term numeracy in countries, such as the UK, Canada, South Africa, Australia, and New Zealand, other names, such as *quantitative literacy* or *mathematical literacy*, are used in the USA and elsewhere. Additionally, other expressions, for example, statistical literacy and financial literacy, are utilised to denote more domain specific uses of mathematics for dealing with mathematically demanding aspects of life.

Although what is meant by numeracy varies between countries, it is now broadly accepted that being numerate extends beyond the mastery of basic arithmetic skills to how to connect the mathematics learnt in formal situations, such as school classrooms, to real world problems. Thus, being numerate also involves the capability to: make sense of non-mathematical contexts through a mathematical lens; exercise critical judgement; and explore and bring to resolution real world problems.

There is a growing understanding that poor numeracy is a social burden that limits successful transitions from school to subsequent work opportunities, with negative consequences for career aspirations, social well being, financial security, and social and political participation (Paulos 2000; Bynner and Parsons 2006; Council of Australian Governments 2008). Such outcomes have led to increased attention to numeracy internationally, resulting in the development of policy and curriculum documents which portray numeracy as a vital skill for informed and participatory citizenship that must be continuously developed over a lifetime. This attention is highlighted by the developing prominence of international testing regimes such as the *Programme for International Student Assessment* (PISA) (e.g., OECD 2004, 2010, 2012) which aims, in part, to determine the mathematical literacy of students towards the end of their compulsory years of schooling, and the *Programme for the International Assessment of Adult Competencies* (PIACC), (e.g., OECD 2013a, b) that analyses the level and distribution of skills, including numeracy competencies, among adult populations.

This article is a synthesis of research that identifies and describes various different themes that have emerged around the idea of numeracy. While these themes overlap student and adult numeracy, adult numeracy will not be a specific focus of this synthesis as this aspect of the field is the focus of a forthcoming issue of ZDM. Through this synthesis, and via this issue of ZDM, we aim to provide insight into the diversity of numeracy understandings and practices internationally. In addressing this aim, we will first outline the development of the concept of numeracy across time and provide examples from the range of interpretations of numeracy across the world. Second, research on numeracy practice will be presented through a number of distinct facets: a critical view; the workplace; the role of technology; and statistical and financial literacy. Third, studies that explore the teaching and learning of numeracy will be examined. Fourth, we will scrutinise the role played by national and international assessment regimes in providing information about the numeracy capabilities of a nation's citizenry and the consequences of making public such data. Finally, we will speculate on the future directions of numeracy in terms of teaching and learning within and outside of school contexts.

2 Understandings of numeracy

While there is an increasing focus on numeracy internationally, there is not yet a widely accepted definition for this construct or of the fundamental characteristics that describe this idea. Thus, the meaning of numeracy still varies widely across international borders; from countries where no direct translation is available in a local language, for example German and Scandinavian (Niss and Jablonka 2014), to variations where becoming numerate is viewed as the acquisition of basic arithmetic facts and procedures, through to richer interpretations that embrace notions of problem solving within authentic contexts. These variations can generally be traced back to historical, cultural, and socio-political influences. Although there is growing acceptance of the definitions of numeracy that underpin international testing regimes, such as those proposed by PISA (mathematical literacy) or PIACC (numeracy), their emergence is a relatively new development. The section which follows outlines the historical development of the idea of numeracy and also discusses examples of local interpretations that highlight the diverse interpretation of this construct internationally.

2.1 International perspectives on numeracy

The origin of the idea of numeracy is attributed by Cockcroft (1982) to the Crowther Report (Ministry of Education 1959). The Crowther Report, *15–18*, (Ministry of Education 1959) was developed by a committee with the task of determining what mathematics was needed for individuals in the UK to continue their participation in further and higher education. In the view of this committee, numeracy was seen as the "mirror image" of literacy where becoming numerate meant not only developing competence with basic mathematical skills but also the acquiring the capacity to apply these skills for some purpose:

> On the one hand is an understanding of the scientific approach to the study of phenomena—observation, hypothesis, experiment, verification. On the other hand is a need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind. (p. 270)

While this definition appears to focus on scientific enterprises, elsewhere the report emphasises the broader importance of numeracy across other human intellectual endeavours:

> Numeracy has come to be an indispensable tool to the understanding and mastery of all phenomena, and not only of those in the relatively close field of the traditional natural sciences. (p. 271)

In response to changes to the demands of the workplace and consequent criticisms of the mathematical capabilities of school leavers in Britain through the 1970s, Wilfred Cockcroft was selected to chair a government commissioned committee established to conduct a comprehensive review of mathematics teaching in both primary and secondary school. The terms of reference of the committee were wide ranging and included an investigation into the mathematics required to participate in further and higher education, employment, and adult life generally. The resulting report, *Mathematics Counts* (Cockcroft 1982) was highly influential in shaping mathematics teaching policy and practice in the UK and beyond. As part of their inquiry, the committee built on the Crowther definition to develop a fuller description of what it was to be numerate:

> We would wish 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical mathematical demands of his everyday life. The second is ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. (p. 11)

The first attribute referred to in this statement ties the notion of numeracy more closely to the skills and capacities required to cope with the requirements of personal, civic and work life—consistent with the earlier Crowther definition. The second, however, is distinct from earlier understandings of numeracy and is an indication of the increasing prevalence and the use of mathematical representations in everyday and work life of that time.

With the deep influence of digital technologies and digitisation on society beginning to emerge, Steen (1999) argued that to thrive in the new times associated with a "data drenched world", citizens must be quantitatively literate. In developing a more detailed description of the use of mathematics to meet the demands of work, home and civic life, Steen (2001) identified seven dimensions of numeracy (using the term quantitative literacy): confidence with mathematics; appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm; logical thinking and decision-making; use of mathematics to solve practical everyday problems in different contexts; number sense and symbol sense; reasoning with data; and the ability to draw on a range of pre-requisite mathematical knowledge and tools. Thus, Steen extends the idea of numeracy to include not only the capability to use mathematics in a "useful" sense and to interpret mathematical information but also forms of thinking and reasoning related to solving problems in the real world. He also considers affective attributes such as confidence with mathematics as an essential characteristic of a numerate person.

Evidence of an increasing international focus on numeracy since the turn of the century, is signalled by the emergence of testing regimes such as the *Programme for International Student Assessment* (PISA) and the *Programme for the International Assessment of Adult Competencies* (PIAAC). These programs have provided definitions for numeracy/mathematical literacy as part of the relevant assessment and analytical frameworks used to develop numeracy assessment instruments.

PISA has been conducted every 3 years since 2000. Through this time, the definition of mathematical literacy used in this program has evolved. The 2000 version of the definition of mathematical literacy placed a focus on the use of mathematics in private, social, and work life.

> … an individual's capacity to identify, and to understand, and to engage in mathematics and make well founded judgements about the role mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen. OECD (2000, p. 50).

This definition was only slightly refined into the form used in the 2003/2006/2009 rounds of PISA.

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well founded judgements, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD 2009, p. 84)

The 2012 round of PISA, however, provided greater emphasis on the capacity to use mathematics in a variety of contexts, and the use of mathematical reasoning, as well as facts and procedures. Additionally, the definition makes visible, for the first time, the use of tools as part of a mathematically literate person's repertoire. The definition introduces the idea that a mathematically literate person should be able to describe, explain, and predict phenomena as well as make decisions and judgements.

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. OECD (2013a, p. 17)

This definition will also to be used in the forthcoming round of PISA assessments in 2015.

While PISA is concerned with students' mathematical literacy (among other literacies), PIAAC is a survey of Adult (16–65 years) skills, including numeracy, that first took place in 2012 (first PIACC results became available in 2013). PIACC identifies numeracy as one of three key information-processing competencies necessary for participating in the labour market, education and training, and social and civic life (OECD 2013b). Within PIACC, numeracy is defined as:

> … the ability to use, apply, interpret, and communicate mathematical information and ideas. It is an essential skill in an age when individuals encounter an increasing amount and wide range of quantitative and mathematical information in their daily lives. Numeracy is a skill parallel to reading literacy, and it is important to assess how these competencies interact, since they are distributed differently across subgroups of the population. (p. 75)

Here, numeracy is viewed as primarily a skill related to interpreting and communicating mathematical ideas and information and is seen as a parallel skill to literacy. This definition has much in common with early views of numeracy such as those proposed by Crowther (Ministry of Education 1959) and does not take into account other dimensions that have emerged in descriptions used by PISA and elsewhere that take into account the role of reasoning, tools and affective attributes.

2.2 Local perspectives on numeracy

In addition to definitions that have attracted international attention, government agencies and education authorities within individual countries have developed definitions of numeracy that align with local needs, demands, and priorities within a local educational context. Three examples are presented below to illustrate the diversity of these perspectives.

In South Africa, for example, mathematical literacy is taught as a stand alone subject in the post-compulsory years of schooling (Grades 10–12). The purpose of the subject is to develop students' capabilities in using mathematics to meet the demands of everyday life, where this is appropriate (SA DoE 2003; SA DBE 2011). Hence, curriculum documents emphasise the use of life-related application of mathematics within this subject as well as the importance using mathematics to interpret and analyse situations in order to solve real world problems.

> Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (SA DoE 2003, p. 9)

By contrast, at a policy and curriculum development level within Australia, numeracy is viewed as a cross-curricular responsibility with the expectation that it is integrated into all subjects. This perspective was reflected in a national numeracy review undertaken by the Australian government (Council of Australian Governments 2008) which recognised numeracy as an essential skill for students in becoming successful learners at school and in their future lives after schooling. The review recommended:

> That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. (p. 7)

The notion that numeracy should be a cross-curricular endeavour is given credence through the Australian Curriculum, which identifies numeracy as a *General Capability* to be developed in all subjects, not only mathematics (Australian Curriculum, Assessment and Reporting Authority 2014). Consequentially, there is a numeracy statement within each subject curriculum which emphasises the importance of developing dispositions and capacities to use mathematics within that subject and in the real world. Connections between problem solving and critical thinking, however, are not made explicit.

The Brazilian view of numeracy is more complex. The notion of numeracy in Brazil is tied to the program of ethnomathematics through the curriculum structure known as the Trivium: literacy, matheracy and technoracy. While numeracy is still seen as a connection between mathematics and the world in a way that promotes the technical skills needed to participate as an informed and contributing citizen in society (D'Ambrosio 2001), matheracy is broader in its conception as it encompasses a connection to the socio-cultural contexts of the teacher and learner (e.g. D'Ambrosio 1999, 2001). Matheracy requires the development of skills and capabilities necessary to apply mathematics in a critical way within specific social and economic environments in order to promote a more equitable and peaceful society (D'Ambrosio and D'Ambrosio 2013). D'Ambrosio (1999), for example, claims:

> It is a high priority that children learn how to deal critically with the major issues of inequity and environmental decay. The best tool to deal critically with these issues is provided by mathematics. (1999, p. 68)

Thus, the focus of numeracy is more than the acquisition of technical mathematical skills that enable effective functioning in personal, civic and work life as there is greater emphasis on the development of additional critical capabilities that empower individuals and collectives to work for the greater good of society at a local level and also more broadly.

3 Facets of numeracy

The intrinsic usefulness of mathematics means that it provides a way of reasoning about, and functioning within, different societies with diverse social norms, values, cultures, and traditions. Unsurprisingly, this gives rise to related but distinct ways of conceptualising and identifying numeracy practices. Socio-political influences, represented and exerted by stakeholders within societies, also shape the vision for a numerate citizenry (Jablonka 2003). Alternative ways of talking about numeracy related capabilities include mathemacy (Skovsmose 1994), matheracy (D'Ambrosio 2003), and critical mathematical numeracy (Frankenstein 2010). While these terms coalesce around a common theme, their originators identify unique conceptual foundations for these ideas. The importance of a numerate workforce is gaining greater recognition within the workplace (e.g., Straesser 2007). Numerate practices are also increasingly mediated through the use of digital technologies, which some suggest requires the revision of what it means to be numerate (e.g. Noss 1998; Zevenbergen 2004; Hoyles et al. 2010). There is also discussion of numeracy related capabilities through lines of enquiry that focus on domain specific knowledge, as is the case of financial literacy and statistical literacy (e.g., Bakker and Gravemeijer 2004; Fox et al. 2005). The next section outlines different facets of numeracy through a discussion of five identifiable but overlapping aspects: critical views of numeracy; numeracy in the workplace; the role of technology in numerate activity; and statistical and financial literacy.

3.1 Critical aspects

A number of researchers maintain that an important aspect of becoming numerate is developing the capability to take a more critical view of the world—from personal, social, and political perspectives. Zevenbergen (1995), for example, draws on Habermas' (1972) tripartite theory of knowledge to distinguish between three types of numeracy: basic technical skills related to mathematics learnt without any reference to life-related contexts; practical skills related to the capacity to apply technical skills appropriately within life related contexts; and critical or emancipatory capabilities in which mathematics is utilised in social or ideological critique. This third type of numeracy is an empowerment related to making socially conscious decisions and the ability to develop arguments that support or challenge positions assumed by authority (Ernest 2002; Madison & Steen 2003; D'Ambrosio and D'Ambrosio 2013).

The idea that becoming numerate is an empowering capability is endorsed by Ernest (2002) who argues:

> The empowered learner will not only be able to pose and solve mathematical questions (mathematical empowerment), but also will be able to understand and begin to answer important questions relating to a broad range of social uses and abuses of mathematics (social empowerment). Many of the issues involved will not seem primarily to be about mathematics, just as keeping up to date about current affairs from reading broadsheet newspapers is not primarily about literacy. Once mathematics becomes a 'thinking tool' for viewing the world critically, it will be contributing to both the political and social empowerment of the learner, and hopefully to the promotion of social justice and a better life for all. (p. 6)

This position is also consistent with that of Frankenstein (2001) and Jablonka (2003) in recognising how mathematical information and practices can be used to persuade, manipulate, disadvantage or shape opinions about social and political issues.

> Critical mathematical literacy involves the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues. It also involves the ability to present data to change people's perceptions of those issues. This critical understanding of numerical data thus prompts individuals to question taken-for-granted assumptions about how a society is structured and enables them to act from a more informed position on societal structures and processes. Frankenstein, (1990, pp. 336–337)

The ethnomathematics research program that emerged from Brazil has strong connections to the critical aspect of numeracy. Influenced by the work of Paulo Freire (1968), who sought to empower the oppressed of society through his work in literacy education, D'Ambrosio (e.g. 1999, 2001) presents enthnomathematics as a framework within which "mathematics can help to fulfil the commitment to children and to promote equity and democracy, dignity, and peace for all of humankind" (1999, p. 131). He sees the role of mathematics in education as (1) enhancing creativity and (2) facilitating full achievement of citizenship. D'Ambrosio (1999) argues that achieving this second aim requires an ability to take responsible decisions.

> To be a responsible consumer an individual must be able to deal, critically, with the optimization of the relation cost/benefit. But it is important to say that optimizing does not mean only to fulfill one's own satisfaction, but also taking into account environmental and social concerns. This is the ethical goal of education. It is a high priority that children learn how to deal critically with the major issues of inequity and environmental decay. The best tool to deal critically

with these issues is provided by mathematics. (p. 68)

D'Ambrosio also asserts that as part of being critical, individuals must be conscious of the consequences of their decisions and actions and have the means to ascertain the wisdom of their judgements. He sees this as essential part of political participation and awareness.

Others have also challenged the values inherent within traditional school mathematics and argued for an approach that aims to realise egalitarianism rather than reproduction. Skovsmose (e.g. 1994), for example, drew on constructs from critical theory and the sociology of education and applied these to mathematics education in order to develop what he termed *mathemacy* for critical mathematics. The purpose of mathemacy is to empower students to become more critical of mathematics, school, and society by sensitising them to social realities such as inequality and disadvantage (Skovsmose and Nielsen 1996). Skovsmose sees mathemacy as broader than mathematics alone, as technological and reflective capabilities are also necessary in achieving its aims.

Like Skovsmose, Steen (2001) contends that the capacity to use mathematics in a critical sense is ever more important in a world where technological innovation and dependency on data are in a state of rapid acceleration into the future. In this "data drenched world", Steen believes quantitatively literate citizens must be capable of thinking critically with data, using available technological tools in order to participate in, and contribute to, critique and influence political decisions that affect their society.

3.2 Numeracy and the workplace

There is now a considerable corpus of research related to the use of mathematics within the workplace including studies that explore a diverse range of settings, for example, nursing (Noss et al. 2002), reception areas in hotels (Kanes 1996), and engineering Strobel and Pan (2011). Research in this area is ongoing as exemplified by a recent special issue of Educational Studies in Mathematics edited by Bakker and FitzSimons (e.g. Bakker 2014; FitzSimons 2014). FitzSimons, for example, identifies transition to the workplace as a challenge in need of further research as, "novice workers need to learn how to adapt to the often tacit social and cultural organisation of their particular occupation and place of work" (FitzSimons 2014, p. 294). While research in this area is extensive, it can be broadly classified into two major themes—one concerned with what mathematics is used in the workplace and how it is used (e.g. Noss and Hoyles 1996; Zevenbergen and Zevenbergen 2009), and the other focused on the role of mathematics in vocational education (e.g., Straesser 2007; Wake and Williams 2000). Regardless of the focus, researchers in this area are in general agreement that the mathematics learnt in school classrooms is rarely transferable to workplace situations without complication (Evans 2000; Hoyles et al. 2010; Straesser 2007) as the context determines which mathematics is used, how it is used, and when it is used (see for example Hoyles et al. 2002). Explanations for the difficulties experienced by workers in making use of the knowledge they have learnt in school classrooms within the workplace include the situated nature of learning (Lave et al. 1984), the visibility (or invisibility) of mathematics within work related contexts (e.g. Hoyles et al. 2010), the backgrounds of teachers of classroom mathematics (e.g. Nicol 2002), and the changing nature of the workplace (Jorgensen 2011).

Some researchers, drawing on the work of Lave (1988), argue that the problem of transfer is strongly related to the situatedness within a community of practice. In this view, the use of mathematics within different practices is characterised by unique ongoing activities, social relationships, and culture based modes of reasoning. This means that the use of mathematics in practice, such as in the workplace, requires the assimilation of mathematical knowledge into more holistic strategies shaped by the nuanced demands of the working context (e.g. Benner 1984). Noss et al. (2002). Thus, developing the capacity to transfer mathematical knowledge to new and different contexts is related to the ability to abstract underlying invariants that are relevant across situations. Through a series of studies Noss and colleagues (e.g. Hoyles et al. 2001; Noss et al. 2002) explored this idea to develop the notion of *situated abstraction* in which mathematical knowledge can be considered to be both situated and abstract.

> Mathematical conceptualization may be finely tuned to its constructive genesis-how it is learned, how it is discussed and communicated-and to its use in a cultural practice, yet simultaneously can retain mathematical invariants abstracted within that community of practice. (Noss et al. 2002, p. 205).

While finding evidence that mathematical concepts existed in both abstract and situated forms, nurses within the study found it difficult to demonstrate understanding of mathematical concepts the further a situation was distanced from the context of their practice.

The visibility (or invisibility) of the mathematics embedded in the practices and processes associated with workplace activity is an issue identified by a number of researchers (e.g. Hoyles et al. 2010; Straesser 2007) as problematic when attempting to understand how mathematics is used in the workplace and thus how to improve its utilisation. Mathematics is often invisible as its presence is hidden at a level below what can readily be seen or noticed within underlying mathematical models, representational or technological tools and other artefacts. The decontextualised nature of how mathematics is predominantly taught and learned in formal educational settings exacerbates this invisibility as it portrays mathematics as disconnected from utility and denies the learner the opportunity to understand how mathematics underpins many workplace activities (e.g. Wake and Williams 2001). Additionally, mathematics is interpreted in different ways when it crosses boundaries between different situations, settings, and contexts (e.g., Kent and Noss 2002), compounding the complexity of the selection of mathematics to use in any specific context.

Teachers of mathematics continue to grapple with the issue of how to help students learn to use mathematical ideas and concepts in contexts away from the classroom. According to Nicol (2002), the personal histories of teachers may be a factor in limiting their attempts to do so as they often come to mathematics teaching from teacher preparation courses where more pure than applied treatments of the discipline dominate. Thus, teachers' own understanding of how mathematics is applied in out-of-school contexts may make it difficult for them to provide students with the learning experiences necessary for them to adapt the knowledge they learn in school to the outside world. While this is a clear challenge, a number of attempts have been made to address this issue through projects that placed teachers in out of school workplaces. For example, Hogan and Morony (2000) conducted a project in which teachers shadowed an employee in a range of different workplaces, in order to help them develop a better understanding of what it means to use mathematics for a practical purpose. Feedback from teachers indicated they felt an enhanced awareness of the different ways in which mathematics is applied in workplaces. This included understanding of the level of tolerances expected in different types of measurement, and the idiosyncrasies ways of applying mathematics—simply because "it works". Some teachers provided reflections on how they might change their own practice based on the experience but it was beyond the scope of the study to determine if and how they did so.

In considering the pedagogical approaches adopted by German teachers involved in pre-vocational education, Straesser (2000) reports that these fall into two general categories—modelling and peripheral participation. The first of these takes place primarily through classroom instruction; the second is related to students learning "on the job" in specific workplaces where the "teaching" is typically done by a mentor who models numerate behaviours in situ. When learning takes place in a classroom, mathematics is typically presented as a separate body of knowledge from which "bridges" must be built to the out-of-classroom context, often in the form of mathematical models. In extreme cases, it is left to the individual to build these bridges themselves when they are required to use mathematics in whatever workplace they find themselves. When learning takes place in the workplace itself, the use of mathematics (or not) is determined by the immediate challenge posed by a specific problem encountered in the process of attempting to complete a task. In this situation, the learner may begin by participating in a peripheral fashion with the expectation that, over time, they will be capable of taking full responsibility for similar tasks.

3.3 Technological aspects

The importance of digital tools in supporting the use of mathematics in private, work, and civic life has been noted by Zevenbergen (2004). Others, such as Hoyles, Noss, Kent, and Bakker (2010), who use the construct of techno-mathematical literacies, have identified the new mathematics based competencies required by societies in which digital technologies are becoming ubiquitous.

A number of research studies have been concerned with the ways in which the use of mathematics in the workplace is being reshaped by technology. Hoyles et al. (2002) conclude, as a result of a national project that focused on workplace skills in the UK that:

> All the sectors exhibit the ubiquitous use of Information Technology. This has changed the nature of the mathematical skills required, while not reducing the need for mathematics. On the contrary, in many cases, a competitive and IT-dependent environment means that many employees are using mathematics skills that their predecessors, or they themselves in the past, did not require. (p. 10)

In a consequent three and a half year study that responded to the perception amongst employers and policy-makers of deficiencies in the technical skills of the labour force, Hoyles and colleagues (e.g. Hoyles et al. 2010; Bakker et al. 2006; Noss et al. 2007), investigated how to improve the mathematical and technical skills of workers. Acknowledging the globalisation and technologicalisation of the means and practices of production, the realities of mass production, the growing mass customisation of commodities, and the increasing dependence on technology for communication with customers and markets, Hoyles et al. (2010) developed the notion of techno-mathematical literacies in which technology has a vital role in mediating mathematical knowledge in different, developing, and emerging contexts within the workplace. This knowledge is more than simply additional mathematical or technological skills but the capability but to bring these together to interpret information and make decisions or provide advice to clientele either directly or by referring them to another worker with knowledge specific to an enquiry (Kent et al. 2007; Bakker et al. 2006). They argue that basic arithmetic skills are of lesser importance compared to conceptual understanding of how artefacts such as graphs and spreadsheets can be used to highlight relationships between factors that can be leveraged to improve production processes and productivity. Thus, it is simply not true that the availability of technology reduces or removes the need for employees to have the technical or mathematical skills necessary to understand products and processes: rather new kinds of techno-mathematical literacies are required.

Zevenbergen (now Jorgensen) (2004, 2011), for example, reports that young employees working in emerging industries (e.g. leisure, hospitality and information technology) deal with workplace problems that involve mathematics in uniquely different ways from more senior colleagues as they are "more likely to approach tasks holistically, to use estimation, to problem solve, to use technological tools to support their work and thinking, to use intuitive methods, and to see tasks esthetically" (2011, pp. 88–89). In particular, the disposition of younger workers to make use of technology means that they place less importance on skills such as mental computation, believing digital tools can be used to relieve them of such mundane aspects of their work, and greater emphasis on more strategic aspects of their work (e.g. planning, problem solving). Thus, technology has influenced young workers' ways of working, acting, and reasoning which in turn results in their reshaping of the structuring practices and the deployment of skills within their workplaces. As a consequence, Jorgensen argues, this allows young workers to solve problems in more inventive ways than their more experienced co-workers.

While research has identified the vital role of digital tools in the practices and process of the workplace, far less research attention has been leveled at the use of digital tools in promoting numeracy within school settings. Previous research efforts into the role of digital tools in enhancing learning within content domains such as number (e.g. Kieran and Guzma'n, 2005), geometry (e.g. Laborde et al. 2006), algebra and calculus (e.g. Ferrara et al. 2006) have been encouraging, however the findings of these studies were not specifically related to the use of digital technologies in the application of mathematics in real world contexts. The role of digital tools in numeracy teaching practice has been addressed by Geiger and colleagues (e.g. Geiger et al. 2014b) as part of a sequence of studies that focused on the use of a rich model of numeracy to enhance classroom numeracy practice (e.g., Goos et al. 2011; Goos et al. 2014). They argue that the effective integration of digital tools into classroom teaching practice can support or enhance students' numeracy capabilities such as: the collection, recording, and analysis of real world data; comparing the features of relevant data sets; critiquing a situation or making judgements.

3.4 Statistical literacy and financial literacy

While descriptions of numeracy are typically broad in relation to the types of mathematics that can be utilised in home, civic, and work life, there are lines of research within the field that are more specific to the types of mathematics employed—these include research into statistical literacy and financial literacy.

Interest in statistical literacy as a construct and an educational goal are often associated with the Cockcroft report where it was stated:

> …the need in the modern work to think quantitatively, to realise how far our problems are problems of degree even when they appear to be problems of kind. Statistical ignorance and statistical fallacies are quite as widespread and quite as dangerous as the logical fallacies that come under the heading of illiteracy (Cockcroft, 1982, para. 36)

The need to educate communities in the use of statistical information and how it can be used to identify important issues that require attention as well as to influence opinion or even deceive has since given rise to the exploration of how topics such as average, sampling, variation, inference, probability, and data handling are used in society (Watson and Callingham 2003). As Steen points out:

> As information becomes even more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen today is as vulnerable as a illiterate peasant of Gutenberg's time. (Steen 2007, p. xv)

According to Gal (2002), becoming statistically literate is achieved through developing two capabilities: (1) an ability to interpret and critically evaluate statistical information within diverse contexts; and (2) the capacity to discuss and communicate the interpretation and evaluation of statistical information. These capabilities allow an individual to form opinions or concerns and to make judgements through data based evidence.

Despite the importance of developing statistical reasoning and thinking skills, there are a number of challenges faced by those in statistical literacy education including: the complex nature and counter intuitiveness of some aspects of statistical reasoning and thinking; the reliance on a learner's knowledge of other forms of mathematics; the misleading nature of some contexts in statistical situations; the messiness associated with data drawn from authentic contexts; and the open endedness of interpretation based on initial assumptions (Ben-Zvi & Garfield 2004).

Financial literacy is another expansion of the broader concept of numeracy into practices that have specific domain knowledge demands. Huston (2009) argues that financial literacy can be conceptualised as having two dimensions: understanding (personal finance knowledge); and use (personal finance application). In this view, a financially literate person must have both the knowledge necessary to complete financial transactions and planning and the ability and confidence to make financial decisions.

While research into financial literacy is limited within the field of mathematics education (Sawatzki, 2013), government agencies and financial institutions are conducting investigations into the financial literacy of target populations. For example, in Australia, an evaluation of the financial education program *Making Cents* among low socio-economic status school communities conducted by the New South Wales Department of Education and Training (2009) revealed the program was most successful when parents were actively engaged with their students in the project. However, the evaluation also found that 82 % of participants experienced difficulty with basic calculations needed for financial transactions (e.g., calculating change or reading bank statements).

Despite the obvious need for education programs in this area, Huston (2010) argues that to date, financial education programs have yielded mixed findings in relation to measurable benefits. She maintains that a clearer understanding is needed of what relevant knowledge, skills, and modes of reasoning are necessary to be financially literate. Further, efforts must be deployed in order to develop methods for assessing the success of financial literacy education programs.

4 Ways of implementing numeracy teaching and learning

While there is increasing international importance attached to the development of numeracy skills and capabilities, especially among young people, how to best foster the development of numeracy remains an issue of debate and continuing research. Within many educational jurisdictions, the learning and teaching of numeracy is expected to take place within traditional mathematics courses and so becoming numerate is seen as an outcome of subject specific instruction. Alternatively, some curriculum authorities have developed numeracy specific school subjects as part of a suite of mathematics course offerings for students, such as the case in South Africa (Venkatakrishnan and Graven 2006; SA DoE 2003).

Steen (2001), however, argues that for students to become numerate, they must engage with tasks that demand the use of mathematics in multiple contexts, and so effective numeracy instruction must take place in all school subjects, not just mathematics. Others have provided illustrations of what this might look like in a school context. Kissane (2012), for example, provides a summary of numeracy related projects within the Australian context, including the *Numeracy across the curriculum* project (Hogan et al. 2004) and *Numeracy: families working it out together* (DEST 2003). He also provides examples of numeracy tasks in curriculum areas other than mathematics, pointing out that both demands and opportunities exist and that teachers need to make connections between numeracy and the curriculum area, even when it is not explicit in the curriculum. This approach appears to have some resonance in Europe where a recent report on the state of mathematics education in Europe found:

> Over the last decade—and most notably since 2007—the great majority of countries have made revisions to their mathematics curricula to focus more on the competences and skills to be achieved rather than on the content to be covered. In addition, current mathematics curricula have reduced subject content in favour of more cross-curricular links and increased focus on the application of knowledge and problem-solving. (European Commission 2011, p. 143).

There are, however, critics of this trend. For example, Lee (2009) argues against any integration of subjects and claims that constructs such as numeracy should be considered an "educational by-product … [that results from] … studying mathematics, physics, chemistry, biology, business studies and various other subjects in which numbers and mathematics concepts find application" (p. 218),

Each of these approaches has implications for the structuring of curriculum and how it is organised as the types of connections made between subjects (or not) will result in different ways to deliver curriculum objectives (Goos and Askin, 2005). The ways in which curriculum can be structured vary through a continuum anchored by subject centred approaches in which focus is maintained on the specific knowledge and practices of a discipline in isolation from other bodies of knowledge, to those that require the integration of a range of discipline specific skills and modes of reasoning in order to address tasks that are problematic in nature. It would appear that there is an increasing level of challenge associated with greater degrees of integration, although examples exist where fully integrated curriculum practices have been successful in some schools (Lingard et al. 2001). The challenges of integration are associated as much with the complicated nature of curriculum reform as with the complexity of designing genuinely integrated approaches to teaching and learning across the disciplines (Wallace et al. 2007).

Two broad categories of integrated curriculum approaches have emerged in numeracy research: working through interdisciplinary enquiry to combine two or more disciplines in a single program; and deliberate, mindful planning that takes advantage of numeracy opportunities that exist when teaching within disciplines other than mathematics.

4.1 Interdisciplinary enquiry

Interdisciplinary enquiry clearly refers to tasks, teaching programs or approaches to instruction that connect two or more academic disciplines. Nikitina (2006), for example, presents interdisciplinary teaching and learning as a continuum anchored between the extremes of no connection at all between disciplines through to total integration into a single (perhaps unique) subject. Interdisciplinary enquiry has been reported to offer great potential for engaging adolescent learners (Venville, Wallace, Rennie and Malone, 2002). Venville et al. concede, however, that the practice of interdisciplinary enquiry brings with it challenges that educational institutions often struggle to address when attempting to move away from existing discipline-based approaches. These challenges include the structure of schooling, much of which is designed to protect disciplinary interests, and factors such as discipline-based teacher training, assessment, and parental preferences for a traditional discipline-based curriculum contribute to maintaining.

In reporting on a large trans-national educational program, the Common Problem Solving Strategies as Links Between Mathematics and Science (COMPASS) project, Maass, Garcia, Mousoulides, and Wake (2013) argue that it is only in the world of school that the application of knowledge to solve problems is connected to a single "academic" discipline while interdisciplinary knowledge application is required to deal with problematic situations in the real world. They observe that in many countries curriculum documents are encouraging of interdisciplinary but the single-discipline training of most teachers and the limited number of resources available to support learning across the curriculum remain obstacles to the implementation of this approach. Maass et al. (2013) claim to have addressed these challenges by bringing together mathematics and science learning within the COMPASS project. The project leaders consisted of experts in the fields of interdisciplinary tasks, real-life-based tasks, modelling tasks, and ICT mediated tasks, and the design and implementation of teacher professional learning programs. These researchers report that teaching changed as a result of the program to a more student centred and application oriented approach to teaching. Teachers and students indicated that the project materials were relevant, interesting and motivating; however, opinion was divided on the appropriateness of enquiry based learning for day-to-day teaching.

In a discussion of the multidisciplinary nature of numeracy, some researchers, for example Miller (2010) and Ward (2005), stress the importance of developing students' capacities to effectively communicate the results of problems that are solved using mathematics. Miller (2010) argues that the ability to communicate the solution to a problem, after interpreting the result of calculations and checking the viability of the proposed solution within its original real world context, is as important a skill as the capability to complete the calculations. This ability draws on learning in mathematics, English, and the substantive disciplines (e.g., science, history). The argument is supported through examples that illustrate the need for students to draw on skills that are generally developed in English when writing about quantitative phenomena. English skills, for example, can be applied to organising writing for a science report or political essay by adapting the standard essay structure. Consistent with this view Ward (2005) argues that children's literature can be used as a means "to connect the abstract, symbolic language of mathematics with their own personal world" (p. 133).

The Archimath Programme (Soygenis and Erktin, 2010) is an example of another attempt to foster numeracy development through the combination of different disciplines, in this case architecture and mathematics. The goal of the course (Year 4–8 students) was to develop student awareness of the built environment, to initiate an effort to improve it, and to illustrate to students the utility of mathematics in the real world. The course designers aimed to leverage off concepts common to both architecture and mathematics such as space, proportion, and measurement. Analysis of students' responses to activity sheets revealed that although they were capable students of mathematic they struggled with representing three dimensional objects, had little knowledge of the built environment, and were challenged by tasks that required the use of scale. Each of these issues required attention as part of developing their understanding of knowledge associated with the built environment.

4.2 Taking advantage of numeracy opportunities within disciplines other than mathematics

Across-the-curriculum approaches also include those that attend to numeracy opportunities within disciplines other than mathematics. This means that the teachers must have the capacity to recognise when a numeracy opportunity arises and the skill and disposition to take advantage of such opportunities.

Quinnell, Thompson and LeBard (2013) argue that a disabling anxiety related to the use of numerical skills or an inability to transfer mathematical competencies across disciplines into a science based context are major limiting factors in students' capacity to make use of the processes of scientific inquiry at tertiary level. They propose a model for science pedagogy that maps the points where students become disengaged, which also coincide with the reliance of scientific processes on numeracy skills. This finding implies that approaches aimed at improving the quantitative skills of science students should not focus only on basic mathematical skills and routines alone but also on students' confidence and opportunity to use these skills in more complex situations.

Commenting on the necessary knowledge and capabilities necessary to study the social sciences, Crowe (2010) argues that such quantitative skills must be developed within subjects themselves as school mathematics instruction does not always foster students' capabilities to "make reasonable judgements of and inferences from information presented to them" (Crowe, 2010, p. 105). In order to make these judgements and inferences students must be capable of: interpreting raw numeric data; discerning the meaning of percentages in a specific context; understanding the meaning and implications of an average; and interpreting graphs and charts. These skills enable students to ask questions of data and look beyond superficial interpretations.

Offering a different perspective on learning within the social sciences, Lake (2002) proposes a five step method, based on an adaptation of the SOLO taxonomy, that he contends makes "explicit the expectations and mechanisms for the interpretations of relevant information and providing opportunities for the active development of those skills" (p. 6). In this approach, analysis starts with the context (the what). The next step (the point) focuses attention on notable data points (e.g. range, maximum, minimum, outliers). In the third step (trends), students look for relationships within a single data set. The analysis here is qualitative and then quantitative if the latter is needed. In the next step (relation) students interrelate the data from more than one data set. The final step (meaning) requires students to interpret the data and possibly evaluate its validity and reliability. Lake describes the ability to interpret and evaluate the data in this manner as indicative of students who have a deep understanding of issues related to social studies.

Phillips (2002) observes that the introduction of the *Numeracy across the Curriculum* strand of England's Key Stage 3 Strategy has sometimes been interpreted as requiring history teachers to teach mathematics in history, for example by asking students to draw a pie chart of a day in the life of a monk. This activity focuses on the task of drawing the pie chart but has no relationship to any historical question. He argues, however, that mathematics should be seen as a tool to aid historical understanding. In support of this argument he offers a number of examples where mathematics is used enrich students' understandings of historical events and concepts, such as calculating the cost of ammunition during World War 2 to illustrate the effect of the war of the British economy or the use of data to highlight financial implications of the slave trade.

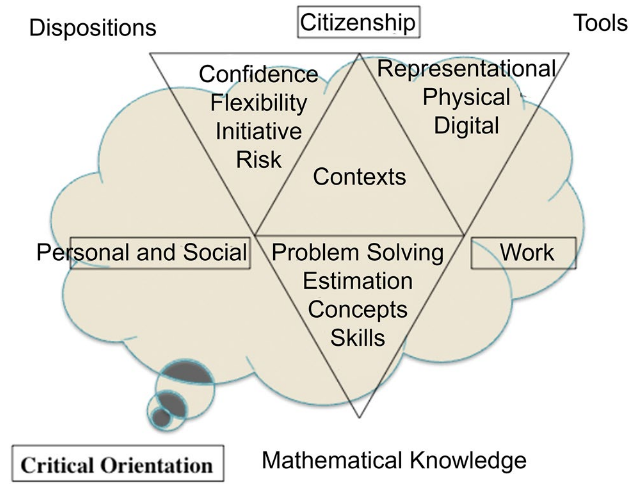

Fig. 1 A model for numeracy in the 21st century (Goos et al. 2014)

In responding to a perceived lack of numeracy skills among students in a microeconomic theory course at tertiary level, O'Neill and Flynn (2013) introduced an increased emphasis on quantitative reasoning—a construct related to numeracy that focuses on mathematical thinking and analysis. Their approach included three elements, logical thinking, making decisions, and mathematics in context, which they attempted to realise by focusing on three procedures, working with graphs, creating mathematical models, and explaining the results and meaning of quantitative results. Students' pre- and post-course responses to a survey instrument designed to ascertain attitudes to quantitative reasoning indicated an improvement in attitudes to using quantitative reasoning relevant to an economics context. Further, there was also moderate improvement in students' attitudes towards using quantitative reasoning in a non-economics context.

In a series of research and development projects, Goos and colleagues (including Geiger, Dole and Forgasz in different combinations) investigated the effectiveness of a teacher professional learning program aimed at enhancing numeracy teaching practice across a range of disciplines (e.g. Geiger et al. 2013; Goos et al. 2011; Goos et al. 2014). This program was based on a multi-faceted model of numeracy and involved teachers at both primary and secondary levels. The numeracy model (Fig. 1) incorporates the four dimensions of *contexts*, *mathematical knowledge*, *tools*, and *dispositions* that are embedded in a *critical orientation* to using mathematics. These dimensions are described more fully in other publications (e.g., Geiger et al. 2014a), but summarised in Table 1.

The model was constructed as an accessible instrument for teachers' planning and reflection and has been

Table 1 Descriptions of the elements and critical orientation of the numeracy model

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position

validated in earlier work when used as a framework for: auditing the numeracy demands of mathematics curricula (Goos et al. 2012); analysing teachers' attempts to design for the teaching of numeracy across the curriculum (Goos et al. 2011); mapping teachers' learning trajectories in effective numeracy pedagogy (Geiger et al. 2011), gauging students' perspectives on their numeracy learning (Geiger et al. 2014a); and using digital tools to enhance numeracy teaching and learning across the curriculum (Geiger et al. 2014b).

5 Assessment

The assessment of numeracy, as distinct from judgements about the attainment of purely mathematical skills and capabilities, has a relatively recent history. The large scale international assessment of numeracy can be traced back to the *Adult Literacy Survey* (ILS) (OECD and Statistics Canada 2000), conducted in 1994–1996. This international comparative survey was aimed at the numeracy skills of adults aged 16–65. Since this time, a number of assessment frameworks for the international comparison of mathematics or numeracy performance, for example, *The International Mathematics and Science Study* (conducted 1995, 1999, 2003, 2007, 2011) and *Adult Literacy and Life Skills Survey* (conducted 2005), have been developed for different target groups and different purposes. Such surveys have used different definitions of numeracy/mathematical literacy to frame assessment programs. For example, varying emphasis has been placed on the degree of sophistication of the mathematics employed and on the type and authenticity of contexts utilised (for a broad discussion of such issues see Gal and Tout 2014). Currently, there are two international testing regimes related to numeracy that have risen to international prominence, the *Programme for International Student Assessment* (conducted 2000, 2003, 2006, 2009, 2012) and the *Programme for International Assessment of Adult Competencies* (conducted 2012). PISA makes use of the term of mathematical literacy for the construct that is assessed while PIAAC uses the notion of numeracy.

The purpose of PISA is to ascertain the effectiveness of educational systems in relation to reading, mathematics and science literacy of 15 year old students with some countries taking the additional options of problem solving and/or financial literacy. In the case of mathematical literacy, PISA is designed to assess if students can make use of their mathematical knowledge in life related contexts as a measure of their readiness for their active participation in society. To date, 70 countries have been involved in the program. This assessment program is conducted every 3 years with a different major domain each time; for example, mathematics literacy was the major domain in 2012 while science will be the main focus of the 2015 round.

PIAAC collects and analyses data related to the skill level of adult populations and how these skills are used in different contexts. Twenty-four countries participated in the first round of PIAAC in 2012. This survey provides information about key information processing skills, *literacy*, *numeracy* and *problem solving in the context of technology-rich environments*, possessed by individuals within countries and also offers a comparison of skills across countries.

While the purpose of both of these testing regimes is to inform government policy making, limited research has been completed on the impact of such assessments on national education reform. Breakspear (2012), however, drew on the results of a survey of country practices to conclude that PISA has become a reliable benchmarking instrument for student performance internationally and has also had an influence on policy reform in participating countries, particularly those elements related to assessments and accountability. Further, the identification of high performing nations such as Finland has led to attempts by countries to enhance their performances by cross national policy learning and borrowing.

Large scale international data sets produced by programs such as PISA and PIAAC have provided impetus for never-before-available international insight into the types of mathematical knowledge that have been acquired by the citizens of nations (e.g., Kieran 2005) and opportunities for future research within mathematics education (e.g. Neubrand 2005), however, such studies are not without criticism. Unsurprisingly for a large scale statistical enterprise, there have been questions raised about PISA from the perspective of technical and methodological validity (e.g., Kreiner, 2011). Further, concern has been expressed about how data are used as a simplistic measure for international comparison by governments and the use of findings by education systems to promote simplistic performance based reforms at school level. Williams (2005), for example, has cautioned against accepting the relationships suggested by PISA data analysis as causal as this could lead to the naive and unfounded direction of education policy.

The assessment of students' numeracy capabilities is also taking place at more local levels. PISA has itself developed tests for schools to make their own judgements about their performance against international benchmarks. The *PISA-based test for schools* is an assessment instrument that can be used to provide descriptive information and analysis on 15 year old students comparable to main PISA scales. The test can also provide analysis that links student performance to other factors such as socio-economic background and students' attitudes and interests.

Individual nations have developed national numeracy assessment strategies. The approaches to, and focus of, these assessments varies from country to country. Data generated by such assessment provides for the identification of areas of improvement, and potentially, where educational resources might be best deployed. However, how such data are used can be a source of public concern and criticism. Australia, for example, conducts the *National Assessment Program—Literacy and Numeracy* (NAPLAN) which consists of national assessments administered to all students in Grades 3, 5, 7 and 9. The expressed purpose of this program is to provide a system-level snapshot of students' literacy and numeracy capabilities, however, schools also use data generated via this program to plan for improving student learning. At the same time, individual school results are available to the public at large via the *My School* website (http://www.myschool.edu.au/) where these can be compared and scrutinised. This availability has been criticised for setting up a competitive culture between schools where parents make choices about where to send their children based on NAPLAN results leading to a "brain drain" away from socio-economically disadvantaged, lower performing schools.

Assessments have also been developed to ascertain students' numeracy skills at a more domain specific level. Dole and colleagues (e.g., Hilton et al. 2013a; Hilton et al. 2013b; Dole et al. 2007; Dole et al. 2012), for example, have developed a two tiered diagnostic instrument aimed at identifying situations in which students could and could not apply proportional reasoning in cross-curricular contexts. The instrument has proved valuable: as a diagnostic tool for classroom teachers within their study; for the design of classroom activities; and the planning of teacher professional learning workshops. Taking a different approach to exploring the impediments to students' numeracy learning, Lowie and Diezmann (e.g. Lowrie and Diezmann, 2009; Diezmann and Lowrie 2012) have investigated the importance of graphics for providing information in classroom learning tasks. Because such tasks are also employed in national and international testing instruments for numeracy, it is important that students are capable of interpreting information graphics in order to be successful. Through a series of studies they found that many students lacked proficiency in basic spatial skills which impact on their test performance. Consequently, they argue that graphical, linguistic, and contextual components should be considered in isolation and in integrated ways as elements of task design in teaching and learning contexts as well as in test development.

6 Conclusion and directions for future research

The concept of numeracy is relatively new and so research into the nature of numeracy and how best to promote numeracy capabilities is only beginning to emerge. A confounding factor in the pursuit of this research agenda is the diversity of opinion on what is meant by numeracy. Current interpretations vary from the acquisition of basic mathematical skills through to richer conceptions that identify critical problem solving capabilities within real world situations, alongside basic skills, as essential knowledge for informed participation in personal, work and civic life (e.g. Steen 2001; Goos et al. 2014). More radical perspectives position numeracy as an essential capability for challenging social injustices and for taking actions that promote a more equitable and democratic society (D'Ambrosio, 2001, Frankenstein 2001; Skovsmose, 1994). While this diversity contributes to the debate about the role of mathematics within society, it means that research findings can only be interpreted via reference to the definition of numeracy that anchors each study. Thus, the results of most numeracy research can often be limited to the specific socio-political cultural context in which a study is situated.

Given the diversity of what is understood as numeracy, it is unsurprising that there is a range of themes under which numeracy research is conducted. The connection between numeracy and the workplace has enabled exploration of a broad range of occupations and professions (e.g. Kanes 1996; Noss et al. 2002; Wake and Williams 2000). Most research identifies the visibility (e.g. Straesser 2007) and situatedness of mathematics in the workplace (e.g. Benner 1984) as vital factors in determining if and how mathematics is brought to bear on workplace problems. In attempting to understand how workers might generalise their mathematical knowledge across contexts while recognising the situatedness of such knowledge, Hoyles, Noss and others (e.g. Hoyles et al. 2001; Noss et al. 2002) have developed the notion of *situated abstraction*. More recently, the work of Jorgensen (e.g. 2011) provides evidence that young workers are less focused on the use of basic mathematics skills in completing workplace tasks and take a more holistic problem solving approach than their experienced co-workers. Her research indicates that there may be generational differences in the way mathematical capabilities are employed in the world of work. While previous studies have provided insight into the nature of numeracy in the workplace, there appear to be few investigations that have attempted to clarify what role of the mathematics taught in school classrooms has in assisting novice workers to transition successfully into new workplaces (FitzSimons 2014) or how school mathematic might change in order that such transitions are more effective.

Numeracy capabilities are often connected to the use of digital tools in the workplace (e.g. Hoyles et al. 2010; Zevenbergen 2004). The notion of techno-mathematical literacies has been introduced by Hoyles and colleagues (Hoyles et al. 2010; Kent et al. 2007; Bakker et al. 2006) in order to describe and explain the intricate mediating relationship between technology and mathematical knowledge. Zevenbergen (2004) also observes that the "new" workplace is one in which the mundane work associated with mathematical calculations is now deferred to digital technologies and workers are increasingly taking technology integrated approaches to the solution of problems. However, despite the near ubiquitous role technology now assumes in the world of work, there appear to be few studies that have investigated the potential of digital tools to advance the numeracy capabilities of students as part of schooling.

Proponents of the domain specific aspects of numeracy, statistical literacy, and financial literacy make strong cases for the importance of research in these areas (e.g. Watson and Callingham 2003; Sawatzki, 2013). While statistical literacy is well established as a field of study, it would appear that there has been limited attention to financial literacy from mathematics education researchers. Given the impact the ability to effectively manage financial transactions and to make financial decisions has on the lives of individuals and families this appears to be an area worthy of greater research focus.

The question of how to best promote numeracy capabilities remains an open question. Research into interdisciplinary or cross-curricular approaches, however, is showing great promise. While this research has demonstrated that teachers are capable of designing rich numeracy task and activities and to take advantage of numeracy opportunities in the range of subjects across the curriculum (e.g. Goos et al. 2014; Geiger et al. 2014b) there appears to be little research that outlines the characteristics of effective numeracy tasks or how these tasks promote student learning.

Various programs of large scale numeracy assessment continue to attract the support of a large number of countries around the world. The results of programs such as PISA and PIAAC are influencing educational policy within participating nations (Breakspear 2012). The collection and analysis of data in order to provide a picture of students' numeracy capabilities is also taking place within individual countries using locally developed assessment instruments. While the information gathered both nationally and internationally has great potential to provide direction for schools in how to improve their approaches to developing students' numeracy and the targeted deployment of educational resources, concerns have been raised about how efforts to improve numeracy might be distorted by promoting competition between nations and even schools. As pointed out by Neubrand (2005), large scale assessment programs provide great opportunity for mathematics education research, however, if researchers simply rely on the data generated by such assessment programs, then the vital role of teachers in making judgements about their students' numeracy capabilities will be neglected. Thus the assessment of students' numeracy development at school level is an area that is also in need of greater research effort.

The preceding synthesis demonstrates that numeracy is a diverse and rich field of research despite its relatively short history. As a consequence, there remain important issues to be researched within each theme in this field. It is important to remember, also, that an important aspect of research is to construct evidence that helps shape education policy in a positive and productive way. There are examples throughout the world when policy decisions and consequent activity in schools appear to be naive reactions that have resulted from superficial interpretations of the data generated via numeracy assessments. It is to be hoped that ongoing research into how to become increasingly numerate will provide a stronger influence on future policy directions.

7 Contributions to this issue

This issue in cludes contributions from authors to one or more of the areas outlined in this synthesis. The issue begins with three papers that explore the nature of numeracy, the challenges associated with becoming numerate, and the influence of the idea of numeracy on curriculum. Callingham, Beswick and Ferme (2015) explore the notion of numeracy from the perspective of professional capital and in doing so seek to extend understandings of what it means to be numerate. Bennison (2015) explores a similar theme in considering the development of teachers' numeracy identity. This is followed by an article from Venkat and Winter (2015) who argue that 'boundary crossing' between mathematical and contextual activities is a critical feature of numeracy teaching. Rosa and Orey (2015) then provide a perspective on numeracy from the Brazilian ethno-mathematical tradition. This is followed by a critique of the notion of numeracy by Jablonka (2015) who contends that the evolution of numeracy is driven by a weakening of the insulation between discourses—a process of 'declassification'.

The issue then turns to articles which focus of the teaching and learning of numeracy in school classrooms. Geiger, Forgasz and Goos (2015) outline and describe the learning trajectories experienced by teachers as they attempt to incorporate elements of critical thinking into numeracy tasks. The professional learning of teachers is also explored by Liljedahl (2015) in study that investigated the development of teachers' understanding of numeracy through the activity of designing numeracy tasks. Taking an early childhood perspective on numeracy, Tsamir, Tirosh, Levenson, Tabach and Barkai (2015) examine number composition and decomposition activities in kindergarten and Mulligan (2015) explores the idea of data modeling in the early years of schooling.

Moving beyond schooling, Straesser (2015) and Wake (2015) take different perspectives on the issue of numeracy in the workplace and Tout and Gal (2015) trace the evolution and future of large scale international assessments of numeracy. The issue concludes with a commentary provided by Askew (2015).

These articles represent a balance between theoretical contributions, reports on empirical studies, and informative descriptions of the "state of the art". The articles have been developed out of a diverse range of contexts resulting in a "landscaping" of the field of numeracy internationally. We hope this provides a rich but coherent picture of an increasingly important issue.

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