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ORIGINAL ARTICLE

Language-related learning of mathematics: a comparison of kindergarten and primary school as places of learning

Marcus Schütte

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Abstract The study described in this paper presents research results from two separate projects, as well as two theoretical pillars that form the foundation of these projects. The first of these pillars focuses on the interactional relationship between mathematical and linguistic learning; the second defines an understanding of learning as a process that unfolds through institutions, whereby the most important element in the education of a child is the child him/herself rather than the specific place of learning. That is, for the purposes of this paper, the same linguistic issues arise independently of whether the child is in primary school or kindergarten. The results of both projects are used in the reconstruction of linguistic conditions of the learning of mathematics. These analyses can lead us to conclude that improvement in children's linguistic competences should be only a secondary goal; the real key to the more effective promotion of children's education in terms of subject-related learning seems to lie in the improved education and further education of teaching staff. The results should thus form a theoretical basis for the development of unified support and training concepts for children and educators.

Key words Primary school and kindergarten · Subject-related learning · Formal and mathematical linguistic register · Mathematical discourse · Differences in framing · Implicit pedagogy

M. Schütte (✉)
Institut für Erziehungswissenschaft, Technische Universität Dresden, Weberplatz 5, 01219 Dresden, Germany
e-mail: marcus.schuette@tu-dresden.de

1 Introduction

International comparison studies such as PISA provide a suitable starting point for this examination of mathematical learning processes—these studies unanimously come to the conclusion that, across the spectrum of the German school system, the principle of early selection fails to lead to higher levels of achievement, and in fact causes a situation where inequality of opportunity among school children is high by international comparison. Eight of the 11 OECD countries with the highest equality of opportunity succeed for example in achieving a high level of literacy among school children (cf. OECD 2005). The key to establishing school systems that are successful in international comparisons therefore seems to lie not in early selection, but in the targeted integration of a diverse pupil population. The simple formula could be phrased thus: equality of opportunity leads to quality across the spectrum. To this end, dealing constructively with linguistic-cultural diversity and the integration of pupils from so-called 'uneducated' families becomes of the utmost importance. At present, such approaches are found in German schools to a limited extent at best.

One explanation for the German approach to diversity in schools was developed by Gogolin (1994). Gogolin shows how, up to the end of the nineteenth century, the German school system was rigidly hierarchically organised, which led to participation in the education system being largely defined along lines of social class. The school system was organised according to the conviction that it was better that children's education should be completed in only one language—German. This basic preconception still lingers today, according to Gogolin, not least because the roots of the approach have long been forgotten. The monolingual education of a diverse pupil population consequently goes unquestioned in the modern German school system; if there is any kind of discussion, it is rather about how to maintain this norm (cf. Gogolin 1994).

The great problem today is that this policy no longer affects only small fringe groups within the pupil population—after many years of high levels of immigration, almost one-third of all pupils in German schools now come from immigrant families. Teaching urgently needs to be adapted to this linguistic-cultural plurality in order to prevent a large proportion of pupils being excluded from successful participation in the German school system, thus endangering the success of the school system as a whole (cf. OECD 2006).

Such evidence of the monolingual self-understanding of German schools means that it is no surprise that proposed solutions tend to be universally associated with overcoming children's 'deficits in the teaching language', in this case German. Obviously, mastery of the teaching language has a significant influence on a pupil's level of achievement in class, including in mathematics (cf. Deutsches PISA-Konsortium 2004; Heinze and Rudolph-Albert 2008; Schütte 2009; Schütte and Kaiser 2011). This mono-causal perspective, however, fails to ask how a diverse pupil population's access to education could be increased through innovative teaching concepts that do not assume competency in German as a precondition for learning. There is great potential for achieving change through approaches that perceive every subject lesson as a place where language is learnt, and which integrate the linguistic competences that all children bring with them into the classroom into this learning process [cf. Riebling 2013; see also I and Chang in this volume (2014)].

In the international literature on mathematics education, approaches have been described in recent years that accord a special significance to language and communicative competence in relation to the learning of mathematical content (cf. among others Pimm 1987). More recent approaches have gone a step further. For example, Moschkovich (2002) places the emphasis on the discursive element of mathematics learning. In her 'situated-sociocultural perspective' on the learning of mathematics, paradigm change involves moving the focus from the deficits of learners towards the resources and competences of a diverse pupil population. According to this perspective, mathematics learning takes place in a social context, into which the participants bring different ways of seeing the situation, which are interactively negotiated (cf. ibid.). Relevant works in German include particularly those by Maier (1986, 2006), who observes that teachers in mathematics lessons—above all in primary school—mostly attempt to introduce concepts through visualisation. According to Maier, mathematical concepts can only be understood to a limited extent through such visualising introductions; this is mainly because mathematical objects are not real, but abstract (cf. Maier 1986), and the dilemma can only be resolved on a linguistic-symbolic level (cf. Schütte 2009, 2010).

This context is reinforced by the fact that, partly also because of the introduction of educational standards, linguistic negotiation in mathematics classes is becoming ever more important in German schools. For example, pupils are now expected to achieve process-specific competences such as communicating, arguing, representing mathematical ideas, modelling or problem-solving (cf. KMK 2004). Pupils are further required to extract meaning from themes and language in texts, apply this to corresponding exercises and produce oral or written compositions on this basis. This increasing importance of language in subject learning, even in early learning, leads us to further consider learning processes that take place beyond the limits of primary school.

According to Fthenakis (2009), in any investigation of children's development it is necessary to examine all the different places in which a child learns (on the learning of mathematics in families see Tiedemann 2012). Following this idea, beginning in the school year 2002/2003 (cf. Lisker 2010) diverse attempts were made in all the German states to diagnose and compensate for the linguistic deficits of children reaching school age—without asking, however, whether the children already had linguistic competences that could be used to successfully develop subject learning beyond existing norms. This kind of perspective follows the monolingual orientation described above. The results of an IQB ('Institut zur Qualitätsentwicklung im Bildungswesen'—Institute for Educational Quality Improvement) comparison study of the achievement level of German primary-school pupils (cf. Stanat et al. 2012) reveal that these recent efforts have not achieved the desired successes. Few concepts methods have been developed to promote interdisciplinary linguistic competences within subject-specific, in this case mathematical, learning. Reviews of immigration research in educational science (cf. Gogolin 2006), as well as current reviews of research in mathematics education (cf. Heinze et al. 2011; Schütte and Kaiser 2011; Schütte 2010), conclude unanimously, however, that it is not the mastering of general linguistic competences that is most significant for successful subject learning in mathematics, but the gaining of competences in a subject-related academic language. The interacting positive and negative influences between subject-related and linguistic competences are accorded inadequate attention in current educational concepts; the research programmes seem to be lacking that might establish the foundations for targeted educational programmes. In relation to the German school system in this context, it is to be criticised that there are no comprehensive programmes or examples of practical educational applications where the existing linguistic abilities of pupils learning German as a second language are appropriately valued (cf. Gogolin 1994; also Schütte 2009). Of course this excludes some attempts made by individual schools that offer bilingual teaching, or which—still rarer—aim at a 'lifeworld multilingualism' (Gogolin 1994, p. 16, translated by the author; cf. Meyer and Prediger 2011).

The results of the study described in this paper reveal common elements in the linguistic shaping of mathematical learning processes that transcend both place of learning and the age of the child. The results seem to support the hypothesis that the key to the more effective promotion of children's education in terms of subject-related learning lies primarily in the improved education and further education of teaching staff, which will lead to an improvement in children's linguistic competences only as a secondary outcome. Reducing the problem to one of how to improve linguistic competences that do not correspond to 'the norm' is, as described, an inadequate reaction.

The following section builds a theoretical framework for the study, providing one possible way of analysing language-related mathematics learning. On the basis of this framework, Sect. 3 outlines the methodological framework of two projects in different places of learning, one in primary school and one in kindergarten. Section 4 presents the results on language-related mathematics learning from both projects. The fifth section outlines the theoretical gains while offering proposals for the education of education professionals as well as the reshaping of children's learning processes.

2 Theoretical framework: subject-related and formal linguistic registers and mathematical discourse

In any analysis of the linguistic shaping of mathematical learning processes, Pimm (1987) is a useful point of reference. In relation to learning in schools, Pimm sees teaching staff as 'native speakers' of mathematics (p. 2), compared with pupils to whom mathematics seems as incomprehensible as a foreign language that they have yet to master. In this context Pimm speaks of a 'mathematics register' (p. 74). According to Halliday (1975), a register can be understood as a collection of meanings that are appropriate for a particular function of the language together with the words and structures by which these meanings are expressed. One can speak of a mathematical register if meanings are concerned that belong to the language of mathematics, and if the question is what a language must express if it is used for mathematical purposes. In this sense, mathematical register consists of more than just terminology (cf. ibid.). Pimm (1987) sees the task of learners as being to master a mathematical register step by step, thus becoming able to adopt verbal perspectives like a native speaker of mathematics. In relation to this theoretical approach one can ask how learners engaging in processes of negotiation of meaning in the learning of mathematics can be supported by adults to build up a linguistic mathematical register alongside mathematical discoveries.

Moving beyond subject register while retaining a focus on the challenges encountered by children acquiring a second language, theoretical works by Gogolin (2006) and Cummins (2000) can be referenced in relation to the linguistic shaping of mathematical learning processes. According to Gogolin (2006), the normative expectation for pupils in German schools is to be able to master the kinds of language that are cultivated in class. In this concept of 'academic language' (ibid., p. 82ff.), Gogolin bases her theoretical approach on a concept developed by Cummins (2000, p. 57ff.) in the context of second-language acquisition—'Cognitive Academic Language Proficiency'. Cummins differentiates 'academic language proficiency' and 'conversational language proficiency', indicating that children learning a second language quickly gain linguistic abilities that are useful in their everyday lives but require significantly more time to achieve competences in the academic language of the classroom. A defining characteristic of this academic language is its conceptual written form, which allows a high density of information as well as a distancing from situations—this fails in fundamental ways to correspond to the everyday oral communication of many pupils. Up to now, there are no empirical studies in German that have been able to precisely characterise academic language in the classroom. Gogolin and Roth (2007), however, do mention specific areas that could be relevant in mastering such a language. These include the passive, impersonal expressions, the subjunctive, constructions with 'lassen' ('to have something done'), making substantives, composites and attributives.

According to Bernstein (1977) and Zevenbergen (2001), the linguistic abilities demanded in school correspond to a formal language that is more compatible with the abilities of middle-class children.[1] This formal teaching language is characterised by exact grammatical structure and syntax as well as complex sentence structure. In mastering this formal language, children develop a sensibility for the structure of objects and language that helps them to competently solve problems in the real world as well as in school. Pupils can more successfully take part in the linguistic discourse

[1] On the concept of a formal academic language, see Schütte (2009, 2010).

of teaching if they have ability in the formal academic language used in the classroom. An analysis of the linguistic shaping of mathematical learning processes should therefore address the extent to which children are given a thorough introduction to a formal academic language, or the extent to which they are presented with examples of such a language, allowing them to adequately express mathematical information independently of context in later situations of collective negotiation of meaning in classroom interactions.

Other voices, above all from international research in mathematics education, are critical of such approaches, which they see as limiting the study of the learning of mathematics from a linguistic perspective. Moschkovich (2002) insists on the value of the situated-sociocultural perspective. Approaches that understand the learning of mathematics as 'constructing multiple meanings of words' (Moschkovich 2002, p. 193) and are based on Halliday's (1975) concept of register tend to focus on differences between children's abilities to express concepts linguistically using different registers. This deficit-oriented approach yields the notion that there is a 'target register', mastery of which is sufficient for an understanding of mathematics and success in school, as well as a less significant 'everyday' register. The situated-sociocultural perspective, according to Moschkovich, represents a change in focus from the difficulties and deficits faced by learners to the resources (see also Gibbons 2010) and competences of a diverse pupil population. According to this perspective, the learning of mathematics always takes place in a public socio-cultural context, and represents a discursive activity. However, there is more than only one 'correct' mathematical discourse to be striven for, as often seems to be suggested by authors basing their arguments on the concept of register. Instead, learners participate in mathematical discourses in different communities, and make use of diverse resources from different registers in order to achieve successful mathematical communication (cf. Gee 1999; Moschkovich 2002):

> Although register may be an inherently social concept, Gee's definition of Discourse reminds (and perhaps forces) us to include more than words and meanings… and that aspects other than language, such as interactional and non-language symbol systems, should be included in discourse analysis. (Moschkovich 2002, p. 199)

The concept 'mathematical discourse' makes it clear that, in this perspective, interactional or 'non-language' aspects assume a central importance in understanding the learning of mathematics, in contrast to approaches focused on register.

Diverse works based on interactional approaches of interpretive classroom research in mathematics education are of interest in this context. Krummheuer (1992), for example, argues that mathematics learners in primary school are engaged in 'collective argumentation' (ibid. p. 143), and that mathematics learning takes place through increasingly autonomous participation in this collective argumentation (Krummheuer and Brandt 2001). In interactions in classroom situations, participants interpret what is going on in widely differing ways because of their diverse abilities and backgrounds. Regarding the mutual negotiation of these subjective interpretations, and the constructing of new meanings, Krummheuer (1992, p. 22) writes of 'definitions of the situation' that are constantly adapted and changed. The generating of changed interpretations represents for Krummheuer (1992) a decisive psychological condition for the cognitive development of the individual, since changing one's own interpretations is the basis for the construction of new meanings, and thus constitutive of learning processes. The goal should be to achieve a 'taken-as-shared meaning' among the participants (Krummheuer 1992, p. 34). Only on the basis of these shared meanings, which usually only require a minor functional adjustment of individual interpretations, can the interaction develop further. The situation where interpretations are temporarily adjusted to fit with each other is termed 'working consensus' (Krummheuer 1992, p. 25; on the concept of 'working consensus' see Goffman 1959, pp. 9–10). The generation of 'simple' definitions of situations does not necessarily, however, lead to new things being learned. Krummheuer (1992, p. 24ff.) calls standardised and routinised individual definitions of situations 'framing', with reference to Goffman (1974). The 'framings' of different individual pupils, both in relation to each other and to those of teachers, often do not agree, however (cf. Krummheuer 1992). Teaching can be understood in this context as a practical crossover point where the framings of two different interactional practices meet. The teacher uses framings taken from his or her knowledge of the subject area and from educational interactional practice in interpreting subject-related aspects of teaching; pupils, on the other hand, use framings from their lives outside school or their previous school career. Only the fundamental transformation or construction of framings, not the changing of definitions of situations, can represent a learning process here (see also Schütte 2009):

> With the learning of new framings, the individual opens up a whole new area of social reality, gaining a

new perspective on reality […]. (Krummheuer 1992, p. 45, translated by the author)

According to Krummheuer, differences between the framings of the participants, which hinder the generation of collective arguments but simultaneously represent the 'motor' of learning, need to be coordinated more and more by an individual with advanced skills in the subject-related interaction.

3 Common methodological elements of the two projects

Both projects are qualitatively oriented and can be categorised as interactional approaches of interpretive classroom research in mathematics education (cf. Krummheuer and Brandt 2001; Voigt 1984). The linguistic shaping of the mathematical learning processes is subjected to interactional analysis using video sequences (cf. ibid.; Schütte 2009). The analysis of interactional episodes is oriented towards a reconstructive-interpretive methodology and one of the central elements of the research approach of Grounded Theory, comparative analysis (cf. Strauss and Corbin 1996). Comparative analysis is a method for the comparison of defined groups, and can be used at all stages of the research process. Such analysis generates research products that can be portrayed as elements of a local, context-specific theory. In the following, comparative analysis is deployed after the presentation of the results of the analysis of the second project. The goal is to use the specific results of the analysis and the developed theoretical elements of the individual projects to enable conclusions about the language-related learning of mathematics in different places of learning and in different age groups.

The research question that the following will address is:

> How are mathematical learning processes shaped linguistically in kindergarten and primary school through the verbal and non-verbal actions of the individual with advanced skills in the interaction?

4 Results from two research projects

In the following I present results on language-related mathematics learning from two projects, both of which focus on children with a broad range of backgrounds.

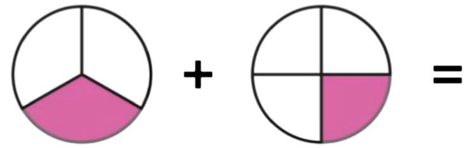


Fig. 1 Picture on the board ⅓ + ¼

The first project focuses on primary school mathematics learning and the second on early mathematics learning in kindergarten.

4.1 Learning in mathematics considering linguistic-cultural diversity in primary school

The following study analyses the linguistic shaping of primary school mathematics teaching by teachers in scenarios where they are introducing new mathematical concepts. In these phases, the linguistic shaping of the teaching is particularly significant, since information is being built up that is subjectively new to the pupils. Three year 4 classes from two Hamburg primary schools, in which around 80 % of the pupils were from families with an immigrant background, were studied over 4 months. The goal was to enable a theoretical description of the opportunities available for pupils to learn mathematics in a primary school mathematics class characterised by the linguistic and cultural plurality of the pupils. Specific sequences from an everyday primary school mathematics class are presented as an illustrative example, along with a summary analysis.

4.1.1 LCM (least common multiple) teaching scenario

The LCM scenario involves the teacher, Ms. Teichmann, who is the class's maths teacher as well as the class tutor, and 25 pupils (nine female, 16 male), 17 of whom are from families with an immigrant background. The majority of the pupils come from families with a relatively low level of formal education and fairly low socio-economic status. Ms. Teichmann is a qualified grammar school teacher and has no background of immigration. In the previous mathematics lesson, pupils had been building on the basic skill of multiplication. The class reported on here involved the introduction of a new mathematical concept, the least common multiple. (See Sect. 6 for an explanation of the transcription notation).

1	<T:	OK people so why do we do that with this . . LCM\
2	<T:	[writes ‘LCM’ on the board]
3	T:	I still haven’t really explained what that **is** only how it **works .**
4	T:	er has anyone read on the sheet what it means/
5	Vahit:	little common m #
6	# T:	but you don’t know what it is
7	<Vahit:	making it bigger making it smaller
8	< T:	the small smallest
9	Nurkan:	alphabet
10	Pm:	eh/
11	<T:	.. the smallest maybe you haven’t had that yet
12	<T:	[T. writes ‘Least Common’ on the board]
13	<P:	cuttable/
14	<T:	least.. common
15		yes yes.. I’ve thought of something else.
16	>T:	[T. writes ‘the’ above ‘Least Common’]
17	>T:	you need to write **the** there; otherwise it doesn’t work.
18	T:	**Multiple**..a multiple of two is six for example\
19		what is a multiple of . aah pfff five/
20	Pm:	[some pupils put their hands up, including Niklas]
21	<T:	[nods at Niklas]
22	<T:	yes/
23	Niklas:	Thirty

After calculating further multiples, Ms. Teichmann informs the class that they will encounter these kinds of calculations later, in Years 5 and 6 in secondary school, and so the current practice will be good preparation <70–76>. After Ms. Teichmann asks in <86–89>: “why do we need that/that’s a good question isn’t it/… why do they torture children for two years with the LCM and HCF there’s also\ highest common factor...”, she starts to draw two big circles of identical size on the board. She then says:

At this point the following drawing is therefore on the board (Fig. 1):

78	<T:	.. you need to\
79	<T:	[puts an ‘add’ sign between the two circles]
80	P:	plus\
81	P:	eh/ a circle plus a circle\
82	<T:	[T divides the left circle into three and the right into four segments]
83	<Pm:	[several pupils call out indistinguishably]
84	>P:	four plus three seven\
85	>P:	ah Mercedes\
86	<Pm:	Mercedes Mercedes Benz
87	<T:	[T shades in a part of both circles in pink]
88	>P1:	oh OK\
89	>P:	Now you understand\
90	P:	Mercedes Benz Cabriolet\
91	P1:	OK\
92	P:	a third plus a quarter\
93	<T:	like this\ [writes an ‘equals’ sign next to the
94		right-hand circle] read out the task\

127	<T:	..there's the task\ Pelin what's the task\
128	<Pelin:	[does not respond]
129	Pelin:	one plus one\
130	T:	Nesrin\ what is the task/ ..
131	<T:	what is this that I've drawn on the board/
132	<T:	[points at one of the circles]
133	Nesrin:	two\
134	T:	what/
135	Nesrin:	one\
136	T:	no one is the whole thing
137	Nesrin:	two\
138	T:	no that's not right either\ two is too many\
139	<T:	that is two\
140	<T:	[points to the two circles]
141	>Pm:	[class is restless]
142	>T:	shh everyone quiet\

In <144>, Otto goes on to say "a third". The teacher paraphrases this and writes $1/3$ in the shaded third of the left-hand circle. Several children say "a quarter" <152>. The teacher writes this onto the drawing too, and then expands the fractions $1/3$ and $1/4$ to $4/12$ and $3/12$. Lots of pupils contribute imaginative solutions for the addition of these fractions, for example $2/7$. The teacher only engages with these answers so far as to say that they are not possible, and adds the fractions to obtain $7/12$. Finally she provides a generalisation and concludes:

235	<T:	no/ you can't do that- a big slice of pizza
236	<T:	[points to the left-hand circle]
237	>T:	and a small one and a smaller one-. add together\
238	>T:	[points to the right-hand circle]
239	T:	that's unequal right/
240	<T:	basically you have to chop them into pieces so that they're equal\
241	<T:	[makes chopping motion with hand]
242	>T:	.. so/ these pieces are the same\
243	>T:	[points to the left-hand circle]
244	<T:	[points to the right-hand circle]
245	<T:	these pieces too\ only here there is one fewer\
246		OK/ here there are only three-
247	>T:	and here there are four pieces\
248	>T:	[points to the left-hand circle]
249	P:	ah now I understand\
250	T:	and that's why you need it\. if you have any fractions-
251		so that you can add these slices of cake together\ OK/ you can't
252		just say three and four is seven and from up here we take two
253		and then I have two sevenths\ two sevenths is something
254		completely different\ no/ you can't do that\

4.1.2 Analysis of the linguistic shaping

The concept ‘least common multiple’ seems already to have been named in class, since at least some children seem aware of it—Vahit’s attempt to fill out the words in the concept LCM in <7> comes very close to the correct technical term.[2] The pupils, however, do not seem to be familiar with the meaning of ‘LCM’, since Ms. Teichmann says in <1> “why do we do that with this… LCM\ I still haven’t really explained what that is”. Consequently, one might expect that the concept and its meaning would be concretely addressed in what follows. However, the participants seem to encounter misunderstandings. The sequence with Pelin and Nesrin shows that some of the children can follow the teacher’s explanations only to a certain extent. The teacher’s approach can be described here using the above theoretical observations.

In relation to the subject-related mathematical linguistic register, the introduction of new mathematical constructs allows us to reconstruct that neither the meanings of such concepts nor the links in content between new mathematical concepts and already-known everyday concepts are produced by the teacher—or are only implicitly produced by her. At no point is the content of the mathematical concepts ‘denominator’, ‘numerator’, ‘fraction’, ‘fraction line’ or ‘multiple’ explained verbally in the ‘official’ teaching conversation. Their meanings remain implicit and are integrated without further analysis into known calculations. Pimm’s (1987) formulated goal that pupils should learn to speak like native speakers of mathematics will become very difficult to achieve, since the mathematical ‘native speaker’—the teacher—presents no model of this active speech.

A similar picture can be seen in the way the teacher engages with elements of the formal linguistic register. The ‘elegance’ of the spoken language of the pupils is only insisted upon insofar as, for example, the correct article (neuter, i.e., ‘das’ in the original German) is placed before LCM and the abbreviation of the new concept is written out in full words. This is apparently intended as a concession to the fact that a large proportion of the pupils do not have German as their first language. More complex linguistic elements are, however, not explained. For example, the teacher does not outline the meanings of the different components that define the mathematical concept. Although the meaning-carrying components of the concept ‘Least Common Multiple’ represent everyday concepts, and all the children are able to deal with the concepts of ‘small’ and ‘common’ in everyday register, an understanding of these everyday concepts does not inevitably lead to an understanding of the mathematical concept of the LCM. Competences in academic language are not successfully taught in this kind of teaching.

This form of teaching can certainly be criticised from a lot of angles, and will presumably cause difficulties in learning for all the children, not just those who may live and learn in more than one language. However, this paper is not concerned with an assessment of the methodological educational procedures of the teacher; rather, I will focus on structural problems. Since the present scenario takes place in a class where the majority of the children are growing up with a background of immigration, the children’s difficulties in following the lesson could be attributed to their lack of linguistic ability. According to the analysis of the mathematical subject register, one explanation for the misunderstandings encountered in the introduction of the new concepts could be that the children were only able to follow the teacher’s explanations to a certain extent because of deficits in (academic) linguistic ability, and therefore encountered difficulties in constructing meaning in the mathematical register.

But we should be careful not to be over-hasty. If we turn to the above-mentioned approaches by Krummheuer (1992), as well as those of Aukermann (2007), we can find explanations that are based on more than just the linguistic deficits of learners and the fact that teachers do not adequately take account of these deficits in teaching. In the above sequence, for example, we can reconstruct how the teacher seems to ‘frame’ the task or the picture on the board on the basis of her own education in the subject in terms of the concept of the addition of rational numbers. She thus sees the dividing up of a circle and the shading in of some of the resulting sections as a graphical representation of a fraction. Differences in framing can be reconstructed both among the pupils and between the pupils and the teacher. Some pupils, for example Pelin and Nesrin (<129–141>), seem to view the circles as shapes, and add them together in a conventional way on the basis of their framing of the addition of natural numbers. Only Otto seems to ‘frame’ the task similarly to the teacher (<144>). Because of the implicitness in the actions of the teacher, and since the teacher does not ascertain the framings the pupils are using as a basis for their interpretations of the task, the pupils’ different framings of the task and the individual definitions of the situation that result are not at all addressed. The teacher therefore does not engage with Nesrin’s interpretations, missing the chance to coordinate or reduce the framing differences between the majority of the class and her. This presents no opportunity for the pupils to bring their definitions of the situation closer to those of Otto and the teacher. As a result, even after being corrected several times by the teacher, although Pelin

[2] In this paper, ‘technical term’ is used to denote a mathematical concept. The mathematical concept together with the mathematical meaning behind it is expressed with ‘concept’ in the following.

changes her answer she never appears to change the framing that she uses as the basis for her answer. An interpretation of the task based on a framing related to fraction calculation thus seems refused to her. According to this analysis, the misunderstandings are not rooted in the linguistic deficits of the children, but rather in the lack of interpretive competency on the part of the teacher to recognise and coordinate differences in framing among the different participants in the class.

Krummheuer refers to this kind of interactional situation as 'ignored framing difference' (Krummheuer 1992, p. 74ff.). Possible framing alternatives for the pupils are not addressed and are simply ignored. The result of such a process of negotiation will be that the working consensus between the participants remains superficial, since the teacher is accepted as the authority in relation to the validity of the content, and the teacher does not ascertain how to adapt her definition of the situation to those of her pupils. One effect of this obscuring of differences in framings is that the interactional situations give an impression of a consensual process of negotiation of meaning. From a mathematical perspective this seems questionable: the quality of such processes of 'understanding without understanding' is doubtful, since a large number of participants cannot develop similar ways of understanding the situation. The process of negotiation is also disadvantaged by a certain vagueness as to that which is actually being negotiated, and what is mathematically correct is replaced by 'intersubjectively achieved relief in a process of understanding' (Krummheuer 1992, p. 113). From an interactional perspective, it can also be noted that this obscuring of differences in framing can be expected to generate no taken-as-shared meaning that transcends the individual frames of negotiation of the participants. This will mean that the 'achieved understanding' will evoke no sustainable processes of learning through subjective conviction.

4.1.3 Implicit Pedagogy as background theory

As a fundamental common goal, we can reconstruct the implicitness of the taught content on different levels. This implicitness is reflected in the use of different mathematical and formal linguistic registers. But it is not possible, for example, to reconstruct any explicit efforts on the part of the teacher to ascertain the framing that she should use to interpret a given pupil's answers.

This approach to the introduction of new mathematical concepts could be explained in terms of a theoretical concept I refer to as 'Implicit Pedagogy' (see Schütte 2009, p. 187ff., and 2010, p. 212ff.). According to this concept, the task of teachers is above all to establish a learning environment for learners, and to ensure that in this environment every child can develop their natural individual abilities and 'talents'. Implicit Pedagogy is based on the idea that the abilities that pupils bring with them into the classroom mean that they are capable of grasping meaning independently, or that basic content-related and linguistic contexts and structures become clear to children 'as if by themselves'.

However, criticism of such implicit procedures should not imply support for behaviouristic approaches where learning is always directed by the teacher. Nor should it be understood as contradicting active-discovery approaches to learning, according to which children work with an educational object in a learning environment designed by the teacher in order to complete mathematical learning steps. Children should be allowed a certain amount of freedom to discover mathematical structures. In beginning to address content, however, precisely because of the heterogeneous nature of the pupil population, linguistic structuring and linguistic reflection at the end of the active-discovery construction phases gain a particular meaning (cf. Krauthausen and Scherer 2010, p. 42).

What is crucial to observe for the purposes of the present paper is that learning of new material hardly seems possible in the presented teaching scenario, since the teacher seems to unconsciously assume that the new mathematical concepts are already immanent and that no further explicit linguistic negotiation is necessary in the class conversation or in the reflection of the individual construction processes, for example in the differentiated examination of the meaning-carrying components of the concept 'least common multiple'. This form of teaching can be understood as a 'pathological form of open teaching concepts', in which the teacher no longer performs the role of the individual with advanced skills in the interaction who helps the pupils to (linguistically) progress in their (linguistic) development. The teacher and the teaching thus lose the decisive influence over pupils' success in school. The social conditions experienced by pupils are reproduced, and differences in achievement are thus often prematurely set into stone by the school system as they are understood as a consequence of socio-economic and social differences.

In relation to the above remarks on places of learning, the question emerges as to whether such phenomena are only evident in (primary) school learning or also in the learning of mathematics in kindergarten or the family. In the following illustrative example, a scenario with children of pre-school age and an attendant adult is analysed to see whether the linguistic shaping of the situation follows the principle of Implicit Pedagogy [for more detail on pre-school language-related mathematics learning see also Johansson et al. in this volume—(2014)].

4.2 Learning in mathematics considering linguistic-cultural diversity in kindergarten

The analysis below is taken from a scenario involving two kindergarten-age children, with an attendant adult. It is extracted from results from the project erStMal (early Steps in Mathematics Learning), carried out by the IDeA Centre in Frankfurt. IDeA stands for Individual Development and Adaptive Education of Children at Risk; the Centre is an interdisciplinary research centre that has taken up the task of researching the development conditions of children with so-called 'risk factors' and to develop appropriate long-term support mechanisms for practical application.[3] The erStMaL project, among other things, seeks to achieve a social-constructivist-oriented theory of the development of mathematical thinking in children (see the concept of the 'interactional niche in the development of mathematical thinking': Krummheuer 2012; Schütte and Krummheuer 2013). To this end a longitudinal study was initiated, in which 144 children aged between three and ten were placed in different groups and confronted with play and discovery situations developed by the authors (cf. Acar Bayraktar et al. 2011).

4.2.1 The 'animal conga line' scenario

This section presents a selected sequence from the 'animal conga line' scenario. The participants are Ayse (4.2 years old), the only daughter of a Turkish-German couple, and Kai (4.3 years old), who is growing up in a German monolingual environment, as did the attendant adult B (Fig. 2).

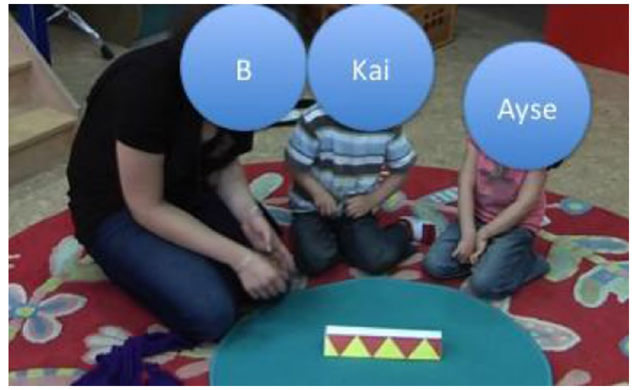


Fig. 2 Initial situation of the scenario

The play and discovery situation comes under the content area of data analysis. The children are given three animals and a platform. The aim is for the children to work together to find different orders in which the small animal figures can walk over the platform. The used learning environment is constructed according to a unified design pattern. The mathematical content is described as follows:

- finding of different orders, counting of possibilities
- determining of possible permutations using the three elements
- positioning in order: nth position, $n-1$th position,…

The scenario can be analysed in three parts:

[3] The IDeA Centre was set up in cooperation with the German Institute for International Educational Research, the Sigmund Freud Institute and the Goethe University, Frankfurt (http://www.idea-frankfurt.eu/de/forschung/programmbereiche/ressourcen-und-grenzen-erfolgreichen-lernens/erstmal).

B: and we'll leave that there\ okay/ and now in the circus – the animals want to perform a stunt on the platform\ [places the three animals next to the platform in the order monkey-elephant-tiger, the monkey is in front in the direction the animals will move in] and so they need to walk in **one** row over the platform\ and keep their balance\ can you help me/


(lines 2–4)

Ayse: [looks at B] yes\+

B: OK put them on the platform so they can walk over it in a row\

Kai: [takes the tiger]

Ayse: [Pushes the elephant next to the platform]

B: hm/ one animal needs to be right- [touches the left end of the platform] it can go more up here \ [taps with index finger at a point about 2 cm further to the right and about 4 cm further right on the platform] look\ on the white\ one animal needs to stand right in front/


(lines 13&14)

Ayse: [stands the elephant on the left end of the platform and pushes it around 2 cm further to the right and then back again]

Kai: [moves the tiger towards the left end of the platform]


(lines 16–18)

Ayse: [lets go of the elephant]

Kai: [stands the tiger on the right end of the platform and pushes it around 3 cm to the left]

Ayse: [stands the tiger on the extreme right end of the platform] it needs to go here\+


(lines 22&23)

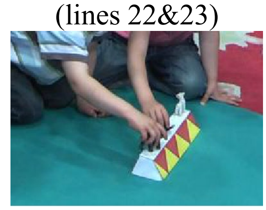
B: a**ha**\ great\ and then/ comes/

Kai: [stands the monkey on the left of the tiger on the platform] the monkey/ [takes the elephant and moves it around 4 cm to the left]

Ayse: [stands the monkey in the middle of the platform]

B: and then the elephant\

Kai: [pushes the elephant around 3 cm to the right]


(lines 28–30)

Ayse: [pushes the elephant back to the left end of the platform again] so\

B: great\ so which animal is now standing right **in front**\

Ayse: [points to the tiger] erm him \

B: aha\ and where is behind/

Kai: there\ [points to the elephant]

B: there\ aha\ OK\ [moves the right index finger from left to right across the platform] yes and they're walking to there too\ no/ [moves the elephant with a bouncing motion so it is next to the monkey on the right. Moves the monkey with a bouncing motion so it is next to the tiger on the right.] OK so then the three of them walk over it like that\ dumdedumdedum/ and then at the end they can/ [stands the tiger around 3 cm to the right of the platform] jump down- [moves the monkey with a bouncing motion to the right across the platform, then moves the elephant with a bouncing


(lines 38&39)

45		motion to the right across the platform and places it in front of the
46		monkey] OK\ and then there's still the big fat elephant/
47		dumdedum\ hm\
48	Ayse:	[points to the monkey] the monkey has fallen over\
49		[stands the monkey back up]
50	B:	[points to the platform] can you find nother order in which they
51		can walk over it/
52	Ayse:	[lets go of the monkey] yes/
53	B:	[puts the animals down to the left of the platform]
54		they need to stand behind again\ I think/ can you put them on it
55		again/ so which animal should be right in front **now**\
56	Kai:	[takes the monkey]
57	Ayse:	[moves the elephant to the right across the platform until it stands
58		just before the edge] one\
59	B:	hm/
60	Ayse:	[pushes the elephant bit by bit to the middle and then to the left
61		end of the platform, then places it on the right end of the platform]
62		the elephant/
63	B:	yes\
64	Ayse:	[places the tiger on the left end of the platform] and the tigers here/
65		[moves the monkey towards the middle of the platform]
66		and the monkeys– erm\ [stands the monkey briefly in the middle
67		of the platform and then behind the platform] in the middles\
68		[looks at B]
69	B:	no\ but all the animals need to walk across the platform\
70	Ayse:	[stands the monkey in the middle of the platform]
71	B:	that's right\
72	Ayse:	[lifts the elephant up above her head] kchch\
73		[stands the elephant just before the right end of the platform]
74	B:	great\ so now the monkey's in the middle again\ hm/
75	Ayse:	[looks at B] jumbed off\
76	B:	hm\
77	Ayse:	[moves the monkey with a bouncing motion to the right end of the
78		platform] chchu\ [places the monkey down next to the platform]
79	Kai:	[moves the tiger down in front of the platform and then with
80		a bouncing motion from the left to the right end of the platform,
81		stands it around 4 cm from the right end of the platform]
82	B:	and can we find nother order in which they can walk over it/
83	Ayse:	[looks at B] yes\
84	Kai:	yes\ [stands the elephant around 2 cm to the right in front of the
85		platform, takes up the monkey and holds it]
86	B:	I think there's still one animal who hasn't been in front\
87	Kai:	[stands the monkey on the right end of the platform] him and ri–
88		the monkey right in front/ [takes the elephant and stands it in the
89		middle of the platform] the mon- and the elephant in the middle-
90		[stands the tiger on the left end of the platform] and the lion needs
91		to go here\+
92	B:	great\


(line 48)


(line 53)


(lines 57&58)


(lines 67&68)


(line 70)


(lines 79&80)


(lines 87–91)

a. <1–53> Opening section—explanations of the platform object
b. <54–168> Introduction of three animals
c. <169–405> Formulation of the task and generation of three permutations

Below are the transcript and analysis of the third section.

4.2.2 Analysis of the linguistic shaping

The participants produce four permutations, and the development of the following three will be analysed:

I Tiger–Monkey–Elephant
II Elephant–Monkey–Tiger
III Monkey–Elephant–Tiger

Permutation I is developed by the children in more-or-less synchronised fashion after the attendant adult asks the children to make the animals walk across the platform in a row. It remains debatable how the concept of the 'row' can be understood: does this intend to describe the kind of arrangement, standing in a line, or the ranking, for example sorted from small to large? In comparison with the above-described primary-school scenario, in relation to the (mathematical) linguistic register we can reconstruct how the concept of 'row' is not explicitly explained by the adult. As the interaction continues, the adult seems to use the concept of 'order' synonymously with the concept of 'row'. Ayse and Kai only start to construct the first permutation after B has given them fairly direct support, including pronounced gestures, to show that they need to make the animals walk across the platform in a row. We can therefore assume that either the children interpret the situation differently to the adult, or they cannot connect any further actions with the concept of 'row'. We can reconstruct how B is trying, at least through her gestural support and the movement of the animals, to generate the first notion of an 'order' in the minds of the children. However, this context can only be recognised by participants who have already mastered the concept of an order, and who can therefore see that the goal here is to achieve different permutations by lining up the animals in different sequences. In the approach observed here, this context remains implicitly concealed from the children.

In the example of the first generated permutation, a spatial orientation of 'in front' and 'behind' is negotiated that all can agree on, which will be seen in the unchanging direction in which the animals walk across the platform in all the cases. The participants thus seem to have generated taken-as-shared meaning in terms of the spatial orientation of in front and behind, without a common framing regarding the task to be completed. The children seem to frame the sequence in terms of everyday framings of in front and behind, or in terms of basic mathematical concepts of spatial orientation. Similarly to the LCM scenario, in this sequence, too, there seems to be a difference in framing, which is also uncoordinated by the adult. The three participants do succeed in achieving taken-as-shared meaning through an apparently similar definition of the situation, but the adult frames this interpretation, unlike the children, in terms of a combinatoric background where the order of the animals is varied to achieve different permutations.

Arranging the second permutation also requires significant support from B. Only the concrete demand to move the animals across the platform again, and to address the question of which animal should stand 'right in front' this time round, leads to the children each picking up an animal. The concepts of 'row' or 'order' are given as little explanation here as they were in the first permutation.

Permutation III, however, represents an individual achievement by Kai. He completes the actions fluently and even his intonation suggests that he is executing a plan that is clear in his own mind. He calls the tiger a lion; the specificity of the individual kinds of animal no longer seems important to him. This could be a first indication of a possible modulation to a stronger mathematical framing (cf. Goffman) of the situation. Kai would thus begin to transform his everyday framing of the situation and start to view the animals as elements he needs to place in rows in different orders.

As a common structural characteristic in the linguistic shaping of the learning environment by the adult, here too we can reconstruct the phenomenon of the implicitness of content and the teaching approach on different levels. This is reflected, as described above, in the use of different mathematical and formal linguistic registers. To a certain extent it becomes clear also in the coordination of different framings, since the gestural support and the movement of the animals can be seen as an attempt on the part of the adult to coordinate different framings. The implicitness is also made clear in the fact that these attempts lead to taken-as-shared meaning among the participants in terms of in front and behind, while linguistic coordination of the different framings on the part of B is hardly evident.

5 Comparison of the different places of learning and age ranges—theory construction

If we look at the two analyses in terms beyond those of place of learning and age ranges, on a structural level we can notice several things that coincide with the results emerging from the theory of Implicit Pedagogy in primary school (cf. Schütte 2009). In the analysed scenarios the adults do not act as linguistic role models. This means that the teacher thematically explains neither concepts nor their meaning-carrying components in the official classroom dialogue; nor does she use such concepts herself in relevant sentences, which might train the linguistic use. Similar behaviour is seen in the attendant adult in the conga line scenario, where the concept of 'order' remains implicit too. However, the adult in part establishes herself as a role model on a non-verbal level, where there are only weak links between linguistic and mathematical learning, when she supports the children in finding the different orders by pointing her finger. The children are therefore unable to learn linguistic skills related to the mathematical concepts—in the sense of learning with format (Krummheuer 1992)—from a role model. That is, they cannot gain autonomy through copying the verbal actions of an individual with advanced skills in the interaction. Thus they are hindered in the process of negotiation of meaning of the mathematical concepts. The mathematical concept and its linguistic structures are apparently supposed to become clear to the children automatically. In such learning situations it is perhaps not surprising that children who grow up in multilingual environments, or in families with relatively little formal education, can encounter difficulties in understanding lesson content. This indeed seems to be the case in the scenarios analysed here. The linguistic deficits of Pelin and Nesrin in the first scenario, and of Ayse in the second, seem to be the cause of misunderstandings. This would all coincide with the current discussion on linguistic abilities of children in school. One solution to this problem would be to train the linguistic abilities of children and thus remove the deficit. However, as we have seen, this does not go far enough.

The above analysis has shown that such misunderstandings can be explained partly by different interpretations of the situation stemming from different framings among the participants (on this, see Aukermann 2007). That linguistic difficulties represent an additional problem for the participating children must remain only an assumption. Significantly, different framings of situations can be reconstructed not only with the children with supposed linguistic deficits, but also with Kai, who is growing up in a monolingual and relatively highly educated environment. He and Ayse seem to frame the situation in a similar way to each other, and with similar differences to the framing of the adult. We can therefore formulate the hypothesis that framings of children with clearly differing linguistic abilities can nevertheless be very close to each other, and children without linguistic deficits can potentially develop framings that are just as far removed from those of the teacher. This means that a perspective focusing exclusively on the possible linguistic deficits of children is inadequate. It is certainly desirable for all participating children to be given an introduction to formal and mathematical linguistic aspects, and for the teacher to act as an explicit role model in this regard. But even when children have a linguistic role model, they need a teacher who engages with their interpretations and attempts to modulate the basic framings of all pupils. For example, in the 'animal conga line' scenario it can be seen that the framings of the adult and Kai are brought closer together. It is entirely possible that Kai's linguistic abilities facilitate this process. But this modulation should be coordinated by the adult in such a way that Ayse, too, is able to benefit from Kai's first step. Without this kind of modulation of framings it is certainly possible that taken-as-shared meaning of the situation can be negotiated, as we can see for example in the agreement the participants reach over 'in front' and 'behind'. With reference to Krummheuer (1992, p. 44–45), however, learning is only to be understood as a new construction or reconstruction of the framings that lie behind definitions of the situation.

Because of the increasing diversity of pupil populations, all places of learning will increasingly be characterised by a plurality of different interpretations in negotiations of meaning during teaching. The fundamental framings that lie at the base of children's interpretations may, despite this diversity, have more in common with each other than with the framing of teachers. Indeed, the framings of children, which emerge from an everyday environment that is at least partly shared, may hardly coincide with the technical framing of teachers. The goal should therefore be to ensure that teachers are sensitive to this variety of interpretations of lesson content, and to build on this by developing interpretational competences in recognising differences in interpretations that occur because of different framings, and in addressing these differences in teaching to allow modulation of framings.

6 Transcription notation

bold	Spoken with emphasis
smaller	Spoken quietly, whisper
sooo	Spoken slowly and at length
[action]	Action that takes place between two temporally separate sections of the transcript
(…)	Word not clear/incomprehensible
	Points show how long the incomprehensible section lasts, in seconds
/	Voice inflected up
-	Voice wavers
\	Voice inflected down
. .. ...	Pauses in speech, in seconds
<L: The house is smaller\	The speakers talk (partly)
<S: is smaller	at the same time
>M:	A change in the direction of the arrow shows the
>L:	beginning of a new section

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