ZDM Mathematics Education (2014) 46:201–212
DOI 10.1007/s11858-013-0560-0

ORIGINAL ARTICLE

Mathematics professional development as design for boundary encounters

Paola Sztajn · P. Holt Wilson · Cyndi Edgington · Marrielle Myers · Partner Teachers

Accepted: 14 November 2013 / Published online: 27 November 2013

Abstract This theoretical paper examines a process for researchers and teachers to exchange knowledge. We use the concepts of communities of practice, boundary encounters, and boundary objects to conceptualize this process within mathematics professional development (MPD). We also use the ideas from design research to discuss how mathematics professional development researchers can make professional development the focus of their research. In particular, we examine the question: How can MPD be conceptualized and designed around research-based knowledge in ways that promote knowledge exchange about students' mathematics and mathematics learning among researchers and teachers to improve the practices of both the research and the teaching communities? We propose that MPD is a premier space for researchers and teachers to exchange knowledge from their communities, impacting both researchers' and teachers' practices without reducing the importance of either.

This report is based on work supported by the National Science Foundation under grant number DRL-1008364. Any opinions, findings, and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the National Science Foundation.

P. Sztajn (✉) · C. Edgington · M. Myers
North Carolina State University, 317 Poe Hall,
Campus Box 7801, Raleigh, NC 27696, USA
e-mail: psztajn@ncsu.edu

P. H. Wilson
University of North Carolina Greensboro, 468 SOEB,
PO Box 26170, Greensboro, NC 27402, USA

1 Introduction

This paper examines a process for researchers and teachers to exchange knowledge. It addresses Krainer's (2011) concern that a fundamental yet under examined issue regarding researchers' and teachers' knowledge "is the question of how researchers' and teachers' knowledge is interrelated and exchanged" (p. 50). From a social perspective on learning, we consider knowledge as competent performance in a valued enterprise, knowing as participating in the pursuit of this enterprise within a community, and boundary encounters as a mechanism for communities to exchange knowledge (Wenger, 1998). The theoretical analysis presented in this paper examines mathematics professional development (MPD) as an instance of a boundary encounter between researchers and teachers. We propose that MPD is a premier space for researchers and teachers to exchange knowledge from their communities, impacting both researchers' and teachers' practices without reducing the importance of either.

Our work focuses on professional development designed around research-based knowledge on students' mathematics. Professional development organized around research knowledge has been criticized for promoting a deficit view of teachers (Ponte 2009). Whereas we recognize that professional development has too often been conceived as a knowledge delivery mechanism, we contend that conceptualizing MPD as boundary encounters offers the possibility of focusing professional development on research-based knowledge while organizing it to promote knowledge exchange.

The theoretical discussion in this paper is presented from the researchers' perspectives. We identify ourselves as MPD researchers: the subset of researchers within the larger mathematics education research community for whom MPD is the research focus. For us, MPD as conceptualized in this paper represents an encounter between MPD researchers and teachers and positions MPD researchers to exchange knowledge with teachers. To foster knowledge exchange, MPD researchers face the challenge of designing professional development that has research-based knowledge at the forefront, cares for teachers (Sztajn 2008), and values their knowledge.

Beyond drawing from the concepts of communities of practice and boundary encounters, we also use design research (Cobb, Confrey, diSessa, Lehrer, and Schauble 2003; Gravemeijer and van Eerde 2009; Kelly, Lesh, and Baek 2008; McKenney and Reeves 2012) to examine our approach to research MPD (Sztajn, Wilson, Edgington, Meyers, and Dick, 2013). We contend that using design research to investigate MPD advances the goal of knowledge exchange between different communities of practice through careful attention to the knowledge that teachers bring to the MPD. Thus, we situate the theoretical discussion presented in this paper within the context of our 4-year design research project.

In what follows, we first examine the relation between researchers' and teachers' knowledge. Next, we introduce the Learning Trajectory Based Instruction project to offer a context in which we situate our theoretical discussion. From a learning perspective, we define concepts of communities and boundaries, use them to conceptualize the work MPD researchers and teachers do together, and offer examples from our project to explain the various definitions. Then, from a research perspective, we discuss design research as an appropriate methodology for conducting research in MPD settings while promoting boundary crossing. Finally, we share teachers' perspectives on the idea of researchers and teachers working as partners in a MPD encounter. We conclude the paper drawing implications for mathematics education researchers.

2 Perspective: researchers' and teachers' knowledge in mathematics professional development

Two distinct perspectives have been put forth when considering researchers' and teachers' knowledge in MPD. On one end, there is the perspective that knowledge needed to improve mathematics teaching is situated in the research community, and the role of MPD is to transmit this knowledge to teachers. This perspective aligns with the technical rationality (Schön 1983) and the research–development–dissemination model of innovation (Krainer 2011). On the other end, there is the perspective that the knowledge needed to improve teaching is situated in the teaching community and generated through reflection on the practice of teaching (Schön 1983). In this case, the role of MPD is to foster teachers' reflective practices through the examination of teaching within professional learning communities (DuFour et al. 2006).

We take both ends of this dichotomy to be myopic when they situate knowledge for improving teaching either with the research or the teaching community. These two extremes lack recognition that both teachers' and researchers' knowledge, albeit different, are important to improve mathematics teaching and learning. Thus, whereas we do not share the often criticized view that the role of MPD researchers is to "fix" poor teaching (Dawson 1999, p. 148), we also do not share the view that MPD organized by researchers with research-based learning goals is intrinsically problematic because it positions teachers as learners (Jaworski 2011).

As an alternative to the dichotomy of situating knowledge needed to improve mathematics teaching and learning with either researchers or teachers, various MPD initiatives have turned to establishing process instead of content goals (Simon 2008) for the work researchers and teachers do together. Jaworski and Goodchild (2006) proposed that researchers and teachers are partners in MPD communities of inquiry. They acknowledged that each group brings their own specialized knowledge on mathematics teaching and learning into MPD communities, and suggested that researchers and teachers study the developmental process while promoting development. Kieran, Krainer, and Shaughnessy (2012) used the concept of teachers as stakeholders to strengthen the idea of teachers as partners. They argued that, because teachers have the greatest potential to transform education and improve student learning, researchers and teachers should work collaboratively in MPD settings to develop common goals and generate common questions.

Our conceptualization of MPD recognizes the importance of knowledge situated in both the researchers' and the teachers' communities while also attending to the emerging understanding of the value of researchers and teachers working as partners in MPD. It seeks to examine the possibility of designing professional development that attends to process goals for researchers and teachers working together, while also explicitly attending to content learning goals that both researchers and teachers as professionals bring to the MPD. We start from two premises. First, the research community has knowledge about students' mathematics and mathematics learning that has the potential to be useful to teaching. Second, the teaching community has knowledge about students' mathematics and mathematics learning in context that is of utmost importance for mathematics education researchers. Therefore, when MPD brings researchers and teachers together, there is an opportunity for knowledge exchange among these communities. The theoretical question we examine in this paper is: How can MPD be conceptualized and designed around research-based knowledge in ways that promote knowledge exchange about students' mathematics and mathematics learning among researchers and teachers, and improve the practices of both the research and the teaching communities?

3 Context: the Learning Trajectory Based Instruction project

To examine our theoretical question, we briefly introduce the context of our work, the Learning Trajectory Based Instruction (LTBI) project. LTBI is a multi-year project to bring MPD researchers and elementary mathematics teachers together to work around researcher-developed representations of student mathematics learning. The project is focused on learning trajectories, defined as "a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, toward increasingly complex concepts over time" (Confrey et al. 2009, p. 347). The project is also designed around a conjectured model of instruction that attends to the importance of learning trajectories in the use of open instructional tasks that elicit students' mathematical thinking and a set of pedagogical practices that centralizes this thinking to organize teaching (Sztajn, Confrey, Wilson, and Edgington 2012).

The first implementation of the project's MPD was initiated by a request from members of the teaching community. Teachers from one school approached Confrey's research team and indicated their interest in the emerging topic of learning trajectories. This interest was communicated to our group of MPD researchers, and we met with a group of representative teachers from the school for 6 months to discuss learning trajectories and to develop plans for the LTBI project. With the support from the school principal, the project started the subsequent summer, and all project meetings happened at the school. It engaged 22 kindergarten through fifth grade teachers from the partner school and the first three authors of this paper. All teachers in the school were invited to participate. Participation was voluntary. Teachers received a stipend for project activities beyond regular school time.

Researchers and teachers in the LTBI project worked together for 1 year around the equipartitioning learning trajectory developed by Confrey and colleagues (2009). There were 50 face-to-face hours of professional development meetings. The first 30 h happened over the summer and explored the trajectory itself. The remaining 20 h happened during the school year and explored aspects of the conjectured model of instruction.

From its onset, participating teachers in the LTBI project had a clear content learning goal: learn about learning trajectories developed in the research community. At the same time, MPD researchers had the learning goal of understanding the ways in which teachers came to learn about these trajectories and whether or not they make sense and are useful to teachers. Therefore, the LTBI challenge was to conceptualize and design MPD around research-based knowledge in ways that promoted knowledge exchange about students' learning trajectories among researchers and teachers and improved the practices of both the research and the teaching communities. In what follows, we use the LTBI project to provide examples for the two frameworks that guide our theoretical analysis: boundary encounters and design research.

4 Professional development as boundary encounters

Our analysis of professional development as boundary encounters starts from Wenger's (1998) definitions of knowledge as competent performance in a valued enterprise and learning as the process of participating in the practices of communities and constructing meanings and identities in relation to these communities. When MPD researchers and mathematics teachers come together in MPD settings, their practices are aligned with those of different communities that value different enterprises. Thus, they bring different knowledge to MPD setting. For example, within the research community, knowledge involves competence in the design of scientific investigations that help understand focused phenomena. Within the teaching community, knowledge involves competence in promoting student learning given complex contextual constraints. When researchers and teachers work together, MPD becomes the space in which two communities come together: a space conceptualized as a boundary encounter.

4.1 Boundary encounters

Community development implies the creation of boundaries between participants and non-participants, as communities define what it means to belong to one community and not another (Wenger 1998). These boundaries make it difficult to exchange knowledge across communities. However, it is precisely the difficulty of knowledge exchange that makes learning at boundaries potentially transformative for communities. Akkerman and Baker (2011) noted that boundaries both divide and connect communities in ways that problematize knowledge. As communities cultivate boundaries, they also cultivate mechanisms of communication and collaboration with the outside. These mechanisms are boundary encounters.

Our interest in conceptualizing MPD as a boundary encounter stems from the realization that within these encounters, members from separate communities communicate about, collaborate around, and potentially transform practice. Boundary encounters allow community members to examine, and potentially change, the ways in which they experience and belong. Most important, participants from separate communities who are involved in boundary encounters negotiate meaning both across the boundary and within their original communities. Thus, MPD as a boundary encounter allows for negotiations of meanings between and within both the research and the teaching communities. It recognizes that researchers and teachers have different knowledge when they work together in MPD, yet offers an opportunity for exchange between the two communities' knowledge.

In the case of the LTBI project, MPD researchers and mathematics teachers came together in a boundary encounter organized around the concept of learning trajectories. The encounter offered members from each community the possibility of learning about the ways in which the other community worked around students' mathematics and mathematics learning. Each community had its own way of organizing practice in regard to student learning, which differed, particularly, in the level of contextualization of the knowledge about students' mathematics. Acknowledging LTBI MPD as a boundary encounter made explicit that researchers and teachers examine students' mathematics and mathematics learning differently; it provided an opportunity for both researchers and teachers to negotiate and revisit the meaning of students' mathematics and mathematics learning in research and in teaching.

4.2 Boundary brokers

Both MPD researchers and mathematics teachers who come together in an MPD setting are boundary brokers: community members who introduce elements of practice from one community into another (Wenger 1998). Brokers enable connections across communities and create opportunities for new meanings to merge in different communities. Boundary encounters are organized around brokers who learn together and from each other, and then return to their own communities with new practices that potentially transform the community. The job of brokering, however, is complex:

> It involves processes of translation, coordination, and alignment between perspectives. It requires enough legitimacy to influence the development of practice, mobilize attention, and address conflicting interests. It also requires the ability to link practices by facilitating transactions between them and to cause new learning by introducing into a practice elements of another. (Wenger 1998, p. 109)

In the LTBI project, MPD researchers and mathematics teachers acted as brokers for their respective communities as they translated, coordinated, and aligned meanings for learning trajectories and what it meant to organize instruction around students' mathematics. For example, researchers viewed learning trajectories as frameworks for understanding students' mathematics, so that instruction could be designed based on both what students knew and what they could make progress toward. Teachers perceived learning trajectories as tools for helping them identify and remediate students who were behind in their learning according to assessment-related expectations for grade-level performance. The coordination of these perspectives required sustained conversations among brokers about the meaning of teaching and the current role of accountability at schools.

4.3 Boundary practices

When boundary encounters are sustained over prolonged periods of time, as in the case of LTBI year-long MPD, they foster members' mutual engagement and allow for the emergence of boundary practices. Boundary practices are a form of "collective brokering" that offer members of different communities "something to do together" (Wenger 1998, p.114). They promote collective enterprises around which members of different communities negotiate diverging meaning, and offer channels for knowledge exchange.

Various boundary practices became part of the LTBI project, most prominently those using the equipartitioning learning trajectory to examine student work, analyze videos of classroom instruction, and study curriculum materials. Through these practices, MPD researchers and mathematics teachers worked together to make sense of students' mathematics learning and the role of students' mathematics in teaching. Researchers and teachers considered the ways in which knowledge about students was present in the teachers' textbooks and how that related to research on learning. These practices involved researchers and teachers providing evidence and justifications—from their own communities—for their claims about students. They required researchers and teachers to negotiate the use of the vocabulary and structures from the learning trajectory to talk about students.

4.4 Boundary objects

When brokers from two communities come together in a boundary encounter that generates boundary practices, they work around representations of knowledge that convey meanings across multiple communities. These representations are boundary objects, which inhabit intersecting worlds and satisfy particular requirements from all of them. Boundary objects have enough in common between communities to make them "recognizable" (Star and Griesemer 1989, p. 393).

In LTBI, artifacts such as student work, videos of clinical interviews with students, and curriculum materials represented boundary objects that shared meaning across the research and teaching communities. They allowed for the emergence of shared boundary practices. The representation of the learning trajectory used in the LTBI project was another boundary object used to generate and organize much of the practice in the project. Researchers and mathematics teachers worked together using the representation of the learning trajectory to describe and make sense of other objects such as student work, videos, and textbooks.

Mathematics professional development that fosters boundary practices around boundary objects requires design: artifacts carry both similar and different meanings across communities and do not necessarily promote shared meaning. Therefore, the design of tasks that facilitate practices around boundary objects is a fundamental feature of conceptualizing MPD as boundary encounters. In the following section, we turn to the idea of design research as a framework to examine the development of tasks in ways that are compatible with MPD as boundary crossing.

5 Professional development as design research

Our definition of MPD as design research within boundary encounters stems from both the importance of designing tasks to promote boundary practices and the recognition that, having their identities in the research community, MPD researchers continue to participate as researchers during MPD encounters—in the same ways that teachers continue to participate as teachers. MPD researchers in boundary encounters seek to investigate learning and knowledge exchange among researchers and teachers. In these settings, MPD researchers design for and participate in boundary practices while making these practices the object of their research. Designing learning tasks while researching learning is the essence of design research methodology.

5.1 Design research

Studies of learning draw on the tenet that people develop knowledge when they engage in authentic, domain-specific tasks (Sawyer 2006). Design research is a methodology created to study learning in the context of analyzing practice around carefully created tasks. Cobb et al. (2003) explained that design research entails the "engineering" (p. 9) of particular forms of learning together with the systematic analysis of the means used to support these forms of learning. It allows for the design of learning opportunities while producing theories of learning related to these opportunities. Further, it shifts attention from developing an understanding of what *is*—used in natural sciences—to an understanding of what *could be*—used in design sciences (Collins, Joseph, and Bielaczyc 2004).

Design research has historic roots in traditions of both radical constructivist teaching experiment and Vygotskian attention to sociocultural settings (Confrey 2006). As such, it is often associated to learning theories deemed more psychological (constructivist or cognitive theories) or learning theories that move toward social aspects of learning without a focus on practice (Wenger 1998, p. 279). Nonetheless, concepts of practice, identity, and community are not in contradiction with the goals of designing tasks and examining learning in context. We consider that design research offers MPD researchers an approach to study learning that is not only appropriate, but also enhances the meaning of boundary encounters among researchers and teachers.

5.2 Design principles

Central to design research is the articulation of a set of design principles (Collins, Joseph, and Bielaczyc 2004), which summarize what is known from previous research. Design principles are the explicit assumptions used to guide the work of researchers in developing and refining learning tasks. They allow design researchers to build on prior knowledge as new tasks are designed and new conjectures examined.

Three design principles guided the work of LTBI (Wilson, Sztajn, and Edgington 2012). First, we built on the understanding that elementary teachers attend to students prior to mathematics (Philipp 2008). This principle established that all LTBI professional learning tasks start with and have an explicit focus on children and pedagogy. The tasks embed opportunities for teachers to consider their knowledge of mathematics, but the development of mathematics content knowledge was not the initial focus of collective practice within LTBI. Second, we built on the research used to develop learning trajectories to consider that artifacts such as clinical interviews with students and written diagnostic assessment items (Confrey 2012) offered meaningful opportunities for detailed examination of students' mathematics. In the LTBI MPD, we examined with teachers a variety of artifacts previously used by researchers to develop learning trajectories. We considered that these artifacts had high potential to serve as boundary objects. The third design principle for LTBI was that teachers have significant contextualized knowledge of students and learning. Thus, we sought to bring this knowledge forth to promote knowledge exchange.

These principles had a variety of implications for the design of the LTBI MPD. For example, the goal of focusing tasks on pedagogy while embedding mathematical opportunities highlighted the potential of learning trajectories as representations that cut across pedagogical and subject matter domains in teacher education. Further, this initial focus allowed teachers to position themselves as experts in LTBI. It established that although researchers' and teachers' practices around pedagogy differed, both practices were important to examine the role of learning trajectories in instruction. Discussions around mathematics content that do not always position elementary teachers as experts emerged in LTBI as a pedagogical necessity.

The use of research artifacts as boundary objects showed that these artifacts carried shared meanings among researchers and teachers. Videos of clinical interviews, for example, promoted opportunities for researchers and teachers to talk about students and their mathematics—a topic of interest to both communities that quickly allowed researchers and teachers to engage in the collective examination of students' mathematics. However, although the videos were of interest to teachers, it soon emerged that teachers did not see their practices represented in them. For teachers, attending carefully to one student thinking was not as connected to examining the role of learning trajectories in instruction as researchers envisioned. As the LTBI project unfolded during the school year, more tasks were designed around artifacts from the teachers' practices such as curriculum materials or lesson plans. Later, once teachers had opportunities to engage in LTBI-related practices in their own classrooms, tasks were designed around artifacts teachers' generated and videos from their own instruction. Thus, there was a shift in the origin of the artifacts used as boundary objects as the program progressed, from artifacts of the researchers' community to artifacts from the teachers' community, because although artifacts from research served to initiate boundary practices, to sustain these practices it was necessary to also include artifacts from teaching.

Finally, the design principle establishing that teachers, like researchers, bring knowledge of students into the professional development had significant importance for the design of learning tasks, which we address next. It established the need to start LTBI tasks highlighting teachers' knowledge about students and then connect their knowledge to the research-based learning trajectories.

5.3 Task design

Creating professional learning tasks around the practice of teaching (Ball and Cohen 1999) is part of MPD researchers' practices. These tasks require careful design because practice-based materials are not self-enacting (Smith 2001). We see task design as a fundamental link between the frameworks of boundary encounters and design research. Task design is important to boundary encounters because it fosters collective practices between researchers and teachers. At the same time, task design is important for design research because researchers engineer the means for the learning they study.

An initial aspect of task design is the selection and adaptation of artifacts from practice that can potentially serve as boundary objects. Because we are interested in MPD organized around research-based knowledge, and our design principle establishes that research artifacts can generate (at least initially) meaningful discussion about students' mathematics among researchers and teachers, an initial task for us as MPD researchers was to take into account our knowledge as brokers with the teaching community and examine the potential of various artifacts from research to serve as boundary objects around which to design tasks.

One artifact we examined was the representation of the learning trajectory. In LTBI, we considered that the detailed representation of knowledge about students' mathematics generated through research on learning trajectories was not always productive for teachers. Created for researchers' practice, these representations supported one-on-one, detailed, slowed-paced interactions with students, which do not align with the fast-paced context of large, diverse classrooms in which teachers practice. Since one practice of brokers is the process of translation, part of the design for the LTBI tasks involved the re-representation of research-based learning trajectories into more condensed information that focused teachers' attention on particular dimensions of the trajectory. In fact, during the MPD, a variety of partial representations of the trajectory were developed with teachers, as the need emerged to highlight particular aspects of the trajectory.

Our work in task design also built on our design principles. The LTBI tasks were designed to first engage teachers in examining their own teaching experiences and practices and later connect these practices to the ideas represented in the learning trajectory. Every professional learning task involved a boundary object and a request to teachers to connect these objects to their work around student mathematics learning or student-centered instruction. These tasks were organized as a four-part sequence: engagement, exploration, formalization, and application. The goal of the engagement part was to elicit teachers' knowledge of the learner and their views on instruction for discussion. The exploration part engaged teachers and researchers in meaning sharing and negotiations of the initial question posed and issues from the engagement discussion. Emerging resolutions were then examined in relation to the researcher-developed learning trajectory or to the model of instruction during the formalizing phase and then used in an application closely related to instruction. Here, we share two examples of professional learning tasks designed for the LTBI boundary encounter. One focuses on the learning trajectory and highlights the exchange of knowledge related to the teachers' learning goals. The other focuses on learning trajectory-based instruction and underscores progress in our own learning goals as MPD researchers.

5.3.1 Task focused on the learning trajectory

One professional learning task sequence that we designed focused on the concepts of factor-based change and reallocation, specific levels of Confrey's (2012) learning trajectory. These two concepts are related to what Confrey calls compensation—the effect of changing the number of parts on the size of the parts, or the reverse. Compensation can be expressed qualitatively (i.e., the more parts created, the smaller is the part) or it can it be quantified through factor-based change and reallocation. Factor-based change represents a multiplicative relationship that describes the changes in the size of a part when the number of parts being created is changed by a factor (i.e., creating twice as many parts reduces the size of each part by half). Reallocation represents an additive relationship that, when the target number of parts decreases, joins originally created parts with an equal-sized portion from the redistribution of a previously created part that is no longer needed (Confrey, 2012). For example, after having shared 12 among 4 to make groups of three, reallocation is used to share 12 among 3 when 1 object from one of the groups originally created when sharing among 4 is redistributed to each of the remaining three groups.

The task sequence focused on the examination of how students, when equipartitioning, transition from qualitatively describing the relationship between the size and number of parts to mathematically expressing that relationship when sharing collections. The sequence began by asking teachers to share the ways in which they had experienced students' compensation when sharing. This initial discussion highlighted the many ways in which teachers had knowledge of compensation. Teachers talked about asking students whether they wanted to share something they liked with more or less people and how children reacted to this question, indicating that if they shared with less people they got more of what they wanted. For some students, these questions brought forth the need to consider fair sharing, that is, one cannot simply take more of what one likes when sharing with others.

Once a variety of examples from the teachers were discussed, the group explored clinical interview videos that showed particular approaches to compensation. These videos highlighted ways in which students went beyond qualitative compensation and into the ideas underneath quantitative compensation. The discussion of the videos was formalized through the introduction of the learning trajectory concepts of factor-based change and reallocation. For some teachers, the idea of reallocation was new and they had not observed it with their students, which generated discussions about whether reallocation was a concept to be taught or to emerge from students themselves.

The application part of this particular task sequence engaged the group in anticipating how different K-5 students might approach an assessment task targeting ideas of factor-based change and reallocation. We also considered implications for follow-up instruction. This example demonstrates how teachers met their goal of learning about students' mathematical thinking through the research-based learning trajectory. They engaged in examining and questioning concepts from research in relation to their practices and knowledge of students.

5.3.2 Task focused on Learning Trajectory Based Instruction

Another example of a professional learning task that had a stronger focus on the project model for instruction was implemented at midpoint during the school year. The role of open tasks in instruction that allows teachers to listen to students' mathematics was an ongoing topic for conversation in the LTBI MPD encounter. For the professional learning task in focus, the goal was to discuss how tasks span multiple levels of the trajectory to engage many students at once—an issue that emerged as the group considered how to move from the researchers' use of the trajectory to understand individual students to the teachers' use of the trajectory to organize whole classroom instruction. This task started with MPD researchers asking teachers to adapt to their grade level one open instructional task, use the task with their students, discuss what happened in the multiple implementations of the task across different groups of students from the same grade level, and prepare a short summary of each grade level's experience to promote cross grade-level conversations in the next MPD meeting.

The next professional development meeting began with engaging each grade-level team in sharing their work and reflections on instruction using their tasks. Teachers used samples of students' written work, lessons plans, and data about students' successes and difficulties as boundary objects to engage the group in conversation. Next, the group explored the instructional issues that emerged across grade levels, and formalized the discussion in relation to the concept of LTBI. As an application, teachers worked on choosing an existing lesson from their textbooks and adapting it to allow for various entry points using the learning trajectory. This example demonstrates how we learned about the teachers' challenges of using an open task and the aspects of the trajectory they drew upon as they implemented the task and considered their students' learning.

5.4 Learning conjectures

Another central tenet of design research is the articulation of a set of learning conjectures (Cobb et al. 2003; Confrey and Lachance 2000). These conjectures are proto-theories to be confirmed or challenged through ongoing and retrospective analysis (Cobb 2000). They are the kernel of the local learning theories to be generated through this research methodology. In considering professional development as a boundary encounter, the refinement of learning conjectures is the MPD "content learning goal" for the researchers. For example, in LTBI, while teachers' goals for the MPD were to learn about learning trajectories, the researchers' goals were to refine conjectures about how teachers come to learn about and use these trajectories. Thus, although researchers and teachers were working together around learning trajectories and connecting these trajectories to their practices, their learning goals for boundary practices were distinct. Each group was seeking to enhance knowledge for their own practices based on their experiences with the practices of a different professional community.

We take learning conjectures to be the researchers' set of expectations about teachers' participation in boundary practices. Initially, these conjectures anticipate how teachers position themselves in the MPD community and what it means to develop expertise in the practice of MPD encounter. Here, we offer an example from the evolution of one conjecture as the LTBI project progressed. We seek to demonstrate the ways we met our learning goals as MPD researchers from the exchange of knowledge in the boundary encounter.

5.4.1 Refining one conjecture on teachers' positioning

At the outset of the LTBI project, we were interested in the ways teachers positioned themselves in the emerging practices of the MPD. We wanted to examine whether teachers would position themselves as experts when researchers and teachers engaged in boundary practices. Our initial conjecture was that teachers' learning about the trajectory would bring their participation closer to the center of the professional community and strengthen their voice in the discourse of the group. As we started working with teachers, because of our focus on student learning, we observed that more often than positioning *themselves* in the discourse, teachers positioned *students*. The discourse act of positioning students, in fact, was so prevalent in the emerging discourse of the boundary encounter that we shifted our attention to this aspect of positioning and whether teachers' positioning of students changed as the LTBI boundary practices were established.

For example, a predominant use of language from teachers' practice was to position students according to grade levels. As the project started, teachers referred to students as being within, above, or below grade level; a language used in their school practice for grouping students for mathematics. Although this language was not the language researchers used to talk about students, as teachers initially learned about learning trajectories, they considered that the trajectories should align with students grade levels and one should be able to describe where a student was on the trajectory given the student grade level.

In studying teachers' discourse within the professional development, we came to learn that teachers continued to use grade-level language throughout the boundary encounter. However, the ways in which this language was used changed as our boundary practice emerged. To provide an example of this change, we draw on over 200 talk turns from the entire project that were coded as referring to grade level—the complete discussion of this empirical analysis within the LTBI project is beyond the scope of this paper. At first, teachers referred to grade levels as a general way to define expectations for what students "should" be able to do based on content goals for particular grades. For example, early on the LTBI project one teacher said:

> So the way she divided them up was what I really would have expected a slightly younger child to do. As a third grader, she should have said, "eight times three".

In this example, positioning the student as a third grader created an expectation that the student should be able to multiply and solve the equipartitioning problem presented. Later in the project, teachers talked about grade level in relation to the new ideas and vocabulary they were learning. For example, one teacher said:

> In kindergarten, there were some students who did benchmarking and composition of splits for the blueberry pie; kids, who at the beginning of the year, lacked memorization skills.

In this statement, although the teacher still referred to students as being in kindergarten, the focus of the talk turn was on the specific strategies students in the kindergarten classroom actually implemented. The student was not positioned mostly in relation to the grade level, but in relation to the strategies from the equipartitioning learning trajectory. This indicated a change from discourse that presented expectations for what students in certain grade levels cannot do, to the use of the learning trajectory as a tool for describing what students can do. In refining our initial conjecture about positioning, we came to state that teachers' learning about the trajectory not only positions themselves as the center of the professional development, but also supports new ways of positioning students as knowers in their classrooms.

6 Teachers' perspective

This paper has used the concepts of MPD as boundary encounter and design research to examine the theoretical question of how MPD can be conceptualized and designed around research-based knowledge in ways that promote knowledge exchange among researchers and teachers to improve the practices of both the research and the teaching communities. Thus far, we have focused the theoretical discussion and examples on our perspective as MPD researchers. Before we draw conclusions and implications, we briefly examine teachers' perspectives to test the feasibility of the boundary encounter concepts and the proposed partnership between researchers and teachers proposed in this paper. None of the theory discussed would be meaningful if teachers did not consider a partnership was possible.

We share three categories that emerged from our conversation with teachers, who were purposefully selected to show the potential of our theoretical frames. These categories are: identities of the brokers, boundary objects and design, and boundary practices. They emerged from a 1.5-h audio-recorded focus-group interview with 4 of the 22 teachers who participated in LTBI. These teachers had maintained contact with the researchers and volunteered to be part of this conversation, which was conducted 1.5 years after the conclusion of the project. The main topic for discussion during the conversation was whether, and in what ways, researchers and teachers could be considered partners in the LTBI project.

6.1 Brokers' identities

In the focus group, teachers talked about all participants in the LTBI project as having "dual roles". From their perspective, both researchers and teachers in LTBI were simultaneously "learners and guides". Teachers used expressions such as researchers and teachers "learning alongside each other," and explained that each group "guided the learning" of the other group. For example, teachers considered themselves learners of learning trajectories. Researchers, in this case, served as guides. At the same time, teachers talked about MPD researchers as learners of classroom realities and constraints that are imposed when research knowledge is instantiated in actual teaching. As one teacher explained, "We were certainly learners, yet we weren't total—we didn't come blank so we brought with us the realities of the age group that we work with, and the constraints that we work with in the field, in the trenches."

From a learners' perspective, some teachers commented on having to learn mathematics in the professional development. When mathematics content knowledge discussions of division or fractions emerged within discussions around the equipartitioning learning trajectory, teachers considered themselves as learners in the most traditional sense. However, when mathematics pedagogical discussions around learning trajectories took place, teachers talked about themselves as "practitioners." As one teacher put it, "There was learning math and learning about the research, so there was us as learners and then there was us as practitioners, I think. And in the role of practitioners, we were colleagues with the researchers, all working together."

6.2 Boundary objects and design

Teachers pointed to a variety of artifacts used in the professional development as important to them in the MPD. In particular, they noted the videos of clinical interviews and of their colleague's classrooms as support for productive conversation. They considered important the work they did with their own students and the artifacts generated through that work that were brought back to the practice of the MPD. They talked about interviewing, conducting assessments, and trying tasks with their *own* students. These instances of taking professional development tasks into their classrooms and bringing back the results to the professional development for discussion generated some of the most meaningful boundary objects from the teachers' perspectives.

However, teachers also noted that it was not just having those artifacts serving as boundary objects for collective practice that was important to them. They considered that the conversations and discussions that were created around these objects were as important as the objects themselves—alluding to the practices that emerged within tasks designed around boundary objects. One teacher commented:

> I liked reaching a point that we did in the sequence, in our work together, where people felt comfortable sharing successes and struggles. And so when we

> videotaped ourselves and we could actually have conversations about that without everyone having to say, "That was really good." There wasn't this level of patronizing about it. We just were focused on what we could learn from those different situations. I felt that was pretty powerful to be able to dialog with colleagues in a nonjudgmental way where we were all focused on learning.

6.3 Boundary practices

When asked about the most significant learning from the encounter, teachers offered practices that challenged their views of mathematics learning. They suggested the value of their emerging focus on what students know, supported by learning trajectory knowledge. One teacher discussed that the most significant idea that she learned was "not to underestimate kids and how much knowledge they bring to the experience—maybe that above all." Another agreed, adding that the practice of giving students space and time to use their own ideas, to "struggle a little longer," was a key learning for her. She added: "It truly is valuable. Kids do really get something out of it…I think that it's important for kids to see where they can go with it without anybody interfering with their thinking and not cutting off their thinking by interjecting something that they might come to on their own."

One teacher focused her comments on student-centered instructional practices. "Knowing what kinds of things to anticipate or responses kids were going to have, and then thinking about, like how to present them to the class, and what order I wanted to select, and who I wanted to select" was a meaningful practice developed in the boundary encounter. Another teacher offered the practice of analyzing students' mathematical work across grade levels to understand the longitudinal development of ideas, and her contribution to that development, as significant: "Looking at like some of the third grade samples or first grade samples and kind of seeing like where our kids were coming from and what they're going on to do, um, was really interesting and it made me more aware of the global picture within the school."

7 Discussion: interrelations and exchange of researchers' and teachers' knowledge in professional development

In this paper, we examined a process for researchers and teachers to exchange knowledge in MPD in ways that impacted both researchers' and teachers' practices without reducing the importance of either. We considered boundary encounters and design research as frameworks that, together, allowed for the conceptualization of MPD in ways that attended to the process goals of researchers and mathematics teachers as partners in professional development while also espousing explicit, albeit different, content learning goals for researchers and teachers. We also provided proof of concept that teachers can see themselves as partners in MPD while learning about research-based knowledge.

To conclude, we highlight some of our learning that we deem important for MPD researchers because they have implications for the ways in which MPD researchers act as brokers and interact with teachers in future boundary encounters designed around research-based knowledge. By sharing these ideas back to research community with the goal of modifying our own practices as mathematics education researchers, we fulfill our role as brokers who bring knowledge from boundary encounters back to their original community.

We initiated this paper arguing that MPD with explicit research-based learning goals could be designed in ways that cared for teachers. We now argue that MPD with research-based goals promotes partnership among researchers and teachers when *teachers* are interested in the research results around which the MPD practices are organized. The goal of learning about research-based knowledge in MPD is a goal for teacher learning and teachers set that goal—not researchers. This statement implies that the design of successful MPD around research-based knowledge is structured around research findings that have the potential to be useful from the teachers' perspective. It is the teachers' interest in the research findings that determines this potential.

When both researchers and teachers engage in practices from their communities that are of interest to the other group, they play the "dual roles" of learners and guides for MPD encounters—as teachers referred to it. In this case, researchers and teachers negotiate meaning as they help others make sense of their practices while also making sense of the practices of others. In the case of MPD organized around students' mathematics and mathematics learning, teachers' practices are highly contextualized in the culture of schooling. Understanding these contexts and culture is part of what researchers and teachers negotiate as brokers. As teachers' expressed it, they taught researchers the reality of classrooms and the difficulties of integrating research knowledge into practice.

Focusing the design of professional learning tasks around pedagogy was an important design principle for the LTBI project from a design research perspective. Placing this principle within the framework of boundary encounters, nonetheless, further highlighted its importance. We came to understand that this feature of design was fundamental in facilitating teachers' participation as partners in the emerging MPD practices. Beyond Philipp's (2008) claim that elementary teachers' attend to students prior to mathematics, we contend that attending to students first positions teachers as experts in MPD and allows for collective boundary practices. As expressed by teachers themselves, they were "partners" when MPD practices were around pedagogy and "learners" when the MPD focused on mathematics content.

On the other hand, attending to the role of MPD researchers as brokers, the expectation that brokers transform their knowledge and take new knowledge back to their communities also positioned MPD researchers as learners. We claim that it is the positioning of researchers as learners in professional development that establishes and makes them partners, as both teachers and researchers are engaged in the MPD to learn. From that perspective, designing research strengthens the researchers' attention to their learning in the MPD.

As brokers, we point to two other contributions from our design research to the mathematics education research community. First, there are our revised learning conjectures, which renew attention to teachers' positioning of students in MPD contexts. Second, there is our emerging understanding of aspects of research that can serve as boundary objects and of the usability of learning trajectories for teachers. Our work is raising important questions and considerations for mathematics education researchers working to improve mathematics learning in classrooms, related to how research-based knowledge can be shared and the utility of their products.

We conclude by arguing that for MPD researchers to serve as brokers from the research community in professional development settings, clear paths are needed to bring knowledge from teachers back into the research community. Wenger (1998) noted that boundary encounters maintained and renewed communities, connecting them to the world. We propose that the knowledge from teachers and teaching MPD researchers learn in boundary encounters with teachers can maintain and renew the larger mathematics education research community—and most importantly, connect it to the world of mathematics classrooms.

Acknowledgments The authors wish to thank our partner teachers for learning and teaching with us.

References

Akkerman, S., & Baker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169.

Ball, D. L., & Cohen, C. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey Bass.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–333). Mahwah: Lawrence Erlbaum Associates.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*, 9–13.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences, 13*(1), 15–42.

Confrey, J. (2006). The evolution of design studies as methodology. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 135–151). New York: Cambridge University Press.

Confrey, J. (2012). Better measurement of higher-cognitive processes through learning trajectories and diagnostic assessments in mathematics: The challenge in adolescence. In V. Reyna, M. Dougherty, S. B. Chapman, & J. Confrey (Eds.), *The adolescent brain: Learning, reasoning, and decision making* (pp. 155–182). Washington, DC: American Psychological Association.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–265). Mahwah: Lawrence Erlbaum Associates.

Confrey, J., Maloney, A., Nguyen, K., Mojica, G., & Myers, M. (2009). Equipartitioning/splitting as a foundation of rational number reasoning using learning trajectories. In *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (pp. 345–353), Thessaloniki, Greece.

Dawson, S. (1999). The enactive perspective on teacher development: A path laid while walking. In B. Jaworski, T. Wood, & S. Dawson (Eds.), *Mathematics teacher education: Critical international perspective*. London: Falmer Press.

DuFour, R., DuFour, R., Eaker, R., & Many, T. (2006). *Learning by doing: A handbook for professional learning communities at work*. Bloomington: Solution Tree.

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *Elementary School Journal, 109*(5), 510–524.

Jaworski, B. (2011). Situating mathematics teacher education in a global context. In N. Bernardz, D. Fiorentini, & H. Rongjin (Eds.), *International approaches to professional development for mathematics teachers* (pp. 2–20). Ottawa, Canada: University of Ottawa Press.

Jaworski, B., & Goodchild, S. (2006). Inquiry community in an activity theory frame. In J. Novotna, H. Moraova, M. Kratka, N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 353–360), Prague, Czech Republic: Charles University.

Kelly, A., Lesh, R. A., & Baek, J. Y. (2008). *Handbook of Design Research Methods in Education. Innovations in Science, technology, Engineering, and Mathematics Learning and Teaching*. New York: Routledge.

Kieran, C., Krainer, K., & Shaughnessy, J. M. (2012). Linking research to practice: Teachers as key stakeholders in mathematics education research. In M. Clements, et al. (Eds.), *Third international handbook of mathematics education* (pp. 361–392). New York: Springer.

Krainer, K. (2011). Teachers as stakeholders in mathematics education research. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 47–62). Ankara, Turkey: PME.

McKennery, S., & Reeves, T. (2012). *Conducting educational design research*. New York: Routledge, Taylor and Francis Group.

Philipp, R. (2008). Motivating prospective elementary school teachers to learn mathematics by focusing upon children's mathematical thinking. *Issues in Teacher Education, 17*(2), 7–26.

Ponte, J. P. (2009). External, internal and collaborative theories of teacher education. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds). *Proceeding of 33th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 99–103). Thessaloniki, Greece.

Sawyer, R. K. (2006). *The Cambridge handbook of the learning sciences*. New York: Cambridge University Press.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Simon, M. (2008). The challenge of mathematics teacher education in an era of mathematics education reform. In B. Jaworski & T. Woods (Eds.) *International handbook of mathematics teacher education—The mathematics teacher educator as a developing professional* (Vol 4, pp. 17–30). Rotterdam, Netherland: Sense Publishers.

Smith, M. (2001). *Practice based professional development for teachers of mathematics*. Reston: NCTM.

Star, S., & Griesemer, J. (1989). Institutional ecology, 'translations' and boundary objects: Amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907–39. *Social Studies of Science*, *19*(3), 387–420.

Sztajn, P. (2008). Caring relations in the education of practising mathematics teachers. In B. Jaworski & T. Woods (Eds.) *International handbook of mathematics teacher education—The mathematics teacher educator as a developing professional* (Vol. 4, pp. 299-314). Rotterdam, Netherland: Sense Publishers.

Sztajn, P., Confrey, J., Wilson, P. H., & Edgington, C. (2012). Learning trajectory based instruction: Towards a theory of teaching. *Educational Researcher, 41*(5), 147–156.

Sztajn, P., Wilson, P. H., Edgington, C., Meyers, M., & Dick, L. (2013). Using design experiments to conduct research on mathematics professional development. *Revista Alexandria, Journal of Science and Technology Education*, *6*(1), 9–34.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.

Wilson, P. H., Sztajn, P., & Edgington, P. (2012). Designing professional learning tasks for mathematics learning trajectories. In Tso, T. (Ed.), *Proceedings of the thirty-sixth annual meeting of the International Group for the Psychology of Mathematics Education*. (pp. 227–234), Taipei, Taiwan.