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ORIGINAL ARTICLE

The textbook-in-use: students' utilization schemes of mathematics textbooks related to self-regulated practicing

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Abstract This paper presents a qualitative study on how students make use of their mathematics textbooks for practicing. The study was carried out in two German secondary schools with 74 students (44 in 6th and 30 in 12th grade). Students' utilization of textbooks for practicing is analyzed using the theoretical framework of instrumental genesis. The results indicate that students' choices of contents from the book for practicing can be categorized into three utilization schemes: position-dependent practicing, block-dependent practicing, and salience-dependent practicing. In terms of position-dependent practicing the relative position of the textbook's contents to teacher-mediated sections guides the students' choice. Block-dependent practicing relates to the use of contents from the book that belong to particular blocks. Finally, salience-dependent practicing is a utilization scheme of the book where students' choice is guided by perceptual salience of the book contents. These findings both show how textbook users are influenced by the way mathematics is presented in textbooks and provide insights into students' conceptions of practicing mathematics.

Keywords Instrumental genesis · Utilization scheme · Textbook · User study · Practicing

1 Introduction

"No evaluation of texts as they are, or texts as they might be, is possible until we consider how they perform in the classroom. One cannot really judge the functional contribution of the text alone, for the text-in-use is a complex social process wherein a book, an institution, and a number of human beings are interlaced beyond the possibility of separation" (Cronbach 1955, p. 188).

Although Cronbach had already drawn attention to the text-in-use as an important issue for textbook research in 1955, the majority of recent publications on textbook research still focus on the text itself (Brantlinger 2011; Bryant et al. 2008; Mauch 2007; Törnroos 2005; Weinberg and Wiesner 2011). However, textbook analysis is only capable of revealing opportunities to learn, and will not provide insights into how these opportunities are taken up in practice.

If textbook use is approached empirically, it is usually the use by the teacher that is studied. Furthermore, most of the studies investigate textbook use with only a few cases (Collopy 2003; Davis 2009; Johansson 2006; Nicol and Crespo 2006; Pepin and Haggarty 2001).

Students' use of textbooks is hardly investigated even though textbooks mainly address students. This might be due to the difficulty in obtaining valid data, as "the use of texts by a student outside of class, working alone, perhaps as part of their homework, following up a lecture, or perhaps because the course is taught 'at a distance,' is even more opaque to enquiry" (Love and Pimm 1996, p. 397). Another reason might be that students' use of textbooks is solely seen as dependent on teacher-mediation in the sense of Pepin and Haggarty (2001, p. 165): "teachers act as mediators of the text. Teachers decide which textbooks to use; when and where the textbook is to be used; which sections of the textbook to use; the sequencing of topics in the textbook; the ways in which pupils engage with the text; the level and type of teacher intervention between pupil and text; and so on."

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However, Rezat (2011) challenged the view that students' use of textbooks is always teacher-mediated. He identified five self-regulated learning activities in which mathematics textbooks are incorporated: (1) solving tasks and problems, (2) practicing, (3) acquisition of new knowledge, (4) interest-driven activities, and (5) metacognitive learning activities. The methodological challenge of obtaining valid data seems to be especially relevant for self-regulated practicing, as this activity is likely to be carried out outside the mathematics class. This paper presents an empirical study on how students make use of their mathematics textbook for self-regulated practicing activities.

In order to investigate students' textbook use it has to be clarified what particular perspective is taken on the mathematics textbook, what it means to 'make use of a textbook' and how this is socially and culturally embedded. This is elaborated in Sect. 2. Section 3 presents the methodology of the study. Results are presented in Sect. 4 and discussed in Sect. 5.

2 Theoretical framework

2.1 The situation of textbook use

This study uses Rezat and Sträßer's (2012) socio-didactical tetrahedron (SDT) to model the situation of textbook use with its social and cultural influences (Fig. 1). The SDT draws on activity theory and uses Engeström's (1987) model of the activity system, which describes the systematic whole of "object-oriented, collective, and culturally mediated activity" (Engeström et al. 1999, p. 9). The textbook is regarded as an artifact which enters into the relation of humans with mathematics and "recreates, reconstructs the whole structure of behavior" (Vygotsky 1997). Rezat and Sträßer (2012, p. 9) argue that this model is capable of "reveal[ing] a picture that contributes to a situated understanding of the complex network of actions and their backings in the situation at hand."

This model provides a structure of the situation of textbook use by drawing attention to teacher and students as the main users of textbooks and the complex interplay between institutional context and important social and cultural influences. As in activity theory, the situation is determined by the goal of the activity.

The power of the model can also be seen as a shortcoming: the model only elicits fundamental constituents of textbook use and their relations. It does not provide a theory of each of the vertices, nor a theoretical understanding of their relations. Awareness of the complexity of the situation of textbook use is not

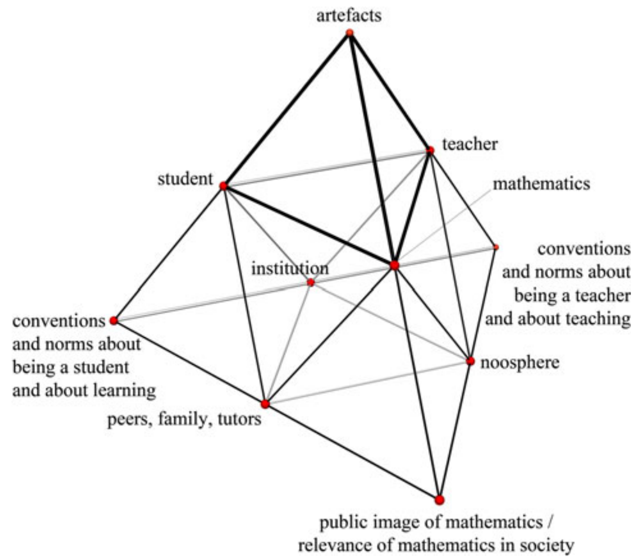


Fig. 1 Socio-didactical tetrahedron model (Rezat and Sträßer 2012)

sufficient: it does not imply an understanding of what characterizes the relations of the fundamental constituents. Consequently, it is necessary to enhance this framework by incorporating other theories aimed at a better understanding of the vertices or characterizing the relations between different vertices in more detail. At the same time it becomes evident that it is impossible to analyze textbook use referring to the whole tetrahedron model within the scope of one paper. Therefore, the scope of the analysis in this paper is necessarily limited to particular segments of the tetrahedron model.

In the present study the SDT provides a model of the activity of self-regulated practicing incorporating one particular artifact, namely the textbook. Self-regulated practicing is understood as a self-regulated activity of students without any active intervention of the teacher. In particular, the teacher does not tell the students which parts of the textbook they are supposed to use. Furthermore, this activity is usually situated outside the mathematics class.

Students themselves refer to this activity with verbs such as 'to practice,' 'to exercise,' 'to review,' and 'to learn.' An investigation of the meanings students attribute to these verbs revealed no substantial differences (Rezat 2011). Therefore, all activities referred to with one of these verbs are subsumed into the activity 'practicing.' According to students' own definition, the general goal of practicing is improvement: improvement of the understanding of concepts and procedures, improvement of the ability to carry out procedures, and, finally, improvement of their grades in mathematics (Rezat 2011).

2.2 The structure of German mathematics textbooks

Analyzing the use of textbooks requires a clear vision of the textbook itself. The particular perspective that is taken in this study focused on the structural level of mathematics textbooks. The structure might be regarded as the interface between the user and the textbook, as it allows identifying and approaching particular parts of the book that the user regards as potentially relevant for his goals.

Drawing on Valverde et al.'s (2002) seminal analysis of mathematics textbooks within the Third International Mathematics and Science Study (TIMSS), Rezat (2006, 2008) analyzed the structures of German secondary school mathematics textbooks of a larger and more recent sample than in the TIMSS. Within the scope of this paper, it is only possible to provide a summary of the results necessary to understand the present analysis. For a detailed description of the theoretical framework and methodology of the textbook analysis refer to Rezat (2006, 2008).

Rezat's analysis reveals that the structures of German secondary school mathematics textbooks are best described in terms of three structural levels: the book-, the chapter-, and the lesson-level. On each level the structure is most adequately characterized in terms of blocks in the sense of what Valverde et al. (2002, p. 141) refer to as "building blocks of the larger structures." For example, the lesson structure is characterized by five types of blocks: introductory tasks and activities; expositions; kernels[1] in a box; worked examples; and tasks and problems. Furthermore, the analysis of German secondary school mathematics textbooks reveals that particular functions are typically attributed to the different block types. More specifically, introductory tasks and activities are intended to engage the student actively, will help to recall relevant past ideas, serve to prepare for the new material, and assist the approach to the new material; expositions explain, develop new ideas, and introduce new concepts; kernels in boxes provide an overview and consolidate what has been developed in the exposition; worked examples are intended to be paradigmatic for the tasks and problems. Most of the textbooks also differentiate tasks and problems according to particular functions. Typical functions which are addressed are evaluation of basic understanding, deepening understanding, and recapitulation of former topics. Within this study the notion of blocks is used to conceptualize what sections students use in their mathematics textbooks.

[1] The notion of "kernels" refers to van Dormolen (1986) and denotes "general expressions that have to be learned as knowledge" (p. 146).

2.3 Utilization schemes of mathematics textbooks

Focusing on textbook use, rather than on the textbook itself, leads to the question of how this is conceptualized. In her meta-analysis of studies on teachers' use of curriculum materials, which include textbooks, Remillard (2005) distinguishes four different conceptualizations of the teacher–curriculum material relationship: (1) curriculum use as following or subverting the text; (2) curriculum use as drawing on the text; (3) curriculum use as interpretation of the text; and (4) curriculum use as participation with the text. Remillard (2005) concludes that an understanding of the teacher–curriculum material relationship is enhanced by studies that see curriculum use as participation with curriculum materials, that is, "that teachers and curriculum materials are engaged in a dynamic interrelationship that involves participation on the parts of both the teacher and the text" (p. 221).

It seems reasonable to assume that Remillard's categories are also applicable to the student–curriculum material relationship. Therefore, the activity of textbook use by students, i.e. the triangle 'student–textbook–mathematics' of the SDT, is conceptualized drawing on Rabardel's (1995) theory of the instrument. This is in line with Remillard's conclusion and the general activity theoretical framework of the study. Rabardel's theory offers concepts to understand the dynamic user–tool interrelationship in terms of mutual participation. By focusing on the cognitive aspects of tool use, this theory allows for a deep understanding of the student–textbook interrelation and how it is incorporated into the learning of mathematics in general.

The differentiation between artifacts and instruments is at the core of the theory of the instrument. The instrument is not a given entity, but is created by the user in the course of using an artifact. Therefore, the instrument is defined as "a composite entity made up of an artifact component (an artifact, a fraction of an artifact, or a set of artifacts) and a scheme component (one or more utilization schemes, often linked to more general action schemes)" (Rabardel 2002, p. 86). The process by which the user develops the instrument is called instrumental genesis. It is characterized by two reciprocal processes: *instrumentalization* and *instrumentation*. *Instrumentalization* is directed towards the artifact. It is related to "the emergence and evolution of artifact components of the instrument: selection, regrouping, production and institution of functions, deviations and catachresis, attribution of properties, transformation of the artifact (structure, functioning etc.), that prolong creations and realizations of artifacts whose limits are thus difficult to determine" (Rabardel 2002, p. 103). Related to mathematics textbooks, this means that instrumentalization refers to functions that the user attributes to the textbook or particular blocks of the textbook: for example, the index can be used to find contents of the textbook related to a particular topic, tasks are useful for practicing, and boxes with kernels are useful for solving tasks from the textbook. It is important to remember that each user attributes these functions individually and that they are not necessarily compliant with the intentions of the textbook authors, or the functions that the teacher or the mathematics education researcher might have attributed to these blocks. *Instrumentation* is directed towards the user and related to the development of utilization schemes, that is, “their constitution, their functioning, their evolution by adaptation, combination coordination, inclusion and reciprocal assimilation, the assimilation of new artifacts to already constituted schemes, etc.” (Rabardel 2002, p. 103). From their early childhood, humans develop utilization schemes of books. On a very elementary level, these schemes comprise holding the book correctly (front and binding on the appropriate side) and turning pages. On a deeper level these utilization schemes refer to the way of reading. A novel has to be read linearly and comprehensively in order to be understood whereas a collection of short stories might be read selectively. The more complex and specific the structure of the book, the more specific and elaborate the utilization schemes of the user might be. Therefore, mathematics textbooks might require very specific utilization schemes, which a user has to develop in the interaction with the book.

In instrumentation processes the affordances and constraints of the artifact affect the users and their development of utilization schemes. This is why an understanding of the structure of the mathematics textbook is essential. The structure affords certain ways of using the textbook and constraints others. For example, the specific structure of mathematics textbooks might afford a selective way of using the textbook, because the composition of blocks enables the user to use a particular block for a certain end. However, whether this is actually the case can only be revealed by an empirical study of textbook use.

Empirically investigating utilization schemes of mathematic textbooks requires clarifying how utilization schemes are conceptualized and how they can be described properly. In line with Rabardel (2002), the notion of a utilization scheme is conceptualized according to Vergnaud (1996), who characterizes schemes generally in terms of four aspects: “operational invariants, inference possibilities, rules of action, and goals” (Vergnaud 1996, p. 222). From these four aspects he highlights the importance of operational invariants as being constitutive for schemes: “These [operational invariants] form the specific parts of schemes that represent objects, predicates, conditions and theorems. The other ingredients of schemes (rules of action, goals, and inference possibilities) have no essential value in articulating practice and theory” (Vergnaud 1998, p. 176). Vergnaud (1998) distinguishes two different kinds of operational invariants: concepts-in-action and theorems-in-action. He defines a concept-in-action as “an object, a predicate, or a category which is held to be relevant,” and a theorem-in-action as “a proposition which is held to be true” (Vergnaud 1998, p. 168). The appendix “in-action” indicates that concepts-in-action and theorems-in-action are usually not verbalized, but “they underlie students’ behaviour, and their scope of validity is usually smaller than the scope of theorems. They may even be wrong” (Vergnaud 1988, p. 144). This means that concepts-in-action and theorems-in-action have to be inferred from students’ actions based on the two questions: firstly, which concepts are relevant to the student in a particular situation; and secondly, which propositions do individual students regard as true in a particular situation?

Referring to Rabardel’s theory of the instrument as a conceptualization of “using” an artifact allows for specifying the research question: “Which utilization schemes of mathematics textbooks do students develop related to self-regulated practicing?”

3 Methodology

3.1 Data collection

Three instruments were used to collect data on students’ use of their mathematics textbooks:

1. A diary
2. Interviews
3. Classroom observation

In order to get a precise, comprehensive and ecological valid account of students’ textbook use in and out of class the students were asked to highlight every part they used in the textbook. Additionally, they were asked to explain *why* they used the part they highlighted in a diary by completing the sentence “I used the part I highlighted in the book because ….” This instrument was developed in order to get precise information about what the students actually use in the book and why they use it by keeping the situation of textbook use as natural as possible. Nevertheless, the method of highlighting sections in a textbook also introduces a particular way of using the textbook, which deviates from the normal use.[2] Consequently, a bias on the data cannot be totally excluded.

In order to develop a deeper understanding of particular uses of the textbooks, additional stimulated recall interviews were conducted with selected students. In these interviews,

[2] In Germany, schools usually provide textbooks. Therefore, writing or highlighting in textbooks is not allowed.

students were invited to recall and explain the situation in which a highlighted part in their textbook was used.

In addition, all the mathematics lessons during the period of data collection were observed and field notes were taken. Firstly, the overall structure of the lesson was recorded in the field notes using a table comprising three columns: time, activity/content, and remarks. Secondly, all utterances concerning the textbook were transcribed literally. Furthermore, a focus was put on all utilizations of the textbook. The use of the textbook both by the students and by the teacher were taken into account. On the one hand, the observation provides an insight into the way the teacher mediates textbook use in the classroom. It makes a difference whether students' use of the textbook is self-regulated or if it is teacher-mediated. On the other hand, the triangulation provides a measure for the validity of the data. Collecting data on how the textbook has been used in the classroom makes it possible to compare the markings and comments of the students with the field notes. The degree of correspondence between these two sources related to the use of the textbook in the classroom indicates how seriously the students took their task. If students carefully documented in-class use of textbooks, it is inferred that they did the same with out-of-class use.

Data was collected for a period of 3 weeks in two 6th grade and two 12th grade classes in two German secondary schools. Within the German tripartite school system, these schools are considered to be for high-achieving students. All four classes were taught by different teachers. While one 6th grade and one 12th grade class were chosen randomly, the other two classes were selected according to the principle of theoretical sampling (Strauss and Corbin 1990). Altogether, 74 students participated in the study: 44 grade 6 students and 30 grade 12 students. Interviews were conducted with 12 students (9 students in the 6th grade, 3 students in the 12th grade). A total of 38 mathematics lessons were observed (15 in grade 6, 23 in grade 12).[3]

3.2 Analysis of individual utilization schemes related to practicing

Instrumentation and instrumentalization are individual processes and each instrumental genesis must be regarded as unique and special. Therefore, individual utilization schemes of mathematics textbooks for practicing were analyzed in a first step. The following section presents an exemplary analysis and demonstrates how operational invariants as the fundamental constituents of utilization schemes are inferred from the data.

[3] The larger number of observed lessons in grade 12 results from one course being an advanced course with twice as many lessons a week as the normal courses.

3.2.1 Data example and exemplary analysis

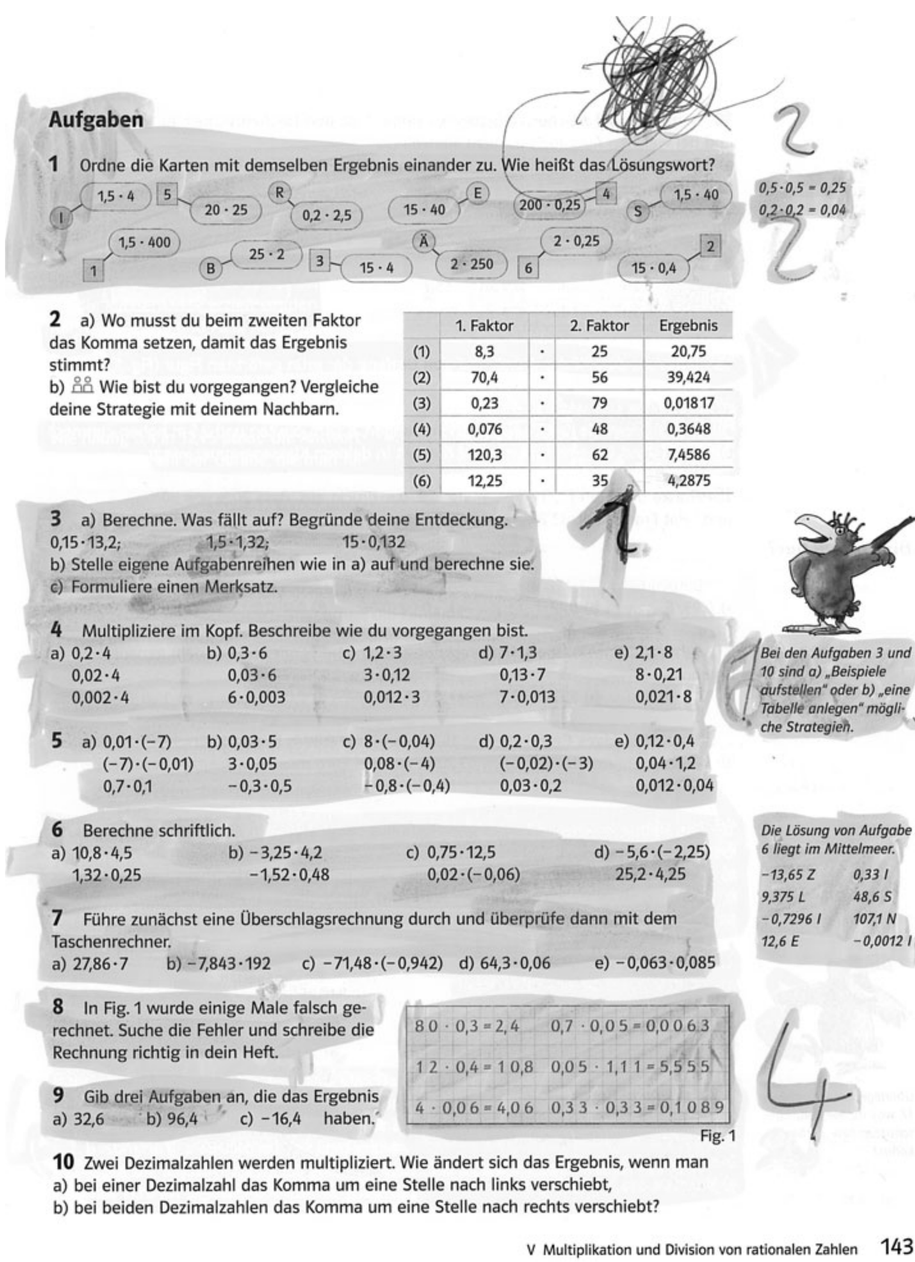
Figure 2 presents a sample of Emma's (6th grade) data. Emma highlights tasks 1 and 3–9. In her booklet she annotates this use with "exercise + class" (see Fig. 3). The field notes reveal that the teacher mediated tasks nos. 3, 4, and 5 on the same page. In the interview Emma explains her way of proceeding as follows:

> Researcher: You have highlighted a lot and here you have described several things as "exercise" and "class" and as "exercise" and "homework." Could you please explain that to me a little further, so I understand what you mean by that?
> Emma: Yes, well, um, exercise and class means, that we, um, did, the, um, exercises in class either orally or written. Then, I did this at home as an exercise, or just for practicing.
> Researcher: Ok, so the same exercise again, just like you did in class?
> Emma: Yes.
> Researcher: So you did more exercises, additionally to the ones you did in class?
> Emma: Yes, um, I did some, um, which we didn't do in class.
> Researcher: Ok, so, why do you do more exercises, more than those you did in class?
> Emma: Well, so I can go through them again, to make sure I can really do them. And um so I can, um, become faster, and, um, be better.
> Researcher: Ok and how do you pick your extra exercises?
> Emma: Well, that differs, um, if we did N° 4 in class, I would maybe do N° 5, because it's similar to N° 4. I pick it, so I… well, I don't like word problems and then I rather do an exercise like N° 5. (Original in German, translated by Inga Gill)

In the interview Emma expresses two rules of action:

1. If we do tasks in the mathematics class, then I will do these tasks again at home.
2. If we do tasks from the book in the mathematics class, then I will also do the adjacent tasks for practicing.

Emma argues that she chooses adjacent tasks because they are similar. It is not absolutely clear whether she infers the similarity from the adjacency or if she looks for similar tasks in the first place and finds them adjacent to the tasks from the lesson. Comparing the tasks she chooses makes the first inference likely, because the tasks do not show any similarity except being related to the same topic. Emma's first rule of action seems to be grounded in a particular concept-in-action of practicing: practicing means repeating teacher-mediated tasks and additionally doing tasks that are


Aufgaben

1 Ordne die Karten mit demselben Ergebnis einander zu. Wie heißt das Lösungswort?

1,5 · 4 | 5 | 20 · 25 | R | 0,2 · 2,5 | 15 · 40 | E | 200 · 0,25 | 4 | S | 1,5 · 40
I | 1,5 · 400 | 1 | B | 25 · 2 | 3 | 15 · 4 | Ä | 2 · 250 | 6 | 2 · 0,25 | 15 · 0,4 | 2

0,5 · 0,5 = 0,25
0,2 · 0,2 = 0,04

2 a) Wo musst du beim zweiten Faktor das Komma setzen, damit das Ergebnis stimmt?
b) Wie bist du vorgegangen? Vergleiche deine Strategie mit deinem Nachbarn.

	1. Faktor		2. Faktor	Ergebnis
(1)	8,3	·	25	20,75
(2)	70,4	·	56	39,424
(3)	0,23	·	79	0,01817
(4)	0,076	·	48	0,3648
(5)	120,3	·	62	7,4586
(6)	12,25	·	35	4,2875

3 a) Berechne. Was fällt auf? Begründe deine Entdeckung.
0,15 · 13,2; 1,5 · 1,32; 15 · 0,132
b) Stelle eigene Aufgabenreihen wie in a) auf und berechne sie.
c) Formuliere einen Merksatz.

4 Multipliziere im Kopf. Beschreibe wie du vorgegangen bist.
a) 0,2 · 4 b) 0,3 · 6 c) 1,2 · 3 d) 7 · 1,3 e) 2,1 · 8
0,02 · 4 0,03 · 6 3 · 0,12 0,13 · 7 8 · 0,21
0,002 · 4 6 · 0,003 0,012 · 3 7 · 0,013 0,021 · 8

Bei den Aufgaben 3 und 10 sind a) „Beispiele aufstellen" oder b) „eine Tabelle anlegen" mögliche Strategien.

5 a) 0,01 · (−7) b) 0,03 · 5 c) 8 · (−0,04) d) 0,2 · 0,3 e) 0,12 · 0,4
(−7) · (−0,01) 3 · 0,05 0,08 · (−4) (−0,02) · (−3) 0,04 · 1,2
0,7 · 0,1 −0,3 · 0,5 −0,8 · (−0,4) 0,03 · 0,2 0,012 · 0,04

6 Berechne schriftlich.
a) 10,8 · 4,5 b) −3,25 · 4,2 c) 0,75 · 12,5 d) −5,6 · (−2,25)
1,32 · 0,25 −1,52 · 0,48 0,02 · (−0,06) 25,2 · 4,25

Die Lösung von Aufgabe 6 liegt im Mittelmeer.
−13,65 Z 0,33 I
9,375 L 48,6 S
−0,7296 I 107,1 N
12,6 E −0,0012 I

7 Führe zunächst eine Überschlagsrechnung durch und überprüfe dann mit dem Taschenrechner.
a) 27,86 · 7 b) −7,843 · 192 c) −71,48 · (−0,942) d) 64,3 · 0,06 e) −0,063 · 0,085

8 In Fig. 1 wurde einige Male falsch gerechnet. Suche die Fehler und schreibe die Rechnung richtig in dein Heft.

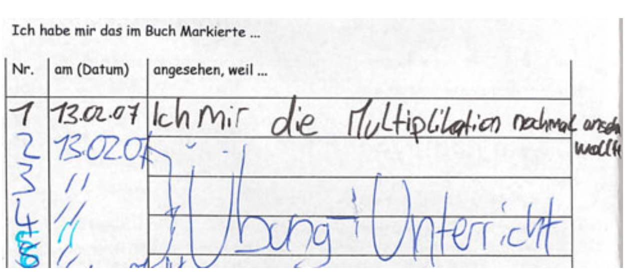
80 · 0,3 = 2,4 0,7 · 0,05 = 0,0063
12 · 0,4 = 10,8 0,05 · 1,11 = 5,555
4 · 0,06 = 4,06 0,33 · 0,33 = 0,1089
Fig. 1

9 Gib drei Aufgaben an, die das Ergebnis a) 32,6 b) 96,4 c) −16,4 haben.

10 Zwei Dezimalzahlen werden multipliziert. Wie ändert sich das Ergebnis, wenn man
a) bei einer Dezimalzahl das Komma um eine Stelle nach links verschiebt,
b) bei beiden Dezimalzahlen das Komma um eine Stelle nach rechts verschiebt?

V Multiplikation und Division von rationalen Zahlen 143

Fig. 2 Excerpt from Emma's (6th grade) mathematics textbook (Hußmann et al. 2006, p. 143) with highlighted used parts. Translation: *1* Pair the cards with the same result. What is the mystery word? *2* (a) Where do you have to place the decimal point in the second factor, so that the result is correct? (b) Explain your strategy and compare with the one of your neighbor. *3* (a) Calculate. What do you notice? Explain your discovery. (b) Make up your own task sequences like in *a* and solve them. (c) Write the rule down. *4* Multiply mentally. Explain how you did it. *5* (no text). *6* Calculate by using written methods. *7* Do a rough calculation first, and then check with your calculator. *8* Figure shows several mistakes in calculating. Find the mistakes and write down the correct way. *9* Write down three problems, which have the following result. *10* Two decimals are multiplied. How does the result change, if (a) the decimal point of one number is moved one further to the *left*, (b) the decimal point of both numbers is moved one further to the *right*? (Translation: Inga Gill)


Ich habe mir das im Buch Markierte ...

Nr.	am (Datum)	angesehen, weil ...
1	13.02.07	Ich mir die Multiplikation nochmal ansehen wollte
2	13.02.07	Übung + Unterricht
3	//	
4	//	
5	//	

Fig. 3 Excerpt from Emma's (6th grade) booklet. Translation: No. *1* I wanted to look at the multiplication once more; Nos. *2–5* exercise + class. (Author's translation)

similar to teacher-mediated tasks. Her way of proceeding in order to identify similar tasks is based on a theorem-in-action: if tasks in textbooks are adjacent then the tasks are similar. Consequently, Emma's utilization scheme of the mathematics textbook in this one particular situation can be characterized in terms of these two operational invariants.

3.3 Typology

A study of individual instrumental geneses has to tackle the question of the generalization of its findings. Different from quantitative studies, there are no accepted standards for generalizing findings from qualitative studies. Nevertheless, qualitative studies also aim at going beyond the particular and the individual by unveiling general patterns and structures which are manifested in individual actions in order to develop theory. The way that was chosen in this study helps to reveal and to describe these general structures by means of a typology. The method of constant comparison (cf. Strauss and Corbin 1990) was used to reveal similarities and dissimilarities between the cases. Finally, it led to the discovery of repeated patterns and structures in students' instrumentations of mathematics textbooks related to practicing. The utilization schemes were compared in terms of their operational invariants as these are regarded as constitutive features according to Vergnaud (1998). The findings presented in this paper refer to such types of instrumentation, which were found in different individuals' instrumental geneses. Their epistemological status might be described as 'empirically grounded' (Kluge 2000) in the sense that these types are grounded in and derived from data but are also idealized. Each type is illustrated by referring to paradigmatic cases.

4 Results

Three typical utilization schemes have been found: position-dependent practicing, block-dependent practicing, and salience-dependent practicing.

4.1 Position-dependent practicing

Emma's instrumentation of mathematics textbooks for practicing was described earlier (Sect. 3.2). It is characterized by two operational invariants: (1) practicing means to do the same tasks that the teacher mediated and tasks that are similar to teacher-mediated tasks; and (2) adjacent tasks in mathematics textbooks are similar. Emma's instrumentation is a paradigmatic example of a utilization scheme called position-dependent practicing.

Emma's data (6th grade) also shows that this scheme is associated with tasks belonging to the same block type. Figure 4 shows that she skips the tasks from a different block type ("Bist du sicher?"/"Are you confident?"), which is meant for students' self-regulated practicing and for self-evaluation.

Teacher-mediated tasks are a precondition for this scheme. This becomes evident in one 6th grade class: the teacher in this class does not mediate tasks from the textbook for 1 week and during that time the position-dependent practicing scheme does not occur in the data. However, after the teacher had mediated tasks from the textbook this scheme is shown by students.

This instrumentation of mathematics textbooks was observed equally in the 6th and in the 12th grade.

4.2 Block-dependent practicing

Besides position-dependent practicing, students also choose specific blocks for practicing. Figure 5 illustrates Laura's (6th grade) utilization of the textbook for practicing. She only uses the boxes with the kernels and comments on her use with the following statement: "because I frequently look at the rules." In the interview she explains: "so that I do not forget and the rules in the book are well phrased and that is why I read them frequently" (original in German, author's translation). Her behavior and her explanation elicit that two concepts-in-action of a specific block, namely the kernels, determine her utilization scheme: (1) "the rules are well phrased"; (2) the rules are in boxes. The first one is explained in the interview; the second is derived from her behavior of only looking at the boxes.

Other students also always use particular blocks for practicing. For example, Lilli (6th grade) and Leonie (12th grade) use tasks from a test on the chapter-level for practicing. The case of Lilli (6th grade) illustrates that other people might also mediate the choice of particular blocks for practicing. In the interview she explains:

> Lilli: Well, I am not so good at Maths, and um, that's why I sometimes study a little (Maths) and some days a little more, and on other days a little less (…) most of the times together with my father.
> Researcher: And how do you pick what you are using for studying?
> Lilli: I usually ask Mr. H. [the teacher] what I can do. (Original in German, translated by Inga Gill)

The fact that Lilli studies with her father draws attention to the bottom of the SDT and the influence of peers, family, and tutors besides the teacher on students' utilization schemes; this will be taken up later in Sect. 4.4.

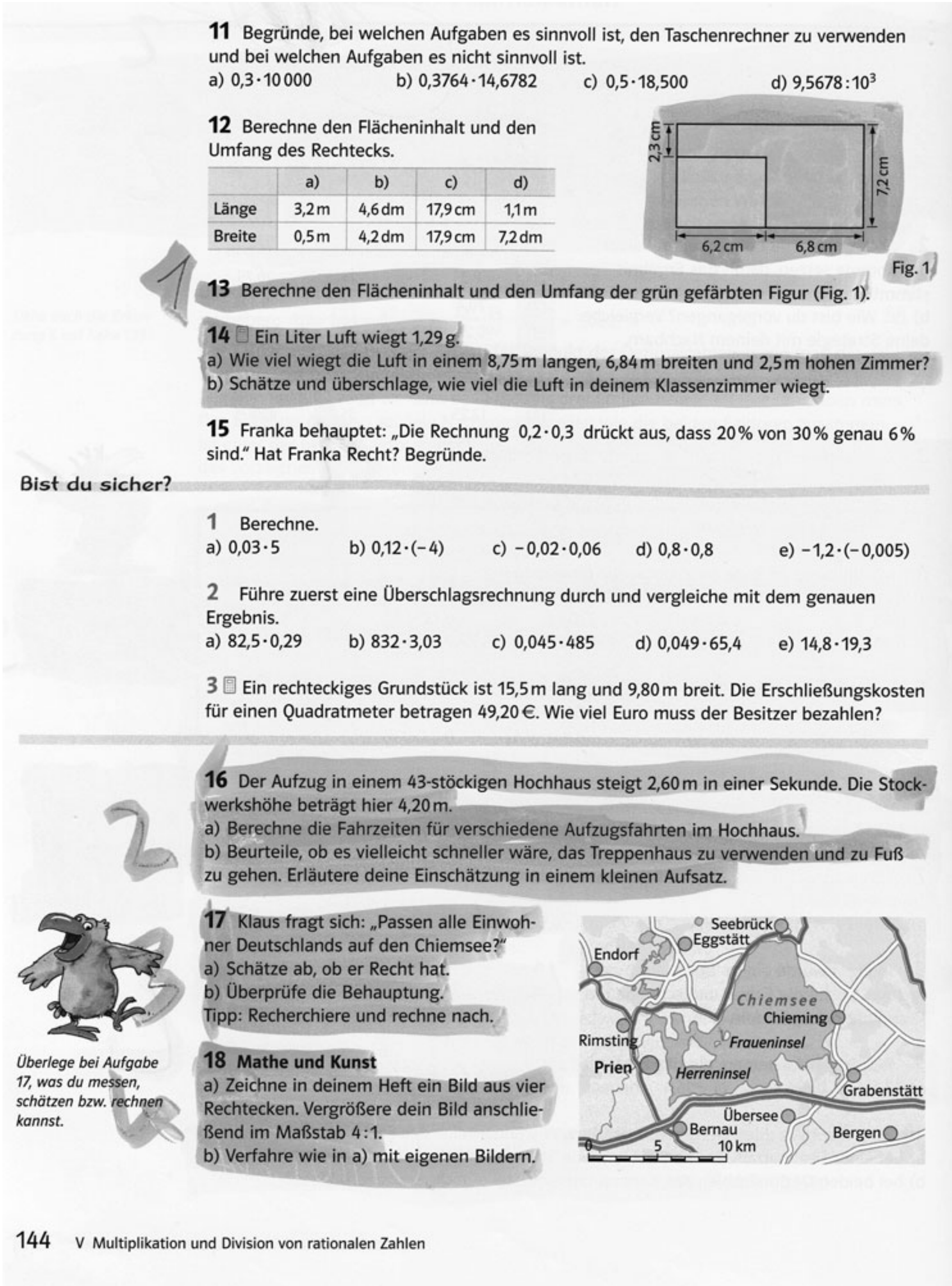
Block-dependent practicing relates to blocks at all three structural levels of the book. For example, in grade 12, Yvonne uses a block with tasks and problems on the chapter-level. This block is specifically intended for students' self-regulated revision of the contents of the whole chapter.

4.3 Salience-dependent practicing

A third utilization scheme of the textbook for practicing is dependent on salient features of tasks on a surface level. It seems to be particularly related to visual features of the task.

The teacher of one of the 12th grade classes announces that tasks similar to the task in Fig. 6 might be relevant for an upcoming test. Jennifer chooses task No. 9 in Fig. 7 from her book.

Whereas the teacher-mediated task can be found on page 21, the task Jennifer chooses is from page 31 (Fig. 7). Accordingly, her choice is not position-dependent. More likely, it seems to be related to visual features of the task, in this case the combination of words with a table. In the case of salience-dependent practicing, the concept-in-action of practicing is similar to position-dependent



11 Begründe, bei welchen Aufgaben es sinnvoll ist, den Taschenrechner zu verwenden und bei welchen Aufgaben es nicht sinnvoll ist.
a) 0,3·10 000 b) 0,3764·14,6782 c) 0,5·18,500 d) $9{,}5678:10^3$

12 Berechne den Flächeninhalt und den Umfang des Rechtecks.

	a)	b)	c)	d)
Länge	3,2 m	4,6 dm	17,9 cm	1,1 m
Breite	0,5 m	4,2 dm	17,9 cm	7,2 dm

13 Berechne den Flächeninhalt und den Umfang der grün gefärbten Figur (Fig. 1).

14 Ein Liter Luft wiegt 1,29 g.
a) Wie viel wiegt die Luft in einem 8,75 m langen, 6,84 m breiten und 2,5 m hohen Zimmer?
b) Schätze und überschlage, wie viel die Luft in deinem Klassenzimmer wiegt.

15 Franka behauptet: „Die Rechnung 0,2·0,3 drückt aus, dass 20 % von 30 % genau 6 % sind." Hat Franka Recht? Begründe.

Bist du sicher?

1 Berechne.
a) 0,03·5 b) 0,12·(−4) c) −0,02·0,06 d) 0,8·0,8 e) −1,2·(−0,005)

2 Führe zuerst eine Überschlagsrechnung durch und vergleiche mit dem genauen Ergebnis.
a) 82,5·0,29 b) 832·3,03 c) 0,045·485 d) 0,049·65,4 e) 14,8·19,3

3 Ein rechteckiges Grundstück ist 15,5 m lang und 9,80 m breit. Die Erschließungskosten für einen Quadratmeter betragen 49,20 €. Wie viel Euro muss der Besitzer bezahlen?

16 Der Aufzug in einem 43-stöckigen Hochhaus steigt 2,60 m in einer Sekunde. Die Stockwerkshöhe beträgt hier 4,20 m.
a) Berechne die Fahrzeiten für verschiedene Aufzugsfahrten im Hochhaus.
b) Beurteile, ob es vielleicht schneller wäre, das Treppenhaus zu verwenden und zu Fuß zu gehen. Erläutere deine Einschätzung in einem kleinen Aufsatz.

Überlege bei Aufgabe 17, was du messen, schätzen bzw. rechnen kannst.

17 Klaus fragt sich: „Passen alle Einwohner Deutschlands auf den Chiemsee?"
a) Schätze ab, ob er Recht hat.
b) Überprüfe die Behauptung.
Tipp: Recherchiere und rechne nach.

18 Mathe und Kunst
a) Zeichne in deinem Heft ein Bild aus vier Rechtecken. Vergrößere dein Bild anschließend im Maßstab 4:1.
b) Verfahre wie in a) mit eigenen Bildern.

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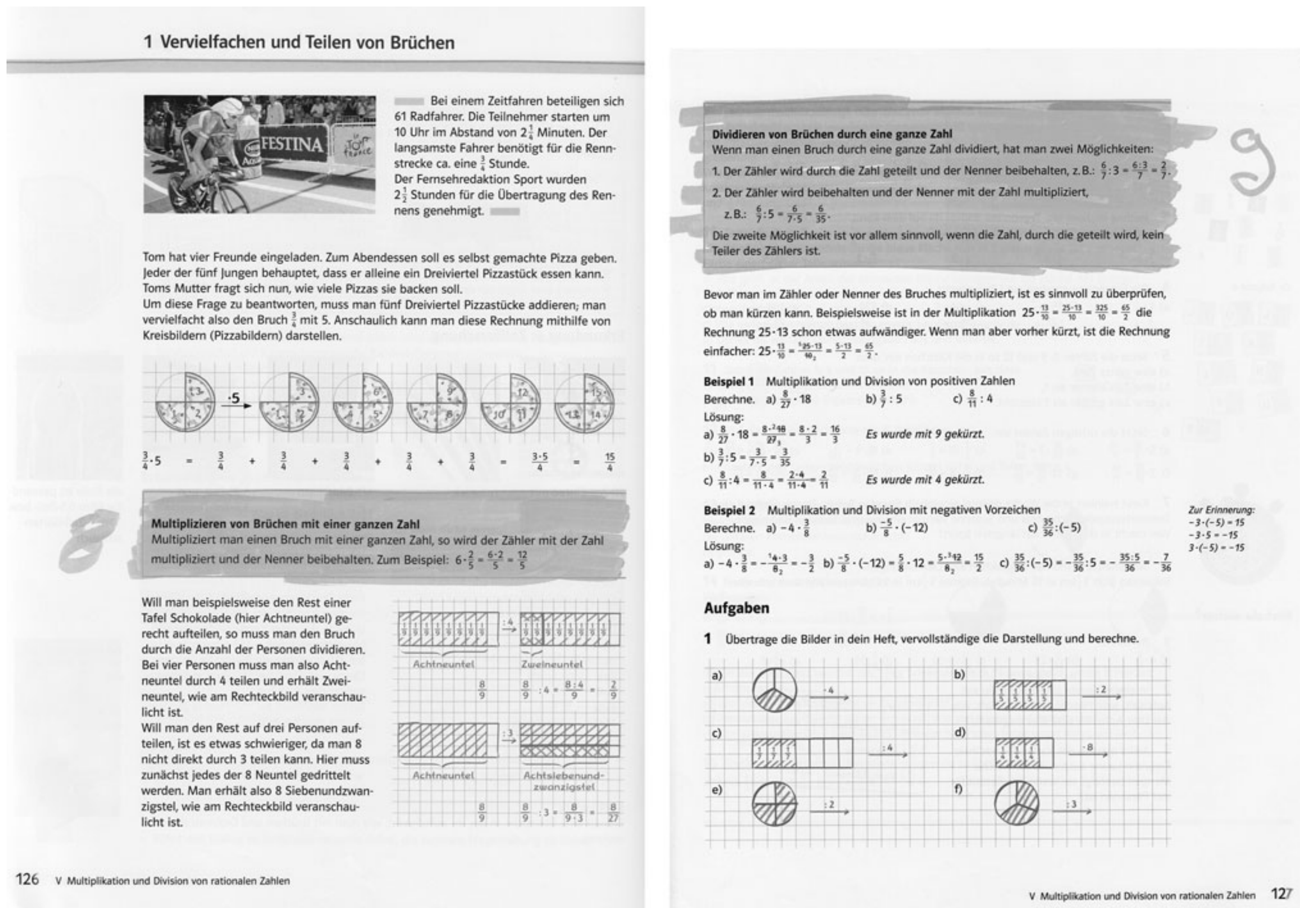
Fig. 4 Emma's (6th grade) use of tasks according to a position-dependent practicing scheme (Hußmann et al. 2006, p. 144). Translation: *13* Calculate the area and the circumference of the *green figure* (Fig. 1). *14* One liter of air weighs 1.29 g. (a) How much does the air in a 8.75 m long, 6.64 m wide and 2.5 m high room weigh? (b) Estimate and do a rough calculation of how much the air in your classroom weighs. *15* Franca claims: The calculation 0.2 × 0.3 shows that 20 % of 30 % is exactly 6 %. Is she right? Explain / are you confident? *1* Calculate. *2* Do a rough calculation and compare with the exact result. *3* A rectangular property is 15.5 m long and 9.80 m wide. The development costs are 49.20 € per square meter. How much does the owner have to pay? / *16* The elevator of a skyscraper with 43 floors rises 2.60 m per second. Every floor is 4.20 m high. (a) Calculate the duration for different elevator rides in the skyscraper. (b) Estimate whether taking the stairway would be faster. Explain your ideas in a little essay. *17* Klaus is wondering: Would all inhabitants of Germany fit onto the surface of Chiemsee? (a) Estimate whether he is right or wrong. (b) Check the argument. Tip: do some research and calculate. *18* Maths and Art. (a) Draw a picture of four rectangles. Enlarge your picture afterwards with a scale of 4:1. (b) Do the same as in (a) with your own pictures. (Translation: Inga Gill)

practicing: practicing means to do tasks that are similar to teacher-mediated tasks. However, the similarity is not inferred from the relative position of the task, but from surface properties. Therefore, another theorem-in-action guides the choice of the tasks: if tasks have similar (visual) surface properties the tasks are similar.

The salience-dependent practicing scheme is similar to a way of using the textbook which Lithner (2003, p. 35)



1 Vervielfachen und Teilen von Brüchen

Bei einem Zeitfahren beteiligen sich 61 Radfahrer. Die Teilnehmer starten um 10 Uhr im Abstand von $2\frac{1}{4}$ Minuten. Der langsamste Fahrer benötigt für die Rennstrecke ca. eine $\frac{3}{4}$ Stunde. Der Fernsehredaktion Sport wurden $2\frac{1}{2}$ Stunden für die Übertragung des Rennens genehmigt.

Tom hat vier Freunde eingeladen. Zum Abendessen soll es selbst gemachte Pizza geben. Jeder der fünf Jungen behauptet, dass er alleine ein Dreiviertel Pizzastück essen kann. Toms Mutter fragt sich nun, wie viele Pizzas sie backen soll. Um diese Frage zu beantworten, muss man fünf Dreiviertel Pizzastücke addieren; man vervielfacht also den Bruch $\frac{3}{4}$ mit 5. Anschaulich kann man diese Rechnung mithilfe von Kreisbildern (Pizzabildern) darstellen.

$\frac{3}{4}\cdot 5 = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{3\cdot 5}{4} = \frac{15}{4}$

Multiplizieren von Brüchen mit einer ganzen Zahl
Multipliziert man einen Bruch mit einer ganzen Zahl, so wird der Zähler mit der Zahl multipliziert und der Nenner beibehalten. Zum Beispiel: $6\cdot\frac{2}{5} = \frac{6\cdot 2}{5} = \frac{12}{5}$

Will man beispielsweise den Rest einer Tafel Schokolade (hier Achtneuntel) gerecht aufteilen, so muss man den Bruch durch die Anzahl der Personen dividieren. Bei vier Personen muss man also Achtneuntel durch 4 teilen und erhält Zweineuntel, wie am Rechteckbild veranschaulicht ist.
Will man den Rest auf drei Personen aufteilen, ist es etwas schwieriger, da man 8 nicht direkt durch 3 teilen kann. Hier muss zunächst jedes der 8 Neuntel gedrittelt werden. Man erhält also 8 Siebenundzwanzigstel, wie am Rechteckbild veranschaulicht ist.

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Dividieren von Brüchen durch eine ganze Zahl
Wenn man einen Bruch durch eine ganze Zahl dividiert, hat man zwei Möglichkeiten:
1. Der Zähler wird durch die Zahl geteilt und der Nenner beibehalten, z.B.: $\frac{6}{7}:3 = \frac{6:3}{7} = \frac{2}{7}$.
2. Der Zähler wird beibehalten und der Nenner mit der Zahl multipliziert, z.B.: $\frac{6}{7}:5 = \frac{6}{7\cdot 5} = \frac{6}{35}$.

Die zweite Möglichkeit ist vor allem sinnvoll, wenn die Zahl, durch die geteilt wird, kein Teiler des Zählers ist.

Bevor man im Zähler oder Nenner des Bruches multipliziert, ist es sinnvoll zu überprüfen, ob man kürzen kann. Beispielsweise ist in der Multiplikation $25\cdot\frac{13}{10} = \frac{25\cdot 13}{10} = \frac{325}{10} = \frac{65}{2}$ die Rechnung 25·13 schon etwas aufwändiger. Wenn man aber vorher kürzt, ist die Rechnung einfacher: $25\cdot\frac{13}{10} = \frac{{}^{5}25\cdot 13}{10_{2}} = \frac{5\cdot 13}{2} = \frac{65}{2}$.

Beispiel 1 Multiplikation und Division von positiven Zahlen
Berechne. a) $\frac{8}{27}\cdot 18$ b) $\frac{3}{7}:5$ c) $\frac{8}{11}:4$
Lösung:
a) $\frac{8}{27}\cdot 18 = \frac{8\cdot {}^{2}18}{27_{3}} = \frac{8\cdot 2}{3} = \frac{16}{3}$ *Es wurde mit 9 gekürzt.*
b) $\frac{3}{7}:5 = \frac{3}{7\cdot 5} = \frac{3}{35}$
c) $\frac{8}{11}:4 = \frac{8}{11\cdot 4} = \frac{2\cdot 4}{11\cdot 4} = \frac{2}{11}$ *Es wurde mit 4 gekürzt.*

Beispiel 2 Multiplikation und Division mit negativen Vorzeichen
Berechne. a) $-4\cdot\frac{3}{8}$ b) $\frac{-5}{8}\cdot(-12)$ c) $\frac{35}{36}:(-5)$
Lösung:
a) $-4\cdot\frac{3}{8} = -\frac{{}^{1}4\cdot 3}{8_{2}} = -\frac{3}{2}$ b) $\frac{-5}{8}\cdot(-12) = \frac{5}{8}\cdot 12 = \frac{5\cdot {}^{3}12}{8_{2}} = \frac{15}{2}$ c) $\frac{35}{36}:(-5) = -\frac{35}{36}:5 = -\frac{35:5}{36} = -\frac{7}{36}$

Zur Erinnerung:
$-3\cdot(-5) = 15$
$-3\cdot 5 = -15$
$3\cdot(-5) = -15$

Aufgaben

1 Übertrage die Bilder in dein Heft, vervollständige die Darstellung und berechne.

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Fig. 5 Laura's use of boxes with kernels and rules (Hußmann et al. 2006, pp. 126–127). Translation of the highlighted parts: *Multiplication of fractions by a whole number*. If a fraction is multiplied by a whole number, the numerator is multiplied by the number and the denominator remains. For example: […] *Division of fractions by a whole number*. If a fraction is divided by a whole number, there are two possibilities to solve it: *1* the numerator is divided by the number and the denominator remains, for example […]. *2* The numerator stays and the denominator is multiplied by the number. For example: […] the second possibility is reasonable if the number, which is used to divide is not a factor of the numerator. (Translation: Inga Gill)

describes as "identification of similarities." Lithner also traces this kind of referring to the textbook back to identifying "similar surface properties" (p. 35).

4.4 Results related to the role of peers, family, and tutors

The case of Lilli (6th grade) drew attention to the role of her father as an influential factor on her utilization scheme. In this section the findings related to the role of peers, family, and tutors are reported, because they are important for understanding students' utilization schemes of mathematics textbooks.

The influence of peers, family, and tutors on students' instrumental genesis of mathematics textbooks becomes evident in several interviews. David (6th grade) explains in the interview that his brother helps him to choose tasks from the book for practicing:

Researcher: Do you have a tutor, who helps you with your repetition at home, or are your parents helping you?
David: Well, actually it's my brother, because he is really good at Maths, he usually helps me.
Researcher: I see! Is he your older brother?
David: Yes, he is 19 years old. He's attending… there's an 11th, 12th and 13th grade for "Gymnasium" at the "Gesamtschule."
Researcher: I see.
David: That's the school he is attending.
Researcher: And you are picking the exercises together? Or how does it work?
David: Yes, I tell him what I would like to repeat and then we look it up in the book.
Researcher: Ok, and then you choose the exercises together, or does he tell you which ones are good for you?

1 Für Düngeversuche sollen aus den drei Düngersorten I, II und III 10 kg Blumendünger gemischt werden, der 40 % Kalium, 35 % Stickstoff und 25 % Phosphor enthält. Welche Mengen werden benötigt?

	I	II	III
Kalium	40 %	30 %	50 %
Stickstoff	50 %	20 %	30 %
Phosphor	10 %	50 %	20 %

Fig. 6 Teacher-mediated task on page 21 (Baum et al. 2001). Translation: in a fertilizing experiment, a 10 kg mix of three fertilizers I, II, and III is used. The mix should contain 40 % potassium, 35 % nitrogen and 25 % phosphorus. What amount of each ingredient is needed? (Translation: Inga Gill)

9 Edelstahl ist eine Legierung aus Eisen, Chrom und Nickel; beispielsweise besteht V2A-Stahl zu 74 % aus Eisen, 18 % Chrom und 8 % Nickel. Aus den Legierungen I bis IV in Fig. 2 sollen 1000 kg V2A-Stahl hergestellt werden. Stellen Sie ein lineares Gleichungssystem auf und lösen Sie es.

	I	II	III	IV
Eisen	70 %	76 %	80 %	85 %
Chrom	22 %	16 %	10 %	12 %
Nickel	8 %	8 %	10 %	3 %

Fig. 7 Task on page 31 (Baum et al. 2001) chosen by Jennifer for practicing. Translation: stainless steel is an alloy of iron, chromium and nickel; V2A steel, for example, contains 74 % iron, 18 % chromium and 8 % nickel. Alloys I to IV should produce 1,000 kg V2A-steel. Determine a system of linear equations and solve it. (Translation: Inga Gill)

> David: Yes, I suggest some and he tells me if… well, if they are good ones.
> Researcher: Ok. You mean good ones for repetition. (Original in German, translated by Inga Gill)

Charlotte (12th grade) explains that she studied together with Pia:

> Well, I, we generally, basically Pia and I have prepared the exam together and then… I think we used this… is that right?… these are also, no, that here is also part of it, right? Well, yes, I think we simply… well, read through this, the examples, to understand it better. (Original in German, translated by Inga Gill)

Although she does not say it explicitly, it is likely that both students influence each other in terms of their individual utilization schemes.

Finally, Helena (6th grade) describes how she goes through the book with her tutor:

> Researcher: Can you explain to me why you are practicing?
> Helena: Well, um, we worked through this in class and then I also have a tutor, and um, we always look into the book and see a few exercises we could use, and I do them to check my knowledge, to see if I have understood it and then so I can do the exercises, so I am better in doing the exercises, also in the exams. […]
> Researcher: And your tutor comes regularly?
> Helena: Yes, once a week. But I also um, look through the pages again at home, without my tutor. (Original in German, translated by Inga Gill)

These are just single cases and it is not apparent in which way peers, family, and tutors actually influence students' utilization schemes of the mathematics textbook. Nevertheless, they all show that social influences on utilization schemes seem to be an important factor, which needs to be analyzed further.

5 Discussion and conclusion

The qualitative study presented in this paper provides a deeper understanding of secondary school students' utilization schemes of mathematics textbooks for self-regulated practicing activities. Textbook use was conceptualized drawing on Rabardel's notion of instrumental genesis comprising two reciprocal processes: instrumentalization and instrumentation. Although all students instrumentalized their mathematics textbook for the same purpose, the analysis of instrumentation yielded three different types of utilization scheme: position-dependent practicing, block-dependent practicing, and salience-dependent practicing. All three schemes are determined by operational invariants, which reflect particular knowledge about mathematics textbooks and about learning mathematics.

On the one hand, these operational invariants reveal that particular knowledge about the textbook is necessary to use a textbook effectively. The block-dependent practicing is a good example to illustrate how knowledge about the structure of the textbook and about the functions of the blocks is helpful for using the textbook effectively. Referring to the analysis of the structure of German mathematics textbooks in Sect. 2.2, the block-dependent use of the textbook is a utilization scheme which is clearly afforded by the structure of the book. The book successfully instruments students who use the book according to the block-dependent practicing scheme. However, the two other practicing schemes reveal that the book is also capable of instrumenting the user differently.

On the other hand, the operational invariants reveal how effective textbook use might be hindered due to particular conceptions about the book. In particular, the theorem-in-action of position-dependent practicing that adjacent tasks are similar might be challenged in terms of accuracy. Is it actually true that adjacent tasks in mathematics textbooks are similar? Of course, this depends on the definition of what is regarded as 'similar.' However, within the scope of this paper it is not possible to go into detail, as this would require a comprehensive analysis of the tasks in the textbooks.

From the perspective of classroom practice, the issue of operational invariants relates to the question of how the instrumental genesis of textbooks and the development of operational invariants might be supported. Do students actually have to learn how to use their textbooks efficiently for self-regulated learning of mathematics? In line with the findings about the use of technological tools (e.g. Artigue 2002), this study reveals that the use of the textbook as an instrument for learning mathematics is a complex learning process itself. The mathematics textbook is a complex artifact, which affords particular ways of being used and constraints others. The teacher could play an integral role in assisting students' instrumental genesis of mathematics textbooks. It is important that teachers are aware of this, because otherwise family members or tutors might take this role. The latter aspect is especially supported by the findings related to the role of peers, family, and tutors.

From a methodological point of view, the findings related to the role of peers family, and tutors exemplify the relevance of the whole SDT for the field of research on the use of mathematics textbooks. Social impacts on utilization schemes seem to be an important factor influencing students' instrumental geneses of their mathematics textbook. Looking at the relevant literature concerning use of textbooks and instrumental genesis reveals that this influential factor is likely to be underestimated. Furthermore, the operational invariants of students' utilization schemes highlight the importance of the SDT's bottom level. In particular, students' concepts-in-action about practicing reflect their individual view of conventions and norms about learning. Consequently, the analysis of students' instrumental geneses of mathematics textbooks is capable of revealing students' individual understanding of what learning mathematics is about. The concepts-in-action about practicing mathematics of the position-dependent and the salience-dependent practicing schemes are especially illuminating in this context. Practicing does not necessarily mean comprehending a mathematical topic better or being able to solve all kinds of tasks related to a mathematical topic, but it means being fluent with the kinds of tasks that teachers mediated.

A second methodological issue is related to the role of textbook analysis within textbook research. Textbook analysis and the investigation of the use of textbooks always have to be mutually related. This became apparent in the present study. Deep understanding of the textbook itself in terms of structure was a prerequisite for understanding the textbook in use. The results raised new issues that can be answered through textbook analysis. Mutually relating textbook analysis and the investigation of the use of textbooks ensures that the questions that are posed are actually relevant in practice.

Finally, this study presents a generic theoretical framework and method for textbook research which is capable of providing a deeper understanding of the use of textbooks. In particular, the notion of the instrument with its cognitive aspects combined with the SDT, which models the large variety of contextual influences on the instrumental genesis, seems to be a fruitful approach. Thus, this study also contributes to the development of research methods in the field of textbook research which has been characterized as "fundamentally underdeveloped" in terms of the philosophical foundation, theoretical frameworks, and research methods by Fan (2011) and Nicholls (2003).

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