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ORIGINAL ARTICLE

Discursive psychology as an alternative perspective on mathematics teacher knowledge

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Abstract Research on mathematics teacher knowledge, including work on mathematical knowledge for teaching, draws heavily on Shulman's categories of teacher knowledge. These categories have been adopted, developed and modified by mathematics education researchers. This approach has led to some valuable insights. In this paper, I draw on discursive psychology to develop a critique of this work. This critique highlights some of the unstated assumptions of much research inspired by Shulman's work, including, in particular, a representational view of knowledge and argues that the resulting theories do not reflect the discourses of knowledge that arise in mathematics classrooms. These ideas are illustrated with discussion of two examples, with the aim of showing how discursive psychology can offer an alternative perspective.

1 Introduction

Research on mathematics teachers' knowledge of their subject, and of their knowledge of how to teach it, has expanded dramatically in recent years. Shulman's (1986, 1987) work on the nature of teachers' knowledge and, in particular, on how teachers' knowledge develops, is generally regarded as a significant trigger for the substantial amount of research on mathematics teacher knowledge that has emerged. Shulman's work is also widely acknowledged as the theoretical starting point for research on what has come to be known as mathematical knowledge for teaching, and most work in this area makes reference to his ideas (see Lerman 2001, or Petrou and Goulding 2011, for a review of several different approaches, or more generally, chapters in Sullivan and Wood 2008).

Shulman's (1986) key point was that teachers with expert knowledge of a subject need, in the process of teaching, to transform this knowledge:

> How does the successful college student transform his or her expertise in the subject matter into a form that high school students can comprehend? When this novice teacher confronts flawed or muddled textbook chapters or befuddled students, how does he or she employ content expertise to generate new explanations, representations, or clarifications? What are the sources of analogies, metaphors, examples, demonstrations and rephrasings? How does the novice teacher (or even the seasoned veteran) draw on expertise in the subject matter in the process of teaching? What pedagogical prices are paid when the teacher's subject matter competence is itself compromised by deficiencies of prior education or ability? (p. 8)

Shulman's insight, as these questions imply, is that the teachers seem to draw on their subject matter in a way that is somewhat different from a practitioner of the subject. Mathematics teachers, for example, draw on a different kind of knowledge of mathematics than that used by, say, mathematicians. Shulman called this knowledge "pedagogical content knowledge" (p. 9).

Research in mathematics education has taken up this idea enthusiastically, with researchers focusing on a range of related problems including: "the elements of knowledge that are essential for effective subject matter teaching" and the "impact [of] limitations in teachers' subject matter knowledge and pedagogical content knowledge […] on their ability to teach effectively" (Borko et al. 1992,

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p. 195); the relationship between "teachers' professionally usable knowledge of their subjects to student achievement" (Hill, Rowan and Ball 2005, p. 377); "the nature of mathematical knowledge for teaching" and "the nature, the role, and the importance of different types of mathematical knowledge for teaching" (Ball, Thames and Phelps 2008, p. 390); and "the ways that elementary trainees' mathematics content knowledge […] can be seen to contribute to their teaching during the 'practical' element of their training" (Rowland, Huckstep and Thwaites 2005, p. 256). Research on these problems has led to important insights into the nature of mathematics teaching. The light that much of this research sheds on the specific nature of mathematics teacher knowledge has, in particular, informed mathematics teacher education and professional development (see, for one example, Koellner et al. 2007).

In this article, I offer some critique of this work, focusing on its epistemological basis. It is reasonable, after all, to look at research on mathematics teacher knowledge, and examine what the researchers might understand by 'knowledge'. This critique is derived from my own interest in discursive psychology (e.g., Edwards 1997; Edwards and Potter 1992), a perspective that seeks to use discourse analysis to examine discourses of cognition, including knowledge, whether in research texts or in everyday life. Indeed, one of the key observations of discursive psychology is that discourses of cognition in everyday life are often rather different from those observed in research texts: the frameworks and models developed by researchers often do not correspond to the practical ways in which cognition is managed in everyday life (Edwards 1997). This focus on discourses of cognition means that the focus of work in discursive psychology is epistemological, rather than ontological; that is, rather than attempting to identify what cognitive processes are really like or what participants really think or know, the focus is on how participants themselves discursively construct cognitive processes in their talk (Edwards 1997).

The business of mathematics teaching and learning is, of course, centrally concerned with cognitive processes such as knowing, meaning, thinking and remembering. One argument I propose in this article, then, is that discourses of cognition in mathematics classrooms are rather different from those found in research on mathematics teacher knowledge. The 'discourse of knowledge' instantiated by mathematics teaching in situ plays out in ways that are not well captured by an approach based on, for example, categories of teacher knowledge. In the next section, I examine in more detail the discourse of knowledge that underpins much research on mathematics teacher knowledge. In the third section, I set out aspects of the discursive psychology approach, using these ideas to continue to discuss the work on mathematical knowledge for teaching. In section four, I present and discuss examples of mathematics classroom interaction in order to illustrate concretely some of the differences between existing approaches and one based on discursive psychology. One insight that this approach permits, for example, is that in the discourse of knowledge in mathematics classrooms, teacher knowledge may be associated with constructions of students as not knowledgeable.

2 The discourse of knowledge in research on mathematical knowledge for teaching

The vast quantity of recent research on mathematical knowledge for teaching makes it difficult to adequately review it all in this article. Since, however, I am interested in the discourses of knowledge within this literature, I focus on some key strands. I begin by discussing the research program of Deborah Ball and colleagues, since this work, while not universally taken up, has been arguably the most influential.

In their review of research on teaching mathematics, Ball, Lubienski and Mewborn (2001) set out the "unsolved problem of teachers' mathematical knowledge" (p. 433). In particular, they argue for the need to develop a theoretical account of mathematical knowledge for teaching and its role in teaching mathematics. This theoretical account, they argue, needs to be based on an examination of teachers' practice. This work is inspired by Shulman's (1986, 1987) distinction between subject matter knowledge and pedagogical content knowledge. The maturing of this research program is apparent in Ball, Thames and Phelps's (2008) article on content knowledge for teaching, in which they present their framework of "domains of mathematical knowledge for teaching" (p. 403). Ball et al.'s theoretical model includes six specific types of knowledge, which they relate to Shulman's categories as follows.

Subject matter knowledge consists of:

- Common content knowledge
- Horizon content knowledge
- Specialized content knowledge

Pedagogical content knowledge consists of:

- Knowledge of content and students
- Knowledge of content and teaching
- Knowledge of content and curriculum

Their model, which Ball et al. present as provisional, has prompted other researchers to investigate specific categories, propose various modifications or suggest additional categories (e.g., Zazkis and Mamolo 2011; Foster, 2011; Petrou and Goulding 2011).

It is apparent from my brief summary that "knowledge" is a key concept in this work. What, then, is knowledge? For the most part, Ball and her colleagues do not discuss the nature of knowledge itself, concentrating on identifying its different forms in mathematics teaching. Nevertheless, a largely implicit epistemology can be discerned, starting with Shulman's antecedent work.

2.1 Shulman's perspective on teacher knowledge

Shulman's work is based on the careful observation of teachers. Adopting a broadly constructivist approach to learning, Shulman refers to Piaget's studies of children's development as a model for the study of teachers' development:

> [Piaget] discovered that he could learn a great deal about knowledge and its development from careful observation of the very young – those who were just beginning to develop and organize their intelligence. We are following this lead by studying those just learning to teach. Their development from students to teachers, from a state of expertise as learners through a novitiate as teachers exposes and highlights the complex bodies of knowledge and skill needed to function effectively as a teacher. (Shulman 1987, p. 4)

This perspective construes knowledge as something that forms coherent "bodies", which have a role in teaching. These bodies of knowledge are also apparent in Shulman's discussion of educational reforms in the United States in the early 1980s:

> The new reform proposals carry assumptions about the knowledge base for teaching: when advocates of reform suggest that requirements for the education of teachers should be segmented and periods of training lengthened, they assume there must be something substantial to be learned. When they recommend that standards be raised and a system of examinations introduced, they assume there must exist a body of knowledge and skill to examine. (Shulman 1987, p. 5)

Shulman ties these two strands of work (observation of teachers and reforms of teacher education) by arguing that his observations have allowed him to identify the "sources and suggested outlines" of the knowledge base (p. 5). There is a sense emerging, then, that knowledge is describable, categorisable and generalizable. Indeed, Shulman goes on to propose various "categories of the knowledge base" (p. 8), several of which are apparent in Ball, Thames and Phelps's (2008) domains of mathematical knowledge for teaching.

Shulman is careful to discuss how knowledge relates to teaching. He discusses, in particular, "processes of pedagogical reasoning and action" (p. 12). He maintains a close connection between reasoning and knowledge, the latter forming the basis for the former:

> This image of teaching involves the exchange of ideas. The idea is grasped, probed, and comprehended by a teacher, who then must turn it about in his or her mind, seeing many sides of it. Then the idea is shaped or tailored until it can in turn be grasped by students. This grasping, however, is not a passive act. Just as the teacher's comprehension requires a vigorous interaction with the ideas, so students will be expected to encounter ideas actively as well. (Shulman 1987, p. 13)

For Shulman, pedagogical reasoning and action includes processes like comprehension, representation, checking for understanding and so on (p. 15). From this perspective, knowledge and reasoning are inter-related but separate: reasoning is the active (constructive) engagement with ideas or facts or experience. This perspective implies a representational view of knowledge, in which knowledge is internally represented in the mind (see p. 16). Part of the process of pedagogical reasoning involves teachers finding ways to represent the knowledge represented in their minds so that students can form their own representations.

A representational perspective is also apparent in Shulman's (1986) sometimes overlooked discussion of forms of knowledge. He proposes three forms of teacher knowledge: propositional knowledge, case knowledge, and strategic knowledge (p. 20). The representational view of knowledge can be seen in Shulman's explanation of these forms of knowledge. For example, propositional knowledge refers to knowledge about teaching "stored in the form of propositions" (p. 10), which Shulman subdivides into principles, maxims and norms (p. 11). Similarly, case knowledge is, for Shulman, not simply a report or memory of a particular event; it is knowledge derived from theory applied to these particulars: "whereas cases themselves are reports of events or sequences of events, the knowledge they represent is what makes them cases" (p. 11), which Shulman subdivides into prototypes, precedents and parables (p. 11). Again, then, this account implies a representational view of knowledge, in which experiences are represented in the teacher's mind in the form of prototypes or precedents or parables. Strategic knowledge (the third form of knowledge) concerns the practical application of propositional knowledge and case knowledge. The representational perspective is again apparent, this time in terms of drawing on existing representations to make sense of teaching as it happens and make decisions about how to act.

I should underline here that my characterization of Shulman's perspective as representational is not meant to imply that his work is flawed or simplistic. His approach was often subtle and based on long observation of teachers. He emphasizes the changing nature of knowledge, and the active role of the teacher in acquiring and transforming knowledge in order to "provoke the constructive processes of their students" (p. 14), a perspective that is consistent with its Piagetian inspiration. It is, however, valuable to clarify the contours of Shulman's epistemology, including its basis in representations and categories of knowledge, not least because many of these contours are apparent in subsequent research in mathematics education research on teacher knowledge.

2.2 Shulman's influence on mathematics education research

Shulman's influence on research on mathematics teacher knowledge is apparent in the frequent references to his work. This influence, not surprisingly, is also epistemological, particularly the underlying representational perspective. Ball and Bass (2000), for example, argue, like Shulman, for the inseparability of subject matter and pedagogy, stating (with reference to Dewey) "how an idea is represented is part of the idea, not merely its conveyance" (p. 85). A representational epistemology is maintained in more recent work:

> As a concept, pedagogical content knowledge, with its focus on representations and conceptions/misconceptions, broadened ideas about how knowledge might matter to teaching, suggesting that it is not only knowledge of content, on the one hand, and knowledge of pedagogy, on the other hand, but also a kind of amalgam of knowledge of content and pedagogy that is central to the knowledge needed for teaching. (Ball, Thames and Phelps 2008, p. 392)

There is a degree of ambiguity about the term 'representation' in much of this work. It generally seems to refer to 'external' representations that the teacher uses in their teaching. The language of representations, however, implies an underlying idea or concept that the external representation represents. Hence discussion of (external) representations also implies an 'internal' mental representation of this underlying idea or concept (see, for example, Goldin 2002, p. 207). Another of Shulman's assumptions, that knowledge can be organized into clear categories, is apparent in Ball, Lubienski and Mewborn's (2001) discussion of "bundles" or "packages" of knowledge or Ball, Thames and Phelps's (2008) reference to a "periodic table of teacher knowledge" (p. 396).

These two features of Shulman's epistemological perspective can also be seen in projects designed to measure teachers' pedagogical content knowledge through the design of quantitative instruments (e.g., Hill, Rowan and Ball 2005). The discourse of knowledge categories, representations and the identifiability and measurability of knowledge suggests an epistemology in which the knowledge necessary for teaching mathematics can be uncovered through careful testing of teachers, analysis of classroom teaching and other forms of analysis. Indeed, one of the main aims of Ball et al.'s program of research is to identify the mathematical knowledge needed to teach mathematics well (through correlation with student outcomes), in order to inform mathematics teacher education curricula and the formal evaluation of teacher candidates' suitability to teach mathematics. This work is not without its challenges. In particular, the multiple categories and subcategories can be difficult to operationalize: it is, for example, difficult to distinguish between them using the research instruments that have been developed (Petrou and Goulding 2011).

Research on mathematics teacher knowledge continues to evolve. A recent focus has been on finding connections between teachers' mathematical knowledge and classroom practice. Research has examined, for example, the role of mathematical knowledge for teaching in relation to the quality of teaching (Hill et al. 2008), teachers' professional development (Koellner et al. 2007), novice teachers (Rowland, Huckstep and Thwaites 2005) and the teaching of subdomains of mathematics (Steele and Rogers 2012). The common denominator in what could be considered 'second generation' research on mathematical knowledge for teaching is the examination of classroom practice, often combined with other forms of data, such as questionnaires, teacher logs or measures of participating teachers' content knowledge. It is worth examining some of this research in more detail.

Perhaps the strongest critique so far of this body of work is based on a situated perspective on knowledge. Putnam and Borko (2000) have argued that "some, if not most, of teachers' knowledge is situated within the contexts of classrooms and teaching" (p. 13). Hodgen (2011) makes a similar point, with direct reference to mathematics teacher knowledge, drawing on analysis of a case study in which a teacher's mathematical knowledge is examined in the context of her professional work, as well as in the context of a formal mathematical interview. He argues that "mathematics teacher knowledge is very much more deeply embedded in practice than the PCK literature generally acknowledges" (p. 35). One response to such a critique is to examine teachers' mathematical knowledge as displayed in different contexts. Koellner et al. (2007), for example, report on a mathematics teacher professional development project in which participants worked together on mathematics problems, before implementing them in their own teaching. They then discussed video recordings of selected incidents collected during their implementation of the mathematics problems. Koellner et al. sought to trace the participants' mathematical knowledge from the setting of their own professional development, into their classrooms and back again. Similarly, others have highlighted the contingent nature of aspects of teachers' mathematical knowledge (e.g., Rowland, Huckstep and Thwaites 2005). Such a response is an important step forward. At the same time, it risks complicating but not solving the problem. A situated perspective draws greater attention to the detail of classroom practice, and away from knowledge as located entirely in the teacher's head. At the same time, it is difficult to shift entirely away from a discourse of knowledge as possessed by the individual teacher. Hodgen (2011) still writes, for example, about the teacher in his case study as having "significant gaps in her knowledge" (p. 36).

There are, then, perhaps two significant, interrelated problems with the Shulman-inspired approach to mathematics teacher knowledge, which a situated approach does not entirely solve. First, it remains difficult to examine mathematics teachers' knowledge, despite recent advances. A situated perspective suggests an approach is needed that can look at the fine detail of classroom interaction. Recent research has attempted to look more carefully at mathematics classroom practice, though generally without specific tools for analyzing interaction. Second, however, the difficulty of examining mathematics teachers' knowledge is in part because of the underlying assumptions and approach of the research. These assumptions include a view of knowledge as categorisable, measureable and as represented in the teacher's mind. A situated perspective underlines how this knowledge is nevertheless context-related and can be changed or reconstructed. What is needed as an approach that goes beyond the situated perspective to examine the discourses of knowledge that arise in classrooms. This is the kind of approach proposed by discursive psychology.

3 Discursive psychology as an alternative perspective on mathematics teacher knowledge

Discursive psychology is both a theoretical and a methodological perspective on human cognition that seeks to understand the locally produced methods through which participants deal with each other's mental processes in interaction. This perspective arises from the origins of discursive psychology in a critique of the prevailing theories and methods of social psychology in the 1970s and 1980s.

Social psychologists study things like motivation, attitudes and collective memory. The prevailing approach to the investigation of such topics involved laboratory-based studies, questionnaires and interviews designed to contribute to the production and refinement of theoretical models of human cognition in social situations. According to the critique developed in discursive psychology, there are a couple of problems with this kind of approach (see Edwards and Potter 1992; Edwards 1997). In particular, the theoretical models rarely correspond to how people generally make sense of each other's intentions, personalities or motivations or with how these cognitive processes are construed in everyday situations. Research on memory or attitudes, for example, struggles to explain the inherent variability of actual everyday accounts of remembered events or expressions of attitude. Discursive psychologists therefore sought to develop an approach to the investigation of cognition that started from the everyday, situated production of accounts of mental processes, and that recognized the fundamental role of human interaction in producing these accounts. This approach uses discourse analysis to examine the discourse of cognition and associated discursive practices in everyday settings. The goal of such work is not to find out what mental processes are really like. Instead, the aim is to understand how these mental processes are understood and interpreted by participants in interaction, and how they are discursively organized.

This kind of critique can be applied to much research in education, including, in particular, the research on mathematics teacher knowledge. As I argued at the end of the previous section, the model of teacher knowledge proposed by Shulman and developed within mathematics education in various different ways (see Petrou and Goulding 2011) does not necessarily correspond to how teachers or students make sense of each other's knowledge, thinking or learning, and hence does not adequately explain how such mental processes are construed in classroom interaction. This critique perhaps explains why, for example, the categories of teacher knowledge are difficult to operationalize, and why they struggle to deal with apparent variation in teachers' mathematical knowledge according to context. One reason for this problem is that researchers are looking for their own constructs in mathematics classrooms. As Askew (2008), for example, has noted "Distinctions between subject knowledge, pedagogic knowledge, semantic/syntactic, product/process can be regarded as being constructed within the discourse of research literature, rather than being discovered as independently existing objects" (p. 29).

In an educational context, discursive psychology amounts to a critique of much that is taken for granted in orthodox constructivist interpretations of teaching and learning. Edwards (1993) draws together this critique:

> Following Piaget's pioneering studies, children's understandings of the world have been taken to be coherent, internal cognitive representations, whose

> nature can be examined via careful experimentation and interview. […] the notion persists that the object of study is the child's mind, an inner world of relatively stable and enduring cognitive representations, whose nature is revealed in the child's responses to questions. Children's conceptual formulations are held to derive from underlying cognitive "theories" of the world, of how things and plants and animals and people exist and operate. […] what children say in the classroom is taken as evidence of underlying and stable cognitive representations, of how children think: as a kind of window on the mind. (Edwards 1993, p. 208)

Edwards' critique also comments on how research methods are implicated in producing children's thinking and knowledge as stable and organized, through the use of interviews and experiments. These methods attempt to control for the natural variability of children's participation. Any changes in children's responses under these controlled conditions are interpreted either as a change in the underlying knowledge schemas (see Barwell 2009, for a mathematics education example) or as 'bad data' caused by weaknesses in the experimental design (Edwards 1993, p, 217). The problem with this approach is that what children say is closely linked to the situation and circumstances in which they say it.

While Edwards is writing about research on children's knowledge, the argument applies equally well to that of their teachers. As Shulman (1987) acknowledges, the inspiration for his approach is the broadly Piagetian idea of observing the growth of knowledge over time. This approach makes the same kinds of assumptions that Edwards summarizes, as I have highlighted in the preceding section. In particular, what teachers say is taken as evidence of underlying, stable (though not fixed), categories of knowledge, held as representations in the teachers' minds.

Discursive psychology has developed an alternative approach that seeks to respond to the kind of problems highlighted in the critique of social psychology. This approach uses methods of discourse analysis including, in particular, principles derived from ethnomethodology (Garfinkel 1967) and conversation analysis (Sacks 1992). More detailed accounts of these methods can be found in several publications in discursive psychology (Potter and Wetherell 1987; Edwards and Potter 1992; Edwards 1997; te Molder and Potter 2005) while aspects of how these methods can be applied in mathematics education can be found in some of my own work (Barwell 2009, 2012; Reis and Barwell 2012). In this article, I highlight a couple of key points that are most relevant to research about teacher knowledge.

First, much of human interaction is concerned with the cognitive process of participants: human talk is often about what participants mean, intend, know or think (Edwards 1997). This is particularly the case in classrooms. One purpose of research in discursive psychology, however, is to examine how participants manage these cognitive concerns. That is, rather than examining what participants actually know or mean or think, research is focused on how participants themselves try to work out these things themselves:

> Discourse analysis does not directly answer the question, What is the conceptual content of children's minds [*or teachers' minds – RB*]? What it does is recast the question as one that an analysis of discourse can answer. It studies what counts for participants as, for example, understanding, thinking, and remembering, so that the psychologist's question can always be recast, methodologically, as a matter of discursive definition. (Edwards 1993, p. 219)

In adopting this approach, discursive psychology is drawing on a key principle of ethnomethodology, the aim of which is to understand how social actors (i.e. people) interpret, construct and orient to social norms that are rarely pre-given and are frequently not explicitly articulated by participants (see Garfinkel 1967). This is an approach that can readily be applied to mathematics classroom interaction. Research on mathematics teacher knowledge would not try to find out what teachers actually know, but would focus on when and how "what teachers know" is a relevant concern for participants in mathematics classrooms.

Second, since human interaction is fundamentally discursive, any accounts of knowing, meaning, intending and so on are inevitably shaped by the immediate temporal, social and interactional context in which they arise. The answer to mathematical questions, for example, may be different according to who is asking, in what circumstances (as Hodgen 2011 discovered). Rather than this variability being a problem for research (how to find out what teachers really know) it becomes its focus: an examination of how the discursive construction of knowing, for example, is shaped in different ways in different circumstances. These circumstances can include the micro-structure of human interaction, including something as simple as the turn-taking structure and sequentiality of talk (see Barwell 2012 for a mathematics classroom example). Moreover, the shaping of accounts of participants' knowing is often rhetorical in nature, designed to deal with issues of accountability, status and the pre-emptive management of disagreement or other 'trouble' (Edwards 1997).

These ideas lead to the following general approach to research in institutional settings like schools:

> [Discursive psychology's] particular focus when approaching discourse in *institutional* settings is on

> how psychological matters are introduced, defined, and made relevant to the business of those settings. Psychological themes are generally pervasive in how such settings work, as they are in mundane talk, but they are sometimes also part of an institution's official normative goals or agenda, such as in educational and therapeutic settings, where how people think and feel are a central focus of concern. (Edwards and Potter 2001, p. 13)

In mathematics classrooms, for example, mathematical knowledge and its assessment is clearly a 'pervasive' concern. Analytic interest is therefore on how this concern is 'introduced, defined and made relevant'.

In the next section, I discuss a couple of examples in an attempt to illustrate both my critique and the way the question of teacher knowledge might be tackled from a discursive psychology perspective.

4 Teachers' mathematical knowledge from a discursive psychology perspective

The first example to be discussed in this section draws on parts of the study published by Hill et al. (2008), one of the most careful and thorough examples of second generation mathematical knowledge for teaching research. For the second example, I refer to data collected in an elementary school classroom as part of my own recent research. I do not have the space in this article to present a detailed analysis of each example; rather, I illustrate how the perspective of discursive psychology can be productive in examining the production of teachers' knowledge in mathematics classrooms.

4.1 Example 1: probability (Hill et al. 2008)

Hill et al. (2008) set out to investigate a series of questions concerning the role of mathematical knowledge for teaching in classroom practice:

- What is the overall strength of the relationship between teachers' MKT and the mathematical quality of their instruction?
- What does MKT afford instruction? How does a lack of MKT constrain instruction?
- What factors mediate the expression of MKT in instruction?
- In which tasks of teaching is MKT—whether strong or weak—most apparent? (p. 432)

The discourse of knowledge traced in Shulman's work is apparent in these questions. Mathematical knowledge for teaching is constructed as an essential quality represented in the teacher's mind, which can be "strong or weak", which is "expressed" through teaching and which influences teaching in some way. Hill et al.'s study, which is impressive in its thoroughness, investigated correlations between the strength of teachers' mathematical knowledge for teaching and the quality of their teaching. Strength of mathematical knowledge for teaching was determined by teachers' scores on a pencil-and-paper measure. Teachers' 'mathematical quality of instruction' was derived from coding and then scoring video recordings of mathematics teachers in action. Their article presents 5 cases derived from the larger study, designed to examine in more detail how mathematical knowledge for teaching influences the quality of instruction.

One of the cases presented in Hill et al. (2008), that of a teacher called Lauren, illustrates teachers who have strong mathematical knowledge for teaching and high quality mathematics instruction. Hill et al. describe Lauren's teaching in some detail (over 12 pages), focusing on her work on a probability topic with her fourth grade class. They highlight several features of her teaching including her use of correct mathematical language, her linking of different mathematical representations, her expectation that students provide explanations for their thinking and that they consider each other's explanations. In one segment of Lauren's lesson, following a review in which students discussed the placement of various events on a likelihood line shown on the blackboard, Lauren introduces the quantitative representation of probability. She adds a second likelihood line to her blackboard and reads from her notes:

Lauren: Sometimes, people use numbers instead of words to describe a probability of an event. In fact, that's what mathematicians use. They use numbers instead of words. Okay, the number we give to an impossible event. Something's that impossible, that can't happen. I'm hearing people whisper. What is it?
Students: [Zero.] [Zero percent!]

> As Lauren writes 0, a student gets Lauren's attention, telling her he "thinks he knows the rest." Lauren asks him to continue:

Lauren: Okay, why don't you tell me…what you're thinking the number would be for certain
Student: A hundred
Lauren: A hundred. A hundred what?
Student: Percent

[…]

Lauren: A hundred percent. So I heard…did you whisper zero percent when I put zero up here?

[Student answers yes.] Okay. I'll go ahead and write…I'll write a hundred percent here (beneath certain)…tell me what you're thinking by a hundred percent. What do you mean by a hundred percent?

Student: For sure. Definite

Lauren: Okay, so what does a hundred percent mean to you?

Student: It WILL happen

Lauren: It's definitely gonna happen. Good. And mathematicians, when they're using numbers, they refer to the number one for an event that is certain. [Directly below certain and parallel to the zero she wrote under impossible, Lauren writes a one] That it's going to happen. Okay, hundred percent chance, it's definitely going to happen

(Hill et al. 2008, pp. 450–451)

In their comments on this lesson segment, Hill et al. highlight the way Lauren makes links between different representations of probability, including the likelihood line, percentages and decimals. They also point out how Lauren is "explicit about mathematical practices" in her references to mathematicians and how she draws attention to the "unique nature of mathematical thought". Finally, they point out how Lauren "repeatedly press[es] students for mathematical explanations".

On the basis of this extensive case study (to which I cannot do justice in the space available), Hill et al. discuss a broader finding from their study:

> We argue that this demonstrates a substantial link between strong MKT and high mathematical quality of instruction. […] The symmetry of this relationship is striking; not only do high-knowledge teachers avoid mathematical errors and mis-steps, they appear able to deploy their mathematical knowledge to support more rigorous explanations and reasoning, better analysis and use of student mathematical ideas, and simply more mathematics overall. (p. 457)

The influence of Shulman is perhaps a little harder to see in my summary of this case study. Hill et al.'s discussion of the detail of Lauren's teaching is more focused on the specific practices that signal its high quality than on direct discussion of her mathematical knowledge for teaching. Nevertheless, Shulman's work is an important formative presence in the research. First, recall that Lauren is identified as having 'high' mathematical knowledge for teaching on the basis of a standardized quantitative instrument, which assumes that the knowledge represented in her mind can be measured in a reasonably accurate and decontextualized way. Second, on the basis of the correlation between Lauren's score on the mathematical knowledge for teaching test and the high quality of her teaching, Hill et al. deduce that because she has this knowledge, she is able to "deploy" it in ways that correspond to effective teaching of mathematics. It is not clear, however, how mathematical knowledge for teaching was identified in the video tapes; presumably this analysis was based on the authors' definition of mathematical knowledge for teaching as "not only the mathematical knowledge common to individuals working in diverse professions, but also the subject matter knowledge that supports that teaching" (p. 431).

From a discursive psychology perspective, interest is in the discourse of teacher knowledge constructed by the interaction in Lauren's classroom. First, and not surprisingly in a classroom, the teacher makes propositional statements. As such these statements are knowledge claims. Moreover, the teacher uses various warrants to support her claims. Rhetorically, a knowledge claim is stronger if it has some external warrant. These warrants include reference to a textbook (according to the researchers) and references to what mathematicians do. She says, for example, that mathematicians use "numbers instead of words" and that they "use the number one for an event that is certain". These formulations add rhetorical weight to the teacher's claims.

Second, there is a clear sequence to the interaction that structures the teacher's and students' knowing. The brief segment quoted above, for example, begins with a transition statement about using numbers instead of words. This statement uses a contrast (numbers vs. words) to distinguish the preceding discussion (words) with what they are about to do next (numbers). This contrast is stated twice. The subsequent exchanges follow a common classroom pattern, with the teacher prompting or initiating student input, which she then responds to, either by repeating, seeking clarification or explicit evaluation. As the sequence unfolds, the teacher records key things on the likelihood line. This form of organization structures both students' and teacher's knowledge quite tightly and in a form that is quite particular to classrooms. Edwards (1993) refers to a similar instance of this form of classroom talk as "a sequential accumulation of separate bits of knowledge" (p. 215). This structure is based on an exchange of knowledge displays, first on the part of the teacher, then, on request, on the part of the students, and then back to the teacher again. A knowledge display is an utterance constructed as a statement of knowledge on the part of the speaker (and oriented to as such by the other participants). One aspect of the construction of the teacher's knowledge in this sequence, then, is that she manages this sequence and selection of information. This form of organization is as much social as epistemological. It is clear that the teacher has certain interactional rights: to initiate new topics, to ask students for more explanation, to decide who speaks and so on. These interactional rights both construct and depend on the teacher's institutional role. As such they are primarily social in nature. The teacher's knowledge is, therefore, highly socially organized.

Third, the sequence includes a moment of what conversation analysis sometimes call 'trouble'. The teacher asks a student to explain what he means by a hundred percent. He gives a plausible response. The teacher reformulates her question and the student reformulates his answer. The 'trouble' is signaled by the teacher's reformulation of the question: an acceptable response would not result in a re-asking. The student recognizes that a simple repetition will not be sufficient and offers a reformulation of his own. The teacher reformulates his answer again and then says 'Good'. In classroom discourse, 'good' is, of course, frequently seen as an evaluative remark, indicating a positive assessment of a student's statement. In this case, however, it serves a slightly different function: it acts as a kind of bridge between the student's still not quite acceptable response and an alternative interpretation that the teacher goes on to provide. At this point, then, the teacher appears to make an assessment that the student(s) do not know something that needs to be introduced (i.e. the representation of certainty by the number 1). She therefore introduces it herself. In response to the trouble, the teacher produces a knowledge display: she makes available something she knows, which is implicitly relevant to the discussion. The teacher's 'knowledge' is, therefore, socially organized and accounts of her knowledge are co-produced through interaction with the students, in which students are sometimes constructed as not knowing.

4.2 Example 2: consecutive numbers

In this section, I refer to an episode from a Grade 6 mathematics lesson observed and recorded in an elementary school in the province of Québec, Canada. The class was working on an extended 'situational problem' on the theme of television schedules. The problem text, which was several pages in length, provided information about a television station output, including types of program and their lengths. The problem also presented several constraints about how the programs may be scheduled. The students' task was to come up with a week's schedule that met the constraints. In the episode transcribed below, the teacher is reading through the problem text with the class before they begin work on devising a schedule in small groups:

Tchr 2	next point (2.5) what's the key information in the next point?
S2	programs
Tchr 2	the **types** of programs right now you don't have to list them all but those are the types of programs that you can choose from (2.5) the third point there is a huge word in the third point what is that you all understand about the third point? actually this is where we are we are not sure if anyone else is can someone read the third point? thanks S1
S1	all the different types of programs to be broadcast over a period of two consecutive days
Tchr 2	stop right there what on earth is two consecutive days
S1	on what?
Tchr 2	two consecutive days let me tell you this you never have gym two consecutive days
Students	straight days
Tchr 2	straight days you never have from my class never has gym on day one day two that's consecutive days you guys always have gym on alternating days
S3	what do you mean
Tchr 2	two consecutive days will be two days in a row like Monday and Tuesday are two consecutive days what are other two day in the calendar that are consecutive? Tell me other days that are consecutive
S1	what do you mean?
Tchr 2	two days in a row
S1	three four
Tchr 2	but are in the calendar so Monday Tuesday Wednesday Thursday and Friday Monday and Tuesday are two consecutive. What are the next two days in the calendar that are consecutive?
S1	Tuesday and Wednesday
Tchr 2	yes Tuesday and Wednesday are consecutive what are the next?
S3	Thursday Friday
Tchr 2	before that what's another consecutive day?
Students	Wednesday and Thursday
Tchr 2	Wednesday and Thursday is consecutive Thursday and Friday is consecutive is Friday and Monday consecutive?
Students	oh no
Tchr 2	no because you have the weekend in between
S1	what about Friday and Saturday?
Tchr 2	Friday and Saturday are consecutive
Students	Sunday and Monday
Tchr 2	Sunday and Monday
Students	February March

Tchr 2	February and March are consecutive months consecutive is in a row
S1	we get [it
Tchr 2	[we get it can you read that sentence again for me?
S1	all the different types of programs must be broadcast over a period of two consecutive days

A Shulman-inspired approach to analyzing this sequence might look for different categories of teacher knowledge proposed by Ball, Thames and Phelps (2008). In terms of subject matter knowledge, for example, minimally, the teacher's common content knowledge includes knowledge of the concept 'consecutive' and different ways it can be applied. This knowledge falls within more general knowledge of the number system and ways of representing it. The teacher perhaps also displays specialized content knowledge in the way she offers the students several ways to understand the concept 'consecutive' for themselves. The teacher's pedagogical content knowledge includes her knowledge of content and students, evident, for example, in the way she anticipates the word consecutive as a likely point of confusion, and more generally, in the way she prepares them to work on a complex text-heavy problem. The teacher's knowledge of content and teaching might be inferred from her selection of the task, decisions made about how to present the task and decisions made about which students' responses to take up in their discussion. More generally, the teacher is no doubt conscious that situational problems are a significant element in the provincial mathematics exams and that students often struggle with them, not least due to the difficulties they sometimes have with interpreting and selecting from complex information.

From the perspective of discursive psychology, the conceptual content of the teacher's talk is, as in the first example, organised by the turn-taking structure of the interaction and, at a broader level, by a structured way of interpreting the text which involves presaging an interpretation, asking a student to read out part of the text and then interrogating the class about particular parts of what was read out. Throughout the exchange, the teacher's knowing is displayed in the form of assessments of what students know, either in pre-emptive form, or else as explicit evaluations of students' utterances.

In the above extract, for example, pre-emptive assessments are apparent when the teacher says there is a 'huge word' in the text about to be read out. This formulation already entails an implicit assessment that there is a word in the upcoming sentence that many students will find challenging. Once S1 has read out part of the text, the teacher stops the reading and explicitly draws attention to the word 'consecutive': "what an earth is two consecutive days?" Again, she is strongly displaying an assessment (note the exaggeration of 'what on earth') that many students in the class will not know what the word 'consecutive' means. In her next turn, the teacher's statement "let me tell you this you never have gym two consecutive days" exemplifies the use of the word 'consecutive', again implying that students will not understand the word. This kind of reading draws particularly on the action orientation of the teacher's utterances: her utterances are not seen as straightforward reflections of knowledge represented in her head; they are seen as constructing and organizing her students' knowledge. In this case, they are constructed as not knowing what the word 'consecutive' means.

The teacher continues with various knowledge displays, including:

- utterances designed to show how the word 'consecutive' can be used ("you never have gym two consecutive days",
- more specific examples ("my class never has gym on day one day two that's consecutive days", and
- definitions using everyday language ("two consecutive days will be two days in a row").

The sequence of interaction featuring these knowledge displays again constructs students as not knowing the word 'consecutive', both through students' interventions ("what do you mean") as well as through the teacher's increasingly specific examples and definitions. This sequentiality highlights a key feature of classroom knowledge discourse from the perspective of discursive psychology—the contingent production of displays of teacher knowledge in interaction with students' own contingently produced knowledge displays. For example, when the teacher repeats "two days in a row", a student responds "three four". The teacher hears this response as partially acceptable, as indicated by her reply: "but are in the calendar so Monday Tuesday Wednesday Thursday and Friday Monday and Tuesday are two consecutive". The teacher then initiates a series of knowledge displays by the students, allowing her to explicitly evaluate and refine them. Overall, then, this sequence involves a contingently developing series of knowledge displays by the teacher and students, with the end point marked by a student saying "we get it". Both students and teacher orient to this structure, as evidenced by the unproblematic way in which the students participate—they do not generally speak out of turn, for example.

4.3 Discussion

The two examples discussed above illustrate some of the differences between a mathematical knowledge for teaching approach and a discursive psychology approach to teacher knowledge. A key and fundamental difference is in the role of representations. My discursive treatment of the two examples is not dependent on making any assumptions about the teachers' or the students' internal mental states; there is no attempt to attribute their actions or utterances to cognitive representations. In particular, no assumptions are made about what the teachers know about probability or "consecutive," nor about what they know about their students. Rather, the focus is on how such a discourse of knowing is constructed through the interaction. This analysis is, moreover, based on the categories and attention of the participants, not those of the researcher. For example, the focus on 'consecutive' arises not from an a priori analysis of the task or of the teacher's knowledge, it arises from the interaction itself: consecutive becomes relevant for these participants in this discussion about this question in this class. In other circumstances, the word may have passed unremarked. Similarly the examples and explanations deployed by the teacher display understandings about her students and what they might or might not know.

A second difference in this approach is in its treatment of the structure of knowledge. For Shulman and his followers in mathematics education, knowledge is structured into categories and researchers can reasonably objectively examine teachers' knowledge and attribute such categories to this knowledge. From a discursive psychology perspective, what the teacher knows is socially organized and discursively structured. In the above examples, the teachers play a key role in structuring their students' knowledge, through their use of questions, their pre-emptive assessments, and their use of explanations and examples. In return, however, their own knowledge formulations are structured by the students' contributions to the interaction. Indeed, in both examples, teachers' knowledge formulations are often inter-related with constructions of students as *not* knowing, such as when Lauren says 'Good' and then makes a propositional statement about mathematicians, or when the teacher in example 2 says "what on earth is two consecutive days" (see also Reis and Barwell 2012).

5 Conclusion

Research on teachers' knowledge about mathematics, teaching and their students has been heavily informed by Shulman's work and approach. This knowledge can be categorized and organized into broader frameworks, such as the diagram shown in Ball et al. (2008) or others discussed by Petrou and Goulding (2011). This work has been influential for good reason: it has deepened our thinking and understanding about teachers' knowledge and led to much greater attention on what it is that teachers need to learn through their pre-service training. Nevertheless, this perspective has some drawbacks. In particular, it is based on a representational model of knowledge and, therefore, requires researchers to make assumptions about the nature of teachers' representations, despite their inaccessibility and apparent unreliability. Moreover, the categories of knowledge that have been developed through Shulman's work are largely external researcher categories. They do not, therefore, tell us much about how teachers and students 'do' knowledge, including the kinds of categories and forms of organization that participants themselves deploy and recognize.

As Edwards (1993, p. 220) has pointed out, the response to such a critique is likely to be to protest that there is more to classroom life than discourse. Classrooms are made up of teachers and students, who have minds and knowledge and beliefs. The point for discursive psychology is that such accounts are constructed by participants through talk:

> it is precisely the status of those things that is the business of education - investigating what the world is like, what we know, and how we know it. The analytical task is to discover how those matters are defined, dealt with, made relevant, and so on by participants. (Edwards 1993, p. 220)

Applying this approach to research problems about mathematics teacher knowledge extends the emerging work that sees this knowledge as situated and distributed. Discursive psychology, however, goes beyond this approach in two ways. First, it recasts the problems as epistemological, rather than ontological. That is, it seeks to understand how teachers and students construct each other as knowledgeable, rather than trying to find out what they actually know. And second, it offers an approach to analysis that is highly sensitive to the fine detail of classroom life.

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