

Being a mathematics teacher in times of reform

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Abstract Within research on mathematics teachers and/or their professional development, the concept of identity emerges as a critique of views of how teaching practice is related to teachers' 'internal states' of knowledge and beliefs. Identity relates teachers' professional lives to teaching practices and to the contexts in which the teaching and/or professional development occurs. However, what might count as the context still needs in-depth discussion. In order to contribute to the development of a theoretical framework for understanding mathematics teachers' professional lives, we will draw on one remarkable teacher's identity as a primary mathematics teacher in relation to one political, sociocultural, and pedagogical context. We use this teacher's experience to discuss how education policies that create what Ball (2003) called 'terrors of performativity' tend to impede the formation of a balanced teacher identity.

1 Introduction

History has seen different conceptions of what it takes to be a good teacher. Some of these conceptions focus on more holistic views as, for instance, the notion of the mathematics teacher as exemplary intellectual (Kutschmann 2000; Otte 1994). Others try to determine the necessary single components of a competent mathematics teacher: domains of knowledge, beliefs, attitudes, and values

(Bishop 2007; Leder et al. 2003; Neubrand et al. 2009; Shulman 1986). Still others are less demanding: they construct 'teacher-proof' curricula and pre-packaged material and act towards what Ginsburg (1986) and Smyth (1998) have called the 'proletarianization of teachers', alluding to what happened to factory workers in the nineteenth century when they came increasingly under factory control. These different conceptions of what makes a good teacher are, implicitly or explicitly, embedded in larger social and institutional frameworks. For some conceptions this is more obvious: values, for instance, are explicitly and closely related to time, space, and social context. For others, identification of the social and institutional frames requires some analytical work: domains of knowledge and beliefs, for instance, appear as more related to the individual teacher, although what counts as knowledge-for-teaching is clearly an institutional concern. Where the social and institutional frameworks are visible, normative statements about a good teacher can be appraised or criticized on the basis of the description of what it takes to work as a teacher under the respective social and institutional conditions. In this paper we discuss the ways in which these social, political, and institutional conditions can become particularly powerful in the context of professional development and curriculum innovation. Our point of departure is a reassessment of the often overlooked or theoretically suppressed institutional and social forces which are effective when the professional development of mathematics teachers is at stake. Within research on mathematics teachers and their professional development, the concept of identity emerges as a critique of views of how teaching practice is related to teachers' 'internal states' of knowledge and beliefs (Brown and McNamara 2011; Hogden and Askew 2007). Identity relates teachers' professional lives to teaching practices and to the social and institutional

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contexts in which the teaching and the professional development occurs. However, what might count as the context is theoretically not sufficiently developed. We ask:

- To what extent does the cultural and political context matter where teachers' professional development is concerned?
- How is the individual teachers' professional development related to institutional policies that aim at improvement of education practice and outcome?

In this paper we analyze dimensions and intricacies of mathematics teachers' professional development by drawing on the case of one Chilean primary mathematics teacher who has shown exceptional performance in the context of her professional development. We are interested in a theoretical discussion of, and conversation between, different theoretical approaches within research on and with mathematics teachers. We therefore use this case, which also engages in matters of education policy, as a starting point for considerations that go beyond the case. After clarifying our understanding of the key concepts of the mathematics teachers' knowledge, beliefs, and identity in the next paragraph, we present the case, before engaging in the analytical and theoretical work. Of course, any construction of a case is principally based on a theoretical framework that is related to the kind of research questions of the research endeavor and the research interest, of the methodology and the data to be constructed, and of the preferences of the researchers in reading the data. The case is not a neutral account of reality, but a theoretically informed and selective construction. Thus, our strategy is somewhat circular—constructing a starting point from one (implicit) perspective, and then discussing theoretical perspectives by referring to the constructed point—but by separating the description from the analysis we are optimistic that we do not produce circular reasoning.

2 Professional knowledge, beliefs, and identities

Any principled analysis of the professional development of teachers of mathematics privileges some aspects over others. Whoever commits oneself in a concrete research endeavor to one theoretical framework, including theoretical concepts and a perspective, does this by sacrificing many other aspects of an issue in order to make possible a precise description of one aspect which is felt particularly important. In the case of the professional development of mathematics teachers, focusing on the developing knowledge, the developing beliefs and values, or the developing identities, is a “natural” research strategy for coming to terms with the complexities of any reality. Of course, the restriction to one of these perspectives only allows partial,

or even fragmental, insights into what it takes for a mathematics teacher to develop professionally. But, is *insight* the right term? Can we look at a teacher's professional development as if it were an objective reality? Or do we, as researchers, erect constructions of teachers' professional development which we subsequently take as objective reality for engaging in discourses of teachers' professional development? Do the various constructions look so different because we are not able to objectively capture the constructed reality or because of the restrictions to one or the other perspective? What are the different tools and materials in the case of the constructions of teachers' professional development? And of what kind are the different constructions?

Let us begin with the knowledge of the teacher of mathematics. A key distinction in this field is the taxonomy of different domains of the teacher's content knowledge (Shulman 1986). The categorization of subject matter knowledge, pedagogic content knowledge, and curricular knowledge—a familiar distinction for researchers and practitioners in some European countries from the 1960s on (cf., Klafki 1958)—has received wide attention in research programs on the knowledge that makes for a good teacher (cf., Ball and Bass 2000; Blömeke et al. 2011; Neubrand et al. 2009). Others recognize the different sources of teacher knowledge: largely a-theoretic craft knowledge developed through experience, principled systematic knowledge acquired in institutional education and training, and prescriptive knowledge transmitted by curriculum guidelines and accountability systems (e.g., Kennedy 2002). In terms of professional development, the major mode of the knowledge dynamics is accumulation. You know more/less, enough/not enough. Although the dynamics could equally call for a focus on the teachers' strengths, a deficit perspective is inherent in many examples of research on teachers' professional knowledge.

With the notion of beliefs we want to refer to what others have also called dispositions, values, conceptions, ideology and the like, leaving the differences between these concepts aside. There is enough evidence to say that this is a weakly structured field (Leder et al. 2003; Pajares 1992). Mathematics teachers' beliefs about teaching, learning, and mathematics affect the teaching of mathematics and some argue that for changing teachers' classroom practices it is essential to influence their beliefs (Prawat 1992; Stipek et al. 2001). In this view, beliefs are regarded as a kind of meta-concept that controls the teachers' professional education and development. This meta-concept has been described as a rather conservative filter in the contexts of curriculum innovation and of professional development. The major mode of the dynamics of beliefs is change. This is different from the mode of knowledge dynamics. It does not make sense to speak of

‘enough beliefs’ or ‘less beliefs’. Mathematics teachers’ beliefs are often qualified as traditional/progressive or negative/positive (e.g., Grigutsch et al. 1998). It is sometimes an authoritarian point of view, from which the properness of mathematics teachers’ beliefs is judged.

Identity is a concept dealt with by many disciplines. Many researchers in mathematics education have first come across this concept by reading the work of Lave and Wenger (1991) on situated learning, possibly defined similarly to the following quote (Wenger 1998, p. 145) highlighting the relationship between human agency and social structure:

Building an identity consists of negotiating the meanings of our experience of membership in social communities. The concept of identity serves as a pivot between the social and the individual, so that each can be talked about in terms of the other.

Research on identities in mathematics education has been tied up systematically by Black et al. (2009a), building on three perspectives: socioculturally looking at practices and participation, discursively focusing on language and power, and psychoanalytically tackling the unconscious. As Black et al. indicate, there are many tensions arising from these perspectives. The major mode of the identity dynamics is formation. Formation is different from change or accumulation as there is not such a thing as ‘enough identity’ or ‘negative identity’. Formation of professional identity has to do with developing a fitting to a situation, with relating individually centered social processes to the socially constructed institutionalized world (cf., Valero 2009). Of course, resistance and subversion are possible reactions if the fitting seems impossible, or undesirable, to achieve.

Table 1 summarizes the dynamics of the teacher’s knowledge, beliefs, and identity as well as the *telos* of each mode.

We will later discuss the practical value that these concepts offer for analyzing the case presented below.

3 The case

Claudia is a mathematics teacher working at a primary school (*escuela básica*) in a small provincial town approximately 10 km from the outskirts of Santiago,

Chile’s capital with about six and a half million inhabitants. Although thus close to the metropolis, when you enter the provincial town with its rural ambience, Santiago seems miles and centuries away. The atmosphere is laid-back and very different from that of a busy city. Primary education in Chile contains the grades 1–8. Grades 1–4 are called the first circle of primary education, grades 5–8 the second circle. When we met Claudia, she taught mathematics in the first circle.

In Chile, education is compulsory until grade 12 and there are ample pre-school and kindergarten opportunities. A distinguished characteristic of the Chilean school system is the coexistence of public schools, private but publicly funded schools, and private schools (*colegios municipales, particulares subvencionados, particulares pagados*). Although some exceptions exist, the private schools are generally regarded as providing far better conditions for learning than public and publicly financed private schools. In terms of class-size, opportunities for learning foreign languages, and the like, this is certainly a fact. However, for many parents social mobility is clearly an argument for choosing a private school for the education of their offspring. The more elite of the private schools apparently meet the need of social distinction. Tuition fees at private schools are high; nevertheless, those who can afford to enroll their children in private schools generally do so. A ranking in the media of primary and secondary schools according to their results in benchmark testing receives much attention from parents and politicians and serves as a guideline for school choice. While the students’ university entry examination results are most important for the ranking of secondary schools, primary schools are ranked according to annual statewide testing by the Ministry of Education (*Sistema de Medición de Calidad de la Educación, SIMCE*) of all fourth graders in language (*castellano*), mathematics, science, and social science, and biannual testing at grade 8. For the schools, SIMCE is high-stake testing. As an indication, consider the four-page report of each school’s SIMCE results that parents (and legal guardians) receive from the school administration. Its introduction (SIMCE 2011, p. 1) holds: “Demand an explanation if the school’s score is much below the reference values we provide. If this explanation does not satisfy you, remember that in Chile you can choose the school for your children or wards.” However, despite a more or less connected bundle of curriculum innovations and the ‘provision’ of an extensive accountability system in the last two decades, the overall achievement of Chilean schools, according to the SIMCE, remains, at least until 2010, unimproved (see Table 2).

In 2010, the then Minister of Education J. Lavín introduced a traffic-light system for categorizing all public schools’ and publicly funded private schools’ academic

Table 1 Dynamics of teacher’s knowledge, beliefs, and identity

	Mode of the dynamics	Telos
Teacher’s knowledge	Accumulation	Sufficiency
Teacher’s beliefs	Change	Properness
Teacher’s identity	Formation	Fitting

Table 2 Progression of SIMCE scores in mathematics, 4th grade (Libertad y Desarrollo 2012, p. 12)

	1999	2002	2005	2006	2007	2008	2009	2010	2011
Public schools	239	235	235	234	231	231	236	237	246
Publicly funded private schools	256	254	255	255	254	254	259	258	262
Private schools	298	298	297	297	298	301	303	299	300

achievement and for making the relative success of schools more easily visible: green for those schools that are officially referred to as “autonomous schools”, yellow for those officially referred to as “emerging schools”, and red for schools officially referred to as “recovering schools”. The categorization considers factors like the relative improvement of each school over the years and the integration of students with disabilities. However, the weight of the last SIMCE results for the categorization is 70 % and a comparison is made with schools with similar student intake (including the private schools). The Ministry sends municipality maps with accordingly colored schools to all parents. The newspapers are used to distribute the information to all others. In addition, the Ministry of Education website provides an interactive Chilean map in which each school of the country is marked according to the traffic-light system and in which more detailed information about each single school (e.g., the detailed SIMCE scores in mathematics and language) is accessible. Red flags are generally dominating the colorful landscape. If a public school is labeled red, it becomes severely restricted in the use of its financial budget. Not surprisingly, most public schools, particularly those which are located in the most problematic neighborhoods, are categorized as “recovering schools”—a euphemism that generally makes the situation worse for them. The map of Claudia’s provincial town shows 21 red, 6 yellow, and 9 green flags. Claudia’s school has a green label.

Meanwhile, as a reaction to strong opposition by leading educationalists, the Ministry of Education has changed its official website and has withdrawn the colored flags for each school, replacing them by the average SIMCE score of the school (though the ‘old’ map is still accessible).

Chile has participated since 2000 in the Programme for International Student Assessment (PISA) and in the 2011 round of the Trends in International Mathematics and Science Study (TIMSS). Although Chile is economically among the leading countries of Latin America and has shown substantial economic growth in the last twenty years, its academic success data (PISA 2009, mathematics: 421 points; TIMSS 2011, mathematics grade 4/8: 462/416 points) remain a worry for the Chilean government and society. It seems as if the government regards the system of quality control according to academic results as one important means to advance academic success.

While the official discourse about academic improvement is intensive, and the mechanisms of quality control detailed, Claudia and her colleagues teach 38 h per week and the classrooms in public schools are crowded. In Claudia’s first grade classroom about 45 children gather. Chile is a country of immense social inequalities and these are reproduced very visibly in and by the conditions learners face at school (Mena and Bellei 1998).

We met Claudia as a participant in an in-service professional development course for primary mathematics teachers that Lorena, Joaquim, and their team at Universidad de Santiago de Chile organized as part of a larger developmental project, financed by the Chilean government (as part of a strategy to promote what Goldin (1990, p. 32) has called a “moderate constructivist view” of mathematics teaching and learning). The year before the in-service course, Lorena, Joaquim, and their team at Universidad de Santiago de Chile had developed didactic units for teaching the mathematics curriculum content of grades 1–4. Brousseau’s (1990) theory of didactic situations and Chevallard’s (1999) anthropological theory of didactics were used as the theoretical framework for the development of the didactic units. These didactic units had then been digitalized into *interactive didactic units* and assembled as software for the teachers’ and students’ use in the classroom. (For the working of the strategy of the developmental project, see <http://www.youtube.com/watch?v=xUjdGdZUtPc&NR=1>; for an excerpt of an interactive didactic unit, see <http://www.youtube.com/watch?v=xLTvM7nu9Vs&NR=1>). Typically, the interactive didactic units embed the mathematics to be taught in a story line, provide a didactically organized sequence of learning activities, engage the learners in (partly simulated) hands-on activities, provide exercises, and facilitate recapitulation and synopsis. The technical environment requires a computer, a data projector, and a whiteboard. Schools which volunteer to participate in the project receive, if necessary, the technical environment from the school administration. The interactive didactic units are devised as a tool in the hands of the teacher, facilitating her structuring of the lesson in a didactically meaningful way.

In the Chilean context, the interactive didactic units signify an attempt to rather radically change classroom practice in mathematics. It is quite usual in Chilean public primary mathematics classrooms to witness a learning style

far from what we might call “progressive education” or “moderate constructivist learning”. Interaction between the teacher and the students is nearly exclusively in the form of a chant: the teacher starts an utterance, and all students complete the utterance, for instance:

Teacher: A quadrilateral has ... [*showing a paper quadrilateral and pointing to the sides*]

All students: ... four sides.

Teacher: And it has ... [*pointing to the vertices*]

All students: ... four vertices.

This sequential structure of the discourse resembles what Sacks (1972) has called adjacency pairs: when the first utterance of the teacher is given, a certain second utterance by the students is expected. This pattern works as a collective order assembly device (Sacks 1995) for the sequencing of the teacher–students interaction. Once established as a teaching–learning pattern, this is, of course, an economic way for the teacher and the students to spend classroom time. For the teacher, the course of the lesson is predictable and can be easily navigated since the students tend not to interfere digressively. For the students, the teacher’s actions are predictable as well and they are not challenged individually. For the pattern to run smoothly, the cognitive challenge of the teacher’s utterances needs necessarily to be low.

While the team at Universidad de Santiago de Chile mainly drove the construction of the didactic units, the Chilean government took the initiative to distribute these didactic units to as many primary mathematics teachers as possible. Thus, the idea of *interactive* didactic units arose as a strategy of distribution. Distributing the interactive didactic units widely was added to the government’s strategy for better achievement in the international comparative studies. However, it is a commonplace in organizational theory and practice that to simply distribute such an innovation, via school administrations’ internet platforms, etc., does not make much sense. Accordingly, a programme of in-service professional development was designed to introduce the didactic functioning of the didactical units and the ways in which the interactive didactic units can be used in the classroom. In the first year as a pilot, 62 primary mathematics teachers, from ten public schools, had been introduced to a series of interactive didactic units in a 1-month (=100 h) in-service course organized as face-to-face learning and a subsequent four-month e-learning course during classroom teaching periods. Claudia was one of these teachers who was particularly dedicated during the course. We followed Claudia into her classroom when she employed the interactive didactic units in her teaching, and we were impressed by the ways in which she could make use of them; ways that demonstrated situation-sensitive adaption rather than

unconditional adoption. Therefore, we decided to engage more deeply with Claudia: we videotaped her teaching extensively (12 consecutive mathematics lessons), and we met for a larger interview (90 min; partly video-stimulated) a year after the videotaped teaching at her school. In the same manner, we videotaped the classroom activities of nine other very active teachers of the in-service course, and interviewed them. We do not present our analyses of the classroom activities and interviews of these other teachers here, but these analyses serve as a background to estimate the particularities of Claudia’s acting in the classroom. We had not the capacity to observe the teaching of, or interview, the remaining 52 teachers.

By analysing Claudia’s teaching and looking for patterns—for our methodology see Gellert et al. (2013)—we found that she continuously tried to question the meaning of what the students are doing. She challenged the students to externalize their (individual) thinking (Bruner 1996) and fostered reflection. She overtly tried to make the knowledge-in-construction collectively available. Of course, these attempts did not always succeed. From the video-stimulated interview with Claudia it can be reconstructed that parts of the pedagogy required by the interactive didactic units had been new to her. During the interview, Claudia referred to the closing moment of one lesson, in which the students did not give the expected answer to her insistent questions:

Claudia: We have 13 hats and 13 guests. What does this mean?

Students: [*no response*]

Claudia: In what are the 13 hats and the 1 guests alike?

In this situation, Claudia decided to leave the questions open because the lesson ended. In the subsequent lesson, she repeated the closing of the activity by emphasizing the linguistic construction ‘as many ... as’. Apparently, Claudia was very successful at not giving control over her actions to the software. Although the interactive didactic units guideline emphasizes the importance of the closing of the mathematical activity at the end of the lesson, she decided to leave the activity open.

While the videotapes of most other teachers showed their first struggles and personal desires to improve teaching by ‘simply’ subordinating their didactic experience to the didactic organization of the interactive didactic units, Claudia was more reluctant to let the interactive didactic units dominate her teaching routines. For Claudia the interactive didactic units might question her instructional routines and we witnessed how she engaged in relating routines with innovation. On a first glance, Claudia’s mathematical interactions with students seemed to be more unsafe and searching. However, her still insecure movements within the didactically “safe space” (Greene 1995)

provided by the interactive didactic units provoked the most mathematically challenging moments for the students, particularly when Claudia engaged with individual students' mathematical thinking and its collectivization. For Claudia, the interactive didactic units served as a tool to organize sequences of mathematical activities. The pre-fabricated interactive didactic units relieved Claudia from task organization. Claudia used this space to establish ties between the mathematics, the individual students, the students as a group of learners, and her didactic support. She tried hard to engage the students in mathematical thinking.

Through the interview that took place a year after the videotaped teaching at her school, it became clear to us that Claudia had already been an exceptional mathematics teacher before passing the in-service course. Apparently, the interactive didactic units had not been much of a conceptual innovation for her. For Claudia, the didactic manner in which the interactive didactic units have been designed, the focus on inquiry questions, provision of tools, and collective reflection, provide a backing for and enhancement of how she always tried to teach mathematics. She said that the interactive didactic units were much in line with the way she tried to engage with the students and that she could make immediate use of them. What Claudia gained from the pre-structured didactic units was a kind of release from the processing of the didactic course through the lessons (sequencing of the content, selection and organization of tasks, ...). She used this liberation to concentrate more deeply on students' actual mathematical activities and the possibilities of collective reflections ("closing moments") at the end of the lessons. To sum up, for Claudia working with the interactive didactic units has been a very positive experience.

The interview with Claudia then turned to how she was teaching at the time of the interview, that is, 1 year after the in-service course had taken place. To our surprise, she indicated that in the ongoing school year she had not made use of the interactive didactic units yet. Her explanation relates to the pace and volatility of education reform (Int. 15/26-35).

Claudia: We have been doing a bunch of things within our school. Other projects.

Interviewer: The Head of Department already told us that the school is with Archimedes and with another project from Singapore. It's true that this school has also worked with a project of English teaching.

Claudia: I don't know whether the other classes, the tiddlers [first and second graders] are working with this project. All the others are in mathematics. The Singapore and the Archimedes are in mathematics. Anyway, we are very busy with these things and we must continue as they

are watching us. For instance, yesterday came Archimedes. The students quickly went into groups! But before, we still needed to be working with the Singapore books. So ...

First, the school had volunteered as a pilot for the introduction of new schoolbooks that had been adapted from a Singaporean series. Then, another university had approached the school for a piloting of curricular material (named "Archimedes", a kind of 'mathematics suitcase'). As the headmaster was interested in the material—as a pilot the school would receive all the materials free of charge—he committed the teachers to use the new didactic approach. The headmaster, who we talked to informally, stated that it is important for his school to always engage with curriculum reform, not least for showing the government that the school is actively trying to achieve always higher in the SIMCE and in order to maintain the green label as one of the few "autonomous schools". And, of course, given the school administration's focus on school achievement and the directness of how the relative achievement is made public, the constant engagement in ever-changing reform attempts is easy to understand.

The curricular innovation of the interactive didactic units as well as the professional development program was quite successful for the Universidad de Santiago de Chile in terms of further funding. Claudia's teaching could be used to show to the government effective mathematics instruction by means of the interactive didactic units. In addition, Claudia's effective use of the interactive didactic units could be backed by the results of a test on numbers and addition, administered to Claudia's and other in-service course participants' pupils. The test results of Claudia's class showed the highest score increase of all classes tested. As a first consequence, the government decided to distribute the interactive didactic units and the corresponding professional development program over larger parts of Chile. As a second consequence, interactive didactic units for the second circle of primary mathematics have now been developed (and later distributed).

In the next paragraph, we analyze the case. We discuss how the attempts at curriculum and professional development affect, and are affected by, the teacher's professional knowledge, her beliefs, and her identity. By explicitly embedding the analysis in a political context, we become clearer about the restrictions and limitations of research that aims at the 'internal states' of mathematics teachers' professional development.

4 Making sense of the case in terms of teachers' knowledge, beliefs, and identity

When the Chilean government launched the distribution of interactive didactic units, one aim was to approach the

problem of a perceived deficiency in the professional knowledge of primary teachers teaching mathematics. International comparative measurement of future primary teachers' mathematics content knowledge (CK) and mathematics pedagogical content knowledge (PCK) had ranked Chile close to economically weaker countries (Teacher Education and Development Study in Mathematics, TEDS-M; see Table 3). In both measures, Chile scored 14th of the 15 participating countries.

The in-service course that aimed at preparing the primary teachers to establish the interactive didactic units as a central teaching aid in their classrooms focused on the teacher's development of mathematics PCK. It engaged in topics of how to start a mathematically substantial activity, of how to maneuver through the possible mathematical structures of a mathematical activity, and of how to take up and socialize the outcomes of individual students' mathematical activities as a form of a collectivization of strategies and insights. These questions are instrumental for the working with the interactive didactic units in the classroom. From the perspective of teachers' professional development, they aim at provoking an accumulation of the participating teachers' mathematics PCK.

For most of the participants of the in-service course and, as we could reconstruct from the videotapes, of the nine other teachers which we followed into their classrooms, this new PCK seemed to be so disconnected from their professional knowledge base that they reacted by adopting a 'proletary' stance (in the sense of Ginsburg and Smyth, see above). They mainly subordinated their teaching routines to the course of the interactive didactic units and let the machine take over. For them, it was rarely possible to connect their craft knowledge gained through experience and the principled knowledge manifest in the interactive didactic units. Claudia seems to be an exception in that her professional knowledge base and her respective teaching patterns provided anchor points so that her professional knowledge could accumulate during the course. For her, the principled knowledge inherent in the interactive didactic units provoked an enhancement of her craft

knowledge. In terms of professional development, Claudia's professional knowledge as a mathematics teacher accumulated as she related the principled knowledge of "didactic situations" and "didactic organizations" to the craft knowledge that controlled her teaching practice.

From the perspective of belief research, the teachers' valorization of a moderate constructivist learning environment is concerned. From this perspective, the in-service course and the subsequent teaching by means of the interactive didactic units aimed at changing the participants' beliefs about the importance of moments of mathematical problematization, of mathematical argumentation, and of the collectivization of learning outcomes. While the perspective on teachers' knowledge focuses, in the terms of Neisser (1976), on the modification of teachers' schemas, beliefs as metaconcepts filter and control this modification (cf., Goodchild 2001). Without a proper style of believing what mathematics teaching and learning is all about, it is rather difficult to make good use of the interactive didactic units. To engage primary mathematics teachers in working with the interactive didactic units can be seen as a strategy to foster the belief in the appropriateness of a moderate constructivist teaching approach in mathematics. Claudia, who already before was committed to such a teaching approach, did not need to radically change her beliefs (as anticipated by the reform initiative) in order to make use of, and benefit from, the interactive didactic units. The distinctiveness of Claudia's beliefs for profiting from the interactive didactic units, which served her as a reconfirmation of the properness of her beliefs in the context of the reform of mathematics teaching, became particularly apparent when the other teachers' use of the interactive didactic units had been analyzed.

The picture turns out to be more complex and the professional development approach more ambitious when the sociocultural environment in which the mathematics teaching is occurring is taken into account. What the introduction of the interactive didactic units is striving for is nothing less than a renunciation of the traditional interaction pattern of the teacher–students chant with its characteristic low level of mathematical challenge, rather addressing forms of enquiry-based learning. The traditional interaction pattern reflects a historically rooted pastoral relationship between the students and the teacher in which the instructional discourse of mathematics is overtly subordinated to the regulative discourse of the moral order (Bernstein 1996). The mathematical knowledge base is treated as unproblematic and the teacher is positioned as a caring relative—primary school students used to call the teacher *tía* (*Tía Claudia* literally means Aunt Claudia)—rather than as a transmitter of knowledge. In fact, the very identity of the teacher is at stake when interactive didactic units try to center classroom life on a subject matter that is

Table 3 CK and PCK of future primary teachers (compiled from Blömeke et al. 2010)

Country	CK mean (rank)	PCK mean (rank)
Taiwan	623 (1)	592 (2)
United States	518 (7)	544 (4)
International average	500	500
Spain	481 (11)	492 (10)
Botswana	441 (12)	448 (13)
Chile	413 (14)	425 (14)
Georgia	345 (15)	345 (15)

made problematic, complex, and challenging. The excerpt above, on ‘as many ... as’, can be regarded as an indication of Claudia still being of two minds about her teacher identity.

The particular attempt of reform is not problematic per se; identity formation might be seen as a promising strategy for professional development. However, as the interview with Claudia shows, the pace of the reform is adequate perhaps for knowledge accumulation but certainly not for identity formation. What Black et al. (2009b, p. 120) call the “increasing desire on the part of policymakers to prescribe certain pedagogic techniques (e.g., pacing, interactive whole-class teaching) which draws on the assumption that providing teachers with a ‘teaching toolbox’ can and will enhance learning outcomes”, is as problematic as, like in the case of Claudia’s school, the coercion of schools by market-forces into a kind of “epidemic of reform” (Ball 2003, p. 215) imposed upon oneself. There is no time to let the interactive didactic units settle. There is no time for Claudia to establish new patterns. The fast pace of the epidemic is, at a large scale, due to the market characteristic of the reforms: the individual engagement in professional development is embedded in a discourse which claims a need to achieve higher in SIMCE compared with other schools (cf., Avalos and De Los Rios 2013). The country has to perform better in order to show that it is ameliorating its status in the global performativity race. The school has to perform better, so that the country will perform better, by outperforming their neighboring schools. We witness a shifting from competency to performativity (Bernstein 1996).

This shifting, which we have presented as a result of the implementation of education policies, can be described in terms of different identity orientations. For this purpose we draw on a grouping of identity orientations by Woods (2003) who explored the links between identity and democracy in the context of governance. While Woods starts from the assumption that meanings are macro-socially shaped and argues for including the “psychological phenomena” (p. 148) in a grouping of identity orientations, our argument goes in the opposite direction: in the context of mathematics teachers’ professional development, we see the need to include the macro-social phenomena in order to more fully understand what is at stake. From Woods’s grouping we construct a triangle of identity orientations for understanding the effects of the Chilean education policies on primary mathematics teachers (see Fig. 1).

The triangle is a conceptual tool and the distinction of the three vertices is analytic in character. It is exactly the interrelation of the vertices which is of concern when the dynamics and the development of identity formations are the focus:

- *Introjected* identities focus on the self and on making sense and internally regulated construction procedures. They are about “inner commitments and dedications” (Bernstein 1996, p. 76) and employ introjection as their organizing procedure. This is the individual element of identity.
- *Categorical* identities (Taylor 1998) refer to the character of teachers as a professional group. This group disposes of a conjunctive experiential space and is, to some extent, cohesive and integrated. Categorical identities give a sense of definiteness (Woods 2003); it is a matter of “identifying with”.
- *Externalized* identities refer to the self as defined by external features that often are short-term, of passing value, or principally instrumental. According to Bernstein the organizing procedure here is projection. In their education market form, externalized identities are constructed as projections on to educational consumables. As Woods (2003, p. 149) adds, they include “performance measures and targets deployed by modernizing governments as policy levers to achieve rapid change in working practices and cultures”. They affect how teachers see themselves and their roles as teachers (Ball 2000).

The triangle displayed in Fig. 1 symbolizes a politicized concept of identity. It expands those conceptions of identity, which aim at linking the psyche and the micro-sociology of group dynamics, by integrating a macro-sociological element considering the influence and impact of cultural and political contexts. We maintain that a teacher’s professional identity is located somewhere in the interior space of the triangle. Nobody can fully escape the projections from (market driven) education policies. Self (introjections) and group-self (conjunctive experience) can never be suppressed. Identity formation can be regarded as the search for a balanced state within the triangle. In the case of an educational reform targeting greater effectiveness and efficiency, this balance, at least temporarily, becomes disturbed. In the case of Claudia the disturbance caused by introducing interactive didactic units was minor,

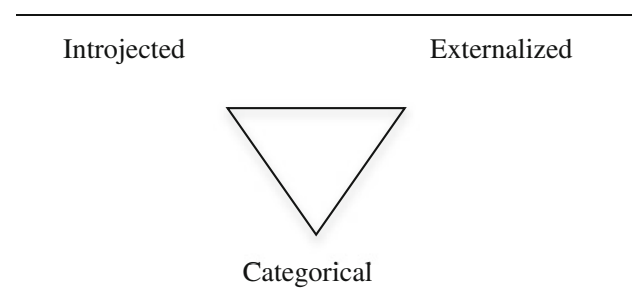


Fig. 1 Identity orientations

because Claudia was already committed to the teaching style promoted. And Claudia was already in a process of transforming her understanding of what it means to be a ‘good’ teacher. There have been indications of this in the video footing, and Claudia is explicitly responding in the interview (Int. 17/27-33):

Interviewer: How do you value the experience?

Claudia: My experience was good, very good, I learned a lot. I tried to apply as much as possible of what I had seen as being an improvement. ... With respect to the lessons, no problem. Quite the contrary, the material was a support, and specific for each lesson. The kids worked more than motivated [sic], they obviously have been much more motivated, the class is far more easy to handle, I did not need to fight for the tiddlers [her first graders] to work.

As the projections from the particular educational policy to introduce interactive didactic units did not contradict her professional self, Claudia’s externalized and introjected identity orientations did not come into conflict with each other. Presumably, the projection was even reconfirming her introjected identity.

Possibly for Claudia, the introduction of the interactive didactic units triggered a release of existing tensions between her introjected and her categorical identities, because one of her colleagues in school, Nelly, also participated in the teacher development course. Although Nelly was not teaching so confidently by means of the interactive didactic units—we see this from analyzing her videotaped teaching and is confirmed by what Nelly said in the interview—Claudia could report on professional exchanges between the two and even other teachers of their school (Int. 4/11-18):

Interviewer: From what we see in the videos, you profoundly understand what is suggested for each lesson.

Claudia: But this is because you get to know. I can’t go to the classroom without knowing what I am going to do or what I like to achieve with the students. The books [descriptions and explanations of the didactic units] we knew in advance. Suddenly I tried together with Nelly, who is parallel, to share the experience: what was difficult or what worked well or what was suddenly a bit more difficult. We always shared, always, and here we are, you even do it with the others who teach same grade.

In other teachers whom we had videotaped using the interactive didactic units in their mathematics classrooms, we could see the disturbance of their professional identity

quite clearly: the teachers seemed unconfident and their instructional practice incoherent. For them, the interactive didactic units came in as just another commodity, effecting, in the long run, a formation of their identity in the direction of the externalized vertex of the triangle. The rapid pace by which these commodities entered classroom life, but also their transience and volatility, impede the mathematics teachers from developing a position, let alone a fitting, to the new views about mathematics instruction. Instead there is a tendency to shift their relationship to educational technology from that of operators to that of consumers. This is not a criticism of the characteristics and the potential of the interactive didactic units. It is a criticism of how this educational innovation is embedded in a particular framework of educational policy (Johansson 2010).

5 Conclusion

By not ignoring the political context of a professional development initiative for primary mathematics teachers, and by constructing a politicized concept of teachers’ professional identities consisting of introjected, categorical, and externalized identity orientations, we were able to support and reinforce the thesis that a focus on the ‘internal states’ of teachers’ knowledge and beliefs produces necessarily a fragmentary and possibly misleading picture when reform and professional development is concerned. We might conclude that the combination of institutional conditions (here: underfunded public schooling) and education policies (here: establishment of an organizational culture of performativity) tend to provoke an unproductive disturbance of the identity formation of teachers and, thus, a fragile base for their professional development. In times of reform, it is essentially the identity of teachers that is at stake. To include externalized identity orientations into the concept of a teacher’s professional identity is a response to the political character of educational reform. It is then possible to explain how the projections of policy makers and educational authorities regarding what it takes to be a good teacher influence, and often contradict, the teachers’ professional identities.

Teachers’ professional knowledge and their professional beliefs are the official targets of educational reform and, of course, they develop over time. However, as Ball (2003) argues, these accelerated attempts at reforming education come as an epidemic, the novelty of this kind of epidemic being that it does not simply attempt to change what teachers do, but what they are. These reforms do neither try to ‘simply’ transmit new knowledge to teachers, nor to change their values and beliefs. They attack the very identity of teachers; in the terms of Bernstein (1996), by shifting official pedagogic identities from introjected

modes to projected modes, “the past is no [longer a] necessary guide to the present, let alone to the future” (p. 77). This is not so problematic in the case of Claudia, but—as we could see in other Chilean mathematics teachers, and as Ball (2003) illustrates by many examples from the UK—has repercussions on those who do not fit to the new situation.

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