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ORIGINAL ARTICLE

Telling and illustrating stories of parity: a classroom-based design experiment on young children's use of narrative in mathematics

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Abstract This paper examines ways to engage young children in constructing and interpreting narratives to develop their understanding of parity. It reports on a teaching intervention that was developed over three research cycles of a classroom-based design experiment, and focuses on the last of these cycles. The teaching intervention set out to investigate how young children (5–6-year-olds) can be supported to draw on narrative in their explanations of whether a whole number less than 20 is odd or even. Evidence of the effectiveness of the intervention is provided through comparison of children's performance on pre- and post-tests in the form of semi-structured individual interviews. Also, authentic examples are provided of how children utilised their power of 'imagining and expressing' to tell stories of whether a whole number is odd or even, using either a counting, partitive, or quotitive model for division. Implications for research and practice are discussed in light of these findings.

1 Introduction

Parity (the quality of being odd or even) is a topic which, although having a central place in the school mathematics curriculum, has not received much attention in mathematics education research. At the teacher education level, Ball (1990) explored student teachers' understanding of divisibility in general, while Zaskis (1998) explored student teachers' knowledge of parity. Both studies identified difficulties faced by student teachers, with Zaskis noting that their conceptions of divisibility by two were somehow distinct from divisibility by other numbers. At the school level, Frobisher (Frobisher and Nelson 1992; Frobisher 1999) assessed primary school children's knowledge of odd and even numbers, and concluded that they lacked conceptual understanding of parity. Hunter (2010) explored odd and even numbers as an example of developing 9–10-year-old children's algebraic reasoning. She concluded that the context of odd and even numbers can be used "to provide children with effective opportunities to make conjectures, justify and generalize", and she highlighted the importance of teachers' role in supporting children to model their conjectures using material (p. 109).

The study on which we report in this paper contributed to this body of research and worked with a younger and less researched student population. It aimed to support 5–6-year-old children to develop their conceptions of parity, with a pedagogical focus on the use of narrative in mathematics. Specifically, the study aimed to investigate how young children can be assisted to use narrative as a cognitive tool to support understanding of parity as three related models (counting, quotitive, and partitive). The study was organised as a classroom-based design experiment and developed a teaching intervention over three iterative research cycles with the first author as the teacher–researcher.[1]

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[1] The first author implemented the interventions in all research cycles, but she was not the usual teacher for any of the classes who participated in the study.

2 Theoretical framework

The design of the teaching intervention was based on three main features, which were developed and refined over the research cycles of the study. Narrative was the core strategy of the intervention, playing two main roles as discussed in the first section below. Parity was the focal mathematical topic; the approach to teaching parity followed in the intervention is discussed in the second section. Finally, although the role of the teacher is reflected partly in the first two sections, we considered important (in part because the first author was also the teacher in the intervention) to explicitly discuss key aspects of this role; this is done in the third section.

2.1 Narrative as a teaching tool and as a cognitive tool

In this paper, we use the term 'narrative' at times as a noun (the story) and at times as an adjective (describing the story-like quality of something). We consider narrative as comprising both words (written or oral) and images (presented or imagined), as in a children's storybook. It should be noted that our use of 'narrative' is distinct from what others may term the 'narrative context' of a mathematical word problem. We view the 'narrative context' (or 'problem context') as the situation or milieu—comprising elements such as characters, a setting, numbers, and a question or conflict—on which a particular mathematical word problem is based. Our use of 'narrative' considers stories more generally, whereby several narratives (distinct stories) can be generated from a single narrative context, and the narratives need not be told in the genre of word problems.

Narrative has received attention in education broadly. Egan (1989) argued that story is the missing link that can bring learning and imagination together, and that stories can make whatever is to be learned into something meaningful, engaging the imagination in the process of learning. Egan (2002) discussed further how teachers "might represent the world in narrative terms to children for whom this becomes a major tool for learning" (p. 72). In this sense narrative can be seen as a *teaching tool*, whereby stories are used by teachers to engage children in a process of learning.

Narrative can also play an important role as a *cognitive tool*, whereby stories are used by children to construct meaning. Bruner (1996) argued for this role when he described narrative as instrumental in "meaning making", noting that people make sense of the world through storytelling (p. 147). He identified two broad ways by which human beings organise and manage their knowledge of the world—logical-scientific thinking and narrative thinking—and observed that schools have traditionally favoured the former. He criticised the limited attention to narrative as being problematic, for, as he said, "if narrative is to be made an instrument of mind on behalf of meaning making, it requires work on our part—reading it, making it, analyzing it, understanding its craft, sensing its uses, discussing it" (p. 41). Bruner did not advocate for narrative thinking over logical-scientific thinking, but rather saw the two as mutually supportive: he pointed out that "[s]cientific explanations are adjuncts to narrative interpretation and vice versa" (p. 92) and noted that "[w]e may have erred in divorcing science from the narrative of culture" (p. 42).

In relation to mathematics education particularly, Mason (2008) has asserted that "human beings are narrative animals" with a deep-seated need to tell stories to others (p. 60). He referred to narrative as a cognitive tool when he identified "imagining and expressing" as a key "children's power", which should be harnessed to develop their thinking or sense-making in mathematics in general and algebra in particular (p. 86). This use of narrative requires a shift of focus from the teacher as storyteller (narrative as a teaching tool) to the child as narrator (narrative as a cognitive tool), though it is the teacher's role to create opportunities for children to tell and retell their own stories to make sense of mathematical ideas (see Back et al. 2010). Bastable and Schifter (2008, p. 176) have questioned the role of narrative context (or problem context) in enabling or hindering children's mathematical insight, when they asked, "In what ways do problem contexts provide a means for students to reason at a general level and when do problem contexts limit their reasoning to specific cases?" (p. 177). Empirical research on how narrative can be used as cognitive tool to support mathematical thinking seems to be in its infancy.

Mindful of the above research on the use of narrative in education generally and in mathematics education in particular, our teaching intervention made use of both roles of narrative. Firstly, narrative was used as a teaching tool, with stories of parity carefully presented using text and images by the teacher to the class. Secondly, narrative was used as a cognitive tool, with children expected to re-present and create new narratives, which they told and illustrated to explain parity of whole numbers less than 20. Our teaching intervention helps to shed some light on Bastable and Schifter's question, as we required children to engage with multiple narrative contexts and to develop their own distinct narratives for the same mathematical problem, thus supporting generalisations about parity.

2.2 The teaching of parity

The design of the teaching intervention was informed by Kaput's (2008) definition of three strands for early algebra:

to generalise arithmetic patterns (in this case, patterns related to doubling or halving whole numbers less than 20), to generalise towards the idea of a function (related again to doubling and halving), and to use language in modelling mathematical processes. Regarding the last of these, Kaput identified an aspect of modelling as '*algebrafying* an arithmetic problem', whereby the constraints of a particular problem are relaxed to explore its more general form. The children in the intervention embarked on a process of modelling by 'algebrafying an arithmetic problem': the starting point for a particular narrative was a selected number, but the underlying concepts needed to be applied to any whole number less than 20. Children were then expected to make general statements (about what was always true) relating to odd and even numbers.

Our approach to teaching parity was also informed by the major shift in learning theory pertaining to arithmetic from a 'set-based conception' to a 'counting-based conception' of number (Yackel 2001). Thompson's (1997) edited book offers a consolidated account of research which supported this shift. Building on this research, the design of the intervention paid attention to strategies that harnessed counting. In particular, 'skip counting in twos' (from zero for the even numbers and from one for the odd numbers) was used. We call this the 'counting model' of parity.

Our teaching approach was informed further by the two meanings of division—division as equal grouping (the quotitive model) and division as sharing (the partitive model) (Anghileri et al. 2002)—applied in the case of division of a whole number by 2. Zaskis (1998) called for the use of definitions of odd and even numbers with young children that capture the essence of these meanings, while using terminology accessible to them, rather than prematurely introducing a parity heuristic. This call is also echoed in Frobisher's (1999) conclusion that "much more quality time needs to be spent developing the idea of partitioning into two equal sets and the equivalent activity of dividing into sets of 2 s" (p. 48). In this regard, we aimed to support children to conceptualise even and odd numbers not only in terms of skip counting in twos (counting model) but also in terms of whether they can be partitioned into two equal groups or whether they can be divided into groups of two.

Recognising the role of narrative as a teaching tool, three narrative contexts were developed for each of the mathematical models of parity (counting, partitive, and quotitive) as explained above. These are summarised in Table 1 and are referred to as 'presented narratives', for they were initially constructed and narrated by the teacher.

The three presented narrative contexts were refined over the research cycles. From the outset, the counting story was built around the visual representation of a number line, but only in the third cycle was it set in a street. Division as sharing is commonly used in primary schools, and a picnic with two characters sharing food was considered to be a relevant context for the sharing story. Dinosaur characters were used in the first two cycles, and by the third cycle children were encouraged to invent their own characters. The context of a 'ball' or of 'dancing pairs', used by Fosnot and Dolk (2001) to explain parity, inspired the context for the pairs story around the quotitive model of division. This was used in all three cycles.

In the first and second research cycles children were only given freedom to change the numbers involved in the presented narratives. In the third research cycle the class activities were opened up to allow the children not only to pick the number being considered, but also the parity model, and the characters for their own stories. The lesson activities aimed to create opportunities for children to tell and illustrate stories (as a means of explaining a mathematical concept). Thus multiple narratives were generated within the classroom community:

- those told by the teacher using the three presented narrative contexts;
- those retold by children using the presented narrative contexts but applied to different numbers; and
- new narratives invented by children where they could change more elements of each narrative context—the numbers, the parity model (counting, partitive, or quotitive) which defined the problem/conflict, the setting, and characters.

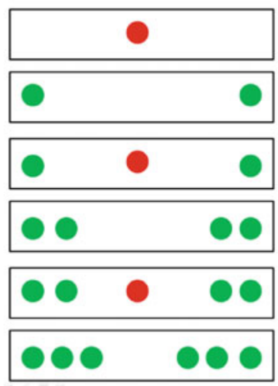
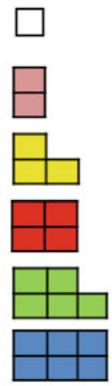
As such a set of different narratives were created which could be compared to each other. The variety and comparison were intended to support abstraction across the stories to explore the underlying mathematical idea (parity).

2.3 The teacher's role

For children to provide adequate mathematical explanations and successfully engage in early algebraic reasoning, appropriate scaffolding, modelling, and teacher involvement are required (Carpenter et al. 2003; Hunter 2010). Accordingly, the teacher in our study set out to present problem-solving situations, encourage mathematical discussion, model mathematical thinking through storytelling, and expected children to provide explanations. Overall, the teacher aimed to support mathematical sense making (e.g. Anghileri 2006) whereby children would not only develop heuristics for identifying odd and even numbers but would also explore multiple ways of thinking about parity.

In particular, the teacher positioned herself as the representative of the mathematical community in the class (Stylianides and Stylianides 2009), with an explicit role to

Table 1 Summary of presented narratives of parity used in the intervention

	The counting story	The sharing story	The pairs story
Starting point	Counting in twos on the number line (counting model)	Division as sharing (partitive model)	Division as equal grouping (quotitive model)
Characters	People	Dinosaurs	People forming couples
Setting	An urban street	A picnic	A dancing ball
Problem/conflict	What number is a particular house in this street?	Two dinosaurs sharing things at a picnic. Do they fight over who gets more?	Pairs going to a ball. Is anyone left out?
Primary visual representation	 Red and white numeral cards	 Sharing cards	 Number shapes

direct the children's attention to the focal ideas thus increasing their awareness of these ideas (Mason 1998). For example, through a line of questioning about different narratives of parity, including "What is staying the same?" and "What is different?", the teacher sought to direct children's attention to common underlying principles related to parity (e.g. their common focus on 'twos') while acknowledging the variation in the narrative contexts.

In all narratives of parity, the teacher expected children to explain and justify their reasoning, and generalise their observations about particular numbers to make statements about odd and even numbers. For example, while the children had freedom to choose a number with which to start their narrative, the teacher prompted them to apply the same narrative to other numbers, thus constructing an 'example space' (Watson and Mason 2005) in which the children were directed. Similarly, while the children had freedom to set certain parameters of their narrative, the teacher's role was to shift their personal example space to consider examples outside of this. The activities were planned to ensure that the children were working with individual numbers, but that they were also seeing them positioned on a number line, always in relation to other numbers. In this way, the arithmetic problem of whether a particular number was odd or even was 'algebrafyed' (Kaput 2008) to reflect on a bigger set of numbers.

3 Methodology

3.1 Research design

This study was conducted as a design experiment (Cobb et al. 2003; Schoenfeld 2006), and aimed to develop and theorise a teaching intervention to support 5–6-year-old children to identify and justify whether a number is odd or even using narrative in mathematics. The teaching intervention, and the theoretical framework that underpinned its design, evolved in dialectic over the three research cycles of the design experiment, with the final version of the framework being as presented earlier. The first research cycle was in a Year 1 class in England, the second in a

Grade R class in South Africa, and the third in a different Year 1 class but in the same school as in the first cycle. The schools were opportunistically selected, as the teacher–researcher (first author) was a parent with her own children in the classes in the first and second cycles. By the third cycle, the researcher was no longer a parent at this school.

In this paper we focus on the third cycle, which was conducted in a large primary school in Cambridgeshire, England, where children's attainment on entry was typical of children nationally (Office for the Standards in Education Report 1997). The focal class was a Year 1 class of 29 children, with their normal class teacher hereafter referred to as Susan (pseudonym).

The data-gathering activities for the third cycle included the following:

- *Pre-intervention activities*: interview with Susan and observations of some of her lessons, as well as pre-test (in the form of video-recorded semi-structured individual interviews) with 16 children who represented the low, middle, and high attainment mathematics groups of the class according to their performance on school mathematics assessments;
- *Teaching intervention*: 8 h of teaching by the teacher–researcher which were video recorded, and all materials presented and children's work created were collected;
- *Post-intervention activities*: new interview with Susan, post-test with 12 children (a subset of those who were interviewed individually for the pre-test), and presentation of results of the intervention to interested parents.

The presumed starting point of the children's learning for the intervention was described by Susan in her pre-intervention interview. She said: "We did very basic stuff on doubling and halving up to ten…. We started off with them drawing three [circles], and then drawing another three [circles]." Susan added that she had not done much on halving an odd number; rather she had referred only to the example of seven and told the class "so you can't do it". She also said she had touched with the class on odd and even numbers by colouring-in a pattern in the hundred square. Susan could not recall how she had explained what made a number odd or even.

Building on the literature and the findings of the two prior research cycles, the prospective endpoint of the children's learning was that, by the end of the intervention, they would be able to identify whether a whole number less than 20 was odd or even, and provide multiple explanations for this by using the counting, partitive, and quotitive models. It was expected that the children would make use of narratives to explain their thinking. The children were also expected to generalise these parity patterns for different numbers, to make connections between parity and doubling and halving, and to be able to make general statements they thought were always true.

The prospective endpoint, interview guide, and coding framework all made use of this organisational scheme:

- Identification of parity;
- Explanations of parity;
- Connection to doubling and halving; and
- Generalisations about parity, doubling, and halving.

Pre- and post-tests for this research were conducted in the form of individual semi-structured interviews (Table 2).

During the interview a resource box was available which included: a whiteboard and marker, two different kinds of counters, digit cards, a ruler, multilink, and a number line. The children were encouraged to draw, write, or show what they were doing using the equipment available.

A coding framework was developed which allowed for full and partial mark allocations per interview question (Table 3). The maximum possible mark was 17; all marks were converted to percentages.

As will be discussed in the following section, the interview protocol helped obtain useful findings about children's use of narrative and understanding of parity. One way in which the interview protocol could be further improved in future studies would be by adding questions like, 'I am thinking of a number, how can you work out if my number is odd or even?', which could offer more information on generalising different models of parity.

Table 2 Interview guide

	Interview questions
Identification of parity	1. Choose a number that is less than 10
	2. Do you think that number is odd or even?
Explanations of parity	3. Why do you think it is odd or even? What makes it odd or what makes it even?
	4. Do you know another way to explain why a number is odd or even? Can you give me another reason for why it is odd or even?
	5. Repeat 2–4 using other numbers to include both odd and even numbers
Connection to doubling and halving	6. Can you double your number? What is double your number?
	7. Can you halve your number? What is half your number?
	8. Repeat 6 and 7 but for another number to include both odd and even numbers
Generalisations	9. Do you know any stories which you can tell to explain if a number is odd or even?
	10. Is there anything that you have noticed, or which you think is always true, about odd and even numbers and doubling and halving?

Our sampling for the pre- and post-tests aimed for a balanced representation of the different attainment groups in the class (low, middle, and high). We started with 16 children in the pre-test who represented the different attainment groups as defined by Susan in relation to their current attainment in her mathematics assessments. We anticipated some drop-out due to absenteeism over the teaching intervention. The resulting sample of 12 children with both pre- and post-test data—which consisted of 5 low attainers, 2 middle attainers, and 5 high attainers—was considered to offer a balanced representation of the different attainment groups in the class and sufficiently large for examination of students' learning in the intervention.

From these 12 children, a few were selected for more detailed qualitative analysis based on the outcomes of quantitative analysis of the pre- and post-tests, to include a child who had:

1. made the most substantial shift in comparison to their peers;
2. made the smallest shift in comparison to their peers;
3. shown a substantial shift in the upper attainment levels; and
4. shown a substantial shift in the lower attainment levels.

We selected three children: Eddie fulfilled both criteria 1 and 4, Catherine fulfilled criterion 2, and Michael fulfilled criterion 3. Detailed case study descriptions were developed for these three children. In addition, all the videos of the lessons were viewed again, deliberately focusing on the three case study children. Each case study included a detailed account of their attainment in the pre-test, an examination of their portfolio of all work produced during the teaching intervention, and compilations of their interactions in the lessons as seen on the video transcriptions.

3.2 The teaching intervention

The teaching intervention was implemented over eight daily 1-h mathematics lessons. The planning for the intervention was detailed and described the activities

Table 3 Coding and marking framework used for each interview

	Code	Full marks	Partial marks
Identification of parity	1	Identifies a number as odd and another one as even (2 marks)	Identifies only an odd or an even number (1 mark)
Explanations of parity	2	*Counting model*	
		Explains that a number is odd and another one is even using the counting model of parity (2 marks)	Explains that a number is odd by providing a list of odd numbers (similarly for even numbers) (1 mark)
	3	*Partitive model*	
		Explains that a number is odd and another one is even using the partitive model of parity (2 marks)	Explains that with even numbers there is no 'fight' while with odd numbers there is a 'fight', but does not refer to sharing between two (1 mark)
	4	*Quotitive model*	
		Explains that a number is odd and another is even using the quotitive model (2 marks)	Explains that with odd numbers there is one left over, but does not indicate the making of groups of two or pairs (1 mark)
Connections to doubling and halving	5	Correctly doubles a number using pairs or equal grouping, or as a known fact (2 marks)	Correctly doubles a number, but only after being told that doubling means 'adding the number to itself' or 'making that many pairs' (1 mark)
	6	Correctly halves an even number (2 marks)	Correctly halves an even number, but only after being told that finding half means 'sharing equally between two' or 'counting the pairs' (1 mark)
	7	Correctly halves an odd number (2 marks)	Explains that half of an odd number is not a whole number or says that the calculation 'can't be done' (1 mark)
Generalisations	8	Makes 3 statements that are always true about odd and even numbers, doubling, or halving (3 marks)	Makes 1 or 2 statements that are always true (1 mark per statement)

and materials to be used in each lesson. The planning underwent multiple revisions over the life of the design experiment, and incorporated both the developing features of the theoretical framework and the accumulated understandings and experience of the teacher about how children interacted with the activities.

The following is a description of key events in the teaching intervention during its implementation with illustrative examples from the work of the three case study children. We offer this description in some detail to give sufficient idea to readers of what actually happened during the intervention and to allow for potential replication of the intervention.

3.2.1 Introduction to twos, pairs, and visual representations

In lesson 1 the teacher explained that all the lessons would focus on twos or pairs. She showed a way of counting in twos, with the class chorusing skip-counting. She then set this number puzzle to be solved by the group:

> Given the pattern 'umbrella, pair of shoes, triangle, chair, and glove', what comes next—an egg box, calendar, or an octopus?

The intention was to create interest amongst children in representations of numbers.

After solving the number puzzle in a plenary, the children made lists or drew pictures of things that they knew came in pairs or groups of two. For example, Michael drew pictures of gloves, shoes, stripy socks, ears, arms, and wings. The children then used skip-counting, whispering the odd numbers and saying the even numbers loudly, to find out how many eyes were in the room. The teacher also read a story *Professor Mouse Collects Numbers* (Roberts and Tyrrell, in preparation), which introduced different visual representations for numbers and stories in mathematics.

In lesson 2, the teacher reviewed a poster of their 'things that come in pairs'. The children then worked on their own invented representation of the numbers 1–12 to create playing cards. They showed groups of two on each card. For example, Eddie was able to accurately represent the numbers 1–12 using one symbol for a unit, and arranging some numbers in two columns. Catherine chose to use paw print numerals where each paw has four claws and she circled pairs of claws.

3.2.2 The counting story

Still during lesson 2, the teacher introduced a new pattern by distributing large numeral cards that were printed on white paper for the odd numbers and red paper for the even numbers, and by asking children to arrange them into the pattern. She then asked the children to describe the pattern. Children described this as a "red–white–red–white–red" pattern, and a "1–2–3–4–5–6–7" pattern, noting that the numbers were in order from smallest to biggest. The vocabulary of 'odd' and 'even' was introduced by a child, and the pattern was now talked about as "odd–even–odd–even". Another child volunteered that "it's a pattern counting in twos: 2-4-6-8". The teacher linked this to the skip-counting in twos and reminded them of the "whisper–loud–whisper–loud" pattern done the day before. The class then chorused skip-counting in twos.

The class was given a problem to solve:

> On one side of the street the numbering went '6–8–10–12–14–16', then it was my house, while on the other side of the street the numbering went '1–3–5–7–9–11–13–15'. What was the number of my house?

The problem was not resolved. The children worked in pairs on a puzzle of a street where the house numbers started at 1, and alternated across the street. The lesson concluded with the teacher reminding the children about the "red–white–red–white" and "odd–even–odd–even" pattern, and asked them to look at their own streets to see the number of their house or flat, and that of their neighbours.

In lesson 3, the teacher handed out the numeral cards and asked the children to arrange them in order. She then returned to the problem of her street, which was carefully retold as a story of parity. The characters in the story were the teacher and her daughter Tessa who were trying to work out the number of their flat. Pictures of each house in Maynard Street, Cape Town were shown in a slide show. The teacher was deliberate about explaining elements that need to be in place in a story: a setting, characters, and a conflict or problem. Particular children helped to break up the number line on the floor to model the pattern of the street numbers using large red and white numeral cards. There was then a short class discussion in which volunteers described their house numbers in relation to their neighbours. The children were encouraged to look at their own streets and try to find a pattern. Through class discussion, the children concluded that the teacher's house was number 18, before revisiting the patterns used to solve this:

- the "red–white–red–white" pattern of the number line;
- the "whisper–loud–whisper–loud" pattern used for skip-counting; and
- the "odd–even–odd–even" number pattern.

The children then completed their street puzzle, this time sticking down the pieces.

3.2.3 The sharing story

In lesson 4, the teacher reminded the children of the counting story before she explained that they would be working on another story for odd and even numbers, which focused on sharing. She then set a visual representation problem, where children had to draw on their whiteboards what they thought should appear on the first blank card (which was 2), and the last blank card (which was 9). Michael chose to draw all the number cards from 2 to 9 (Fig. 1). Two volunteers then made the cards for 2 and 9 using counters, and other children described what they saw as the pattern.

The teacher then introduced the second story of parity: the sharing story (Fig. 2). A dinosaur picnic was given as the context, and a collection of stories—with examples of sharing different numbers of objects between two—were recounted. These stories were first told by the teacher for particular numbers, and then children retold the same stories for different numbers, using the number line and sharing cards as their visual representations of the numbers in each story.

The children then worked on their own sharing stories. They chose their context and how many things were being shared, drew a picture, and concluded whether their number was odd or even. For example, Eddie clearly depicted two robots sharing six cucumbers, indicating that he was referring to six by writing the numeral 6 and concluding that six was even, by writing 'even' (Fig. 3).

The teacher concluded the lesson by emphasising that an even number of things could be shared equally between two characters, but for an odd number of things there was a fight, because one was left over. If there was a way to cut the objects in half then the story had a different ending as they each got "something and a half".

3.2.4 The pairs story

Lesson 6 began with a review of the counting and sharing stories. Children arranged the primary visual representations and volunteers narrated each story. The lesson then shifted to the third story of parity: the pairs story. The story's setting of a ball was introduced by showing some pictures on the whiteboard.

Fig. 1 Michael's solution to the visual number pattern

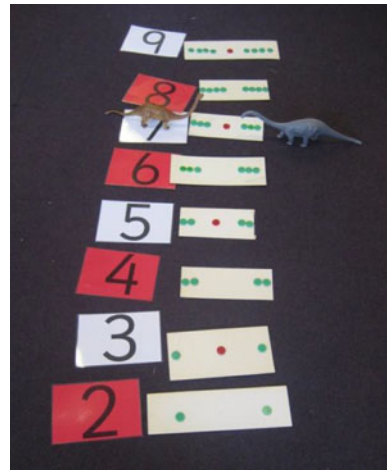


Fig. 2 Two dinosaurs sharing

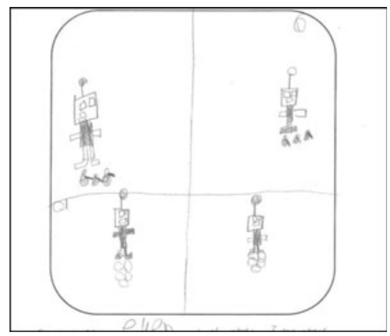


Fig. 3 Eddie's sharing stories for 6 and 10

A short role-play involving seven children followed. The teacher invited pairs of children who had dressed-up, into a ball by reading out an invitation, and they each came into the centre of the circle. Three pairs were able to go to the ball, but one person (Emma) was left out. There was some discussion about the problem this caused, as for everyone to have a partner an even number of people was necessary. The class spontaneously started chanting "Henry! Henry! Henry!" so Henry was also brought into the role-play. Emma and Henry were now the last pair who could come to the ball. In total there were four pairs who could go to the ball (8 is an even number).

To focus the class again, the teacher asked the children to count in ones while clapping and hitting their knees. The teacher also handed out multilink number shapes for the numbers 1–10 and asked the children to put them next to the correct number on the number line. Volunteers described the pattern. The teacher modelled the pairs story by breaking up the multilink shape for 7 into three pairs, with one left over. The children broke up the other numbers and retold the ball story for *this* number to explain if *this* number was odd or even (Fig. 4).

The children then each drew an illustration for their story of whether a number of their choosing was odd or

Fig. 4 Multilink models of numbers broken into pairs

even, using pairs. For example, Michael first worked on whether the number 2 was odd or even. He drew a heart around the two adults to show that they made a couple. He concluded that 2 is even. Michael then showed that 3 is odd. The couple who were inside the ball were circled and shown to enjoy the ball under a glitter ball; the person who was left out was shown crying at the door to the ball (Fig. 5).

At the end of lesson 6, volunteers each told one of the three stories: the counting story, the sharing story, or the pairs story.

3.2.5 Consolidating all three stories

In lesson 7, the teacher handed out each primary visual representation and timed how long the class took to sequence them. Volunteers then told each story.

Each child was given an envelope with a different number of counters in it. They arranged their counters to show the pairs story to establish whether their number was odd or even. They rearranged their counters to tell the sharing story and to see if their number was still odd or even. Children then wrote their own story about a certain number of things, and how they knew whether it was odd or even.

Fig. 5 Michael's illustration that 3 is odd

3.2.6 Concluding lesson

The teacher handed out all three primary visual representations and timed while the children sequenced them. She retold the counting story and asked for volunteers to retell the counting story, the pairs story, and the sharing story. At this stage most children had been able to recount a few versions of each story.

4 Findings

4.1 General findings

The pre- and post-test data were analysed, using the coding scheme described earlier. Figure 6 presents the pre- and post-test percentages for each of the twelve children.

This provided a quantitative sense of the learning that took place for these twelve children. It showed that their mean attainment on the pre-test test was 34 % while their mean attainment on the post-test was 66 %. The average attainment almost doubled from pre-test to post-test. Considering shifts in results from pre-test to post-test for each child, the average of the individual shifts was 32 %. These shifts are shown for each test code in Table 4. The coding and marking framework was presented in Table 3.

Perhaps it was to be expected that after 8 h of teaching the average attainment would improve. Yet it is encouraging that the post-test results were within a narrower range, that all children showed a positive shift in attainment, and that the mean attainment showed a notable increase (from 34 % in the pre-test to 66 % in the post-test).

If we look in more detail now at the test attainment of the twelve children (Table 4), we observe that while only six of the children could identify an odd and an even number in the pre-test, all of them could do this in the post-test. All of the children, except Jake, were able to provide at least two different reasons for why a number was odd or even. Seven of the children (58 %) were able to provide three different explanations that drew on the three different models: counting, partitive, and quotitive. Most children (92 %) were able to identify whether a whole number less than 20 was odd or even, and to provide multiple explanations for this. There were improvements in relation to being able to double a number (from 62 % in the pre-test to 92 % in the post-test), halve an even number (from 23 to 50 %) and halve an odd number (from 0 to 25 %). Children making any general statements about parity improved from 8 % in the pre-test to 45 % in the post-test.

The portfolios of work revealed that, when given the opportunity, all twelve children were able to make use of

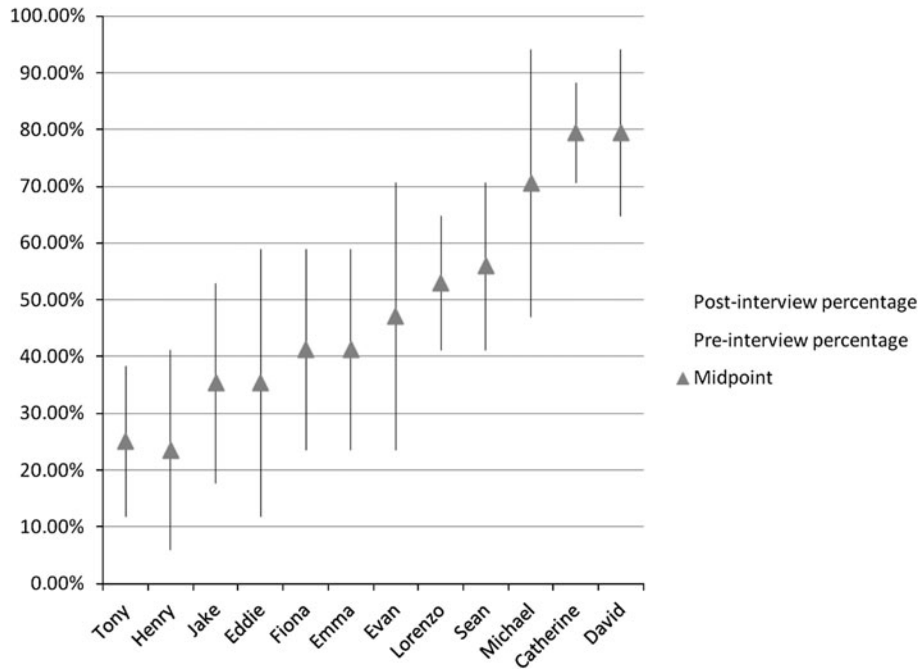


Fig. 6 Results from analysis of the pre- and post-test organised by child

Table 4 Results from analysis of the pre- and post-test organised by test code

Code	Pre-test		Post-test	
	Children with full mark allocation	Children with partial mark allocation	Children with full mark allocation	Children with partial mark allocation
Identification of parity				
1. Identifies an even and an odd number	50 % (6)	0 % (0)	100 % (12)	0 % (0)
Explanations of parity				
2. Gives a reason for parity using the counting model	38 % (5)	50 % (6)	58 % (7)	17 % (2)
3. Gives a reason for parity using the partitive model	31 % (4)	42 % (5)	92 % (11)	8 % (1)
4. Gives a reason for parity using the quotitive model	0 % (0)	8 % (1)	75 % (9)	8 % (1)
Connections to doubling and halving				
5. Doubles a number	62 % (7)	25 % (3)	92 % (11)	8 % (1)
6. Halves an even number	23 % (3)	67 % (8)	50 % (6)	50 % (6)
7. Halves an odd number	0 % (0)	58 % (7)	25 % (3)	8 % (1)
Generalisations				
8. Makes statements that are always true about odd and even, doubling or halving	0 % (0)	25 % (3)	0 % (0)	42 % (5)

The numbers in brackets indicate frequencies

the presented primary visual representations, and to create their own invented representations of numbers. With appropriate direction, they were able to show how these representations could be sequenced (counting model), or be adapted to reveal pairs of units (quotitive model) or two equal groupings (partitive model). All of them could use standard numerals to present a counting model (although some children reversed the numerals when writing them).

Furthermore, all twelve children were able to use stories to explain their reasons for why a number was odd or even. Yet, as would be expected, the number of narratives and the way of narration differed from child to child.

4.2 Case studies

Below we draw on the three case studies for an illustrative account of shifts in children's learning. The strategic selection of cases (as explained in the methodology section) helps give an insight into the impact that the intervention had on different attainment groups in the class.

Eddie showed the most substantial shift in learning both in the lower attainment range and in the class overall. He obtained 12 % in the pre-test (he was only able to halve an even number and be aware that halving was not possible for an odd number) and 60 % in the post-test. By the end of the intervention Eddie was able to refer independently to the counting and sharing stories. It was clear from his post-test that sharing evenly between two groups was Eddie's dominant model for explaining and thinking about parity. He no longer held the misconception that double 'n' was 'nn' (a two-digit number with both digits being 'n'). There was clear progress in his learning in relation to his ability to identify and justify whether a number less than 20 was odd or even.

Michael also showed a significant shift, but within the higher attainment range of the class. He scored 47 % in the pre-test: he was able to identify the odd and even numbers by listing them, and noted that odd and even numbers alternated in the counting sequence. He scored 94 % in the post-test. By the end of the intervention, Michael was confidently able to identify odd and even numbers and provided multiple explanations for parity. He decided that sharing between two was his favourite story, and he told this by embellishing on the basic plot. He could also tell the pairs story and provided examples to show the two possible endings of the story: one for an even number, where everyone goes to the ball, and one for an odd number, where someone is left out. He vividly recalled the role-play for the ball story and assigned names of his classmates to the multilink model being used to recount this story. He could also distinguish and order odd and even numbers and described this in relation to houses in a street. In his narration of the counting story he showed his awareness that the even numbers are "counting in twos", and in his post-test he confirmed his awareness that the even numbers are the doubles. He knew that whole numbers were not the only numbers in the counting sequence, and confidently referred to 'something and a half'. Michael could recount all three stories of parity.

Catherine got the highest score in the pre-test, 71 %, and displayed the smallest shift in learning, with her post-test score being 88 %. Catherine showed in her pre-test that she was comfortable using a range of visual representations to support her thinking. She was able to observe and articulate patterns and had a clear understanding of odd and even numbers in the counting sequence and as equal sharing between two. The teaching intervention introduced the pairs story and seemed to support her to model the different reasons for parity using a story context. She also shifted considerably in her realisation that halving an odd number was possible, and realised that this had two possible outcomes: either there was one left over or there were two halves which could be shared. Catherine could recount all three stories of parity.

4.2.1 The counting story

All case study children gave their own distinctive versions of the counting story. While Eddie needed some prompting to recall this context, Michael spontaneously recounted the problem in detail. Catherine related the problem to her own street—she modelled the same problem but in a new context.

Eddie's counting story (Fig. 7)
E: One goes on that side [gestures to right of whiteboard] and two goes on that side [gestures to left of where one was shown] and three goes on that side [gestures below location to one, looks up for reassurance] [pause].
N: Great. So what do we call the numbers on this side, and the numbers on that side?
E: Even [gestures to left hand side]. And these are odd [gestures to right hand side].
Michael's counting story
M: Well first there was. Well you and Tessa kept on crossing the street so it went

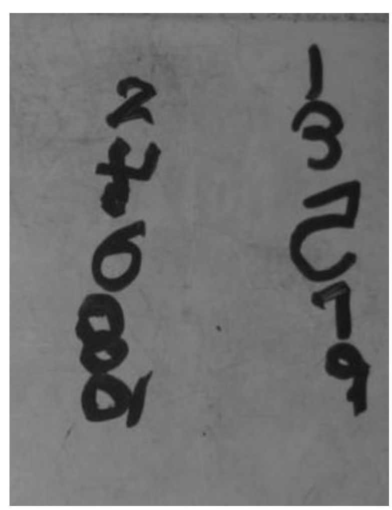


Fig. 7 Eddie's illustration of his counting story

one [places a white one card on his right],
two [places a red two card on his left],
three [places a white three card on his right],
four [places a red four card on his left],
five [places a white five card on his right]
and um we couldn't find out your number. Um but then we did some maths and then the answer was 18.
N: Great. How did we work that out?
M: Because it's counting in twos… and the even two that comes after 16 is 18.

Catherine's counting story

C: Well I looked at my street… at my numbers on my street… and my next door neighbour was 66, and my house was 68, and my other next door neighbour was 70, and my other one across 77. Which is a bit strange.
N: Yes that is a bit strange, isn't it?
…
C: There is not a lot of pattern, because, 66 and then there must be 67 [C draws in 67 on other side of street] on that side where they are all odds. And then before it I came 69 [C writes in 69] and then it was…
…
C: 69 [C redraws 69] and then 77.
N: So that 77 was out of place wasn't it?
C: Yeah.
N: What did you think should be there?
C: 71.
…
N: So your house is 68. Is your house an odd or an even number?
C: Even. And that's even too—[pointing to 66, 68 and 70] 'cause these are all even, and these [points to other side of street with 67, 69 and 77] are all odd.

These examples show how young children can use a presented narrative—a problem of a missing house number in a street—to articulate their ideas of parity as a counting pattern. Eddie could separate odd and even numbers in a small number range, and seemed to visualise alternating sides. His explanation did not include characters or a problem (a missing house number), but he was clear about setting it in a street. The narrative seemed to help Michael to memorise the exact problem context (considering that the problem was presented two and half weeks prior to his narration) and he named characters, setting, and a problem which was to be resolved. Catherine's story was about herself and set in her own street where she was trying to resolve a new problem. For Catherine the narrative seemed to help her generalise the specific presented narrative to consider it in a similar but more personal context, which required a much higher number range.

4.2.2 *The sharing story*

Eddie spontaneously reverted to the sharing story immediately after drawing the numbers in the street, showing he connected the counting and sharing stories:

> It's because… If there is one. How can you share it? Because they [referring to dinosaurs] would take turns biting it there [shows an imaginary tussle with his hands over one unit] and snatching off it. And it's not fair. And even three… [gestures to three on his street]

He used the sharing context as his explanation for why the odd and even numbers alternated. He was not yet able to make coherent connections to the pairs story. It seemed that the idea of sharing (and/or fighting over) food was his dominant or preferred model for thinking about parity. He still did not name the characters in the stories (the teacher assumed that 'they' were dinosaurs, although he also narrated a different version of this story in class). He made clear that the problem was: how can you share it? His reference to 'and even three' and his pointing to all the odd numbers on the one side of his street suggests that he is aware that the same outcome applies to other odd numbers.

Michael (see Fig. 8) chose to use bunnies as his characters for his sharing story.

He provided a coherent narrative which included characters, setting and plot:

> Two bunny rabbits that were friends came out of their rabbit hole and they [pause]. And that rabbit hole was in a garden. And in that garden, a man, the man that lived there had grown ten carrots. And they found the ten carrots [pause] ten carrots lying on the ground, 'cause he had picked them and he was going to come back to pick them [pause] he was going to come back to bring them in the house later. And um they said, and one said "how shall we share these?" The other said, "Let's count them". And then the answer [pause] there were ten

Fig. 8 Michael's illustration of his sharing story to show that ten is even

carrots. And one said, "Good there are ten, that's even. So that means we can have five each." And they did. And then when they had done that they gave a little hug to each other and then jumped back in their rabbit hole.

Michael knew that a process of sharing was required and expressed this through a dialogue with his bunny characters. He calculated half of ten to be five, and made clear that the number was even, and therefore could be shared equally. This was an example of the work he did on narrating this sharing story, as he recounted and illustrated several other versions of the same basic narrative using other settings and other numbers.

Catherine demonstrated her awareness of two different possibilities for divisibility in her explanation of the sharing story:

N: Ok. So why do you think [five is] odd?
C: Because if there would be two people… two people would get two [each] and there is one that's left over. [Arranges counters into 2 pairs, and 1 left over]
C: And if you can halve [units] then you can halve it [the remaining one] and then you have two and half; but if you can't [halve units] then… then that one would be left over. [She gestures at cutting the one left over into two halves]

Catherine was thinking about divisibility as having two possibilities—one where units can be divided, and another where units cannot be divided. She was comfortable explaining both options and did so spontaneously without any prompting. She was aware that if her story characters were people that it was not possible to cut them in half. She showed awareness that this was not the only possible context for sharing—and that "if you can halve" then the story would have a different ending. Her explanation did not have characters and setting; but she had disentangled the mathematical explanation from this narrative context and provided a coherent explanation of a sharing conceptualisation of parity.

These examples show how young children can use narrative to justify whether a number is odd or even. By having the freedom to choose the number of things being shared, they were able to apply the story of parity and provide justifications of parity as sharing equally between two for a range of numbers. How this was done differed by child, with some children continuing to make use of the narrative as a kind of generic example for the idea (as in the case of Eddie and Michael); and other children discarding the narrative to communicate only the mathematical concept.

4.2.3 *The pairs story*

In the post-tests, Eddie was not able to independently recount the pairs story. However, Michael and Catherine both told versions of this parity story.

Michael retold the pairs story demonstrating his awareness of two possible outcomes (for even and odd numbers). He referred to his illustration of "three is odd" (Fig. 5) and extended his story to include the contrasting conclusion that two is even. He also narrated the ball story role-play, in a similar way to Catherine.

Catherine's power of imagination was clear with her accurate reference to the characters in the class role-play as she modelled the process using counters:

[C arranges the seven counters quickly into three pairs with one left over]
C: Well we went out somewhere in the role-play area…
[C rearranges the counters into two columns as she speaks]
And we put things on. And then, and then we… and then two people came and they came into the ball.
[C moves two counters away from the group]
C: And then two other people came in Debbie and somebody else…
[C moves two more counters away from the group]
C: … and then two other people came.
[C moves two more counters away from the group]
C: … and then Emma was left over.
N: Ok so what did that tell us about seven?
C: That it was odd.
N: Great and so then what happened?
C: And then Henry came. [C looks for another counter]
N: [N gives a counter.] Shall we make that Henry?
C: And then Henry came …
[C places counter next to single]
C: … and then they all had partners and then that was eight.
[C moves counters 'Emma and Henry' to join the group at the ball]
N: Ok fantastic. And so what did that show us about eight?
C: Eight was… even.

Catherine immediately arranged the counters into pairs, revealing her awareness that pairs are important to this narrative. Her subsequent recounting of the pairs story showed that she recalled the role-play, its characters, the setting of ball, and the plot. She needed prompting, however, to remember the purpose of the story, which was whether seven was odd or even. Although she seemed aware that it was significant that there were now eight people and she stated this, it was the teacher's role to prompt her to conclude what this meant mathematically.

5 Conclusion

This study developed a teaching intervention to support 5–6-year-old children to identify and justify whether a number is odd or even using narrative in mathematics. The study made a contribution to the research literature on the use of narrative in mathematics education by exploring how narrative may be used to provide a vehicle of mind for articulating mathematical reasoning. To use Bruner's (1996) terms, children were supported to make use of 'narrative thinking' to express their 'logical-scientific thinking' for whether a number was odd or even.

It seems that fundamental to the children's ability to effectively draw on narrative to support their explanations was that they were given opportunities to choose aspects of the narrative context (numbers, characters, a setting, and a question or conflict) for their stories. We think that this has deeper educational significance than simply motivating children. It was not only that children's imagination was engaged in the process of learning (Egan 1989), but also the fact that providing the flexibility for them to retell the stories seemed to have strengthened their mathematical conceptualisations. Unlike the lima-bean problem context referred to by Bastable and Schifter (2008), the children in this study were engaged in multiple narrative contexts for developing the meaning of parity. Although the narrative context varied, the underlying mathematics of the three different models (counting, partitive, and quotitive) remained unchanged. It seemed that telling and reviewing multiple versions of the same story meant that fixed elements—the underlying mathematics—became more transparent and less confused with the variable elements, which otherwise may have become the focus of attention for some children. The teacher played an important role in directing students' attention to, and thus increasing their awareness of, the common elements across different stories (Mason 1998).

The fact that the study focused on three different models of parity meant that the parity concept could be looked at in three different ways. Also, as there were many children in the class, multiple stories for each of the parity models were generated, which offered then a setting for common elements to be identified and the underlying mathematics to be generalised. In this way each story of parity was 'algebrafyed' (Kaput 2008), allowing children to model various narrative contexts for different numbers and using those narrative contexts to communicate three different—but related—models of parity. As such the study also contributes to the research literature on early algebra by exemplifying how the abstract concept of parity may be 'grounded' in situations that are more concrete and familiar to children (Koedinger et al. 2008) and how a process of algebraic modelling and generalisation, that makes use of narrative, may be facilitated among very young children (Kaput 2008).

During this study an inter-dependence became clear between words and images when using narrative as a cognitive tool for meaning-making in mathematics. Telling stories was supported by use of both presented and invented images (visual representations) to which the children could refer. And conversely, telling stories seemed to generate new representations, which the children could imagine and use. In this regard imagination and expression were both verbal and visual and, appropriately, might be referred to as a single "children's power" (Mason 2008). The inter-dependence of words and images when constructing narratives does not seem to have attracted much attention in mathematics education research, where explanation (verbal or written) and visual representations tend to be considered separately. There is a need for future research to closely examine the potential roles of words and images when using narrative to support children's learning of mathematics (not necessarily limited to studies with very young children or studies on parity).

This study also has implications for teaching practice. An approach to teaching parity—informed by narrative as both a teaching and a cognitive tool, considering three mathematical models of parity, and with awareness of the teachers' role—was designed, field-tested, and found to yield promising results. It would be important to examine whether the teaching intervention could be replicated in other contexts and with teachers other than the researchers. These examinations could also cast more light on what is required from the teacher in supporting children's learning of parity and their use of narrative for meaning-making.

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