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ORIGINAL ARTICLE

Mathematical creativity in generally gifted and mathematically excelling adolescents: what makes the difference?

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Abstract Due to uncertainty regarding the relationship between mathematical creativity, mathematical expertise and general giftedness, we have conducted a large-scale study that explores the relationship between mathematical creativity and mathematical ability. We distinguish between relative and absolute creativity in order to address personal creativity as a characteristic that can be developed in schoolchildren. This paper presents part of a study that focuses on the power of multiple solution tasks (MSTs) as a tool for the evaluation of relative creativity. We discuss relationships between mathematical creativity, mathematical ability and general giftedness as reflected in the present empirical study of senior high school students in Israel which implemented the MST tool. The study demonstrates that between-group differences are task dependent and are a function of mathematical insight as it is integrated in the mathematical task. Thus, we conclude that different types of MSTs can be used for different research purposes, which we discuss at the end of this paper.

Keywords Mathematical creativity · Multiple solution tasks (MST) · General giftedness · High level of mathematical instruction

1 Rationale

This study is motivated by our observation that uncertainty exists with regard to the relationship between mathematical creativity, general giftedness, and excellence in mathematics. Our first study in the series was a qualitative study that involved 18 students with different levels of general giftedness (by IQ measures) who study mathematics at different levels of instruction (Leikin and Lev 2007; Leikin 2009). The study employed multiple solution tasks (MSTs—see Sect. 3 in this paper) in order to explore students' creativity in mathematics. When the tool was initially developed it was also employed for the investigation of the development of mathematical creativity in high school students (Levav-Waynberg and Leikin 2012, 2013) and with pre-service primary school mathematics teachers (Guberman and Leikin 2012). The two former studies also examined the relationship between mathematical creativity and the level of mathematical ability of the participants. All the studies led to several hypotheses that we are currently examining in a large-scale study: we hypothesized that (1) between-group differences are task dependent and (2) in the originality–fluency–flexibility triad, fluency and flexibility are of a dynamic nature, whereas originality is of the "gift" type.

The study presented in this paper has two interrelated goals. First, it examines relationships between mathematical creativity, general giftedness and mathematical excellence; and second, it explores the power of different types of mathematical tasks for the identification of between-group differences related to mathematical creativity as reflected in multiple solutions produced by the students.

2 Background

2.1 Creativity

Creativity is a complex concept, considered by various scholars from different points of view (Haylock 1987; Mann 2006; Sriraman 2005). There is no single, authoritative

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perspective or definition of creativity (Mann 2006; Sriraman 2005). There are a variety of views on creativity and they keep changing over time. Based on research literature, Mann (2006) argues that there are more than 100 contemporary definitions of creativity. Here, we consider several examples.

Guilford (1967) distinguished between convergent and divergent thinking (production). Convergent thinking involves aiming for a single, correct solution to a problem, whereas divergent thinking involves the creative generation of multiple answers to a problem or phenomenon, and is described more frequently as flexible thinking. Torrance (1974) suggested a definition of creativity that served as the basis for a battery of tests designed to identify creativity. The definition was based on four related components: fluency, flexibility, novelty, and elaboration. *Fluency* refers to the continuity of ideas, flow of associations, and use of basic and universal knowledge. *Flexibility* is associated with changing ideas, approaching a problem in various ways, and producing a variety of solutions. *Novelty* is characterized by a unique, new/fresh way of thinking and unique/original products of a mental or artistic activity. *Elaboration* refers to the ability to describe, illuminate, and generalize ideas. Of these four components, novelty or originality is widely acknowledged because creativity is viewed as a process having to do with the generation of original ideas, approaches, or actions, and is manifested in novel and original products (for example, a new work of art or a scientific hypothesis).

2.2 Relationship between creativity and ability

One of the intriguing points in educational research is the relationship between creativity and giftedness. While drawing a connection between high abilities and creativity, researchers express a diversity of views. Some claim that creativity is a specific type of giftedness (e.g., Sternberg 2005), others feel that creativity is an essential component of giftedness (Renzulli 1978, 1986), and still other researchers suggest that they are two independent characteristics of human beings (Milgram and Hong 2009). Thus, analysis of the relationship between creativity and giftedness, with a specific focus on the various fields of mathematics, is important for better understanding of the nature of both mathematical giftedness and mathematical creativity.

The relationship between insight in problem solving and creativity has been stressed by several researchers (Csikszentmihalyi 1996). At the same time, insight is viewed as a trait central to the construct of giftedness (Davidson 1986). Davidson (2003) finds that gifted children outperform their average peers in problem solving because of the increased tendency towards insight. High-ability children are shown to understand an insight-based problem immediately and to solve it quickly, whereas children with average ability are more likely to work on the subproblems and not solve the whole problem in its entirety (Overtoom-Corsmit et al. 1990). These connections between insight and high ability and insight and creativity influenced our study design.

2.3 Mathematical creativity

There is a distinction between general and specific creativity. *General creativity* is associated with using problem-solving patterns from one field to solve problems in another field. *Specific creativity* refers to creativity in a particular field which takes into account the logical deductive nature of the field (e.g., Piirto 1999). In this paper we focus on students' specific mathematical creativity. As in the case of general creativity, providing a precise and broadly accepted definition of mathematical creativity is at least extremely difficult and probably impossible to achieve (Mann 2006; Haylock 1987; Sriraman 2005; Liljedahl and Sriraman 2006). Mann (2006) maintained that analysis of the research attempting to define mathematical creativity demonstrates that the absence of an accepted definition for mathematical creativity hinders research efforts.

One of the complexities related to the relationship between mathematical giftedness and mathematical creativity is rooted in the contrast between viewing mathematical creativity as a property of the professional mathematician's mind (Subotnik et al. 2009; Sriraman 2005; Liljedahl and Sriraman 2006) and the opinion that mathematical creativity can and must be developed in all students (Sheffield 2009; Yerushalmy 2009; Hershkovitz et al. 2009). According to Subotnik et al. (2009), creativity is fundamental to the work of a professional mathematician: in the course of their work, mathematicians find and solve problems that are substantive and challenging. Similarly, Ervynck (1991) considers mathematical creativity as one of the characteristics of advanced mathematical thinking. He connected mathematical creativity with advanced mathematical thinking and considered it as the ability to formulate mathematical objectives and find inherent relationships among them.

At the same time, Silver (1997) and Sheffield (2009) address "creativity to all students" and consider problem solving and problem posing as main tools for the development of mathematical creativity in all students. Taking a developmental point of view, Sheffield (2009) suggests a continuum of mathematical proficiency through the development of creative ability in mathematics: innumerates → doers → computers → consumers → problem solvers → problem posers → creators.

We assume that this development is related to personal mathematical talent, and our previous studies (e.g., Levav-Waynberg and Leikin 2012; Guberman and Leikin 2012) demonstrated that at school we develop flexibility but not originality, while development of (relative) originality can only be performed by a small number of individuals (Levav-Waynberg and Leikin 2012).

Naturally, creativity in school mathematics differs from that of professional mathematicians (Sriraman 2005, p. 23). Mathematical creativity in school students is evaluated with reference to their previous experiences and to the performance of other students who have a similar educational history. Silver (1997) suggested that creativity-enriched mathematics instruction can increase students' representational ability, strategic fluency and flexibility, and their appreciation for novel problems or solutions. Thus, a broad range of students can access the core dimensions of creativity—namely, fluency, flexibility, and novelty. In this process, the role of context becomes essential.

Leikin (2009) suggested that viewing personal creativity as a characteristic that can be developed in school-children requires a distinction between *relative* and *absolute* creativity, similar to the distinction between objective and subjective creativity (Lytton 1971) and that of Big C and Little C creativity (Csikszentmihalyi 1988). *Absolute creativity* is associated with discoveries at a global level ("historical works" as termed by Vygotsky 1930/1982, Vygotsky 1930/1984: e.g., Riemann, Lobachevskii, Hilbert, Fermat). Our work deals with *relative creativity* which refers to mathematical creativity exhibited by school students when evaluated in relation to their previous experiences and to the performance of other students who have similar educational histories (Leikin 2009).

2.4 Mathematical creativity and problem solving

Mathematical creativity in school mathematics is usually connected with problem solving or problem posing (e.g., Silver 1997). Kwon et al. (2006) proposed two major components of mathematical creativity: the creation of new knowledge and flexible problem-solving abilities. They demonstrated that solving open-ended problems was useful for enhancing students' creative thinking skills. Chamberlin and Moon (2005) suggested that creatively gifted students have an unusual ability to generate novel and useful solutions to applied problems. Chiu (2009) connected mathematical creativity with the students' ability to solve non-routine problems and to approach ill-structured problems. Some limitations of the evaluation tools presented in these studies may be seen in the close connection between the evaluation tools and the type of activities the researchers considered to be creative.

Following Torrance (1974), Silver (1997) suggested developing creativity through problem solving as follows. *Fluency* is developed by generating multiple mathematical ideas, multiple answers to a mathematical problem (when such exist), and exploring mathematical situations. *Flexibility* is advanced by generating new mathematical solutions when at least one has already been produced. *Novelty* is advanced by exploring many solutions to a mathematical problem and generating a new one.

Ervynck (1991) considered creativity to be a critical component of advanced mathematical thinking. He suggested that there are three different levels of creativity: Level 1 contains an algorithmic solution to a problem, Level 2 involves modeling a situation, and Level 3 makes use of the problem's internal structure. Note that in terms of Torrance's categories of creativity (originality, fluency, and flexibility), Ervynck's levels of creativity seem to describe levels of originality rather than of fluency or flexibility.

The current study represents a step forward in the design of a multidimensional creativity test that takes into account the relative nature of creativity. It draws on the views of Ervynck (1991), Krutetskii (1976), Polya (1973), and Silver (1997) that solving mathematical problems in multiple ways is closely related to personal mathematical creativity, and suggests evaluating mathematical creativity by means of MSTs. The tool for evaluation of creativity is based on the model for the evaluation of creativity introduced by Leikin (2009). Based on Torrance (1974), the model considers three components of creativity—fluency, flexibility, and originality. For the evaluation of originality the model combines Ervynck's (1991) insight-related levels of creativity with conventionality of the solutions as comprising students' educational history in mathematics.

3 The model for evaluation of mathematical creativity

3.1 Multiple solution tasks

A multiple solution task is an assignment in which a student is explicitly required to solve a mathematical problem in different ways. Solutions to the same problem are considered to be different if they are based on: (a) different representations of some mathematical concepts involved in the task, (b) different properties (definitions or theorems) of mathematical objects within a particular field, or (c) different properties of a mathematical object in different fields (see the definition and various examples of MSTs in Leikin 2006, 2007, 2009). Figure 1 demonstrates an example of a MST (Jam problem) and depicts ten different solutions to the problem.

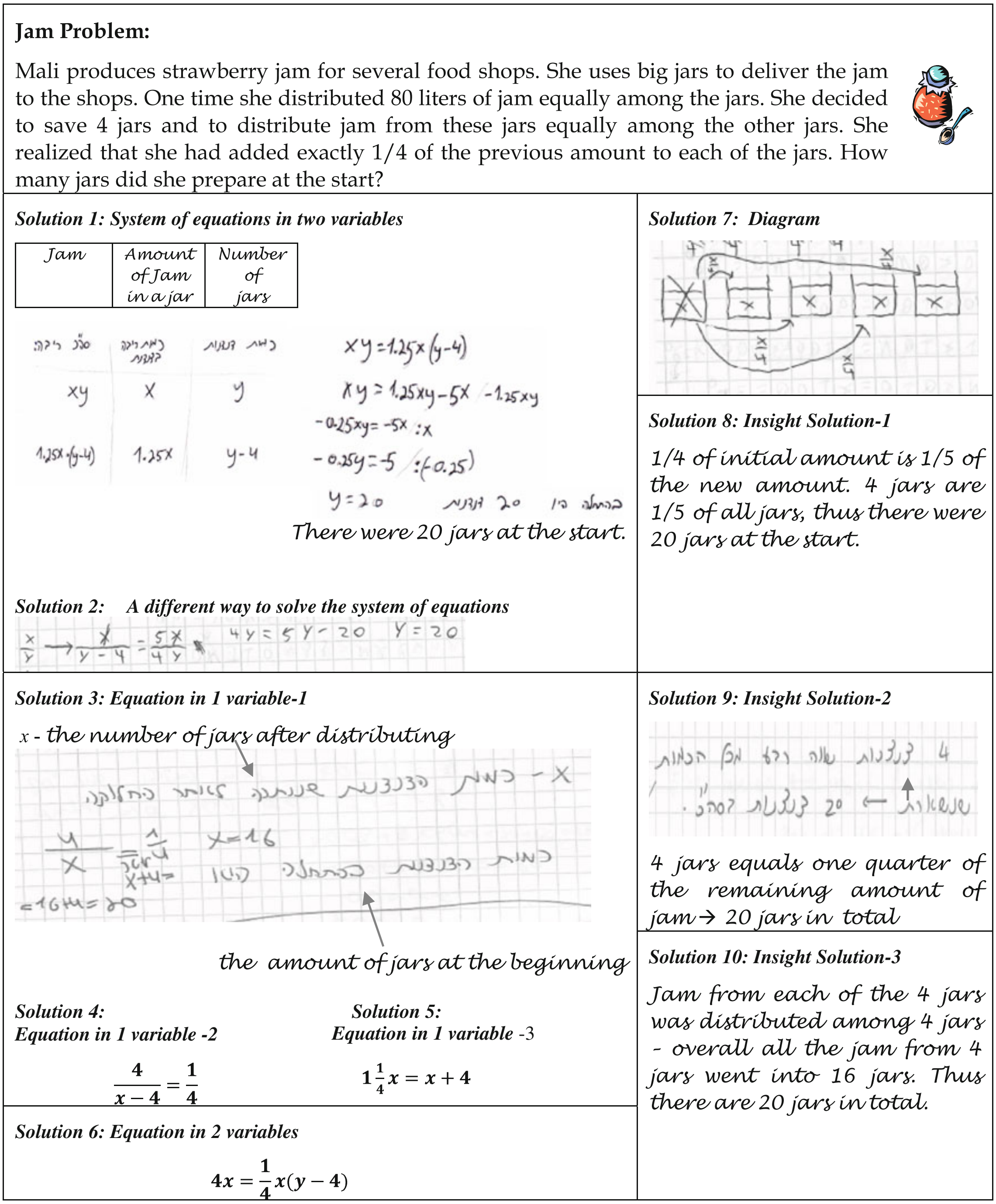


Fig. 1 Multiple-solution task—Jam problem: student-generated solutions

3.2 Solution spaces

Leikin (2007) suggested the notion of *solution spaces* that enables researchers to examine the various aspects of problem-solving performance using MSTs. *Expert solution spaces* include the most complete set of solutions to a problem known at a particular time. They can also be conceived as a set of solutions that expert mathematicians can suggest to the problem. In school mathematics, expert solution spaces include *conventional solution spaces*, which are those generally recommended by the curriculum, displayed in textbooks, and taught by the teachers. By contrast, *unconventional solution spaces* include solutions based on strategies usually not prescribed by the school curriculum, or that the curriculum recommends with respect to a different type of problem. *Individual solution* *spaces* are collections of solutions produced by an individual to a particular problem. With respect to the ability of a person to find solutions independently, we distinguish between *available individual solution spaces*, which include solutions that individuals can present on the spot or with some effort without help from others, and *potential solution spaces*, which include solutions that solvers produce with help from others. Solutions derived from the potential solution spaces correspond to the personal zone of proximal development (ZPD; Vygotsky 1978).

Finally, *collective solution spaces* are a combination of the solutions produced by a group of individuals. Collective solution spaces are usually broader than individual solution spaces within a particular community, and form one of the main sources for the development of individual solution spaces. Both individual and collective solution spaces are subsets of expert solution spaces.

Solution spaces are used here as a tool for exploring the students' mathematical creativity and for ascertaining the potential of a task to evaluate mathematical creativity.

3.3 The scoring scheme

In this section we describe the way in which students' creativity is evaluated in our study. This evaluation model was first introduced in Leikin (2009) and then employed in Levav-Waynberg and Leikin (2012, 2013) and in Guberman and Leikin (2012). Table 1 summarizes the scoring scheme.

Fluency (Flu) refers to the pace at which solving proceeds and the switches taking place between different solutions:

- The fluency embedded in an MST is the number of solutions in the expert solution space. For example, the fluency of the solution space presented in Fig. 1 can be scored with 10.
- The fluency of a student on a written test is detected by the number of solutions in the individual solution space.
- The fluency of a student in an individual interview is the number of appropriate solutions in the individual solution space produced within a given time unit.

To evaluate *flexibility* (Flx), we established groups of solutions for the MSTs. Flexibility embedded in a problem is evaluated according to expert solution space. Students' flexibility is evaluated with individual solution space. Two solutions belong to separate groups if they employ solution strategies based on different representations, properties (theorems, definitions, or auxiliary constructions), or branches of mathematics. We evaluated flexibility as follows:

- $Flx_1 = 10$ for the first appropriate solution (see an explanation of the decimal basis for scoring later in this text).

For each consecutive solution:

- $Flx_i = 10$ if it belongs to a group of solutions different from the solution(s) performed previously. For example, score Flx = 10 can be given to Solution 5 when it is produced after Solution 1 (see Fig. 1).
- $Flx_i = 1$ if the solution belongs to one of the previously used groups but has a clear but minor distinction. For example, Solution 4 can be scored with Flx = 1 if performed after Solution 1 (see Fig. 1).
- $Flx_i = 0.1$ if the solution is almost identical to a previous solution. Solution 2 in Fig. 1 can be scored with Flx = 0.1 if produced after Solution 1.

A student's total flexibility score on a problem is the sum of the student's flexibility on the solutions in the student's individual solution space. The total flexibility embedded in a task is the sum of flexibility scores of all the solutions in the expert solution space:

- $Flx = \sum_{i=1}^{n} Flx_i$ where n is the number of appropriate solutions in the individual solution space of a student.

Originality is evaluated by comparing individual solution spaces with the collective solution space of the

Table 1 Scoring scheme for evaluation of creativity (based on Leikin 2009)

	Fluency	Flexibility	Originality	Creativity
Scores per solution	1	$Flx_1 = 10$ for the first solution $Flx_i = 10$ solutions from a different group of strategies $Flx_i = 1$ similar strategy but a different representation $Flx_i = 0.1$ the same strategy, the same representation	$Or_i = 10$, $P < 15\ \%$ or for insight/unconventional solution $Or_i = 1$, $15\ \% \le P < 40\ \%$ or for model-based/partly unconventional solution $Or_i = 0.1$, $P \ge 40\ \%$ or for algorithm-based/conventional solution	$Cr_i = Flx_i \times Or_i$
Total score	$Flu = n$	$Flx = \sum_{i=1}^{n} Flx_i$	$Or = \sum_{i=1}^{n} Or_i$	$Cr = \sum_{i=1}^{n} Flx_i \times Or_i$

n is the total number of appropriate solutions

$P = (m_j/n) \times 100\ \%$ where m_j is the number of students who used strategy j

reference group. If P is the percentage of students in the group that produces a particular solution, then (relative evaluation):

- $Or_i = 10$ when $P < 15\ \%$ or for insight-based unconventional solutions. For example, Solution 7 (Fig. 1) is scored with Or = 10 as an insight-based solution. Solution 6 is scored with Or = 10 since only one student from the research sample produced this solution.
- $Or_i = 1$ when $15\ \% \leq P < 40\ \%$ or for conventional solutions applied in an unconventional situation.
- $Or_i = 0.1$ when $P \geq 40\ \%$. For example, Solution 1 is scored with Or = 0.1 since it is algorithmic solutions which are usually produced.

A student's total originality score on a problem is the sum of the student's originality on the solutions in the student's individual solution space. The total originality embedded in a task is the sum of originality scores of all the solutions in the expert solution space:

- $Or = \sum_{i=1}^{n} Or_i$ where n is the number of appropriate solutions in the corresponding space.

In the decimal basis we used in scoring, the total score indicates the originality and flexibility of the solutions in the individual solution space of a participant. For example, if the total flexibility score for a solution space is 21.3, we know that it includes two solutions that belong to different solution groups (based on different solution strategies), one solution that uses a solution strategy similar to a former solution but differs in some essential characteristics, and three solutions that repeat previous ones. If the total originality score for a solution space is 12.3, we know that it includes one original solution that is insight-based and non-conventional, two solutions that are partly unconventional, and three algorithm-based conventional solutions.

The *creativity* (Cr) of a particular solution is the product of the solution's originality and flexibility: $Cr_i = Flx_i \times Or_i$. The use of the product of flexibility and originality scores enables evaluation of the most creative solutions, with the highest score (Cr_k =100) given for a flexible and original solution. This also addresses the fact that previously performed solutions cannot be considered as creative. Thus, when a student performs an original solution (Or_m =10) that is similar to one produced earlier, his flexibility is scored $Flx_m = 1$ or $Flx_m = 0.1$, and then the creativity score is $Cr_m = 10$ or $Cr_m = 1$, a score that indicates a different level of creativity for the solution process. Repeating unoriginal solutions scores of Cr = 0.1 or Cr = 0.01 indicates that a student does not see the similarity between the solutions and produces only those solutions that were learned in the classroom.

The total creativity score on an MST is the sum of the creativity scores on each solution in the individual solution space of a problem: $Cr = \sum_{i=1}^{n} Flx_i \times Or_i$. The observation that the creativity of two individual solution spaces that contain identical sets of solutions should be scored equally led to the decision to evaluate flexibility of the first solution to a problem with a score of 10 ($Flx_1 = 10$).

The model for evaluation of creativity applied with a particular set of MSTs constitutes the research instrument in this study. This model, which was validated in previous studies (Leikin 2009; Levav-Waynberg and Leikin 2013), was accepted and employed in this study with a different set of problems.

4 The study

4.1 Research goals

There are two main interrelated goals in this paper:

1. To examine relationships between mathematical creativity, general giftedness, and mathematical excellence.
2. To explore the power of different types of MSTs for the identification of between-group differences related to mathematical creativity as reflected in multiple solutions produced by the students.

4.2 Population

In order to achieve the goals, three groups of target population took part in this study.

Mathematics is a compulsory subject in Israeli high schools, and the students can be placed in one of three levels of mathematics: high, regular, and low. The level of instruction is determined by students' mathematical achievements in earlier grades. The differences between instruction at high level (HL) and that at regular level (RL) are in the depth of the learning material and the complexity of the mathematical problem solving involved. The items we used in our study are appropriate for both the RL and HL curriculum and are learned similarly by students in the HL and RL groups. Additionally, classes for generally gifted students (IQ >130) exist in several Israeli schools. Students in these classes can learn HL and RL mathematics but do not necessarily excel in mathematics. The majority of HL and RL students usually are not identified as gifted.

We used two different factors for the choice of our research sample: G factor, which comprises general giftedness (IQ >130); and L factor, which refers to the level of mathematical instruction.

Three groups of 11th and 12th grade students took part in this study:

(a) The G group consists of generally gifted students who excel in mathematics.
(b) The HL group consists of students who learn math at a high level of mathematical instruction.
(c) The RL group consists of students who learn math at a RL of mathematical instruction.

We report here the solutions produced by 51 (out of a total of 155) participants who were given Variant 1 of the test. Of these 51 students, 6 belong to the G group, 27 to the HL group and 18 to the RL group.

4.3 Test

Three variants of the test were designed in order to examine the possibility of varying the tasks on the tests. Each class that participated in this study was presented with at least two variants of the test, so that students seated next to each other were given different problems to solve. After the examination we demonstrated that the "parallel" problems in different variants of the tests provided us with equivalent information from the viewpoint of correctness and creativity components within each group. Thus, in this paper we present results on one variant only. The comparison between the groups was performed across the "parallel" tasks after examination of the equivalence.

The problems included in the test differed with respect to:

1. *Mathematics topic* to which the problem belongs in the school curriculum
2. *Complexity*
3. *Conventionality* of the problem and conventionality of the solutions, requiring insight in order to produce the solutions (following Ervynck 1991).

The test consisted of five problems (see Fig. 2 for the problems in Variant 1 of the test). Four problems were obligatory while the fifth problem was included as a bonus task. *Correctness* of the solution for a problem was evaluated according to the complete solution produced by the student to the problem. For a complete solution a student received 25 points. The fact that other solutions to the problem appeared to be incomplete did not affect the correctness score. Overall, students could receive a total correctness score of 125 for the five problems in the test. The students were asked explicitly to solve each problem in as many ways as possible. The time allotted for the test was an hour and a half. Creativity components were evaluated according to the model described in Sect. 3 of this paper.

5 Findings

We describe student-generated solutions and the between-group differences revealed through the examination.

5.1 Jam problem

In this section we present the findings of our study for the Jam problem. The Jam problem is chosen because of the richness of the solutions produced by the students in all of the groups and because only this problem revealed differences among the three groups of participants, as we present later on.

5.1.1 Solution strategies

Figure 1 displays the different kinds of solutions produced by the students. Tables 2 and 3 summarize the findings for each of the groups of participants.

Tables 2 and 3 demonstrate that the collective solution spaces of G and HL students are similar: both of them include four different groups of solutions (addressed here as "collective flexibility"). However, individual solution spaces of the three groups of students differed significantly. G students were the most successful in producing multiple solutions to the Jam problem. Five out of six G students succeeded in solving the problem in two ways which were from different groups of solutions. Even though 7 out of 27 HL students solved this problem in two different ways, HL students were less successful than we had expected. Sixteen out of 27 HL students did not succeed in solving this problem at all. We were also disappointed to find that among 18 RL students only one student succeeded in solving this problem. The results are reflected in the range of the number of solutions in the individual solution spaces as well as in the mean values of solutions we found.

5.1.2 Group differences

Table 4 displays the mean values, medians and SD for the different components of creativity examined in this study which students from different groups received for the Jam problem. The number of students in the G group was small for Variant 1 of the test. To compare problem-solving performance on MSTs by students from different study groups we performed the Kruskal–Wallis test with Mann–Whitney test for pairwise comparison of the groups.

As expected, G students received the highest scores on all of the parameters examined: they all solved the problem correctly; most of them produced two solutions that belonged to different groups of solutions and, thus, their mean flexibility was 15.03. HL students received relatively low mean scores with zero medians for all the components of creativity. RL students received the lowest scores.

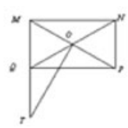
Mathematical topic in the curriculum	Problem	Characteristics
1. Arithmetic problems	$2\frac{1}{4}\times 1.75=$	Routine tasks One of the solutions is insight-based: $2\frac{1}{4}\times 1.75=\left(2+\frac{1}{4}\right)\left(2-\frac{1}{4}\right)$
2. Word problems 	**Jam problem:** Mali produces strawberry jam for several food shops. She uses big jars to deliver the jam to the shops. One time she distributed 80 liters of jam equally among the jars. She decided to save 4 jars and to distribute jam from these jars equally among the other jars. She realized that she had added exactly 1/4 of the previous amount to each of the jars. How many jars did she prepare at the start?	Routine word problem Several non-routine solutions for secondary mathematics (no use of variables) The insight-based solution is based on the view of the problem structure: *4 jars include 1/4 of all the jam. Thus there were 20 jars.*
3. System of equations	$\begin{cases}3x+4y=14\\4x+3y=14\end{cases}$	Routine tasks Multiple solutions – substitution, linear combination and graphing – are learned in school. The system is symmetrical, thus it has an insight-based solution: *The exchange of variables does not change the system which has only one solution: x=y=2*
4. Geometry problems	 Given a rectangle MNPQ where QN =2 NP. The diagonals meet at point O. T is on line MQ so that MQ=QT. Prove that MO is perpendicular to OT.	Geometry problem. All solutions are standard. However, some are less regular *OQ is a median which equals half of the side MT thus MOT is a right angle*
5. Movement problem (bonus task)	Dor and Tom walk from the train station to the hotel. They start out at the same time. Dor walks half of the time at speed $v_1$and half of the time at speed v_2. Tom walks half way at speed v_1 and half way at speed v_2. Who gets to the hotel first: Dor or Tom?	Solution of the problem with algebraic tools requires use of quadratic inequation. Insight-based logical solution: *Suppose $v_1>v_2$ If Dor walks half of the time with greater speed he passes more than half way. Thus Dor walks a longer distance than Tom at a higher speed and thus reaches the hotel first.*

Fig. 2 Problems in Variant 1 of the test

A significant group effect was found for all the examined criteria. G students exhibited significantly higher scores compared with RL students on all the components of creativity, and on correctness, fluency, and flexibility compared with HL students. Differences between HL and RL students were significant only for fluency criteria.

5.2 Between-subject differences on all the problems in Variant 1 of the test

Analysis of the students' solutions was performed for all problems on the test similarly to the analysis demonstrated for the Jam problem.

5.2.1 Solution strategies

Analysis of the methods used by the students when solving the arithmetic problem demonstrated that only HL students (8 of 27) and RL students (3 of 17) used a calculator as a way of solving this task. Only one HL student discovered that and received the highest score for originality for this solution. One of the G students demonstrated a connection between this product and the Pascal Triangle, while two HL students used the Arithmetic Sequence to solve this task. RL students performed this calculation in conventional ways in which the two multipliers were represented as decimal numbers or as regular fractions. Sixteen out of

Table 2 Summary of findings for the Jam problem

Task 2.1 Solve the following problem in as many ways as possible:
Mali produces strawberry jam for several food shops. She uses big jars to deliver the jam to the shops. One time she distributed 80 l of jam equally among the jars. She decided to save four jars and to distribute jam from these jars equally among the other jars. She realized that she had added exactly 1/4 of the previous amount to each of the jars. How many jars did she prepare at the start?

Group of solutions	Way of solving (see Fig. 1)	Description	No. of students who produced the solution			Range (R) of the individual solution spaces	Mean (M) number of solutions
			G (N = 6[a])	HL (N = 27)	RL (N = 18)		
A	1	System of equations 1	2 (34 %)	5 (19 %)	1 (6 %)		
	2	System of equations 2	0	1 (4 %)	0		
B	3	Equation 1	5 (83 %)	8 (30 %)	0		
	4	Equation 2	2 (34 %)	0	0		$M_G = \frac{11}{6} \approx 1.8$
	5	Equation 3	0	0	0	R_G: 1–2	
D	6	Equation in 2 variables	0	0	0		
E	7	Diagram	1 (17 %)	0	0	R_{HL}: 0–2	$M_{HL} = \frac{18}{27} \approx 0.7$
C	8	Insight: fractions/percents	0	0	0	R_{RL}: 0–1	$M_{RL} = \frac{1}{18} \approx 0.1$
F	9	Insight solution	1 (17 %)	2 (8 %)	0		
G	10	Insight solution	0	2 (8 %)	0		
Collective flexibility: number of groups of solutions in the collective solution space			4	4	1		

Similar analysis was performed for all other MSTs in the test. Space limit of the paper does not allow us to depict these results for other tasks

[a] No. of students who were given Variant 1 of the test

Table 3 Fluency: the number of solutions in the individual solution spaces

Jam problem	G (N = 6)			HL (N = 27)			RL (N = 18)		
No. of solutions in the individual solution spaces	0	1	2	0	1	2	0	1	2
No. of students who produced a particular number of solutions	0	1	5	16	4	7	17	1	0

27 HL students and 10 out of 18 RL students performed long multiplication as an alternative way of solution.

When solving the geometry problem, G students produced two or three solutions while the range of number of solutions in the individual solution spaces of HL and RL students was 0–2. Group flexibility for the HL and G students was identical (5 groups of solutions) and differed significantly from the group flexibility of the RL students (2 groups of solutions).

The Half way–Half time problem was the most difficult one, as each group included students who did not succeed in solving this problem. Only three RL students solved this problem, each in one insight-based way only. All G students and 4 of the 27 HL students produced table-based solutions.

The arithmetic problem and the system of equations were found to be the easiest problems in the test: all G students and the majority of the HL and RL students succeeded in solving these problems in at least two ways. The Jam problem and the Half-way problem were the most difficult ones: 16 (for the Jam problem) and 20 (for the Half-way problem) out of 27 HL students and, respectively, 17 and 15 out of 18 RL students failed to solve these problems. The geometry problem appeared to be at a medium level of difficulty. All G students solved the geometry problem in at least two ways, 13 out of 27 HL students and 3 out of 18 RL students solved it in two ways. However, five HL students and three RL students failed to solve it.

Table 4 Different components of creativity for the Jam problem and between-subjects differences

Group 	G			HL			RL			Kruskal–Wallis test[a]	G	HL
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD	H(2)	Mann–Whitney test (*U*)	
Cor	25	25	0	10.19	0	12.52	1.39	0	5.89	17.962***	HL: 2.72* RL: 4.151***	RL: 2.396[b]
Flu	1.83	2	0.41	0.67	0	0.88	0.06	0	0.24	19.729***	HL: 2.948* RL: 4.374***	RL: 2.404*
Flx	15.03	15.05	5.44	5.30	0	7.09	0.56	0	2.36	19.43***	HL: 2.926* RL: 4.341***	RL: 2.386[b]
Or	3.78	0.65	5.26	2	0	4.61	0.56	0	2.36	14.276**	HL: 2.2 RL: 3.633**	RL: 2.365
Cr	37.5	6.01	62.89	18.63	0	46.28	5.56	0	23.57	14.676**	HL: 2.288 RL: 3.702**	RL: 2.341

*** $p < .001$, ** $p < .01$, * $p < .05$, [b] marginal significance $p = .05$

[a] The Kruskal–Wallis test is a non-parametric test used to compare three or more samples. It is a logical extension of the Mann–Whitney test for pairwise comparison. The Kruskal–Wallis test statistic is approximately a Chi-square distribution, with $k - 1$ degrees of freedom where n_i should be >5 (http://www.statisticssolutions.com/resources/directory-of-statistical-analyses/kruskal-wallis-test)

These findings are also reflected in the between-group differences presented in Sect. 5.2.2.

5.2.2 Group differences

G students received significantly higher scores than RL students on all the criteria (correctness, fluency, flexibility, originality, and creativity) for the Jam problem, geometry problem and Half-way–Half-time problem (with the exception of the correctness score for the geometry problem). On all these problems G students were significantly more fluent and flexible than HL students. Additionally, G students were significantly more successful than HL students when solving the Jam problem and more creative than HL students on the geometry problem.

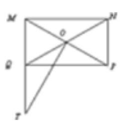
When the scores of G students are significantly higher than those of both HL and RL students, the level of significance is always higher for the differences between G and RL students' scores than the level of significance for the differences between G and HL students' scores. The strength of the significance of the differences finds clear expression in the arithmetic problem and the system of equations for which no significant differences were found between the G and HL groups. Additionally, significant differences between the scores of G students and RL students only were found for creativity for the Jam problem, the system of equations problem and the Half-way–Half-time problem.

The differences found between G and RL students are not surprising. Nevertheless, the initial explanation which might be suggested for these findings—that since G students learn HL mathematics they perform better than RL students—does not hold. In light of the finding that no significant differences were found between the scores of HL and RL students on three out of five problems, with a marginal effect of the L factor on correctness of students' solutions and their flexibility revealed in the Jam problem, we argue that the G factor has a major effect on students' creativity, whereas the L factor has only a minor effect on students' creativity. The effect of L can be observed in the differences between fluency and flexibility of HL as compared with RL students when solving the arithmetic problem and in differences in HL and RL students' flexibility on the Jam problem.

The finding that the G factor and L factor do not have an influence on the students' success when solving the problems (with the Jam problem being an exception) is based on the fact that all problems in the text were chosen from a curriculum which is common to HL and RL instruction. Although one might expect HL instruction to be more creativity directed, this assumption was not proven in our study.

An additional finding depicted in Table 5 supports our hypothesis about task dependency of group differences in creativity: the arithmetic problem differentiates between RL students and HL (including G) students; the geometry problem and the Half-way problem provide distinctions between G students and their counterparts; and the system of equations reveals between-group differences in originality and creativity only between G and RL students.

Table 5 Different components of creativity problem and between-subjects differences

		G			HL			RL			Kruskal–Wallis test	G	HL
		Mean	Median	SD	Mean	Median	SD	Mean	Median	SD	H(2)	Significance of Mann–Whitney test[a]	
$2\frac{1}{4} \times 1.75$	Cor	25	25	0	24.07	25	4.81	20.28	25	9.47	6.6*		
	Flu	2.50	2	0.84	2.04	2	0.59	1.44	2	0.78	11.391**	RL*	RL*
	Flx	23.35	20	8.16	19.63	20	5.18	14.44	20	7.84	11.297**	RL*	RL*
	Or	2.20	0.25	3.95	1.20	0.20	2.60	0.84	0.20	2.34	5.9941		
	Cr	21.84	2.01	39.60	12.30	2	25.93	8.44	2	23.38	6.438*		
	Cor	25	25	0	10.19	0	12.52	1.39	0	5.89	17.962***	HL* RL***	RL[b]
	Flu	1.83	2	0.41	0.67	0	0.88	0.06	0	0.24	19.729***	HL* RL***	RL*
	Flx	15.03	15.05	5.44	5.30	0	7.09	0.56	0	2.36	19.43***	HL* RL***	RL[b]
	Or	3.78	0.65	5.26	2	0	4.61	0.56	0	2.36	14.276**	RL**	
	Cr	37.5	6.01	52.89	18.63	0	46.28	5.56	0	23.57	14.676**	RL**	
$\begin{cases} 3x+4y=14 \\ 4x+3y=14 \end{cases}$	Cor	25	25	0	25	25	0	23.85	25	3	5.654		
	Flu	2	2	0	2.06	2	0.44	1.77	2	0.44	5.268		
	Flx	10.64	11	0.49	10.89	11	0.48	10.7	11	0.47	2.225		
	Or	0.92	1.10	0.75	0.76	0.20	1.96	0.25	0.20	0.26	8.817*	RL*	
	Cr	3.40	2	4.28	2.42	1.10	3.22	1.76	1.10	2.51	3.092	RL**	
	Cor	25	25	0	18.15	25	10.39	19.17	25	10.04	2.595		
	Flu	2.17	2	0.41	1.30	1	0.78	1	1	0.59	11.297**	HL* RL**	
	Flx	21.67	20	4.08	12.60	10	7.64	10	10	5.94	11.485**	HL* RL**	
	Or	4.42	1.55	4.81	2.33	0.20	4.22	0.50	0.10	0.50	9.761**	HL* RL**	
	Cr	44.17	15.50	48.13	23.26	2	42.17	5	1	4.97	9.745**	HL* RL**	
Half-way–Half-time	Cor	11	10	11	5.20	0	10	2.31	0	6.96	8.166*	RL*	
	Flu	0.90	1	0.88	0.24	0	0.48	0.12	0	0.33	11.287**	HL* RL**	
	Flx	8.01	10	7.89	2.45	0	4.80	1.15	0	3.26	11.232**	HL* RL**	
	Or	2.25	0.10	4.18	1.64	0	3.74	1.15	0	3.26	9.557**	HL* RL**	
	Cr	21.51	1	42	16.41	0	37.36	11.54	0	32.58	8.344*	RL*	

*** $p < .001$, ** $p < .01$, * $p < .05$, [b] marginal significance $p = .05$

[a] Std. test statistics values (U) are not presented here, due to space limitations of the paper. The values for Problem 2 are presented in Table 4

5.3 Differences between the creativity components

The between-task differences can also be seen through examination of correlations between the different components of creativity (Table 6). For four of the five problems on the tests we received significantly high correlations between correctness, fluency, and flexibility. However, the system of equations demonstrated that correctness and fluency are different measures and do not necessarily correlate positively. This observation is also supported by the fact that between-group differences on the correctness factor were not found between G and HL students for four of the five problems in this study, whereas differences between G and HL students were significant on the flexibility factor in three of the five problems.

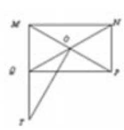
The findings presented in Table 6 demonstrate construct validity of the tool. Convergent validity is expressed in consistently high correlations between fluency and flexibility as well as between originality and creativity. Discriminant validity is expressed in the differences in the level of correlations between correctness and creativity components as well as differences in correlations between originality and other components of creativity. Clearly, originality is different from fluency and flexibility measures. On word problems (Jam and Half-way) the correlation between flexibility and originality is high, and we deduce that flexibility is a necessary condition for originality when solving these problems. Thus this examination supports our previous arguments (Levav-Waynberg and Leikin 2012, 2013) about the constructive validity of the model for evaluation of creativity using MSTs.

Finally, based on the observation that correlations between correctness, originality and creativity appeared to be less strong, we hypothesize that mathematical instruction can lead to the development of fluency and flexibility, which rise when the correctness of solutions rises; however, an increase in the correctness of solutions does not necessarily lead to growth in originality.

6 Summary and discussion

This paper presents a study that examines relationships between mathematical creativity, general giftedness, and mathematical excellence. It also explores the power of different types of MSTs for the identification of between-

Table 6 Correlations between different components of creativity in different problems

N = 51		Flu	Flx	Or	Cr
$2\frac{1}{4}\times 1.75$	Cor	.693**	.758**	.093	.095
	Flu	1	.893**	.020	.014
	Flx		1	.074	.075
	Or			1	.998**
	Cor	.943**	.921**	.568**	.542**
	Flu	1	.939**	.620**	.588**
	Flx		1	.759**	.743**
	Or			1	.998**
$\begin{cases}3x+4y=14\\4y+3x=14\end{cases}$	Cor	−.009	.032	.059	.051
	Flu	1	.340**	.305*	.279*
	Flx		1	.451**	.449**
	Or			1	.991**
	Cor	.858**	.838**	.069	.068
	Flu	1	.969**	.277	.276
	Flx		1	.307	.307
	Or			1	1.000**
Half-way–Half-time	Cor	.922**	.923**	.579**	.511**
	Flu	1	.981**	.579**	.529**
	Flx		1	.601**	.558**
	Or			1	.950**

The correlations between different components of creativity in our model were performed with the HL group of students

Shaded boxes denote significant correlations

Pearson correlation: *** $p < .001$, ** $p < .01$, * $p < .05$

group differences related to mathematical creativity as reflected in multiple solutions produced by the students. To achieve these goals a research tool consisting of a set of MSTs from different mathematical topics that vary in level of insight embedded in the task was designed by employing a model for evaluation of mathematical creativity using MSTs. We examined correctness of students' solutions as well as their fluency, flexibility, originality, and creativity as expressed in multiple solutions to the problems that they performed. Three groups of 11th grade and 12th grade students who varied in the level of general giftedness (G factor) and in the level of mathematical instruction (L factor) participated in this study. The results of this study which, in large part, support the results of a previous qualitative study are, nonetheless, somewhat surprising.

First, our study demonstrates that G students have higher scores on all the examined criteria than HL students, and HL students, in turn, scored higher than RL students on all the criteria. However, neither the G nor L factor significantly influenced correctness of students' solutions (on four of the five test problems). Second, we found that the results of the study are task dependent: we found that the G factor has a significant effect on fluency, flexibility, and originality in problems that are more open for insight-based solutions and provide students with an opportunity for individual (non-algorithmic) mathematical thinking (problems 2, 4, 5).

The calculation problem and the system of equations demonstrated that the G factor did not have an effect on creativity components. Both problems were the most algorithmic ones, less complex than other problems in the study, each having one insight-based solution. Moreover, while the calculation tasks revealed differences between HL (including G) students and RL students on fluency and flexibility, the system of equations did not reveal between-group differences on each one of the examined criteria. We assume that differences in findings related to these two MSTs are due to the fact that equations are often solved and used for solving other problems in mathematics lessons in 11th and 12th grade. By contrast, the calculation problem could be considered as customary for elementary mathematics while having an unexpected element for high school students (as one of the students from the G group noted: "I had not solved such an easy exercise in a long time"). We speculated that the element of surprise associated with the calculation problem, which was absent in the system of equations, is reflected in the differences that we found for these two problems.

Similar between-group differences were found for the geometry and Half-way problems in spite of the difference in students' success in solving these MSTs. The geometry problem was the only one in which the G factor affected students' originality. Students from the G group produced more elegant and irregular solutions than students in the HL and RL groups. The between-group differences on flexibility were identical for the two problems and the level of instruction was not among factors that influenced students' flexibility. For example, in the geometry problem, G students solved the problem in two ways and produced solutions that were based on different auxiliary constructions, while HL and RL students sometimes produced two similar solutions. For the Half-way problem, which was the most complex one, some of the G students produced both insight-based and table-based (conventional but difficult) solutions while the majority of HL and RL students did not solve this problem at all, and those who did solve it produced one solution only.

Even though the L factor has a marginal effect on the between-group differences the Jam problem appears to be the most powerful one for identification of between-group differences. The problem is rich in the variety of solutions with both algebraic and non-algebraic (elementary mathematics) tools (Fig. 1). Significant differences were found between G and HL students, G and RL students, and HL and RL students on the flexibility criterion. This problem, which is standard for junior-high grades, contained some element of unexpectedness for the senior-high students who participated in the study. This fact led to the low success among HL and RL students when solving this problem. Consequently, the differences between RL students and students from the other groups were primarily due to the lack of success of RL students in solving the problem.

Works by Vygotsky (1930/1984) related to creativity in children ("imagination" in Vygotsky's term) address a complex relationship between knowledge and creativity. On the one hand, knowledge is a necessary condition for a person to be creative, while having imagination is a necessary condition for knowledge construction. In our study, the differences in creativity of RL and HL students exposed to the arithmetic calculation problem are related to this relationship. Based on the findings of this study we argue that students' knowledge associated with solving relatively algorithmic problems can be developed in all groups of students and is related to a similar level of creativity on these types of mathematical problems (e.g., systems of equations). Additionally, despite the fact that the formula designed for the creativity criterion does not take into account the measure of correctness, we found correlations between fluency, flexibility, and correctness for four of the five problems in this study. Consistent with the previous observation, we find that mathematical knowledge is necessary for fluent and flexible mathematical reasoning.

On the other hand, problems that require insight (Half-way, Jam problem) require a particular ability level, as the G factor affects correctness of students' solutions as well as flexibility and fluency of the solutions. We also found that correctness of the solutions correlates with originality and creativity (R = .511–.579) for these two problems; hence, we conclude that knowledge is correlated with creativity on insight-based problems. Additionally, our findings related to correlations between different components of creativity in our model indicate that originality is of a different nature from flexibility and fluency, and that the relationship between knowledge and originality is different from that between knowledge and flexibility.

Correlations between the different criteria in the evaluation scheme serve as an additional indicator for the validity of the research instrument. We found that different problems lead to different correlations. In general, correctness, fluency, and flexibility were highly correlated with each criterion, whereas each of these criteria had a lower correlation with originality and creativity. Note that the roles of originality and flexibility in the creativity formula are symmetrical (Table 1, Sect. 3.3). The highest correlation in this study was found between creativity and originality. This finding is consistent with Leikin's (2009) observation that creativity depends mainly on originality, even when we adopt a relative perspective on creativity. This finding lends additional support to the research instrument presented here as being consistent with definitions of creativity at an absolute level: being creative means being original (e.g., Liljedahl and Sriraman 2006).

An obvious limitation of our study is in the group sizes and the standard deviations, which were very high for the originality and creativity scores. We are continuing our investigation with a large-scale sample that is likely to shed more light on the effects of mathematical excellence and general giftedness on mathematical creativity.

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