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ORIGINAL ARTICLE

Online paths in mathematics teacher training: new resources and new skills for teacher educators

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Abstract This paper considers the work carried out by online teacher educators and their professional development. We use the theoretical perspective of the documentational approach that focuses, in this case, on the interaction between teacher educators and the resources they use for their online training work. We thus study the following issues: (1) What kinds of resources do online teacher educators need, and how are such resources modified according to the educators' specific skills and needs? (2) What specific skills are needed for setting up online training for mathematics teachers and how do these skills evolve as teacher education resources are used? We consider both questions simultaneously, while presenting results from a study within a specific teacher training programme in France that proposes "training paths" on a national platform. These "paths" are resources designed for teacher educators. We follow the appropriation of two training paths by two educator teams. The ways in which these educator teams were able to appropriate the paths give insights into the teacher educators' skills and, as well, into the resources they need. By looking at their use of resources (as online mathematics teacher educators), we observe and analyse professional geneses, leading to the development of new skills.

Keywords Documentational approach · Dynamic geometry · Individualisation · Investigation · Mathematics teacher education · Online resources · Online teacher education · Teacher educators' professional knowledge · Training path

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1 Introduction

The development of information and communication technologies (ICT) has led to the development of new teacher training programmes (Trouche et al., 2012). In particular, digital platforms with various communication tools can provide new ways for *collaborative work*—itself recognised as an efficient means for in-service teacher education (e.g. Jaworski, 2008)—that can lead to innovative programmes. Collaborative platforms also allow asynchronous communication that helps deal with schedule constraints. Moreover, they open possibilities for up-scaling: for example, resources designed for a specific training session can be used in others once they are uploaded onto a platform. Nevertheless, drawing on these technical possibilities is not simple, as discussed below.

Firstly, there is the issue of how these resources are used by teacher trainers: a teacher educator cannot immediately turn into an online educator, since online training requires specific skills such as maintaining teachers' continuous work at a distance (Borba and Gadanidis 2008; Llinares

and Olivero 2008; Borba and Amaral 2011). Also, supporting the collaborative work of teams of teachers over time is a difficult task that requires particular interventions and appropriate resources (Jaworski 2008). Likewise, using resources to set up a training session is a challenging task for a teacher educator. In France, as in many countries, there is not usually a national framework that can provide a common reference for training sessions, even those concerning the same topic. A training session seems to depend on the teacher educator's very personal conception and production.

The work presented in this paper aims at problematising these issues. We focus on an online teacher education programme in France: Pairform@nce.[1] This programme offers "training paths" on a national platform. These paths are resources for teacher educators that allow them to set up training sessions anywhere in France. Our team participated in the design of paths for mathematics, and followed their implementation by teacher educators. This implementation led to evolutions of the educators' skills, as well as of the paths themselves, following remarks and suggestions given by the educators to the path designers.

In order to situate our contribution, we begin with a short survey about mathematics online teacher education. We describe the Pairform@nce programme and the theoretical and methodological approach we used for its study. We go on to present and analyse a cross-implementation of two training paths, where the designers of one path became teacher educators for the other path. Finally, we discuss the use of resources by the teacher educators, and examine the evolution of their professional skills—as well as of the paths—due to the cross-implementation.

2 Online mathematics teacher education

In this section, we give a brief overview of the opportunities offered by online resources for innovative practices in professional development, including how these can provide new means for participation and collaboration amongst teachers and teacher educators. We then discuss the role of teacher educators in developing and using training resources.

The programme presented in this paper is one of *blended* training (Osguthorpe and Graham 2003), combining distance education with face-to-face sessions. It is relevant to clarify that *online* education is not necessarily carried out at a distance, and that it can involve much more than simply providing means for distance education and exchanges; whereas *distance* education can be carried out by means other than through the internet—that is why, in our case, we specify that we focus on *online distance education*.

The development of online distance teacher education resources, programmes and research has increased exponentially over the past 5 years across the globe: examples can be found not only in developed countries, but also in developing countries (e.g. Orleans 2010; Baran and Cagiltay 2010; Alsawaie and Alghazo 2010). A quick education database search returned over 500 academic journal articles, most of them published within the last 3 years, reporting on online mathematics education in many parts of the world. Particularly in the United States, distance teacher education programmes proliferate.

However, as Santos and da Ponte (2003) pointed out, though distance education is useful for providing in-service opportunities for mathematics teachers, it can represent a large variety of pedagogical perspectives.

2.1 Online opportunities for participation, collaboration and other innovative practices in professional development

One approach of distance education is to use ICT affordances to promote online sharing, discussions and collaboration. Since the 1990s, teachers in networks, assisted by experts, have designed situations for the use and integration of digital technologies in the classroom, such as dynamic geometry environments (Allen et al. 1996), and for promoting new inquiry-based teaching practices.

By interacting in such networks, teachers can be involved in their own development. In particular, online exchanges and collaborations can lead to analysis and reflection by teachers on their practice (García et al. 2006; Davis 2006), which in turn can assist in the development of knowledge, skills and perspectives for teaching. For example, teachers can be involved in activities of inquiry and exploration, as well as collaboration and sharing, of mathematical (teaching) problems and projects, of resources and documents—and the development of these—and of teaching experiences. In this sense, ICT provides a space for teachers (both pre-service and in-service), teacher educators, mathematicians, researchers, and sometimes also developers of resources, to come together in joint activities. They develop and/or share resources and experiences, through online discussions and/or communities of practice (Wenger 1998), with members from different schools or geographical regions (e.g. Goos and Bennison 2008; Baran and Cagiltay 2010; Parada et al., 2012). A survey reported by Krainer and Wood (2008) points to the importance of *teams, networks and communities* for teacher education.

[1] http://national.pairformance.education.fr. An English equivalent of the word "Pairform@nce" could be "Peertr@ining", highlighting the principle of collaboration among teachers.

Furthermore, online communities and collaboration not only bring together members from different settings (which in turn can enrich the exchanges), but can also give time flexibility for participating. As an example, there is, in the UK, the National Centre for Excellence in the Teaching of Mathematics (NCETM),[2] which aims to support and encourage continuing professional development for teachers of mathematics with the motto "working collaboratively to enhance mathematics teaching", and which provides an online platform with a wealth of resources and tools for teachers, including tips, chat rooms and communities to share experiences.

Another interesting aspect that has been researched is the sharing and discussing of multimedia cases (e.g. video cases) as a method for professional development (e.g. Llinares and Valls 2009, 2010; Alsawaie and Alghazo 2010), sometimes within heterogeneous groups of mathematicians, pre-service and in-service teachers, and mathematics teacher educators (McGraw et al. 2007).

2.2 The role of teacher educators

In most of these cases, the role of teacher educators in the conception and use of resources seems to be central. Most papers, however, do not focus much on the educators, except as part of a larger description; Llinares and Krainer (2006), in a literature review, do devote a section to the learning of teacher educators, pointing out that it is an area which needs closer attention. In fact, the appropriation of resources, not only by teachers themselves but also by teacher educators, especially when referring to training proposals as "resources", is key for the integration of computer technologies (Artigue 1998). Some papers do analyse the use by teachers of resources—for example, a study in the USA points to the lack of exploitation by teachers of online resources (Moore-Russo et al. 2009)—but in general how resources and training sessions are conceived is seldom described, nor is much attention directed towards difficulties in the implementation of training proposals. An exception is the work by Sanchez (2010) who, in order to study the design of an online teacher training device, introduces the notion of "documentary orchestration", which is defined as an arrangement—by the trainer—of resources, with the aim of facilitating the *documentation work* (see Sect. 3.2) of trainees. This notion is directly connected with the documentational approach that we also use in our work (Sect. 3.2).

In this paper, we are concerned with the appropriation of resources and training proposals by teacher educators during the French Pairform@nce training programme.

[2] http://www.ncetm.org.uk.

3 Presentation of the Pairform@nce programme, and our study

In this section, we begin by presenting the Pairform@nce training programme. We then present the theoretical background of the research addressed in this paper and the questions we want to discuss. Finally, we present the methodology, derived from this background, that will allow us to deal with these questions.

3.1 Pairform@nce, the collaborative design of resources as a crucial mode of teacher training

The French national programme Pairform@nce is related to all class levels, from primary to secondary, and all topics (Gueudet and Trouche 2011). It aims to develop in-service teachers' skills for using ICT with students and to promote the *collaborative design of resources*. The Pairform@nce website (see Fig. 1) illustrates this clearly, presenting the objective of integrating ICT in schools for a renewal of teaching, and the way of achieving this objective (by designing resources and learning from collaboration, using both face-to-face and online work).

In practical terms, Pairform@nce is a programme that makes available, on a national platform, a collection of *training paths* dedicated to particular subjects, and a virtual space for their design, use and revision. We conceive these paths as sets of resources on the platform; whereas *courses*, or training sessions, correspond to enacted paths. We do not present here a detailed description of the Pairform@nce programme (such description is given in Gueudet and Trouche 2011), but simply summarise some of its principles that are central for the discussion presented in this paper.

Training sessions, developed from a path, are set up by teacher educators for a group of trainees at a regional level. These training sessions alternate face-to-face workshops, personal autonomous work, and online asynchronous exchanges using a web-based platform.

The most important Pairform@nce principles are:

1. *Collaboration* among teachers: professional development, especially concerning ICT, results from collective activity and experience with peers.
2. *Implementation in class and reflectivity*: a teacher's development programme necessarily implies experimentation of resources on the field, followed by shared reflection.
3. *Continuous work*: working efficiently on resources requires maintaining an ongoing collaboration, intertwining face-to-face and online training activities, as well as classroom experiments.
4. *Possibility of appropriation* of a path by teacher educators: teacher educators can use (to design their

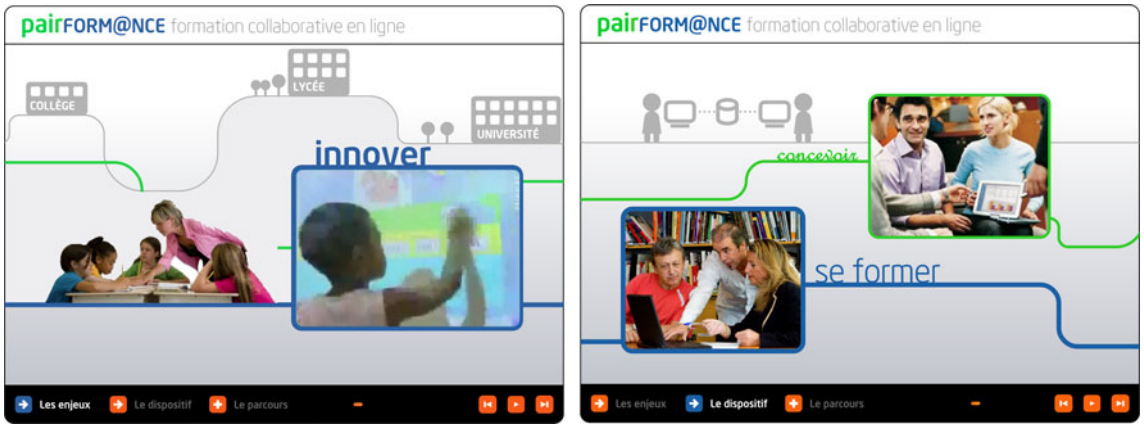

Fig. 1 Presentation of the Pairform@nce principles via a video on the programme website

training sessions) a path that *they have not designed themselves*.

We concentrate mainly in this paper on the fourth of these principles, focusing on the appropriation of paths by teacher educators. The theoretical perspective we adopt to analyse these processes of appropriation is the one termed *documentational approach*. We now briefly present this approach and related research questions.

3.2 The documentational approach: considering the teacher's work, as a work with—and on—resources

In this section, we consider the nature of the process through which a teacher integrates a given resource into her/his practice. For the sake of simplicity, we use the general term "teacher" to refer to "someone who teaches something to somebody in a given context"; this person could be a teacher, or a teacher educator. By resource, we mean a large range of "things" that are involved in the teacher's activity: such as textbooks or software (etc.) that the teacher voluntary gathers, or "things" that "interfere" with the teaching activity, such as a student's reaction. As such, we concur with Adler's (2000, p. 207) conceptualisation and perspective: "It is possible to think about *resource* as the verb re-source, to source again or differently"; thus a teacher's resources are things that she draws on in her activity.

The *documentational approach of didactics* (Gueudet and Trouche 2009) considers essentially that the teacher's work is a work *with*, and *on*, resources. A teacher interacts with resources, selects them, works on them (adapting, revising, reorganising, etc.), within processes where *design* and *enactment* are intertwined. These interactions have always existed, but they are enlarged considerably by the use of the internet, which makes available a profusion of resources. The interactions between teachers and resources

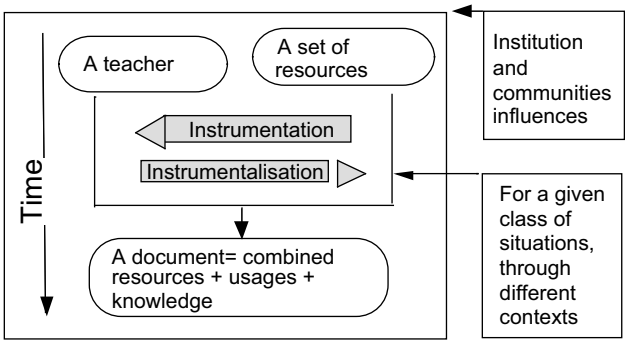

Fig. 2 Schematic representation of a documentational genesis

for performing a given type of task are analysed as dialectic ones (Fig. 2): the resources *act* on the teachers (in helping them conceive their activity in a given way); we name this process *instrumentation*. Conversely, teachers also act upon these resources as they appropriate them; we name this inverse process *instrumentalisation*. The latter is an important process, involving an enrichment of the initial resources, as the teacher incorporates into them aspects of her own experience. This appropriation process *gives birth* to something new, both for the teacher (who always learns something in this process) and for the resources, which are transformed when integrated into the teacher's repertoire. We name this process a *documentational genesis*. The powerful word "genesis" is used because a document is considered as a living entity that develops along phases of continuity and moments of rupture. Since documents incorporate teacher knowledge, the development of documents, and teachers' professional growth, are *interrelated* (Gueudet et al. 2012); we consider this growth as *a professional genesis* (Sect. 5).

The documentational approach is a holistic one, enlarging the consideration of the teacher's work, often reduced to the dialectic scenario/lesson (where a *scenario* is what a teacher considers will effectively happen in her

class or *lesson*). Actually, a teacher mobilises much more than a scenario to prepare her teaching and a document is much more than a lesson: one could say that a document incorporates all the possible lessons that the teacher has in mind when preparing her teaching. This theoretical approach inherits the theories of mediation, particularly Vygotski's (1978), which is widely used in the field of teacher education (Jaworski 2008), by considering that the teacher's activity is a social activity oriented by goals (the object of the activity). More precisely, our approach borrows its concepts from three major frames:

- the focus on teachers *resources* comes from the field of curriculum materials, seeing the essence of teachers' work as *transactions* with curriculum resources (Remillard 2012);
- the essential concept of *genesis*, seen as dialectic processes (that are instrumentation and instrumentalisation) between human and "things", comes from the *instrumental approach of didactics* (Guin and Trouche 1999); this approach, mostly used to study the student learning in a technology-rich environment, considers teaching and learning as processes of building *instruments* from a set of *artefacts*;
- the concept of *document* comes from a new field developing in the digital era, the *information architecture* (Salaün 2011). A document, seen as the support of each information and communication process, has three dimensions, its form, its content and its function, deeply modified by the use of the internet.

Beyond the change of labels (*resource* instead of *artefact*, *document* instead of *instrument*), the move from the instrumental to the documentational approach of didactics could be considered as a real theoretical metamorphosis, due to the metamorphosis of the technological environments. When introducing a new theoretical approach, this has, of course, to be carefully considered, particularly in the field of mathematics education that is already rich in many concepts and theories.

On the one hand, this introduction is relevant if, at an epistemic level, it clearly relates itself to existing theories and helps them to be mutually understood. We argue that in the case of the documentational approach, it is useful for analysing mathematics teaching in the internet era and for bridging didactics with the fields of curriculum studies and information architecture.

On the other hand, it is relevant if, at a practical level, it helps for analysing new phenomena and formulating research questions: it is indeed the case for the documentational approach, framing our analysis and our research questions. Actually, when teacher educators set up particular training sessions, in order to achieve their task they use various resources, organised into the proposed training path. This constitutes a particular documentation work. Hence, our research questions can be formulated as:

- What kinds of resources do online teacher educators need and how are such resources modified according to these educators' specific skills (which means studying instrumentalisation processes)?
- What specific skills are needed for setting up online training for mathematics teachers, and how do these skills evolve as teacher education resources are used (which means studying instrumentation processes)?

In the next section, we present the methodology associated with our theoretical perspective.

3.3 Studying documentation work of teacher educators, an instantiation of a general methodology

Understanding teacher educators' work as work with/on resources implies some strong methodological principles. These include:

- The principle of *continuous* survey: in the case of educators, this involves following their work *during* training sessions, as well as *outside* these sessions, because the documentation work is developed during the preparation, implementation and revision of resources.
- The principle of *extended-time* survey: this implies following the educators' work during a time long enough to be able to grasp the complexity of geneses (documentational as well as professional).
- The principle of "*resource*" survey: this means following the work on resources carried out by an educator when s/he designs, appropriates or modifies them. In this way, professional geneses can be read through the mirror of documentational geneses.

This continuous, extended-time, "resource" survey cannot be completely comprehended by an external observer. It requires the educators themselves to actively contribute and follow their own resources through specific tools. We used this method of *reflective investigation*, described in Gueudet et al. (2012), to follow the work of Pairform@nce educators.

The methodology of our study involved setting up conditions for a documentational genesis of a training path to occur that would provide data regarding the appropriation processes by educators and on the two intertwined processes of instrumentation and instrumentalisation. Thus, we organised a cross-experimentation study, which we briefly describe next.

In the cross-experimentation, we recruited two teams of teacher educators. Each team designed a training path (for the purposes of this paper, the path characteristics and the educators' experience as path designers are relevant, rather

than the designs themselves). Then, each team organised a training session using the path designed by the other team. During this latter stage, we asked the educators to fill in logbooks of their training path appropriation processes (see an extract of a logbook in Appendix 1). They also had to report on what modifications they made to the path when they used it in their training session (technically, it was possible for educators to change any elements of the path). These modifications are observable facets of the instrumentalisation process. In the last step of the experiment, the two teams reassessed the design of their own path and could modify it according to the suggestions that the educators had given during the previous step after using the paths. This last step added a pragmatic purpose to the formulations and justifications of suggested changes. Thus, in this methodology, an educator is, first, the designer of a path A, then the user of a path B (which includes appropriation, modifications and suggestions for evolutions of path B) and finally, again, the designer of path A, modifying it in order to improve it.

In the next section, we present and analyse the data collected through the above methodology. Our method of analysis consists of recording the path modifications proposed by the educators, as well as the ones they actually achieved, and confronting them with the logbooks' data. We analysed each of them in terms of indicators of the documentational genesis of the educator. Each modification led us to identify some resources that were produced (the instrumentalisation process) and to infer some of the associated knowledge (the instrumentation process). The combination of the two can give insights into the processes involved during the professional development of the educators, clarifying possibilities and also explaining difficulties.

This study methodology clearly has limitations. We only consider two teams of teacher educators, who, additionally, had extensive experience in teacher education. Being designers of Pairform@nce paths, they were familiar with its principles. They were also familiar with the paths' designers and were able to communicate with them because of their common involvement in the research project. Hence, our study will not produce general results about the appropriation of training paths; it aims, rather, at identifying documentational geneses for teacher educators, in particular geneses linked with online training skills.

4 Presentation of two training paths

We used two training paths that were designed and tested in 2007/2008. One of the paths is designed to train teachers in the use of *online exercises* for addressing different students' needs (Sect. 4.1). The second path deals with *inquiry-based* teaching using dynamic geometry (Sect. 4.2). These two paths shared some features, due to the common Pairform@nce framework (in particular, a path must be organised in seven *stages*, Fig. 3), but also because of the collaboration between the two designer teams inside the research group. In Sect. 4.3 we will discuss their central features, resources and choices, in relation to teachers and teacher educators.

The paths are available on the Pairform@nce platform as sets of pages that gather texts, links, collaboration tools and other kinds of resources (but no video means for distance communication). We will focus on the features that (1) result from the designers' attempt to address and support the appropriation process of the teacher educators that would use the path to implement a training session; and (2) support the face-to-face and at-a-distance intertwining of the *blended* training (Osguthorpe and Graham 2003).

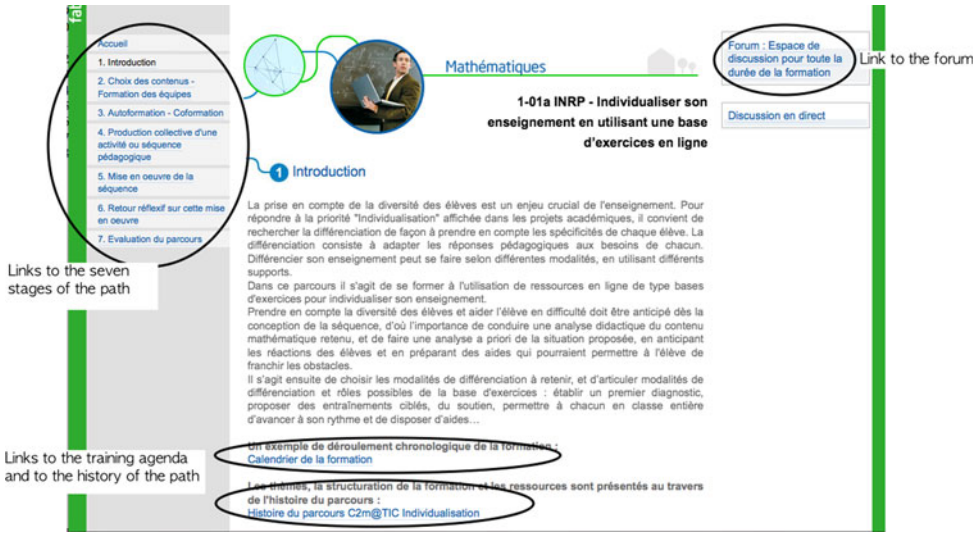

Fig. 3 Extract of the stage 1 page of a path, available on the Pairform@nce platform

4.1 Presentation of the path "individualisation with online exercises"

The objective of this path (named "individualisation" in the rest of this paper) is to enable teachers to develop individualisation by using online exercises with their students. *Individualisation* means taking into account the needs of each student. The purpose of the path is to help mathematics teachers (in lower secondary school) use online exercises, available in web-based repositories, as a means to design lesson scenarios that will provide each student with tasks according to her/his level of knowledge and expertise. The teachers' activities (Table 1) are distributed over a 13-week period, with 3 days of face-to-face workshops (i.e. days of face-to-face meetings for teachers and educators) and periods of work at a distance.

The path proposes that some activities take place during the workshops 1 and 2: in particular, establishing teams of four teachers (if possible two in a given school, two in another), giving demonstrations of the training platform, presenting the important dimensions of individualisation, and discussing lesson examples. The teams of teachers also have to choose a theme for their own lesson and begin designing a scenario.

The distance work is mostly conceived for the design and testing of the team's lesson. Each team is given a working space and a forum on the platform. The path recommends that a teacher educator follows each team, so that she/he can address the questions and comments related to different versions of the lesson scenario (posted on the team's forum).

The designed lessons should be presented and discussed during the third workshop.

4.2 Presentation of the path "inquiry with dynamic geometry"

The objective of this path (named "inquiry" in the rest of this paper) is to encourage the use of a dynamic geometry environment for *inquiry-based mathematical learning*. In

Table 1 Summary of the "individualisation" path

Pairform@nce seven stages	Activity of the teachers	Distance or in presence	Educators' resources
Stage 1 "introduction"	Sharing expectations about the training	Distance	Questionnaire
	Getting information about the training session	Distance	Agenda
	Learning about the different purposes for using online exercises	Presence workshop 1	Slideshow, web page of stage 1
Stage 2 "teams and content"	Choosing the team members	Presence workshop 1	Pedagogical advices about team structure
	Checking if there is need for an optional technical training	Presence workshop 1	Online exercises tutorials
Stage 3 "self-training"	Individual training to theoretical and pragmatic approaches of individualisation	Distance	Scenarios examples, scenario grid, observation reports, bibliography and links to online resources
	Collective discussion about individualisation	Presence workshop 2	Slideshows
	Optional individual technical training	Presence workshop 2	Tutorials
	Choosing the theme of the lesson	Presence workshop 2	
Stage 4 "collaborative design of a scenario"	Designing the scenario	Distance	Scenario grid, observation grid, online exercises
	Sharing of the scenario	Distance	Online database, forum
Stage 5 "lesson trials"	Implementation of the scenario in a class	Distance	Scenario grid observation grid, report grid, online database, forum
	Interaction with the other pair of the team about the other trials in class	Distance	
	Evolution of the scenario after several class trials	Distance	
Stage 6 "reflective feedbacks"	Preparing the scenario synthesis and the lessons report	Distance	Reflection guidelines, forum
	Confronting the different teaching scenarios that have been designed	Presence workshop 3	Scenario grid, report grid, reflection guidelines,
Stage 7 "evaluation"	Sharing the achievement of the training session within the teachers group and with the educators	Presence workshop 3	Final questionnaires

this path, *inquiry* means that a significant responsibility is given to the students, both in the mathematical activity and in the use of the dynamic geometry environment. The impact of this path on teachers' practices has been studied and discussed in Gueudet and Trouche (2011).

The path's general agenda is similar to the "individualisation" one: 13 weeks, including three one-day face-to-face workshops. The teachers' activities (Table 2) are also similar; nevertheless, the path proposes that, between workshops 1 and 2, the teams design a first lesson scenario which will not be tested in class. Moreover, some resources differ; we will discuss this in the following sections.

4.3 Central resources and choices in the "individualisation" and "inquiry" paths

4.3.1 Resources for the teachers and organisation of the training

The paths contain many resources (texts, images, working spaces, files to be downloaded). We focus on the ones that are essential for the distance activities. Both paths propose a detailed agenda of the training, describing the activities planned for the 13 weeks. Both paths also use a distinct model for the description of a lesson. In the

Table 2 Summary of the "inquiry" path

	Activity of the teachers	Distance or in presence	Educators' resources
Stage 1 "introduction"	Sharing expectations about the training	Distance	Pre-training questionnaire Forum
	Getting information about the training	Distance	Agenda of the training session
	Assessing the level of competencies concerning dynamic geometry	Distance	Pre-training questionnaire
			Conceptual map of the path, list of variables about the training session
			Teacher educators' prerequisites
Stage 2 "teams and content"	Examining different dynamic geometry tasks	Distance	Texts about inquiry, dynamic geometry and black boxes
	Identifying added value of dynamic geometry to the inquiry based teaching	Presence workshop 1	Agenda for the first face-to-face workshop
			Pedagogical advice about dynamic geometry
	Choosing the team members	Presence workshop 1	Feedback about the questionnaire
	Designing a first class scenario	Presence workshop 1 Distance	Student task proposal (class resource)
Stage 3 "self-training"	Individual training to inquiry based learning in mathematics and to dynamic geometry usages	Distance	Educators pre-requisites about dynamic geometry usages
Stage 4 "collaborative design of a scenario"	Discussion about the first scenario	Presence workshop 2	
	Choosing the lesson theme	Presence workshop 2	
	Design of the scenario for the team's lesson	Presence workshop 2 Distance	"Resource template"
Stage 5 "lesson trials"	Implementation of the scenario in a class	Distance	Observation grid with two levels of observation (class management, dynamic geometry interaction)
	Gathering elements for improving the scenario	Distance	
			List of guidelines and advice for class trials
Stage 6 "reflective feedbacks"	Preparing the lesson report	Distance	
	Confronting the achievement and difficulties of each lesson	Presence workshop 3	Lesson report
	Improved version of the scenario	Presence workshop 3	Scenario grid
Stage 7 "evaluation"	Sharing the achievement of the training session within the teachers group and with the educators	Presence workshop 3	Final questionnaire

"individualisation" path, it is called a "scenario grid" (Appendix 2), and is associated with two other grids: one for the observation of a lesson and one for the lesson report. In the "inquiry" path, the lesson description is inserted into a more general "resource template", which also should include a student sheet, some post-test reports, and some examples of students' productions. Despite these differences, both designer teams agreed on the need for a common model of scenario description, since the lesson has to be designed through distance work. Therefore, it is essential to facilitate the understanding of the different propositions. Moreover, the model categories emphasise the "individualisation" or "inquiry" aspects, and invite teachers to carefully consider these aspects while designing their lesson. The same model is also used to present lesson examples aimed at familiarising the teachers with the different categories.

The "inquiry" path also includes specific resources to maintain the continuity of the training during the distance work period. Indeed, the "inquiry" designers were aware of the importance of sustaining the contact at a distance (based on previous experience as distance educators in the SFoDEM project—Trouche and Guin 2005). The path recommends that educators send a *workshop agenda* to all the participants 1 week before each workshop, and a *workshop report* describing it and its main discussions 1 week after it takes place.

4.3.2 Resources for the teacher educators

All the resources for the teachers can also be considered as resources for teacher educators. Moreover, key aspects of the training and designer choices, as well as the detailed activities of each user of the path (teachers and educators), are listed in a series of training guidelines and tools for the educators, within a pedagogical advice section (Fig. 4). This section points to organisational notes, resources and comments, which reveal that the designers were concerned with the educator's role and tried to support her/his activity by providing relevant information for the implementation of the path.

Despite all these resources, the implementation of a path is not straightforward. Educators need to carry out important adaptations. These adaptations and modifications reveal the documentational genesis. Additionally, they are likely to improve the quality of the path if they lead to a new design that incorporates the experience of several educators, as we will discuss below.

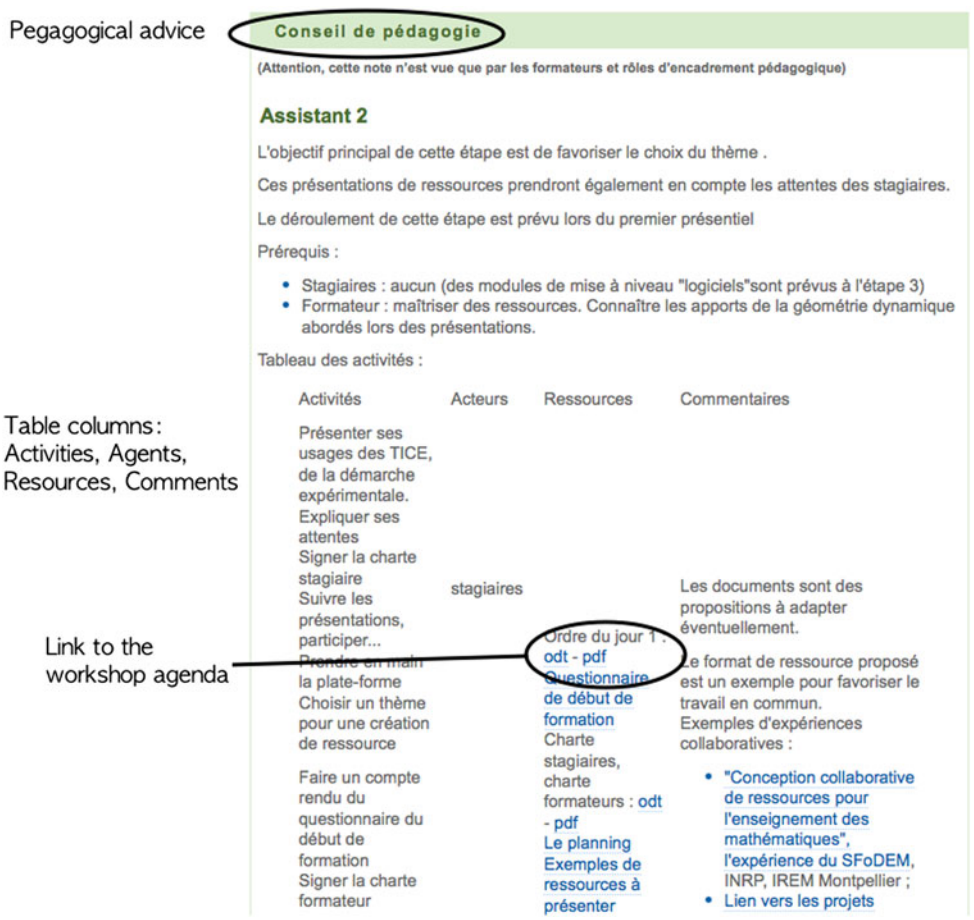

Fig. 4 Pedagogical advice section, stage 2 of the path "inquiry"

5 Cross-implementation of the paths, further design and teacher educators' professional geneses

During the academic year 2008/2009, we organised the cross-implementation of the paths. We set up two training sessions for two different groups of in-service teachers, each guided by teacher educators who did not design the path they were teaching (but designed the one used by the other team). The first team of educators was composed of three secondary school teachers, each with over 10 years' experience in teacher training. They met 15 times (including during the three workshops) over a period of 25 weeks. The second team was made up of four members: a researcher, two teacher educators and a secondary school teacher, all of them also with extensive experience in teacher training for the ministry of education. According to their logbook, they met eight times (including the three workshops) over a period of 22 weeks. The teacher educators also spent a lot of time in collaborative distance work, via e-mail or on the phone.

In Sect. 5.1 we provide results from the analysis of the data collected through the logbooks; that data includes remarks and requests addressed to the designers and the modifications actually performed on the paths by the educators. After the training sessions, the initial designers chose to retain some of the modifications suggested by the educators to create a new version of their paths; we discuss this and present interpretations in terms of documentational geneses in Sect. 5.2.

5.1 Turning a path into a training session

In the activity of transforming a path into an actual training session, the educators have to face a process of appropriation, adaptation and transformation of the path (the documentational genesis).

5.1.1 *The agenda, essential for the appropriation*

Some of the proposed resources were considered very useful, and were used almost without modifications (this is mentioned in the educators' remarks, addressed to the designers). The most significant example is the training agenda (see example in Fig. 5). Whereas the national Pairform@nce programme does not demand that paths include an agenda (nor that the different stages of a path take place at different training moments), our paths do each include an agenda. Both educators' teams declared that this resource was essential for the appropriation of the path, even if they had modify it before uploading it to the training platform. Of course, precise dates (including holidays) were added to the 13-week agenda. In one case, the local authorities cancelled, for financial reasons, a face-to-face workshop, so its training plan was replaced by distance work through the platform.

As in that case, other modifications were required due to external constraints. Nevertheless, here we focus on modifications derived from choices made by the educators, and resulting from their professional knowledge.

5.1.2 *Resources modifications*

Both teams of educators used some lesson examples provided by the path; but they transformed their presentation, using their own design of a lesson model (i.e. the pattern for the presentation of the content and the pedagogical setting of a lesson). As explained above (Sect. 4.3), the lesson models in both paths had different styles; as a result, the educators—designers of the other path—did not use the model proposed by the path. In both cases, they designed a new lesson model, structured like the ones in their own path and adapted to the current path's specific topic. The

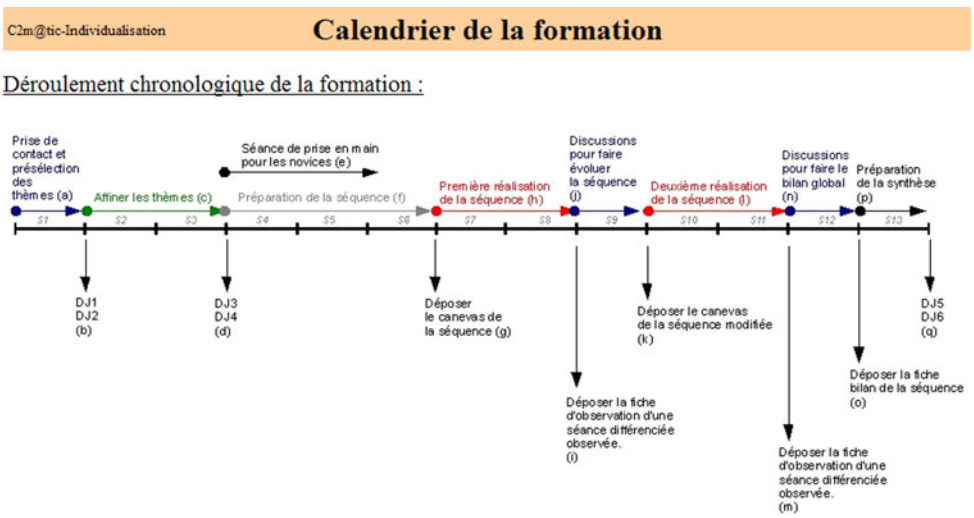

Fig. 5 "Individualisation" path: the 13-week training agenda (each week labelled S1, . . . , S13). Face-to-face work is labelled DJ1, . . . , DJ6 (described as half-days because of their content)

lesson models are central resources in a path, and essential for distance collaboration. Since the educators were already used to working with a given lesson model, they were not able to integrate a different one into their set of resources for addressing a similar objective; we discuss this further below (Sect. 5.2).

5.1.3 Adding resources

Each team added resources to the pre-designed path they had to implement. For example, as a consequence of their professional knowledge on dynamic geometry research, the educators who implemented the "inquiry" path added a slideshow presenting research results about inquiry-based teaching and dynamic geometry. The other team of educators added workshop agendas and workshop reports to the version of the "individualisation" path uploaded on their local platform; this was a consequence of their expertise in the field of distance education. We interpret that these additions result from the process of instrumentalisation. Professional knowledge influences the documentation work and its output.

5.2 Further design of the paths and teacher educators professional geneses

All of the suggested changes and adaptations developed during the training sessions were reported and sent to the initial designers (in July 2009). In September 2009, for technical reasons, the "official" paths had to be moved to a new national platform; this move forced the designers to upload again all the resources and texts of each path, thus providing a natural opportunity for introducing changes to the "official" paths. Designers carried out some of the suggested changes and rejected others. Based on our theoretical perspective, we interpret that some of these choices are indicators of ongoing professional geneses in the teacher educators. We develop here two key examples for distance training organisation.

5.2.1 Genesis of specific skills for distance training

The educators who used the "inquiry" path, though experienced teacher educators, were novices in distance training when they started the Pairform@nce programme. In contrast, the other team already had several years of experience in distance training. Thus, the pedagogical advice section of the "inquiry" path included a recommendation for each workshop to design, and send to the trainees, an agenda at the beginning and a report afterwards. For the designers of this path, such a report is important as a record (a "memory") of the training, and more so when it comprises extracts of the discussions that took place during that training. Also, teachers can have very different ideas about inquiry-based teaching (for example, about the need, or not, for a rigorous proof following an inquiry stage). The report is part of a document, that includes professional knowledge, about the need to discuss different possible points of view—in this case about inquiry—with the trainees. Additionally, sending the report to the trainees provided an opportunity to recall their engagement "lightly" (not as a reminder, nor mentioning a deadline). The report resource proposed by the path was a means for the educators to develop a document.

The teacher educators noted the positive impact of the report in terms of continuous contact maintained with the trainees.

We interpret this as a genesis: the teacher educators developed a document that integrated the agenda, the report, and professional knowledge. In their logbook they wrote: "Sending regular information to the trainees helps to maintain the continuity of the training." During their participation in the cross-experimentation, they developed their skills as distance teacher educators within an instrumentation process: the features of the resources, provided by the path, led to the development of new professional knowledge.

Naturally, because of the cross-experimentation, the educators using the other path—the "individualisation" one—noticed the lack of recommendation for agendas and reports (which they had recommended in the "inquiry" path that they had designed). They suggested its integration in the "individualisation" path, whose designers, having just experimented with the usefulness of such resources, accepted.

In this way, the "individualisation" path designers developed a new document maintaining the continuity of the training, which corresponded somewhat to a new objective of which they were not aware in their original design and before their own experience. Their experience as educators using the "inquiry" path led them to acknowledge the significance of this objective; there they used corresponding resources, and developed new professional knowledge. Thereafter, they could accept to integrate similar resources into their own path, since these matched the knowledge they developed.

In contrast, when the proposed resources corresponded to objectives that the teacher educators/designers were already aware of, and for which they had already developed their own documents, the suggestions of the other team were rejected; this we illustrate next, in the case of the lesson models.

5.2.2 Rejection of the changes in the "lesson model"

Both teams had extensive experience of collaboration, in particular as designers of their own path. During previous collaborations, their common documentation work led them to develop documents, including a particular "lesson model".

As designers, they integrated their lesson model into their path. As teacher educators, they were already used to a specific type of lesson model. During the cross-implementation, educators were confronted with another type of lesson model. The "individualisation" lesson model (Appendix 2) is very straightforward. In contrast, the "inquiry" lesson model is more complex and complete: it includes a basic description but also contains worksheets for the teacher, worksheets for the student, technical recommendations, etc. Apparently, each team of educators/designers was strongly bound to their own style of "lesson model", which led them to reject the one proposed in the path they had to use. The "basic model" can be considered part of a document associated with a knowledge statement of the type: "the model will be more likely accepted by the trainees if it is simple and contains only a few essential categories". On the other hand, the "complex model" would be part of a document based on knowledge ideas stating that: "for sharing a lesson, many details have to be proposed; the user will choose from amongst the given information". Hence the documents developed by each team differ, not only in their "resources" part (i.e. the models) but also in the knowledge they incorporate.

Each team tried to convince the other of the relevance of their own model. In the end, they both refused the other's suggestions. Geneses are complex processes, associating stabilities and evolutions. In the design of a path, an objective was to "provide the trainees with a model to facilitate their common lesson design work". But each team of designers/teacher educators had already developed such a document; thus they naturally drew on their own previous ideas. They adapted their own models to the specific context of the training (introducing categories about individualisation, or about inquiry, respectively) maintaining the professional knowledge attached to their own models.

This cross-experimentation was a very rich experience in regard to teacher training, path design and also research. In the given conditions, the paths were useful resources (or sets of resources) for the teacher educators. Simultaneously, the teacher educators enriched the paths with some of their own resources (in particular, with examples of lessons). The modifications tested by the teacher educators were communicated to the designers. Some of these suggestions were retained and others rejected; the suggestions retained seem to correspond to new objectives, discovered by the designers in their role of teacher educators using another path. It suggests the idea of possible transfers from one path to another, or even of a general "path model". The teacher educators acknowledged the interest of a new type of resource once they had experienced its relevance in the context of a training and if it addressed new objectives. Naturally, investigating the process of appropriation of a path by a teacher educator who is not a path designer requires another study.

6 Conclusion

The research presented in this paper concerns two intertwined themes: the resources of mathematics teacher educators and their specific skills. More precisely, our focus is on online teacher educators, on their professional development and on the design of online resources for them.

Considering simultaneously these two themes is a natural consequence of our theoretical perspective: the documentational approach. The teacher educators interact with resources; in the course of these interactions, the resources are modified—teacher educators act as designers of their own resources. Simultaneously, the teacher educators' professional knowledge is modified—setting up a teacher education device contributes to their professional development. In the particular case of the Pairform@nce programme in France, we could say that a training path is also an educator's path and a resource path.

Firstly, our research questions concerned the *resources* needed by teacher educators and the modifications of these resources that resulted from their use. We observed, in the context of Pairform@nce, that a training path must offer possibilities for adaptations, both to the local context and in accordance with the educators' experience, in an instrumentalisation process. Moreover, the implementation of training sessions leads to an enrichment of the path. The use of online resources allows this enrichment; even the design of several versions of a same training path. In our research and development project, this has been made possible by the specific conditions of cross-implementation. Outside of this particular context, a "path editor", taking care of the path's evolution since its initial design, seems necessary. This editor should be able to distinguish the new resources emerging from the experiences of educators, which could help further appropriations by future educators.

In relation to teacher educators, our research questions addressed the specific skills required in online training, and the evolution of these skills. We observed in our work that some of the key competencies that appear in online and distance conditions are crucial: particularly, taking care of *maintaining the collaborative work and the common memory* of the trainees (via precise agendas and reports). In our experiment, novice online educators were able to develop such professional knowledge, via the implementation of a path designed by experienced online educators, in an instrumentation process. More generally, the instrumentation process that results from the implementation of a training path is a central issue for a programme like Pairform@nce. It means that a training path can be designed by specialists of a given topic, and that its implementation will contribute to the development of specific skills by the teacher educators who use it.

Nevertheless, a prerequisite is the adoption of resources proposed by the path, something that is not straightforward. In our case study, it corresponded to the emergence of new objectives that the teacher educators were not aware of; in contrast, when the educators/designers had already considered an objective, they preferred their own resources. Further research, on a larger scale, is needed to inquire whether adoption of resources and identification of new objectives are always associated.

We conclude that complementary roles (editors, designers, educators) and complementary competencies (in mathematics, in mathematics education, in distance teaching) are desirable in online distance mathematics teacher education. Online teacher education certainly permits the up-scaling of successful programmes—but this up-scaling is neither natural, nor automatic: it requires specific conditions that research with a documentation perspective can help to identify.

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Appendix 1: Extract from the teacher educators' logbook on the training path "individualisation" (our translation)

	Used resources	Productions, indicate if they are adapted from resources of the path	Development	Comments
…				
Analysing the resources of the training path on individualisation			Period from 9/18/08 to 10/19/09: video conferencing and emails	It seems that the proposed resources do not explain enough of the individualisation aspect where pupils work on the same mathematical object, but at different levels
Adapting the path to constraints	Our notes made during two video conferences	First face-to-face meeting agenda Adaptation of the training session schedule	10/16/08 audio-conferencing with Skype from 7:30 p.m. to 11:30 p.m.	The number of face-to-face meetings and the lack of trainees from the same school let us foresee that cross observations among trainees will be difficult to achieve
			Booking of the computer lab at IREM for the first face-to-face meeting	We do not find an example of agenda for the face-to-face meeting
			10/18/08 video-conferencing with Skype 8:30 p.m. to 11:30 p.m.	
			Individual work	
			Email exchange	
First face-to-face meeting	The agenda of the day Every resource of stage one of the path The examples of the planned sessions from the ZEP-IREM group in Montpellier The examples of use from the Mathenpoche group of the IREM in Lorraine The leaflet to activate a session with Wims	The slideshow adapted with our names and platform	10/23/08 from 9 a.m. to 4:30 p.m.	See the agenda of the first face-to-face meeting The technical settings of the first face-to-face meeting were difficult (breakdown of the computer lab during all the morning, moving of the group on another site…) Presentation of the slideshow "individualisation" was not possible

Appendix 2: Lesson model in the training path "individualisation" (our translation)

Description of the lesson				
Theme/class				
Teaching objective				
Prerequisite				
Online repository used				
Other ICT tools used				
Number of course sessions and duration				
Session	Type of session (1)	Pedagogical structure (2)	Precise objective	Material used (3)
S1				
S2				
…				

1. Introduction, revision, construction, training, remediation, evaluation, etc. The sessions that include Individualisation are mentioned in italics.
2. Whole class, half-class, place, individual work, work in pairs, etc. In the case of group work, indicate the composition of the groups: homogeneous groups, heterogeneous groups, groups formed after a diagnostic, etc.
3. Indicate the precise references: extract of a textbook, or file (insert a link to the file in this case).

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