ZDM Mathematics Education (2011) 43:951–963
DOI 10.1007/s11858-011-0360-3

ORIGINAL ARTICLE

Elementary school teachers' growth in inquiry-based teaching of mathematics

Olive Chapman

Accepted: 14 August 2011 / Published online: 27 August 2011

Abstract This article reports on a self-directed, school-based, practice-based professional development (PD) experience aimed at helping elementary school teachers to develop knowledge and expertise in inquiry-based teaching of mathematics. It discusses the characteristics of the self-directed orientation of this PD that supported the teachers' learning, the nature of the inquiry-based knowledge they constructed, and the impact on their teaching. It highlights the centrality of agency, practical knowledge, and situated learning in this PD approach. The findings suggest that this approach can help mathematics teachers who want to be the architect of their own learning to transform their classrooms in meaningful and desirable ways.

1 Introduction

The current learner-focused perspectives of mathematics education require teachers to use effective pedagogy that will actively engage students in developing conceptual understanding of mathematics and mathematical thinking. Inquiry-based (IB) pedagogy offers opportunities to achieve this in the mathematics classroom. However, the challenge for teachers is how to adopt it as the basis of their teaching. Several obstacles arise in teaching and learning with inquiry, because it requires skills unfamiliar in traditional mathematics classrooms. In addition to holding a deep understanding of mathematics for teaching, teachers require, for example, the ability to embrace uncertainty, foster student decision-making by balancing support and student independence, recognize opportunities for learning in unexpected outcomes, maintain flexible thinking, and tolerate periods of disorganization (National Research Council, 2000).

This paper reports on a self-directed, school-based, practice-based professional development (PD) experience aimed at helping elementary school teachers to develop knowledge and expertise in IB teaching of mathematics. The goal is to identify the characteristics of the self-directed orientation of this PD that supported the teachers' learning, the nature of the IB knowledge they constructed, and the impact on their teaching.

O. Chapman (✉)
University of Calgary, Calgary, Canada
e-mail: chapman@ucalgary.ca

2 Related literature

Helping teachers to change or grow professionally in relation to different aspects of knowledge specific to the teaching of mathematics has been the aim of many recent studies of teachers and their learning (e.g., Even & Ball, 2009; Krainer & Wood, 2008; Tirosh & Wood, 2008). Researchers have examined a variety of PD processes, models, and tools to understand and identify promising characteristics to support mathematics teachers' growth to teach in new ways to improve students' proficiency in mathematics. One theme emerging from this research is the importance of practice-based PD, i.e., learning in and from practice. The general view of a practice-based model is that its learning activities should be purposefully connected to the curriculum the teachers are teaching, student learning or work in their classrooms, content situated in an environment that models effective teaching, pedagogy of their classrooms, and a collaborative environment with colleagues.

In Tirosh and Wood (2008), several researchers (e.g., Chapman; Markovits & Smith; Maher; and Yoshida) discussed a variety of "tools" or processes that embody such learning activities. A common feature of these processes is providing realistic or actual events and contexts of classroom situations to provide opportunities for teachers to explore important mathematical and pedagogical ideas that relate to their practice. These tools or processes include the use of narratives (Chapman), cases (Markovits & Smith), video recordings (Maher), and lesson study (Yoshida). In these four types of processes, teachers study episodes of their practice by creating and analyzing narratives of their teaching; analyzing cases of sample teaching or problem situations; analyzing self-created videos of their teaching or researcher-created videos; and creating, teaching, and analyzing *research lessons*, respectively. All of these approaches were shown to be effective in helping teachers to grow in their mathematical pedagogical knowledge for teaching.

In addition to such practice-based activities, studies have focused on the community aspect of the practice-based PD. For example, Males, Otten, and Herbel-Eisenmann (2010) examined teacher collegiality by focusing on the ways in which a study group of middle-grade mathematics teachers interacted as critical colleagues in a long-term PD project with a focus on classroom discourse. Nickerson and Moriarty (2005) explored the conditions that afforded or constrained the development of teachers' professional communities for a school-based PD aimed at increasing teachers' mathematics content knowledge and helping them improve their practice. Also, Koellner-Clark and Borko (2004) examined how community was established in a PD institute that focused on algebra content knowledge for middle school mathematics teachers. These studies have provided insights into factors that support the development of the learning communities. For example, Koellner-Clark and Borko suggested that giving tasks that provided access to all participants on the first day allowed active participation from all participants and the characteristics of the community emerged. Nickerson and Moriarty identified the relationship that the mathematics specialists had with the school administration and other classroom teachers as one critical factor that influenced the emergence of teachers' professional communities. Males, Otten, and Herbel-Eisenmann found that challenging interactions were related to instances in which the teachers interacted as critical colleagues and were marked by particular features including the use of personal experience as a form of evidence.

Other studies have focused on the learning activities of long-term practiced-based PD communities and the impact on specific aspects of the teachers' knowledge and practice. One aspect has been about students' thinking. For example, van Es and Sherin (2010) investigated how a "video club" in which teachers watched and discussed excerpts of videos from their classrooms influenced their thinking and practice. They found changes of increased attention to students' mathematical thinking in their thinking and classrooms. Kazemi and Franke (2004) reported on the activities of a teacher workgroup they initiated. They organized a monthly workgroup of ten teachers at an elementary school. The workgroup mainly studied the work of students to improve teachers' understanding of mathematical thinking of students. Prior to each meeting, all the teachers were given the same problem to pose to the children in their classrooms, with the understanding that the teachers could adapt the problem as needed. The research found that the teachers became gradually more attentive to the details in the students' thinking and started to develop possible instructional trajectories. Francisco and Maher (2011) reported on the experiences of a group of elementary and middle school teachers, who participated as interns in a 1-year, after-school, classroom-based research project on the development of mathematical ideas of middle-grade students. The teachers observed the students working on investigations that provided a context for the students' formation of particular mathematical ideas and different forms of reasoning in several mathematical content strands. As a result, they gained insights into students' mathematical reasoning.

Practice-based PD activities and learning communities, then, have been studied in different ways that explain their potential to provide meaningful and effective opportunities for practicing teachers to grow in their teaching of mathematics. This article offers another way of understanding such promising PD approaches for mathematics teachers by investigating the PD experience of teachers who took ownership of their learning in a practice-based learning community. The focus is on the "self-directed" aspect of the PD aimed at helping the teachers to grow in their knowledge and adoption of IB teaching of mathematics.

3 Theoretical perspective

The study of this PD experience is based on two key ideas: (a) inquiry and (b) self-directed, practice-based learning community.

3.1 Inquiry

Inquiry can be linked to Dewey (1905) who advocated "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends" (p.6). More recent perspectives of inquiry include Wells (1999) "dialogical inquiry" defined as:

> A willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them. At the same time, the aim of inquiry is not "knowledge for its own sake" but the disposition and ability to use the understandings so gained to act informedly and responsibly in the situations that may be encountered both now and in the future. (p.121)

In this study, inquiry is considered on two levels. First, inquiry is considered as a way of engaging students in the learning of mathematics in the classroom. In this context, some common notions associated with inquiry are: learner-focused, investigation/research, question-driven, communication, reflection, and collaboration. For example, IB teaching allows students' questions and curiosities to drive the curriculum, honors previous experience and knowledge, makes use of multiple ways of knowing, and allows for creation or adoption of new perspectives when exploring issues, content, and questions. Students are given the opportunity to direct their own investigations and find their own answers.

Second, this study involves inquiry as a way of learning for the teachers, i.e., a way to further develop their pedagogical knowledge and teaching of mathematics. As Dewey (1905) suggested, teachers should engage in inquiry or "reflective action" that would transition them into inquiry-oriented classroom practitioners. Schon (1987) also advocated that teachers should engage in a process of posing and exploring problems or dilemmas identified by the teachers themselves as an integral part of their professional practice and growth. The perspective in this study is that teachers' inquiry should be a systematic, intentional study of their own practice to create something new or different in terms of their knowledge and teaching.

For both levels of inquiry, a predetermined version of inquiry was not imposed on or presented to the teachers. Instead, for the first level, the PD was intended for the teachers to develop their understanding and approach to inquiry. To do this, for the second level, the teachers engaged in a self-directed, practice-based PD.

3.2 Self-directed, practice-based PD

Practice-based PD was discussed earlier in the literature review as directly related to the participating teachers' practice. This section addresses the perspective of the self-directed aspect of the PD. A self-directed focus can be framed in different ways, but the main feature is that all decisions are made by the teachers. In this study, self-directed is characterized by agency, practical knowledge, and situated learning.

3.2.1 *Agency*

In a self-directed PD, teachers need to take control of their learning, and thus the importance of agency in making sense of this PD. According to Bruner (1996), agency involves one taking more control of one's own mental activity, which assumes that "one can initiate and carry out activities on one's own" (p.35). "Agency implies not only the capacity for initiating, but also for completing our acts, it also implies skill or know-how" (p.36).

> The agentive view takes mind to be proactive, problem-oriented, attentionally focused, selective, constructional, directed to ends. … Decisions, strategies, heuristics – these are key notions of the agentive approach to mind. (p. 93)

Attributing this perspective of agency to the self-directed PD gives teachers the authority to think for themselves, to make decisions, and in general to be the architect of the PD in terms of its goal, activities, process, and outcome. While this can be done on an individual basis, a collaborative social context is important to aid the process. As Bruner explained, "the agentive mind is not only active in nature, but it seeks out dialogue and discourse with other active minds" (p. 93) to construct new meaning or knowledge.

3.2.2 *Practical knowledge*

Practical knowledge (PK) plays an important role in teachers' practice. It is what teachers know as a result of their experience as teachers that guides their practice. Fenstermacher (1994) referred to it as "know how" that teachers accumulate through experience and reflection. "[It] refers broadly to the knowledge teachers have of classroom situations and the practical dilemmas they face in carrying out purposeful action in these settings" (Carter, 1990, p. 299). It "encompasses first hand experience of students' learning styles, interests, needs, strengths and difficulties, and a repertoire of instructional techniques and classroom management skills" (Elbaz, 1983, p. 5). In general, PK is related to how to do things, the right place and time to do them, or how to see and interpret events related to one's actions. So, for example, teachers can use PK to adapt to situations in the classroom, to shape situations in the classroom, and to make selections when choices are available. Thus in a self-directed PD, in which agency is important, PK should also be important. It provides a basis for teachers to work from their perspectives, for personal experiencing, and in a way that makes sense to them to support their learning.

3.2.3 *Situated learning*

Given the importance of teachers' PK to their practice, further development of it is likely to be central in a self-directed PD. Since PK for the most part is situated knowledge, this makes situated learning an important means for teacher engagement and growth. From situated perspectives, knowledge is situated in the culture of a particular community and consists of socially shared knowledge, skills, and beliefs (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991). Knowledge continuously develops through interactions during an activity or experience. Based on Brown et al. and Lave and Wenger, situated learning usually involves engaging in tasks which parallel real-world situations. It emphasizes the context and application of knowledge. Thus, knowledge needs to be presented in an authentic context, i.e., settings and applications that would normally involve that knowledge. From a situated perspective, teacher knowledge is constructed through repeated teaching experiences and reflection on those experiences. Collaborative social interaction is also a critical component of situated learning—learners become involved in a "community of practice." In a practice-based PD, situated learning, then, is of importance for meaningful teacher engagement and learning.

The preceding brief discussion of agency, PK, and situated learning is intended to highlight three key constructs being associated with a self-directed, practice-based PD. These constructs are also in harmony with the inquiry perspective of learning as a learner-focused process. Thus, together, they provide a meaningful basis to consider and interpret the nature of the PD experience that is the focus of this article.

4 Research method

The research method is a case study (Stake, 1995) grounded in a naturalistic paradigm that focuses on the experiences of the participants in a natural setting. The goal is to understand their realities by identifying significant patterns in their thinking and actions while participating in the educational activities. This study sought to gather information related to such a goal where the realities involved how the participants engaged in, developed, and used an inquiry approach in their learning and teaching. Specifically, it focused on identifying: (a) characteristics of the self-directed orientation of this PD that supported the teachers' learning, (b) the nature of the IB knowledge they constructed, and (c) the impact on their teaching.

4.1 Research context

In the Province of Alberta, Canada, one of the recommendations of the Alberta Commission on Learning (2003) accepted by the Alberta Education Ministry was to "require every school to operate as a professional learning community dedicated to continuous improvement in students' achievement" (p. 8). School administrators and the teachers' association also embraced the initiative. This resulted in schools adopting it in a variety of ways based on what worked for them individually. The public school in which this study was conducted had its teachers volunteer to be in subject-area study groups of their choice. Fourteen teachers volunteered for the mathematics study group. I became involved in the study group when one of the teachers invited me to join it as an expert in mathematics education. The teachers had already worked in their study group for a semester before I joined them and had already established a working community in which they shared and discussed their teaching. At the point of joining them, they were interested in transcending that approach to learn something new, but still wanted to maintain their autonomy. I became interested in researching their PD experience once it was established that it was going to be a self-directed approach.

Three of the teachers assumed the role of group leaders and were responsible for organizing the group's meetings and activities. The group met once every 3 weeks for about 1.5–2 h in their school after their last class. They were able to use one half day and one full day of their school's PD days in each term to work in their group. They also organized themselves so that they could take turns in small groups to observe their *research* lessons and sometimes met during lunch breaks to plan and reflect on the lessons. Although the study group continued beyond the first year, the focus here is only on the first year since it consisted of the key activities of the self-directed approach that framed what occurred in subsequent years.

The teachers engaged in a self-directed PD process, in which they decided on what to do and how to do it. My role as "expert" was to make suggestions but not to impose an approach or direction. The role was to provide support through a nonthreatening, non-authoritarian presence, by responding to their needs rather then imposing direction, and not deliberately influencing the process of events by dictating what they should do or how to do it.

4.2 Participants

The participants were 14 practicing teachers with representation from grades 1 to 6. They ranged from 3 to 20 years of teaching experience, with most being over 10 years. They were generalist teachers with bachelor's degrees in elementary education. They chose the mathematics study group because they thought it was the area in which they needed the most help, to transform their teaching to bring it closer to the current curriculum and the anticipated revised version with greater emphasis on inquiry. This curriculum was significantly connected to the National Council of Teachers of Mathematics [NCTM] (1989, 2000) standards and principles. While there was some progress in the teachers making changes in their practice, in general, they were well behind in implementing this reform perspective in their classrooms. While the textbooks offered inquiry-oriented tasks, their mostly teacher-centered pedagogy restricted the tasks' use and learning opportunity for the students. However, some of the participants had taken some isolated workshops that exposed them to inquiry-oriented activities, which had formed the basis of their sharing prior to this study. Their interest now was about transforming their teaching in a more holistic way and not only to incorporate isolated activities.

4.3 Data collection and analysis

Data collection focused on two aspects of this self-directed PD: the way it evolved for the teachers and the way it impacted on their learning and practice. As such, the notes made by the researcher and the three lead teachers of discussions about, for example, what to do; how, when, or where to do it; why to do it; and planning, conducting, observing, and evaluating the research lessons were key sources of data. The note taking was less intrusive for the teachers than audio/videotaping and was seen as an integral part of their work to keep track of it as a means of looking back and making decisions on moving forward. It was for this purpose that the lead teachers made notes. However, some of their discussions and sharing of students' work were audiotaped. All relevant documents or notes pertaining to the development of the PD were also obtained. This included participants' notes on observation of videos, plans of research lessons, and researcher's and observers' notes during the observations of the research lessons. Field notes of classroom observations when each teacher, by herself, was adopting the IB teaching model (developed during the PD) in her teaching were also obtained. Three open-ended group interviews and one with each of the participants were conducted in the latter part of the year to probe their thinking about the PD and their learning through it, and their use of communication and their IB teaching model. These interviews were audiotaped and transcribed.

Data analysis was guided by the research questions. The first round of analysis focused on identifying the characteristics of the self-directed orientation that supported the teachers' learning. As an initial step in data analysis, a research assistant reviewed the data and created a chronological record of activities within the PD along with a summary of each learning activity. At the same time, she identified decisions made by the teachers that resulted in the learning activities and learning outcomes. The researcher then revisited the data to confirm the description of each step of the PD model in terms of the thinking and actions of the teachers. She then analyzed it for themes related to self-directedness, taking into account examples of the use of agency, PK, and situated learning from the teachers' perspective.

The second round of analysis focused on the nature of the IB knowledge the teachers constructed and the impact on their teaching. The researcher and research assistant conducted open-ended coding of the data to identify attributes of the teachers' thinking and actions that were characteristic of their conceptions of IB teaching. The coding focused on significant statements and actions that reflected the teachers' knowledge, judgments, intentions, and expectations regarding inquiry and communication in their teaching. Transcripts of classroom observations were also analyzed for communication and inquiry features of the lessons, for example: (1) types of questions and prompts that were inquiry oriented; (2) what the teachers attended to in students' responses during discourse; and (3) the inquiry structure and features of the lessons compared to their IB teaching model. The coded information was categorized based on themes that emerged from them and used as a basis to draw conclusions relating to growth in the teachers' thinking and practice. Verification procedures for the findings were based on those for a naturalistic study and included prolonged engagement, using data from a variety of sources, triangulation of coded information from the various data sources, elimination of initial assumptions/themes based on disconfirming evidence, and member checks with the teachers.

The findings reported here do not consider the unique ways in which each teacher developed and applied her learning about IB teaching. The focus is only on what was common to the teachers in terms of their experience in the PD, the knowledge constructed, and impact on their teaching.

5 Outcomes of the self-directed PD

The self-directed PD was effective in helping the teachers to grow in their thinking and teaching and to develop useful knowledge of mathematics teaching. Similar to findings of other practice-based PD studies, the collaborative, collegial community and the presence of a mathematics education expert were important contributors to this. However, the focus here is not on these factors, but others associated with the self-directedness.

5.1 Factors of the self-directedness that supported the teachers' learning

The following five factors of the self-directedness of the PD were identified as supporting the teachers' learning. They represent choices and decisions the teachers made and thus what they considered to be meaningful to support their growth.

5.1.1 An emergent process of inquiry

The process of inquiry the teachers' engaged in as a basis of their learning consisted of a sequence of steps (Table 1) emerging from decisions they had to make to achieve their goal. Each step emerged when needed and was discussed and defined only in relation to that need. For example, step 2 became a need after the teachers decided on their pedagogical problem (see Sect. 5.1.2) and realized they needed to understand IB communication and IB teaching, i.e., the two key constructs in the problem. This emergent approach allowed them to contextualize and personalize each step so that they all could make sense of it in a similar and relevant way that supported their learning collectively and individually.

In Table 1, the steps 4–6 cycle was done three times before the teachers finalized their IB teaching model (Table 4). Step 7 involved applying the model to different grades and topics with teachers working in small groups of two grades (i.e., 1 and 2, 3 and 4, 5 and 6) to plan, conduct, and observe the lessons. Teachers also started applying it to their individual teaching and made notes of their own observations to present in step 8.

5.1.2 A common pedagogical problem

The teachers had a global goal of learning more about IB teaching and adopting it in their practice. In discussing what this meant to them, they realized that they needed a specific topic (i.e., pedagogical problem) to focus their inquiry. To identify a topic of common interest that they could relate to their individual teaching, they decided to focus on the introduction section of their elementary mathematics curriculum, which outlined the perspectives of mathematics, learning, and mathematical processes that were required to enact it as intended. They had not attended to this section before; so, they decided to read it in order to identify what might be meaningful to explore in their practice. What stood out for them were the mathematical processes emphasized in this section and throughout the curriculum, that is, communication, connections, estimation and mental mathematics, problem solving, reasoning, and visualization. After reading and discussing the description of each and reflecting on their teaching in relation to each, they concluded that *communication* in an inquiry-teaching context was the key process that they would like to study as a starting point. As one teacher explained and the others agreed:

> Our students and their parents were used to "doing math" calculations but did not always have the experience or understand the importance of explaining and thinking "through math." Thus it seems like a logical starting point for all levels of our learning community and our teaching.

Thus, their pedagogical problem to inquire became how to transform their teaching to use communication that allowed students to think and be actively engaged in their learning in an inquiry learning context. This self-determined goal provided a meaningful basis for their learning.

Table 1 Process of inquiry

1. Pose a pedagogical problem
2. Investigate/understand key constructs in problem
3. Hypothesize an IB teaching model
4. Test hypothesis
i. plan a *research lesson*
ii. conduct and observe lesson
iii. analyze and evaluate lesson
5. Revise hypothesized IB teaching model
6. Repeat step 4 with revised model [#4–6 form a cycle]
7. Apply IB teaching model
8. Share, discuss, evaluate outcome from step 7

5.1.3 Relevant practical knowledge

Building on relevant PK was also important to the self-directedness and teachers' learning. When faced with a situation of learning more about something or planning to enact something, the teachers would choose to start with their PK or practical situations where they could connect to PK as in the following two examples:

(a) Using own PK. In planning the first research lesson on the topic "explore and classify 3-D objects according to their properties," the teachers decided to draw on their PK by brainstorming in small groups the possible IB activities to teach the topic. Group 1 suggested: observe objects in the classroom; discuss why these objects have certain shapes; post pictures of objects in the real world around classroom and use to identify shapes; name geometric objects; link to objects in class; refer to chart with formal names; investigate attributes; relate to real world—why things have certain shapes. Group 2 suggested: describe geometric objects in groups/pairs; list names of objects they come up with and descriptive words on a chart; build a model of one object (a skeleton representation); discuss, comparing skeleton and actual object; introduce formal names. Group 3 suggested: pose a problem, e.g., build a house with this object; discuss attributes; explore attributes; classify attributes; describe common features. These approaches were influenced by the PK of the grade levels of the teachers in each group. After considering these approaches, the research lesson the teachers decided on for a Grade 1 class to match the IB teaching model they were developing consisted of the following: students will talk/experiment/observe 3-D geometric objects; discuss what they noticed; predict which will roll only, slide only, or roll and slide using pictures of the objects; test prediction and record findings; discuss solutions and support answers; use Venn diagram to sort pictures of shapes and make general statements about “What I know about 3D shapes”; solve problem: “Think of self as a builder. Suppose I want to build a house on a mountain, what would I need to know about shapes?”; and for homework, find things at home and around the school that roll or slide.

(b) Using practical situations. In step 2 of their inquiry process (Table 1), the teachers decided that instead of beginning with reading theory about IB teaching and communication, they preferred the option of studying a video case as the basis of their learning. This allowed them to see what these IB ideas looked like in practice and to access the PK reflected in the teachers’ actions in the video. Their selection of videos was based on what was readily available to them, i.e., “Mathematics with manipulatives” (Burns, 1988). They chose two of these videos, “Pattern blocks” and “Cuisenaire rods”. Each video consisted of six inquiry-oriented lessons that covered the different elementary grades and stands of the curriculum.

5.1.4 Personalized models/guidelines

Constructing and using personalized models/guidelines were central to the teachers’ self-directed inquiry process and learning from it. Instead of studying and adopting available theoretical models/guidelines, the teachers decided to develop their own based on their own theorizing and testing. This decision impacted both on their engagement in inquiry as a basis of their learning and a basis of developing an inquiry approach to their teaching as in the examples that follow.

(a) Guidelines for observing videos. While the Burns’ (1988) videos came with suggestions of how to use them for PD, the teachers did not use those. Instead, they discussed what they thought they should look for in the videos based on their PK and the description of the mathematical processes in the curriculum, which resulted in the following themes to guide and record their observations: learning goal of lesson; students’ role; teacher’s role; questions posed by the teacher to encourage and extend students’ thinking; learning environment; nature of tasks; and key inquiry features of the lesson. The first lesson observed was used to practice noticing these themes. With each lesson observed, what they noticed increased. By the end of the first video, they had identified a pattern of similar features in the lessons for these themes. Thus, the second video became more about looking for these patterns.

(b) Guidelines for observing research lessons. The teachers decided on a plan to guide the initial observations to make it easier for them to compare and discuss their findings. They prepared an observation sheet with the following words in one column and the other blank for recording notes: notice, make sense, predict, how know, make connections, describe/explain, generalize/summarize, and other. These were key words to prompt the observers to notice specific aspects of the behavior of both the students and teacher as they interacted during the lesson.

(c) Model of IB teaching. Instead of studying and adopting a theoretical model of IB teaching, the teachers decided to develop their own based on their own theorizing and testing. They felt that this would allow them to create something that made sense to them in terms of how to enact it in their teaching, which was more important to them than acquiring formal theoretical knowledge about inquiry. The model they developed (Table 4) is discussed later.

5.1.5 Accessible mathematics topic

To create their IB teaching model, the teachers started with mathematics topics that they thought they all could make sense of with adequate depth and relate to individually in the context of their teaching. Having a commonly accessible topic allowed them to focus on the features of the IB teaching model and see the relationship between the topic and IB teaching and learning. Thus, they started with a topic based on the Grade 1 teachers’ schedule, i.e., *explore and classify 3-D objects according to their properties*. Their approach to creating a lesson plan for this topic has been described in Sect. 5.1.3. As with the previous four factors, this one was also important to the effectiveness of their self-directed PD.

5.2 Inquiry pedagogical knowledge constructed

There were several interrelated aspects to the knowledge that the teachers constructed during the PD over the year. However, only two aspects that relate to the primary aim of the PD are considered here, i.e., growth in knowledge of IB communication and IB teaching of elementary school mathematics. While the teachers held this knowledge in a situated way with unique features relative to them

Table 2 Questions and prompts

1. What do you notice?
2. What else do you notice that is different?
3. Who can explain how (or why) this makes sense?
4. What do you think the answer (or pattern or outcome) could be? How do you know?
5. How do you know it will (will not) work?
6. Where (or when) would you use this ___?
7. Suppose I want to___, how can I start?
8. Who can describe it so that I can do it?
9. Present your idea.
10. Explain the problem to your partner (the class).
11. What do you know about ____ (e.g., this topic)?
12. Can you make a general statement about ____?

individually, the focus here is on what they developed collectively during the PD.

5.2.1 *IB communication*

The knowledge the teachers were able to construct about IB communication was centered on key questions to engage students in inquiry, e.g., questions that allowed students to see for themselves patterns, structures, properties, or relationships of mathematics concepts embodied in a situation, problem, or object. They also focused on questions that allowed students to think about the mathematics or their thinking and share their thinking. The initial set of questions/prompts they adopted was obtained from the IB mathematics lessons of Burns' (1988) videos. Table 2 presents these questions/prompts in general form that can be applied to a mathematics concept or procedure, e.g., What do you notice about these numbers? What do you notice about these shapes? Suppose I want to subtract these two numbers, how can I start?

The teachers' knowledge and understanding of this style of questioning was reflected in their use of it in planning and executing their research lessons. For example, in an early IB lesson (see Table 4) on *estimation with mass*, the questions/prompts used by the teacher during whole-class discussions included.

> What do you notice about the objects? What do you notice when you pick them up? What are you talking about when you say heavy/light? Turn to some one in your group and explain everything you know about weight. Explain to someone in you group what you will be doing next. What is one important thing we need to know before we can get started? How did you [your group] decide which [object] goes into which circle? Explain what you think happened. How come so many think the ball with the holes is lightest? Why do you think one group disagree – can anyone explain this for me? Why do you think some groups disagree? Why do you think not everybody say the same thing?

This teacher with over 20 years of teaching experience was using this style of questioning for the first time. Based on the positive outcome on the learning of both students and teacher, this experience was the turning point for her and inspiration for the other teachers to engage students in this way.

By the end of the year, the teachers' thinking and use of this form of questioning reflected their growing understanding of it conceptually and how to enact it in their teaching. They felt that they had acquired a good understanding of "questioning techniques that guide and enrich student thinking" and "thought provoking questions to motivate students to discuss and understand mathematics at a deeper level." The following examples of unplanned questions/prompts from whole-class discussions in one of the Grade 3 teachers' class indicate the growing scope of her questions:

> What do you do when you read a large number? What did you think of first when you read the riddle? Who used the place value mat – can you tell us why you chose to use that? What tools do we need to use for math today to determine our height? How do you know it is a rectangle – make me believe that it is a rectangle. Is there anything that you can think of in your life that makes you think of 17? Who experienced a math situation since we met in class yesterday? Where would we find the number one million used in our world? But just talking about numbers, does anybody really know where numbers came from and why we have numbers? If I decided to sit here and count from one to one million, how long do you think it would take?

As a result of engaging in such questioning, the teachers also developed knowledge of what it meant to observe students' actions, and listening to and probing students' thinking to make sense of what they were doing and thinking. As one explained, they gained understanding of "student-centered strategies for listening to students and observing their problem-solving behaviors."

5.2.2 *IB teaching model*

The second key aspect of the knowledge the teachers constructed as a primary goal of the PD involved a model of IB teaching. They represented the model in the form of a jigsaw puzzle to indicate that it was not linear and could be pulled apart and reorganized in different ways with missing pieces and, thus, called it "The Jigsaw Inquiry-Teaching Model" (Fig. 1).

The model requires students to: (a) make *predictions* about possible outcomes related to a mathematics topic (e.g., a concept, process, or problem); (b) engage in *free exploration* of the topic (i.e., not guided by the teacher); (c) engage in *focused exploration* (i.e., guided by the teacher through questioning or specific inquiry tasks); (d) work on *application* of the topic; (e) engage in *comparison, evaluation,* and *reflection* of their learning; and (f) consider *extension* of the topic to other situations or related topics. The model also highlights a key question and prompts to indicate the importance of questioning to the process.

Based on their hypothesizing and testing of the model (steps 3–6, Table 1), the teachers were able to conceptualize it in a way that was meaningful and workable for each of them. As one teacher explained, they had developed understanding of "strategies that allow students to assume ownership of their knowledge and knowledge construction." Tables 3 and 4 are the teacher's outline of two of the lessons planned based on the model (for 2 different Grade 1 teachers) selected here based on length and to show variations to the model.

Testing of the model allowed the teachers to develop new understanding of their students as learners of mathematics. They were surprised and impressed with what the children were able to do, the richness of their thinking, and the depth of their learning of the mathematics topic involved. It was this recurring unexpected depth of the students' thinking and learning of the topic and engagement in the activities that the teachers used as evidence for the success of the lesson and the meaningfulness and effectiveness of the knowledge they were developing.

Although the teachers' mathematics knowledge for teaching (MKT) is not a focus of this article, it is worth noting that there was also growth in it as a result of using this IB teaching model. This was influenced by the unexpected complexity when the teachers were unpacking a mathematics topic to plan the inquiry lesson and realized the need to further develop their own understanding of it to engage students meaningfully in the inquiry of it. This opened opportunities for me to offer activities to further their development of MKT as in the case of their planning of lessons for the following topics:

1. "The meaning of three."
2. "Students will understand (a) why the value of a digit changes depending on its position in a number; (b) the meaning of regrouping among hundreds, tens and ones."

This resulted in me engaging them in an exploration task, in which they were to develop a numeration system

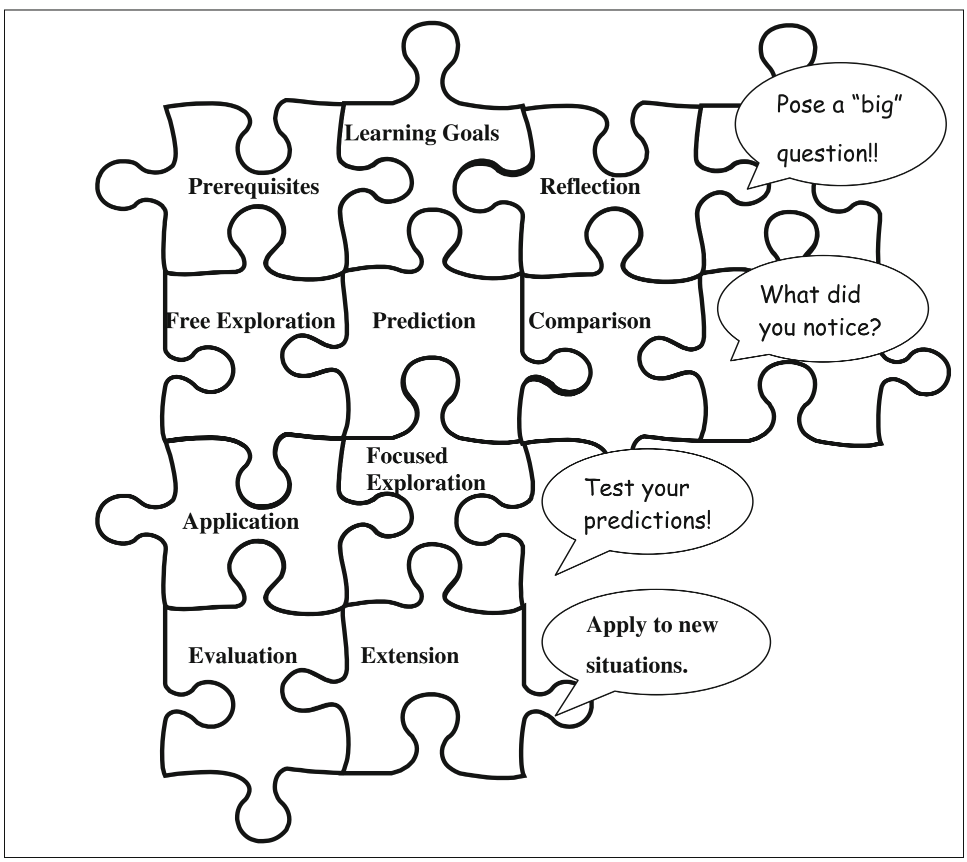


Fig. 1 The Jigsaw inquiry-teaching model

Table 3 Lesson 1

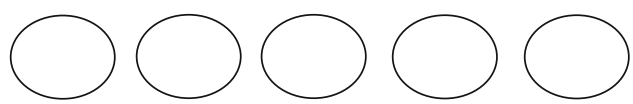
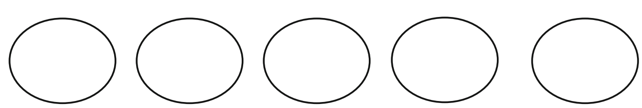
IB model	Lesson outline
Inquiry learning goal	Students will determine which of two or more given objects has the greatest/least area by covering and explaining their reasoning
Prerequisites	Students familiar with utilizing non-standard measurements
Introduction/ connection	Literature connection: read *Big, Bigger, Best* (Stuart Murphy)
Pose problem/ challenge	I have two floor plans and I need to know which is the biggest and will provide the most space for students
Free exploration	Small groups of students look for tools to use to cover the floor plans
	Try to solve the problem through discussion and experimentation
Comparison	Share with another group and explain or justify why your answer is the same or different than their answer
Focused evaluation	Large group sharing of answers and strategies
Discussion	How I know which is bigger?
Application	How and where do you think that people in our world might use this kind of measurement?
Extension	At home compare two rooms for size. Which would take more flooring? How do you know?

for aliens with a total of three fingers and to represent each of the first 30 numbers in sequence with *unifix* cubes of one color.

The children's difficulties and unexpected ways used to tackle tasks also contributed to the teachers' MKT. The teachers would share and discuss such situations and solicit my help when needed. For example, one teacher shared the following:

> Where I thought the fraction question was going to go, it didn't go there. They all came up with pie chart and showed the 1/3. The question was: *If you put your hand into a bag of M & Ms [candies] and took out some M & Ms and 1/3 of them are red, what would that picture look like?* So I thought, "Oh, you can get some nice pictures here! M & Ms, some of them might have 24 and some of them might have 12." No! I got a pie chart divided into three equal pieces, (laugh), 1/3 red, and the other two coloured green or blue or whatever colours there were, right. … [one group said] "you should see the M & Ms. Let's draw a hand!" … so they drew the hand and they drew some M & Ms. [One student explained] "It's got to be three, and I don't know why, I don't know why exactly, but it's got to be three" because you are

Table 4 Lesson 2

IB model	Lesson outline
Free exploration/ prediction	Groups of 4 each with 5 balls of various sizes/mass (golf balls, ping-pong balls, tennis balls, wooden beads and lead marbles)
	Predict and order lightest to heaviest
	Record lightest to heaviest on strip
	Strip 
Comparison/ discussion	Post-prediction strips
	Discuss process (How did they decide order?)
	Discuss product (Why do you think there are differences?)
	Question—"How can we figure out who is right?"
Focused exploration	Use scales and beans to find exact order of balls
	Record actual number of beans for each ball on other side of sheet
	Actual  # beans _____ _____ _____ _____ _____
Share/discuss/reflect	What do you notice?
Application	Discuss places and times when they would use this skill of estimating mass in their world
	Question—"Where or when would you use this process?"
Extension	Find two things at home that you can bring for students to guess which is lighter and heavier

counting by threes, right? … This one group eventually came up with that. … But the others went to the pie chart.

This resulted in a discussion about the students' thinking, the teacher's role, the task, and how they were connected.

Over time, the teachers included more focus and discussion of the mathematics they taught and the relationship to learning and inquiry to further develop their knowledge. They then started to consult a range of mathematics texts and NCTM teacher journals. Van de Walle (2005) also became a popular resource for them.

5.3 Impact on practice

Each of the teachers had opportunities, first, to be involved in group planning and testing of lessons for her grade and, later, to individually plan and test lessons to discuss with the group. This gave the teachers the support they needed to turn the knowledge they were constructing into action. Thus, all of the teachers were able to make significant changes to their teaching over the year, growing at different rates, and developing their own unique style of inquiry based on the model they developed. In addition to their collegial support, increased achievement of their students on external achievement tests and positive feedback from students' parents added to their motivation to continue to grow in transforming their teaching. Their classrooms were transformed in many ways that included greater focus on exploratory tasks, active engagement of students, integration of students' experience, and greater focus on group work and whole-class discussion. Providing specific details on how the IB model evolved for each teacher is beyond the scope of this article.

In addition to incorporating the IB model in their teaching, the teachers also made significant changes by using the knowledge they constructed of communication, in particular, the questioning style described in Sect. 5.2.1. Their ways of engaging students in IB communication was focused on getting them to talk about what they noticed; their thinking (e.g., about a mathematical idea, students' misconceptions or alternative approaches, problem-solving strategies, and their thinking); and their in- and out-of-class experiences with mathematics. Over the year, because the teachers had become curious about students' thinking and saw the potential to teach and learn through it, they were able to pose questions in an impromptu way to capture/expose it during discussions. As some explained:

> I try to get inside the children's heads. [T1]
>
> I want my kids to know you have to explain the why, not just an answer, you need to make sense of this. So I use that term [make sense] a lot in my questions. [T2]
>
> I really want to know the process. So my questioning is more around not just the answer but how they got there. It tells me a lot more about the kids…. But when you ask questions like that it opens up a whole new can of worms for the kids. That is when they have to think about the mathematics and it leads to more inquiry and discussion. [T3]

At the end of the year, there was significant difference in the teachers' practice in terms of their growth, but they saw it as the beginning of an ongoing journey to becoming an IB teacher.

6 Discussion and implications

In this study, a group of teachers embarked on a journey to make changes to their practice to reflect IB pedagogy. Their self-directed approach allowed them to carve a path and create an IB process that worked for them in achieving their goal. The teachers were able to create a model of IB teaching that reflected common notions associated with inquiry, such as learner-focused, investigation/exploration, question-driven, communication, reflection, and collaboration, as identified in the theoretical perspective. It also reflects the qualities of recent reform perspectives of teaching mathematics (e.g., NCTM, 1991, 2000). Similarly, the inquiry path (Table 1) the teachers took to transform their teaching also reflected common notions of inquiry from a research perspective. The unique feature of this path is its self-directedness, which is discussed next in terms of how agency, PK and situated learning, as discussed under the theoretical perspective, were manifested in characterizing it.

6.1 Agency

The teachers engaged in an inquiry learning process based on their sense making of it as opposed to adopting a predetermined, theoretical version, thus asserting their agency (Bruner, 1996). They were instrumental in determining the inquiry process for both their learning and teaching. They underwent the experience in a way that maintained their autonomy of the process. They made decisions that gave them a sense of ownership to the goal and meaningfulness to the process.

Thus, self-directed for the teachers meant that they took control of the process in terms of what they learned and how they learned it. Their use of the expert, when they considered it to be necessary, involved seeking possibilities of what to do, but they decided on the action to take.

The self-directed approach also emerged as a focus on self, linked to agency. The teachers always began with self.

The starting point had to be about them and not the "expert", about the reality of their practice and not abstract theory, about the mathematics they knew and worked with and not what they ought to know or work with. They wanted to start with their own sense making, what they were able to bring to the situation. They wanted to start from a place of knowing and not deficiency, strength and not weakness, and PK and not theoretical knowledge. In general, they wanted to start with and use what they knew in a way that empowered them to take control and responsibility of their learning.

6.2 Practical knowledge

As can be expected, the teachers' PK (Elbaz, 1983) played a significant role in the self-directed process. Their preference was generally to build from and on their PK or through that of others (e.g., via videos) as opposed to through theory that seems too abstract and disconnected to be a meaningful stating point. This use of PK was an important connection to self and way to empower self to make the learning through and about inquiry personal, real, relevant, important, and meaningful. It allowed them to personalize the inquiry they engaged in, to see inquiry in their way of thinking, and to "inquiry-ize" their practice. They dealt with inquiry not as something "out there," but as related to the personal embodied in their PK. The teachers accessed their PK through stories of experience, i.e., they shared, resonated in, and reflected on stories of past and present pedagogical experiences to facilitate their decision-making, planning, and enacting of the IB learning and teaching processes they engaged in and created.

6.3 Situated learning

Situated learning in the context of a learning community was also central to the teachers' self-directed approach. Their learning was situated in the context of their individual and collective teaching. Thus they generally started, not with theory, but with experience, their own and actual situations of others, which embodied what they wanted to inquire and learn about. They created an *authentic environment*, i.e., the tasks paralleled real-world situations (Brown et al., 1989). The situatedness was also reflected in their decision to learn about something only as it became necessary to make progress in their inquiry and achieve their goal. For example, they initially sought only the knowledge they needed to interpret IB teaching and communication. They later consulted external sources and engaged in mathematical activities to expand their MKT, but this was still linked to the particular situations involved. Their learning community was also genuine from a situated perspective being based on a common goal and process they could relate to individually and collectively, a common vision of student learning needs, collaboration within and across grade levels, and collaborative decision-making.

6.4 Implications

As a case study, this study does not provide generalizable outcomes. However, it offers an example of a self-directed PD and illustrates its potential and factors that are important to frame it theoretically and to facilitate teachers' growth in inquiry-based teaching of mathematics. It highlights the potential effectiveness of teachers' inquiry of inquiry as a means of transforming their thinking and teaching to a more desirable perspective. In particular, it suggests that there could be significant benefits from PD to support elementary teachers' growth in IB teaching of mathematics if the PD is based on the following factors:

(a) *Inquiry of IB teaching from the teachers' perspective.* Inquiry has to be an emergent process that allows the teachers to contextualize and personalize each step of it, so that they can all make sense of it in a similar way that supports their learning collectively and individually. The two processes of inquiry, i.e., inquiry by teachers and the IB teaching, should mirror each other or be similar conceptually in the way they emerge.
(b) *Agency, PK, and situated learning* as key constructs to frame the PD from a self-directed perspective. This allows the teachers to make the PD about them and maintain autonomy of the PD process/activities and their learning.
(c) Development of personalized models or guidelines for inquiry and IB teaching. This allows the teachers to develop their own ways of making sense of key ideas being investigated (e.g., IB communication, IB teaching) through their own sense making. It allows them to get inside and dwell within these ideas in a way that makes them meaningful and useful aspects of their knowledge.
(d) *Understanding of inquiry-oriented questioning*. This requires teachers to identify for themselves characteristics and examples of key questions that make sense to them to allow them to make sense of and develop knowledge of how to integrate such questions in their teaching.
(e) *A common pedagogical problem*. This has to be a topic/problem of common interest that the teachers can understand collectively and individually and relate to their individual teaching. For example, if teachers of different grades are involved, choosing a mathematical process, such as communication, connections,

problem solving, or reasoning, as opposed to a specific mathematics concept, to frame the topic/problem could be more effective to accomplish this. In particular, the choice of communication can be an ideal anchor for inquiry of IB teaching. It can allow the teachers to consider it in relation to the triad relationship among content, teacher, and students through communication, which is necessary for their implementation of IB teaching practices.

(f) *Accessible mathematics topics*. This means starting with mathematics topics that all of the teachers involved can make sense of with adequate depth and relate to individually in the context of their teaching. This allows them to start from a position of perceived strength than one of deficiency or weakness, build on their PK, gain the confidence to take risks in their practice, and overcome initial challenges in the inquiry process. It also allows them to focus on the features of IB teaching and see the relationship between the topic and IB teaching/learning without getting lost in the topic and thus distracted from their intended goal.

To conclude, the study suggests that teachers could engage in meaningful inquiry based on their own sense making of inquiry. Thus, it provides an example that can be used to help teachers who may be interested in a self-directed PD to see that it can be doable, meaningful, and effective.

References

Alberta Commission on Learning. (2003). *Every child learns. Every child succeeds.* Edmonton, AB: Alberta Learning Publication.

Brown, J. S., Collins, A., & Duguid, S. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.

Bruner, J. (1996). *The culture of education*. Cambridge, Massachusetts: Harvard University Press.

Burns, M. (1988). *Mathematics with manipulatives*. New Rochelle, NY: Cuisenaire Company of America Inc.

Carter, K. (1990). Teachers' knowledge and learning to teach. In W. R. Houston, M. Haberman, & J. P. Silkula (Eds.), *Handbook of research on teacher education* (pp. 291–310). New York: Macmillan.

Dewey, J. (1905). *How we think*. Mineola, NY: Dover Publishers.

Elbaz, F. (1983). *Teacher thinking: A study of practical knowledge*. London: Croom Helm.

Even, R., & Ball, D. L. (Eds.). (2009). *The Professional education and development of teachers of mathematics: The 15th ICMI Study*. New York: Springer.

Fenstermacher, G. D. (1994). The knower and the known: the nature of knowledge in research on teaching. *Review of Research in Education, 20*, 3–56.

Francisco, J. M., & Maher, C. A. (2011). Teachers attending to students' mathematical reasoning: Lessons from an after-school research program. *Journal of Mathematics Teacher Education, 14*, 49–66.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*, 203–235.

Koellner-Clark, K., & Borko, H. (2004). Establishing a professional learning community among middle school mathematics teachers. In *Proceedings of the 28th Conference of PME* (Vol. 2, pp. 223–230).

Krainer, K., & Wood, T. (Eds.). (2008). *The international handbook of mathematics teacher education: Individuals, teams, communities and networks*. Rotterdam: Sense publishers.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Males, L. M., Otten, S., & Herbel-Eisenmann, B. A. (2010). Challenges of critical colleagueship: Examining and reflecting on mathematics teacher study group interactions. *Journal of Mathematics Teacher Education, 13*, 459–471.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2000). *Inquiry and the National Science Education Standards: A guide for teaching and learning*. Washington, DC: National Academy Press.

Nickerson, S. D., & Moriarty, G. (2005). Professional communities in the context of teachers' professional lives: a case of mathematics specialists. *Journal of Mathematics Teacher Education, 8*, 113–140.

Schön, D. A. (1987). *Educating the reflective practitioner*. Oxford: Jossey-Bass.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: SAGE.

Tirosh, D., & Wood, T. (Eds.). (2008). *International handbook of mathematics teacher education: Vol. 2: Tools and processes in mathematics teacher education*. Rotterdam: Sense Publishers.

Van de Walle, J. A. (2005). *Elementary and middle school mathematics: Teaching developmentally*. Toronto: Pearson Education.

van Es, E. A., & Sherin, M. G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*, 155–176.

Wells, G. (1999). *Dialogic inquiry: Toward a sociocultural practice and theory of education*. Cambridge: Cambridge University Press.