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ORIGINAL ARTICLE

Communicative characteristics of teachers' mathematical talk with children: from knowledge transfer to knowledge investigation

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Abstract In-service teachers actively collaborated in a developmental research project. The main aim of the research project was the advancement of one central aspect of teacher professionalism: teachers' diagnostic competencies. Conditions of understanding and possibilities of enriching teachers' talk are of special interest because mathematics teaching is particularly affected by speech and communication (Söbbeke and Steinbring in Mathematik für Kinder—Mathematik von Kindern, pp. 26–38, 2004). One research focus was on the support of a productive enhancement of the teachers' talk with one child. Is the teacher's talk mainly a kind of knowledge transfer similar to traditional instruction or can it be seen as an investigation of the child's own views and ideas of elementary mathematical knowledge? These teachers' talks with one child should offer more reflective communication between teacher and child and result in a changed view of the child's mathematical understanding. Using an elaborate interpretation based on a theoretical instrument of analysis, called "**For**ms of teachers' **m**athematic**al** **In**teraction (Formal-In)", we describe the development from the first diagnostic talk with one child, at the beginning to the last talk at the end of the research project. Using an elaborate analysis of short episodes of teachers' talk distinguishing the interactive and the epistemological dimensions, we can describe how both dimensions influence each other. The theoretically identified characteristics of teachers' talk together with compatible video cases can be used in theory-based (in-service) teacher training aimed at enhancing professionalism.

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1 Research background and central characteristics of mathematical talk

During the last few decades, the importance of using *videographed episodes of mathematics teaching* and interactions to raise teachers' awareness of their own teaching and talking activity in and about mathematics (i.e. Benke, Hošpesová, and Tichá 2008; Maher 2008) has increased in research studies investigating mathematics teachers' professional development. Reflecting with colleagues about one's own teaching experience is a central and necessary issue for the professional development of teachers. "Systematic reflection on mathematical interactions that focus on the students' learning and understanding processes, as well as on one's own interaction behaviour, represents an essential professional competence of teachers" (Scherer and Steinbring 2006, p. 166). Particularly, "Communities of Practice" (Lave and Wenger, 1991) offer teachers the opportunity to form a collective, which already possesses knowledge that can simultaneously be further developed (Matos, Powell, Sztajn, Ejersbø, and Hovermill 2009, p. 171). By observing videographed teaching episodes, teachers can enlarge their consciousness of forms of communication in mathematics teaching, which otherwise would remain unconscious and unreflected.

Teaching is not a routine task of transferring mathematical knowledge prepared by the teacher directly to the students. Steinbring (2008, p. 372) points out that "school mathematics, as finished given knowledge, is not the *actual* *subject of teaching* in an unchanged way. Mathematical knowledge emerges and develops only in an effectively new and independent way within the instructional interaction with the students. Thus, finished, elaborated mathematics is not an independent input of the teacher into the teaching process which could then become an acquired output by means of students' elaboration processes." Hence, there might be the risk that teachers act on the assumption that they support their students' learning processes by just offering learning environments and asking open questions. But the posing of open questions is no guarantee for students' successful and active mathematics learning. In different parts of interview transcripts, we observed that, for instance, the posing of open questions by the teacher is not sufficient to initiate an explorative interaction for investigating the child's own understanding of mathematical knowledge. Further, due to their direct personal involvement in the mathematical teaching process, teachers tend to follow their personal views of knowledge. These views of how to communicate with students about mathematics are strongly shaped by their own former experiences as students at school and as pre-service student teachers. Their spontaneous reactions are based on their own—mostly unconscious—experiences and routines: Even in learning environments, the students have no opportunity to investigate mathematics on their own, because the teacher's talk is characterized by leading, funnelling and product orientating (Bauersfeld 1995; Wood 1998).

In this complex setting of teachers' joint reflection of video episodes of their own mathematical interaction, of routine teachers' talk in classroom and the specific nature of mathematical knowledge, our particular research focus in this article is investigating essential characteristics about mathematical talk in one-to-one diagnostic interviews. We will elaborate on a theoretical construct consisting of a two-dimensional grid, an interactive (ID) and an epistemological dimension (ED). This construct will be based on relevant literature and used for analysing four small episodes.

In addition to the well-known components of teachers' professional knowledge (Ball, Thames, & Phelps, 2008; Shulman, 1986), Steinbring (1998) emphasizes the *epistemological* characteristics of mathematical knowledge for teaching. The aspect closely relates to social and interactive processes of communication and influences the way of mathematics teaching.

In mathematics teaching, the teacher is personally involved in the all-embracing classroom interaction and has almost no chance to consciously observe the ongoing events from an external position. Therefore, a reduction of the complexity of social interaction processes supports raising awareness and starting to slowly change one's own ways of communicating and interacting. A positive transformation of one's own everyday teaching style is extremely challenging.

Within a classroom, this is more difficult than in a one-to-one situation as in diagnostic talks. Diagnostic talks with one child offer a reasonable reduction of the communicative complexity in a way that the teacher can concentrate and express interest in the child's mathematical understanding and explanation better. Similar to clinical interviews, where the teacher tries to find out something about the student's mathematical ideas, conceptions and working procedures, during diagnostic talks between one teacher and one pupil, the ongoing mathematical communication influences the reciprocal understanding between the communication partners. The setting of a diagnostic talk that we have consciously chosen as a situation of mathematical communication is because the teacher can focus his attention more clearly on one child in particular.

1.1 Characteristics of diagnostic talks

In mathematical diagnostic talks, the teacher tries to investigate the particularities of a child's mathematical knowledge, imagination and ways of proceeding. The setting of a diagnostic talk as a one-to-one situation offers possibilities for the teacher to intensively turn towards one student and to "scout" out his/her understanding of mathematical problems. Guidelines for an ideal clinical interview, as for instance developed by Krainer (1988), offer a supporting orientation for conducting such mathematical talks. This means that the interviewer should appear on the same level as the child, exploring the child's explanations, showing interest in the child's ways of thinking and leaving the child enough time for consideration. The teacher is meant to deepen his or her knowledge about the child's mathematical thinking and become aware of the existing boundaries and the lack of clarity concerning the child's mathematical knowledge.

1.2 Perspectives on mathematical communication

Several interpretations of mathematical communication emerge from reform documents such as the NCTM Standards (1989, 1991). Mathematical communication with students consists of three components: questioning, listening and responding. Several studies (Buschman 2001; Moyer and Milewicz 2002; Nicol 1999) have shown teachers' difficulties in asking the right questions—those that help the "students to think more intelligently about the important issues under study" (Wassermann 1992, p. 19)—in listening to the students' explanations and in responding or reacting appropriately to their statements.

In this context a major issue, which we wish to strongly emphasize, is often neglected. It is important to interrelate the child's answer with the teacher's previous question. The starting of the interaction process is not only caused by the teacher's open question, but also by the child's reaction. A teacher's intended explorative question together with a typical classroom-based student's answer could lead to a rather traditional classroom instruction process contrary to what the teacher aimed for. But research has also shown that students can benefit from teachers learning to use children's thinking as a way of informing their instructional decision making (Jacobs and Philipp 2004; Wilson and Berne 1999; Carpenter, Fennema, & Franke, 1996). The way in which teachers hear their students is influenced by their own ways of understanding the subject matter and by their commitment to their students. Teachers can improve their knowledge about students' thinking, but they will always hear students "through" their personal and social resources (Ball 1997; Wallach and Even 2005, p. 397).

Given this emphasis on mathematical communication, it is important to firstly describe the interplay between teacher and child in a communicative and interactive situation. As already mentioned earlier, teachers' investigating questions can trigger different reactions in the child. This is a reason for considering and analysing the *communicative interplay* between teacher and child. This interplay forms a basis to better understand how teachers can develop practices that foster mathematical communication. Further, we want to describe the effects that communicative interplay has on the understanding of mathematics and how in this interplay specific types of discourse arise. These results offer us a sound basis for later developing theory-based teacher professional training units using contrasting video episodes of teacher's talk to reflect with the teachers their ways of communicating about mathematics within their classroom.

1.3 Perspectives on the epistemology of (school-) mathematics

Mathematics, seen as a *science of structures and patterns* (Wittmann 2003), can be realized in school in a way that the children discuss and justify different interpretations of a mathematical problem by identifying, within the concrete material, relations and patterns that might lead to an elementary process of gradual generalization (examples for more clarification will be found later in the episodes of teacher–student talk). For realizing processes of investigating the child's own ideas about mathematical knowledge, the understanding of mathematics as a rich relational and emerging structure is an inevitable precondition.

Another perspective looks at mathematics as a *static body of knowledge* where you can only decide if it is right or wrong. Starting from this contrasting view, school mathematical knowledge—as a readymade product—is first of all interpreted and conveyed by the teacher, and then passively received by the learners. In such settings, teachers tend to base the transfer of mathematical knowledge to the child by asking closed questions without emphasizing students' own strategies and ideas. The communication in class, which is focused on the right solution, supports the supposed unambiguity of mathematical signs (Steinbring 2005; Voigt 1994). Often, the results are funnelling patterns and univocal communication (Wertsch and Toma 1995; Wood 1998).

1.4 The analysis grid "Formal-In"

We particularly look at diagnostic talks and consider questions such as: In what ways do teachers facilitate and guide the diagnostic talk communication? What is an appropriate *model* for investigating the functioning of a mathematical diagnostic talk: is the teacher the only one who is responsible for a successful talk? or: Is the interactive interplay between teacher and student a central component for a well-balanced characterization of the emerging type of mathematical talk? To address these and similar questions, the authors provide a framework of two perspectives classified as *investigation of the child's knowledge* and *knowledge transfer*, connected with two dimensions, *the interactive* (*ID*) and *the epistemological dimension* (*ED*), to analyse different forms of diagnostic talk communication. The *ID* consists of four constructs: *explorative*, *moderating*, *instructive* and *intervening* interactions. Interactions of the type *explorative* and *instructive* refer more or less explicitly to the mathematical knowledge as the specific content of communication in question, whereas *moderating* and *intervening* interactions refer to the "form and play" of communication. Further, *explorative* and *moderating* interactions are classified as investigation (of student's own mathematical knowledge) and *instructive* and *intervening* interactions as transfer (of pregiven mathematical knowledge). This leads to an arrangement as shown in Fig. 1. This theory-based model and instrument of interpretative analysis, called "**For**ms of teachers' **m**athematic**al** **In**teraction (Formal-In)" connects the *ID* with an *ED*.

The special nature of mathematical knowledge is expressed by means of the *ED* where we look at the specific use of mathematical learning material (i.e. representations of mathematical ideas by using different concrete material or by written ways) in the interaction about mathematics. Are they used as concrete objects—focussing on the observable properties of the material—or do they represent something else, i.e. do they have a symbolic function intending to express a relational structure (see Fig. 1)?

Fig. 1 The analysis grid "Formal-In"

Epistemology \ Interaction	Instructive	Intervening	Explorative	Moderating
Concrete use				
Symbolic use				

Throughout the article, the authors both develop and use these two dimensions in this theoretical construct for analysing two teachers' communications and their corresponding diagnostic talk practices. The explanation and elaboration, as well as a more general model, evolve during the process of analysing exemplary data from teacher–student talks. This research activity is still ongoing and will contribute to a further refinement of the instrument of interpretative analysis together with a clarification of the conditions under which this theoretical instrument can be successfully applied.

2 Research project MathKiD and methodology of interpretative interaction analysis

2.1 The design of the research project

"**Mathe**matics ta**lk**s with children—**i**ndividual **d**iagnosis and supporting" (MathKiD) is a collaborative research and in-service teacher-training project. The starting point of the project is the development of diagnostic competencies of teachers as one key aspect of teacher professionalism (Helmke, 2009; Kultusministerkonferenz, 2005) connected to a more conscious awareness of their interactive and communicative behaviour in mathematics teaching. A main question of this research project is: In which ways do teachers' professional communication and interaction processes develop in the course of several diagnostic talks? Five teachers with their students (grade 1 or 2) of two elementary schools volunteered to participate in this research project for 1 year. In the following, the names of students and teachers are pseudonymized. All of the teachers were introduced to diagnostic mathematical talks. They all participated in five professional teacher-training workshops as part of the MathKiD project. During the course of the research project, interactions between the teacher and one (different) child of his/her class were videographed about six times. The teacher and the child talked about "pure" mathematical situations or game situations with implemented mathematical requirements. The talks were supposed to permit diagnostic assessments of the child's mathematical abilities. During the year, the teachers of each school met three times for a moderated joint reflection, in which videographed episodes out of their own diagnostic talks were carefully observed and analysed with the help of corresponding transcripts and guided by the intervention of a moderator (project leader). Short video episodes from their own diagnostic talks were the object of the teachers' critical analysis.

The short description of the research project MathKiD already makes clear that quite a number of interesting research issues could be pursued. In this article, we will concentrate on the specific discursive style of two teachers during some of their diagnostic talks. We will use the analysis grid "Formal-In" to classify the type of the teachers' diagnostic talks at the beginning and end of the collaborative project. One of the teachers conducted six and the other four diagnostic talks with children of their own class; both teachers participated in six joint reflections. For reasons of comparability, we will use diagnostic talks with the same mathematical content.

2.2 Learning environments

The learning environments used in the teacher–student talks emphasize game situations with implemented mathematical requirements or "pure" mathematical situations. In this article, we only refer to two learning environments, which form the mathematical content of the analysed diagnostic talks. The interaction between teacher and students about mathematical issues in the two learning environments was not intended as a teaching–learning process, but should offer opportunities for students to develop and communicate their own mathematical ideas and understanding.

"Collecting coins" (Hengartner, Hirt, Wälti, and Lupsingen 2006, pp. 27–30) is a game situation with implemented mathematical requirements. In this game, you throw your dice and move the shown number forward on the playing field. At special places on the playing field, there are structured or unstructured amounts of coins, which you can win. When one of the players has reached the end of the playing field, the game ends. The winner of the game is the player who has collected the most coins. The *mathematical* goal of the game is to structure the coins, which are won in a way that you can always find out very easily and quickly how many coins you have already won and to be able to compare your amount of coins with that of your opponent.

The empty number line offers "pure" mathematical situations. This means of visualizing numbers only consists of a horizontal "empty" line and represents a model for the sequence of numbers. Number operations can also be displayed by means of arcs above the line. It offers the support of an arithmetical model. The "usual" and completely scaled number line and the empty number line differ in one central point: the empty number line is not scaled and there is no other pre-given visible landmark on the empty number line. The spatial distance between the marks can correspond to two pairs of numbers having an equal arithmetical distance, but is not a necessary requirement.

2.3 Method of interpretative interaction analysis

The qualitative data is carefully evaluated in an interpretative way and analysed using the analysis grid Formal-In. Before starting the interpretative analysis, we will accomplish a kind of "paraphrase" of the interaction in the observed episode. We have the possibility to observe the video of the episode—supported by the corresponding transcript—as many times as we want to examine closely various alternative interpretations and to refute or support them. The following step is to qualitatively interpret the episode (for the research approach of qualitative and interpretative analyses of mathematical interaction processes, see e.g. Krummheuer 2000) and to identify the aspects of the ID and the ED. To describe the change and development in the two different dimensions, we use the analysis grid.

We break down the transcripts of the diagnostic talks into *units of meaning* according to the content that is discussed in this episode. For our interpretative analysis, it is not sufficient to look at just the teacher's single and isolated questions. For our analysis, not only is the teacher important, but also the children's reactions to the teachers' questions and general remarks according to their academic socialization and schooling experience. Only a couple consisting of a teacher's *verbal* (*and/or gestural*) *action* and a child's *verbal* (*and/or gestural*) *reaction* produces a communicative element that can be qualitatively interpreted. Therefore, for our analysis, we choose units of meaning comprising teachers' *verbal actions* and children's *corresponding verbal reactions*. The end of the unit of meaning (UM) is marked by the clarification of the issue in question.

We analyse units of meaning because we would like to observe whether the teacher or the student "springs spontaneously to an immediate conclusion" in the interactive situation. In the course of interactive situations "… normally, whenever we hear anything said we spring spontaneously to an immediate conclusion, namely, that the speaker is referring to what we should be referring to were we speaking the words ourselves. In some cases this interpretation may be correct; this will prove to be what he has referred to. But in most discussions which attempt greater subtleties than could be handled in a gesture language this will not be so" (Ogden and Richards 1972, p. 15). In particular, the teacher has to be conscious about the fact that only in a few cases does his/her "immediate conclusion" reflect a correct interpretation of the child's utterance.

An essential characteristic of *instructive* as well as *explorative* interactions is their explicit reference to the mathematical content in question. In contrast, *intervening* as well as *moderating* interactions do not explicitly refer to the mathematical content, but rather depict the form (and "play") of communication.

Furthermore, *instructive* and *intervening* interactions are assigned to the *transfer* dimension of mathematical knowledge. These communicative forms are linked with the teacher's understanding of mathematics and the mathematical knowledge he/she requires the student to learn. The teacher proposes rules according to which the child is meant to act and in this ways he/she falls back to traditional communication forms in teaching. The aim of such an instructive talk might be to reach the "correct" solution by means of the interaction. Especially in teaching and learning processes, it happens that the "communication about the content of mathematical knowledge is often 'transformed' into communication about the information intended by the teacher" (Steinbring 2005).

Investigation (of students' knowledge) on the other hand is characterized by *explorative* or *moderating* interactions. The teacher explores the child's understanding about mathematics, while the child has to produce and interpret meanings of the mathematical signs. These aspects should be realized in an ideal clinical interview or diagnostic talk. The focus is on the child's interpretation and explanation of a mathematical task.

Thus for a theoretical description of the four elements of the ID, we present the following specifications:

- *Instructive interactions* resemble traditional classroom teaching, in which the child mainly has to follow the teacher's instructions.
- *Intervening interactions* describe interplay between child and teacher, in which communication is limited to assist or to bring back the child to the teacher's intended solution.
- *Explorative interaction* is characterized by an interplay between teacher and student in which both use the verbal interaction as springboards for deeper investigations and explorations (Brendefur and Frykholm 2000, p. 127). The questions asked by the teacher provoke the child to develop a deeper exploration of the mathematical content.

- *Moderating interactions* are characterized by the fact that control of the process takes place within the interplay between the child and teacher. In moderating situations, the teacher accompanies the child's solution process by listening and reflecting on the child's messages or by encouraging the child with statements like "you told me that …".

The ED distinguishes between the use of the learning material and/or visual representations in a *concrete* and *empirical* or in a *symbolic* and *relational* way. The child's use of the material in a concrete way is characterized by activities concentrating on the concrete properties of the material (empirical) features; coins, for instance, are simply taken as counting objects and not as an embodiment of a mathematical structure. The child's use of the material in a *symbolic* and *relational* way is characterized by activities with the concrete material, with the intention to take them as a means for symbolizing something else, i.e. relations, patterns and structures. The concrete coins are no longer objects for counting, but can be used to build a pattern, for instance the number six in the form of a dot-six on a dice. A child projects such an interpretation into a collection of specifically placed coins.

3 Two teachers' different forms of mathematical talk with young students: interpretative analysis using the instrument "Formal-In"

With the help of the analysis grid Formal-In, we will examine the transcript excerpts of the diagnostic talks regarding the aspect of the development of the ID and the ED. Our intention is to analyse whether the teacher encourages the child to follow established conventions and calculation procedures of mathematics, or whether he or she explores the child's mathematical ideas even if the child does not stick to the established mathematical conventions.

3.1 Case 1: Mrs. Olders

Initial dispositions and definitions

Mrs. Olders has 30 years of teaching experience. She has already participated in several collaborative studies on mathematics education research before and is very interested in and dedicated to mathematics teaching. During her project collaboration, she taught a first grade and conducted six diagnostic talks with her children (first grade). In her first diagnostic talk with Tom (first grade), they communicated about "collecting coins". The topic of her fourth diagnostic talk with Stefan (first grade) was the empty number line. She conducted two more diagnostic talks, which we will not consider here because we want to compare the communication and interaction of Mrs. Olders and Mrs. Dierks. Therefore, we chose comparable diagnostic talks, each time the first and the fourth one. Mrs. Olders participated in three non-moderated and three moderated joint reflections, in which her diagnostic talk with Tom was discussed and reflected.

3.1.1 Analysis of Mrs. Olders' first diagnostic talk "11 coins"

Short description of the content of the episode[1]

After about 6 weeks of the school year (school started in August), Mrs. Olders conducts a diagnostic talk with her first grade student Tom. Both Tom and Mrs. Olders have won 11 coins each (Fig. 2).

Tom is expected to count the amount of Mrs. Olders' coins. He compares his and her amount of coins and says, "your six is smaller somehow" (4).

Mrs. Olders enquires and Tom explains that his coins lie in a line with gaps. She asks Tom to find a new way of arranging the coins. He then puts his coins in a "zigzag" pattern (15) (Fig. 3).

Tom negates Mrs. Olders' question on whether he can now see the coins more quickly and is asked to arrange them in a way that "one can see very quickly, ah, it's

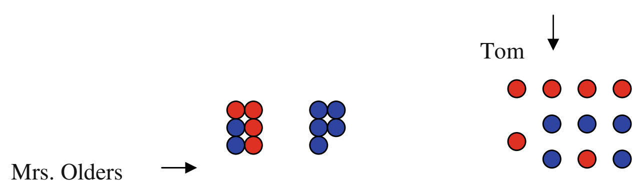


Fig. 2 Patterns of coins put down by Mrs. Olders and Tom

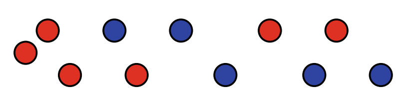
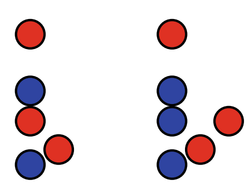


Fig. 3 Tom changed his pattern of coins

Fig. 4 Tom formed "11" with coins (seen from the observers' perspective)



[1] The following interpretative discussion is based on especially chosen and carefully transcribed episodes of the corresponding diagnostic talks. The numbers used (i.e. "1" or "5–11") in the ongoing presentation of student's and teacher's statements and comments are original from the transcript, in which every verbal/non-verbal contribution is numbered. The transcripts used in this article can be requested from the authors.

eleven" (20). After about 45 s, Tom has put two ciphers, one constructed with five coins and the other with six coins and thus has constructed the number sign "11" (Fig. 4).

Analysing units of meaning

First unit of meaning (contributions 1–11)

In the first UM, Mrs. Olders focuses on Tom's statements and enquires about them. Tom receives the task of counting her coins just as he has counted his (1–4). This is classified as a *moderating* interaction as she asks Tom to "count as you did with yours" (4). Here, one can see that Tom sees six (coins) as part of a structure and thus uses the coins as objects in a *symbolic* function.

Mrs. Olders encourages Tom by "so where is my Six?" (5) and "what do you mean is smaller?" (7) to think about his statement (4). An *explorative* interaction between Mrs. Olders and Tom develops in which Tom uses the coins in a *concrete* way by saying that there are "still small gaps" (8) between the coins. Mrs. Olders re-interprets this as the coins lying "a bit closer" (9) to each other (Fig. 5).

Second unit of meaning (contributions 12–23)

Tom is expected to find a new arrangement for 11 coins. The developing interaction between Mrs. Olders and Tom is characterized as *moderating* as she relativizes her request: "put a bit different" (12) and "put yours like this" (14). Tom reacts by using the coins as a *concrete* counting tool.

Similar to traditional classroom teaching, Mrs. Olders reacts to Tom's new arrangement of coins with a suggestive question (18) and thus questions this. At this point, the interaction between Mrs. Olders and Tom is *intervening*.

She asks Tom to put his coins "a bit more cleverly" (20). Tom develops a third arrangement of the 11 coins and puts them to show the number sign "11". This sequence is a *moderating* interaction again as she follows Tom's way of proceeding and refers to his arrangement of the coins. Tom symbolically places the coins in shape of the numeral 11. He thus uses the coins in a *symbolic* way, but different from the way that coins are used in mathematics instruction in order to represent numbers in a structured way (Fig. 6).

Summary of the analysis: epistemology, interaction and interplay of dimensions

During this episode, the coins are mostly used in a concrete way and as a means of counting.

Explorative and moderating interactions focus more on the child and his understanding of mathematics and are thus characterized as investigation. In this respect, this sequence is an investigation of the child's perception of mathematics.

In the interplay between the two dimensions, it is striking that a symbolic use of the learning materials happens exclusively during the phases of investigation, while during the phases of transfer, which are characterized by instructive and intervening interactions, the learning materials are only used in a concrete way.

3.1.2 *Analysis of Mrs. Olders' fourth diagnostic talk "empty number line"*

Concerning this diagnostic talk, it is her fourth out of a total of six, but the first one on the mathematical meaning of the empty number line. With this specific number line, there exists the distinctive feature that in an enactive level a lot of action with the number cards is possible, but sometimes fewer verbal statements and more gestural actions of the child occur. However, the number cards enable the children to make the ways of continuation visible, which they sometimes are not yet able to express verbally.

Fig. 5 Application of the analysis grid Formal-In (contributions 1–11)

Interaction / Epistemology	Instructive	Intervening	Explorative	Moderating
Concrete use			"still small gaps", "closer" (5-11)	
Symbolic use				six is seen as part of a structure (1-4)

Fig. 6 Application of the analysis grid Formal-In (contributions 12–23)

Interaction / Epistemology	Instructive	Intervening	Explorative	Moderating
Concrete use		"count them all separately" (18-19)		Coin as means of counting (12-17)
Symbolic use				Placed number sign with coins (20-23)

Fig. 7 Empty number line

Short description of the content of the episode

After about 7 months of the school year, Mrs. Olders conducted a diagnostic talk with her first grade student Stefan. Stefan is expected to arrange the number cards 1, 5 and 10 on the empty number line and then to justify his arrangement. He places 1 first, then 5 and then 10. He changes the position of 10 in such a way that the card 5 is approximately at the middle, between 1 and 10. He justifies the position of 10 by arguing that between 5 and 10, there still needs to be room for the numbers 6, 7, 8 and 9, and between 1 and 5 for the numbers 2, 3 and 4 (Fig. 7).

Analysing the unit of meaning (contributions 1–13)

Stefan is requested to arrange the cards 1, 5 and 10 on the empty number line. This task and Stefan's reaction could also have taken place in traditional classroom events and therefore this interaction is characterized as *instructive*.

In the following, the interaction turns to be *explorative* as Stefan justifies the position of the card 5. According to his answer, he could not only imagine a scaled number line or a ruler, but also the complete sequence of numbers. He justifies the position of the card 5 by stating the missing numbers between 5 and 10, as well as between 1 and 5. This interaction is not explorative in the sense in which it would emerge in connection with the symbolic use of material. Stefan uses the materials in a *concrete* way; he assigns number cards to positions according to "fixed" rules. In the following, an *intervening* interaction develops by Mrs. Olders who moves the number card 5. Stefan's interaction at this point complies with a stimulus–response scheme. At the end, there is an *instructive* interaction as Stefan reacts in the same way as during traditional classroom events (Fig. 8).

Summary of the analysis: epistemology, interaction, interplay of dimensions

Stefan uses the number cards exclusively as *concrete* material. A symbolic use of the learning materials cannot be observed at any point, although Mrs. Olders attempts to create explorative situations. For Stefan, there seems to be a fixed concept according to which he assigns positions to the number cards. He takes a local, but not a global, point of view towards the sequence of the numbers. His interactions are based on his fixed concept and hence many of the interactions become classroom or transfer situations.

Mrs. Olders' proposal does not lead to irritations or conflicts on the part of Stefan. She wants to show Stefan that the empty number line—in contrast to the (completely scaled) number line for example—does not require accuracy, but according to Stefan he needs to be precise and he does not change this understanding. Perhaps, the number line serves as a sort of "frame" for him or he is trying to "re-construct" a number line.

Although Mrs. Olders asks open questions, this is not sufficient to convince Stefan to verbalize his thoughts. Stefan shows "fixed" knowledge. Exploration of Stefan's conceptions about the number line does not take place.

In the interplay between the two dimensions, it is striking that in the phases of transfer the materials are used only in a concrete way; but here, the learning materials are used in a concrete way also during the phases of investigation. The depth of the investigation and the insights about the child's thinking clearly differ from phases of exploration in which the materials are used in a symbolic way.

3.1.3 *Comparing Mrs. Olders' first and fourth diagnostic talk*

About 5 months have passed between the first and the fourth interview. Both interviews deal with arithmetic topics. In the coin collection game, coins have to be placed in a way that they can be counted quickly, and for the empty number line, number cards are arranged.

In the first diagnostic talk, the concentration on Tom's interpretation continues through the later development of the diagnostic talk. During the fourth diagnostic talk, Stefan's perception of the topic empty number line as well as the topic number line seem to lead to an unclear situation, instead of an explorative situation. On the one hand,

Fig. 8 Application of the analysis grid Formal-In (contributions 1–13)

Interaction / Epistemology	Instructive	Intervening	Explorative	Moderating
Concrete use	placing number cards (1-2) moving the number card (11-13)	changing the position of the number card (5-10)	counting the missing numbers (3-4)	
Symbolic use				

Mrs. Olders does not vary her questions in a way that Stefan has to depart from his perception. On the other hand, several instructive interactions comparable to classroom teaching develop during the interplay between Mrs. Olders and Stefan.

When comparing the two interviews, initially it seems irritating that the first interview is rather an investigation and the fourth displays a mixture of both types, considering that Mrs. Olders has now been participating in the project for quite a while and has already conducted several diagnostic talks. The contents of the communication are different and influence the interaction. Tom adapts himself to the interaction with Mrs. Olders and concentrates on the questions she raises, while Stefan adheres closely to his interpretation of the number line.

Final dispositions towards mathematical communication

Mrs. Olders' way of interacting in diagnostic talks seems to depend on the mathematical content that is the topic of the particular talk. There are explorative situations in both diagnostic talks, but only in the first diagnostic talk these are accompanied by a symbolic use of the materials. Is this because she is more certain in her own understanding of the mathematical content when it comes to structuring the coins, and because for this reason she can initiate interactions which cause the child to explore mathematics in a different way? Mrs. Olders' mathematical understanding of the empty number line seems to focus on constructing a self-made number line. This might be a reason why there is no opportunity offered to explore this content further to the child.

3.2 Case 2: Mrs. Dierks

Initial dispositions and definitions

Mrs. Dierks has 7 years of teaching experience and before this she has worked in administrative contexts for 12 years. During her project collaboration, she taught a second grade and conducted four diagnostic talks with some of her pupils (second grade). In her first diagnostic talk with Christian (second grade), they communicated about "collecting coins". The content of her last and fourth diagnostic talk with Andreas was also "collecting coins". She participated in three non-moderated and three moderated joint reflections and in one we discussed and analysed her diagnostic talk with Christian.

3.2.1 Analysis of Mrs. Dierks' first diagnostic talk "eleven with two coins"

Short description of the content of the episode

During the course of the diagnostic talk, Christian is expected to place the coins he has won in a structured way first and then to represent them in an additive number system based on the decimal place value system. Accordingly, there is a change of representation of *coins, quantities and patterns* to *place value representation*.

Christian is expected to arrange the 11 coins he has won as 1 ten represented by the *blue* side of a coin and 1 one represented by the *red* side of a coin.

Summary of the Analysis of the unit of meaning (contributions 1–31)

Our careful interpretative analysis of the UM in this episode identified just one type of interaction without any variation in the analysis grid. Therefore, we will only present a summarizing description of this type of communication without considering the details of a substantial analysis according to the two dimensions "interaction" and "epistemology" of this diagnostic talk.

During the whole episode, the learning materials are used in an exclusively empirical concrete way, and the interaction processes contain only knowledge transfer (the cell "concrete use" and "instructive" in the analysis grid). The type of talk evolving here is very similar to traditional mathematics instruction with emphasis on knowledge transfer by the teacher.

3.2.2 Analysis of Mrs. Dierks' fourth diagnostic talk "a multiplication task for 19 coins"

Short description of the content of the episode

After about 10 months of the school year, she conducts her fourth and final diagnostic talk with a student of her second grade. The episode starts with Andreas winning three coins, which he adds to his already won 16 coins that are placed in a "4 by 4 pattern", so that there are "$4 \cdot 5 - 1$" coins. Andreas is expected to consider if he can place a multiplication task with 19 coins. Mrs. Dierks answers this question herself by "$1 \cdot 19$" and "$19 \cdot 1$". Following this, Andreas is required to place a multiplication task with a remainder and he decides on "$2 \cdot 9 + 1 = 3 \cdot 6 + 1$", by seeing two rows of threes as the six on a dice.

Analysing units of meaning

First unit of meaning (contributions 1–19)

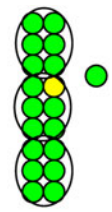
At first, Andreas determines the new overall number of coins he has won in an instructive interaction. Subsequently, Mrs. Dierks initiates an *explorative* interaction, in which Andreas does not refer to the coins, but exclusively to the arithmetical calculation of the quantity with the help of the strategy "place values extra". In an *instructive* interaction similar to a classroom situation, Andreas is expected to place a multiplication exercise with 19 coins, but cannot find one. This leads to Mrs. Dierks answering

Fig. 9 Application of the analysis grid Formal-In (contributions 1–19)

Interaction / Epistemology	Instructive	Intervening	Explorative	Moderating
Concrete use	19 coins (1-3) multiplication task for the result 19 (6-9)	multiplication tasks for the result 19 (11-20)	(6 +3) + 10 = 19 (4-5)	
Symbolic use				

Fig. 10 **a** Andreas placed his 19 coins. **b** Application of the analysis grid Formal-In (contributions 20–32)

a



b

Interaction / Epistemology	Instructive	Intervening	Explorative	Moderating
Concrete use	multiplication task with remainder (20-24)			
Symbolic use			2 · 9 = 3 · 6 (24-32)	

the question herself for a suitable multiplication task to 19 with "1 · 19" and "19 · 1". Andreas uses the coins in a *concrete* way or does not refer to the coins at all (Fig. 9).

Second unit of meaning (contributions 20–32)

Andreas comes up with the exercise "2 · 9 + 1" for 19. This solution develops within an *instructive* interaction, which has been initiated by Mrs. Dierks. Andreas does not refer to the coins. Instead, he uses an appropriate arithmetical expression, that is, he uses the objects in a *concrete* way in a certain sense and he does not give them a new meaning. Later she lets Andreas place his coins and explain his arrangements. Thereby an *explorative* interaction develops, in which Andreas uses the coins to interpret a new structure in the arithmetical expression that he has used before. In order to do this, he places "9 · 2" coins in which he sees "3 · 6 = 3 · (3 · 2)" coins. Here, Andreas uses the coins in a *symbolic* way (Fig. 10).

Summary of the analysis: epistemology, interaction and interplay of dimensions

During this episode, the coins are used mainly in a concrete way or not used at all. The interaction processes in the first UM are characterized by transfer situations and those in the second by an explorative situation.

In the interplay between the two dimensions, it is striking that the symbolic use of the material occurs only during the investigation phase, while in the transfer phase the coins are used only in a concrete way.

Mrs. Dierks' talk with Andreas proceeds differently than the one with Christian. Yet, the conversation with Andreas might have proceeded even more differently if Mrs. Dierks had, for instance, talked with him about multiplication tasks and their "neighbour exercises" within the game "collecting coins".

3.2.3 *Comparing Mrs. Dierks' first and fourth diagnostic talk*

About 8 months have passed between the first and the last (the fourth) diagnostic talk. The episode of the first diagnostic talk shows a pure transfer situation in which the learning materials are used exclusively in a concrete way. During the episode from the fourth diagnostic talk, a change concerning Mrs. Dierks' manner of mathematical teacher interaction becomes evident. At first, there is a transfer situation, but subsequently she is able to explore the child's statements. On comparing these episodes, to some extent a development of Mrs. Dierks' style of conversation during the course of the project can be seen. Because of this, the student receives the opportunity to use the objects in a symbolic way.

When it comes to Mrs. Dierks' view on mathematical contents, however, only the first signs of a change can be observed. The child's mathematical ideas are only partly explored, as it seems to be difficult for her to take a flexible perspective towards the mathematical interpretations. This becomes particularly clear in the episode from the fourth diagnostic talk. Apparently, Mrs. Dierks connects the meaning of a "nice" pattern of coins exclusively with "pure" multiplication exercises. In this situation, she does not seem to be aware of the fact that this is not possible with a prime number such as 19 at first. It can be assumed that not only the way of interaction, but also the epistemology of the mathematical knowledge in the respective episodes is closely connected to her personal view on mathematics and learning mathematics. Partly, she also seems to lack some parts of the necessary mathematical knowledge.

Final dispositions towards mathematical communication

Mrs. Dierks' perspective on mathematics and learning mathematics limits the extension of her style of conversation and reduces her references to the level of mathematical content. Her style of conversation changes from transferring to investigating, but she cannot allow for a currently implicit mathematics-related interpretation by the student. This becomes particularly clear in the interview with Andreas. Here, she accepts only multiplication exercises without remainder and not one with remainder, as well as no complementary exercises (i.e. switching between addition and subtraction exercises with the same numbers or switching between multiplication and division exercises).

4 Conclusions

On the basis of the interpretative analysis of the two teachers' diagnostic conditions, the child's framing of the mathematical problem as well as the teacher's flexible mathematical knowledge has shown great influence on the kind of interaction in the course of the episodes analysed here. In the cases when the teacher demonstrates secure and flexible mathematical skills and understanding—as it could be observed in some of the documented communication events—and if the teacher is able to see and point to the symbolic function of the learning material, a basis and chances for the development of interaction phases with a character of *investigation* is offered.

Despite that mathematical classroom interaction is of a very complex form, Gellert and Krummheuer (2005) emphasize two contrasting types of interaction: interactionally steady flow versus thickened interaction. "Teacher development may be seen as a path towards better opportunities for students' learning of mathematics, that is, to facilitate thick interactions that interrupt the interactionally steady flow of everyday mathematics lessons" (Gellert et al. 2009, p. 51). Steady flow interaction can be compared with phases of investigation, which primarily refers to explorative and moderating interactions; the ED is not taken into consideration. Within the analysed transcripts, we could identify symbolic use of learning material in a similar way as mathematical signs only in those cases in which interactions had been classified as explorative or moderating. This seems to be a necessary condition and it does not mean that every interaction phase leads automatically to a symbolic use of material and objects. At a first glance, such situations contain a greater part of children's speech and the pupils are offered more possibilities to develop and to verbalize their own ideas.

In those interaction episodes that after analysis display phases of (mathematical knowledge) transfer, the learning material is used primarily in a concrete and empirical way. Again, this does not imply that the learning material referred to is used concretely only in phases of transfer in interaction.

In further research and interpretative analysis within the project MathKiD, the analysis grid "Formal-In" will be refined and sharpened. Up to now, this instrument has been used for analysing and theoretically characterizing mathematical interaction and its continuation during mathematical diagnostic talks between one teacher and one child according to its two interrelating dimensions of *Interaction* and *Epistemology*. The communicative interplay between the teacher and one pupil is seen in a different and new way through this grid. With this theoretical grid, we can search for hints and instances, explaining reasons for the emergence or non-emergence of phases of investigations. Analysing several talks of teachers with this instrument, showing a broad spectrum of communicational behaviour, is inevitable for strengthening and consolidating the analysis grid. The success of the phases of investigation is not directly and not only dependent on the skill and fortune of the teacher asking the right questions, as can be seen from the interrelation between verbal actions and reactions occurring in units of meaning. Ultimately, the type of communication depends on both partners interacting in a developing social situation.

Each of the participating teachers in the collaborative project MathKiD conducted an ordinary mathematics lesson at the beginning and at the end of the 1-year cooperation that have been videographed and transcribed. One main intention of the teachers' introduction into the particularities and the developing accomplishment of diagnostic talks with one child was to make the teachers more conscious and explicitly aware of what otherwise often remains hidden, their own discursive and communicative behaviour in mathematical interaction with young students in school. The joint reflection of small video episodes of their own talks, together with their colleagues, should offer opportunities for this intention and support the development of alternative ways to communicate about mathematics. The realization of the kinds of communication between teachers and students depends on complex conditions and factors. A first factor might be the awareness and the personal wish of a teacher to understand his/her spontaneous way of communicating in a better manner and to search for alternative ways to communicate with students. On this basis, the joint reflections on interaction and communication with other teachers can support such intentions for a personal change of communication. Further, in the project, within moderated joint reflections about specific interaction scenes, an external moderator tried to explicitly ask for possible alternatives or focused on contrasting interaction examples. Changes in the ways of communication are not single causal events and cannot be directly forced, but might be initiated by offering occasions for change as tried in the project.

This increased consciousness and the starting attempts to change their own manner of communication should not be limited to diagnostic talks, but should be broadened to everyday mathematics teaching. In this regard, the analysis grid "Formal-In" will be used to carefully analyse transcribed episodes from the two videographed lessons of the teachers to search for hints and traces in which changed forms of interaction with students in the normal mathematics classroom can be reconstructed. Gellert (2007, p. 34) emphasizes that teaching can positively change if a more precise perception of teaching interaction leads to changed communication activities. There is a reasonable hope that a more careful perception of one's own mathematical interaction with children in diagnostic talk will shift forms of teachers' interaction from "transfer of mathematical knowledge" more to "investigation of the child's mathematical ideas and interpretation". The growth of consciousness of about one's own forms of mathematical communication in diagnostic talks will assumedly have effects on teachers' mathematical interaction in mathematics teaching.

The analysis of the interactive interplay with the ID and ED is used firstly to better understand discourse situations and secondly to discuss with teachers different types of "Formal-In", to focus their attention on communication behaviour and to slowly change their specific discourse style more towards *investigation* by reflecting on videographed episodes of diagnostic talks or classroom situations.

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