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ORIGINAL ARTICLE

Developing expertise: how enactivism re-frames mathematics teacher development

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Abstract In this article, we present a re-framing of teacher development that derives from our convictions regarding the enactive approach to cognition and the biological basis of being. We firstly set out our enactivist stance and then distinguish our approach to teacher development from others in the mathematics education literature. We show how a way of working that develops expertise runs through all mathematics education courses at the University of Bristol, and distil key principles for running collaborative groups of teachers. We exemplify these principles further through analysis of one group that met over 2 years as part of a research project focused on the work of Gattegno. We provide evidence for the effectiveness of the group in terms of teacher development. We conclude by arguing that the way of working in this group cannot be separated from the history of interaction of participants.

Keywords Mathematics teacher development · Awareness · Purposes · Collaborative group · Enactivism · Expertise

1 Introduction

We are interested in the journey of mathematics teachers from novice to expert and, in designing professional development opportunities, we have made use of Varela's (1999) arguments to develop a way of working in which novices can develop expert knowledge in the same way experts extend their knowledge. Varela's ideas are, in his own formulation, enactivist (1991, 1999; Stewart, 2011). We begin by setting out the enactive view of cognition and distinguishing our approach from others in the professional development literature. We show how enactive insights into developing expertise run through all mathematics education courses at the University of Bristol and draw out principles informing how we work with any collaborative group. We then analyze the work of one such group, providing evidence for our claims regarding novices gaining expertise. We conclude by arguing discussions of professional development need to be re-framed to take account of the context and community within which they take place.

2 Theoretical underpinnings: enactivism and 'purposes'

The enactive conception of knowledge is essentially performative, 'knowing is doing' (Maturana & Varela, 1987, p. 248); the terms can be taken as equivalent, so, we could say 'doing is knowing'. We acknowledge someone knows something if they respond in a given situation in what we deem to be an adequate manner. Maturana (1988) therefore links knowledge with effective behaviour. What this means for professional development is that we are interested not only in what people say they do, but also in what they do. What they know is literally what they do. The complex decision-making of expert teachers seems effortless and novice teachers can find it difficult to see what the expert is doing. However, for novice teachers, post hoc articulation can reveal patterns of some useful behaviours that could be repeated in awareness in the future.

Varela, Thompson & Rosch (1991) introduce the enactive approach to cognition by saying that perception and action are 'fundamentally inseparable in lived cognition'

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(p. 173)—perception is not the passive receipt of information, but an active process of categorisation made possible by our history of interaction with the world. The insight here is that 'the world is not something that is given to us but something we engage in by moving, touching, breathing, and eating' (Varela, 1999, p. 8). As we live, we literally create our world and are created by interaction with it. This is the biological basis of being. Over time, if novices are able to analyse their experiences, they literally come to *see more* linked to their actions. The complex immediate decision-making of the expert practitioner develops over time (Brown & Coles, 2000). The culture of a classroom can be viewed as emerging over time from the patterns of social interactions (co-ordinated behaviours) between teacher and students. Enactive cognition is, therefore, not an individualistic theory.

So, how does learning take place? What helps the process of developing such co-ordinated behaviours? In everything we do, we make distinctions; categorising is the basic mental function (for biological evidence of this claim, see St. Julien, 1997). We learn through adapting to feedback from the distinctions we make with our environment in a continual process (see Maturana, 1988).

Our theoretical perspective implies that there can be no fixed conception of what constitutes expertise in mathematics teaching. Behaviours that are effective in a classroom are constrained by the histories of interaction of teacher and students. What is effective for a teacher in one situation may not be effective in another. What counts as effective is any behaviour that is adequate in terms of the motivations or purposes in play at that moment.

From an enactive standpoint, what characterises expert (as opposed to effective) behaviour is not so much the adequacy of a particular response but the manner of analysis of behaviour that leads to an on-going co-ordination of effective behaviours over time. To paraphrase Varela (1999), 'deliberate analysis' (pp. 27–32) refers to the way experts are able to act without deliberation and yet, after the event, reconstruct the awarenesses that led to action. This post hoc deliberation provokes 'intelligent awareness' as it allows experts to unpick, if necessary, the reasons an action was taken, and hence open themselves up to alternative possibilities in the future. Using deliberate analysis in this manner steers a middle path between unreflective spontaneity of action (which is not open to analysis) and deliberateness (which characterises the way beginners are often in the position of needing to rationally decide each course of action). The expert avoids both extremes, both acting spontaneously and able to analyse afterwards; enactivism implies a commitment to the view that beginners can learn in this way too:

> Indeed, even the beginner can use this sort of deliberate analysis to acquire sufficient intelligent awareness to bypass deliberateness altogether and become an expert (Varela, 1999, p. 32).

Novices in any field are not able to make the distinctions of experts; they cannot see in the same detail, and therefore do not have the same nuanced repertoire of possible actions available. Not only that, but novices typically do not have the same resources available to aid their own development since their possibilities for analysis are limited by their possibilities for perception/action. To support deliberate analysis with novices, they need some sort of motivation linked to action. Simply wanting to get better does not seem to be focused enough. For a teacher educator working with prospective teachers to support their development, talking at the level of behaviour does not work because the prospective teacher would try to apply any tip blindly in their school culture, provoking comments like 'it didn't work'. Nor do discussions at a philosophical level such as 'what is mathematics?' seem to support effective behaviour. What does seem effective is making use of certain kinds of questions asked by prospective teachers, when these questions are provoked by interactions in the world of the classroom. We label as 'purposes' (Brown, 2005) questions such as 'How will I know what they know?'; 'What will they have done before?'; 'How can I share their responses?'. What seems to be common to these questions, and how we recognise them as purposes, is that they are in some kind of middle position between the specific detail of behaviour, and philosophical speculation. The questions, though likely to be about a specific task or experience, are easily generalised to other instances. We now recognise such purposes as motivations that are directly linked to action and can, because they are relevant over time, accrue a complex set of behaviours in the journey from novice to expert.

The expert practitioner seems to act in the moment from a complex range of possibilities that are all embodied (see a discussion of Damasio's (1996) explanatory principle of *somatic markers* applied within mathematics education, Brown & Reid, 2006). Somatic markers are created in the body by positive or negative affective responses to actions linked to specific purposes—according to whether that action furthered the particular purpose or not. A positive somatic marker attached to an action means that, in the same context, that same action is likely to arise as a possibility (and the opposite for a negative somatic marker). We need to have a purpose in order for actions to accrue the somatic markers that are necessary if we are to become fluent in making the complex decisions needed to survive in a classroom.

A crucial element, then, in being able to engage in 'deliberate analysis' is that action is supported and guided by purposes; since it is in relation to a given purpose that behaviour (post hoc) can be seen as effective or not (either by self or other). In Davis et al. (2009), there is a description of four factors in teacher development; the concept of purposes is a key element of one of these factors 'Teacher Positioning' (p. 153). Finding a purpose could be linked, conceptually, to Hodgen's (2005) notion of 'the facilitation of desire' (p. 5) as a key to learning. We view learning as an opening up of possibilities for action, 'extending the space of the possible' (Davis & Sumara, 2006, p. 57).

We now turn to a brief survey of professional development literature within mathematics education and show how our enactivist stance commits us to alternative ways of working with teachers.

3 Professional development

Liljedahl (within Brown & Coles, 2010, p. 377) categorises the literature on professional development of mathematics teachers into three strands: *content* (pertaining to teachers' knowledge and beliefs including teachers' mathematical content knowledge), *method* (focus[ing] on a specific professional development model), and *effectiveness* (look[ing] at changes in teachers' practice as a result of their participation in some form of professional development programme). We locate this article within the literature on 'methods' of professional development; while also, in our conclusion, arguing that enactivism implies a re-framing of teacher development and hence of these categories.

There are overlaps between our practice and collaborative action research (Raymond & Leinenbach, 2000) in that participating teachers are encouraged to develop thinking and practice around a common issue, and investigate their concerns in their own classrooms. A difference is we do not see ourselves as explicitly 'stimulating classroom reform' (p. 285); we work with whatever agenda a teacher brings, we have views on "good teaching" but do not believe starting from these supports a teacher's journey into researching and developing effective practice.

We also distinguish our methods from professional development models where planning lessons is part of a communal activity, within a cycle of design–teach–reflect, e.g., aimed at creating exemplary lessons (Huang & Li, 2009), or as part of lesson study (e.g., White & Lim, 2008). From our perspective, schools and teachers are too different to want to try and do the "same" lesson. We are not so interested in the content of lessons as in supporting teachers to develop so that they can effectively analyse and adapt their own practice, including lesson content.

The model, within mathematics education, that is closest to our way of working (partly because it, too, is not solely focused on the content of lessons) is Jaworski's (2003) conceptualisation of co-learning partnerships. Jaworski takes as a starting point the definition from Wagner (1997) of co-learning to mean:

> researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling (Wagner, 1997, p. 16).

However, she extends this notion of co-learning to include 'insider researchers', i.e.:

> those practitioners who also engage in research into teaching, and hence develop their own teaching. Although, predominantly, this means teacher-researchers … [i]t can include educator-researchers exploring processes and practices in teacher education (Jaworski, 2003, p. 250).

This is an important extension. Wagner's definition of co-learning implies a division between researchers and practitioners where their interests are seen as separate and the learning is partly 'about the world of the other'. Jaworski draws parallels between co-learning partnerships and the model of 'sustained interactivity' between researcher and practitioner in which:

> the goal of research [develops] from one of primarily informing the practitioner to one of jointly constructing knowledge through shared activity (Huberman, 1999, p. 291).

In other words in both co-learning partnerships and programmes based on the model of sustained interactivity, beginnings of the work are characterised by the researcher 'primarily informing the practitioner'. It is on this point that our approach diverges. Our enactivist principles imply that as you act, so you see the world. We never have a phase of work in which we 'primarily inform' in working with a new group of teachers, as we are wary of the patterns and expectations this sets up. Beginnings are important, and we deliberately place ourselves outside the frame of providing answers. We want to engage in deliberate analysis—with novice and expert teachers; and believe, following Varela (1999) that lack of experience is no barrier to being able to engage in this way of working.

While we find no practice that matches our own within mathematics education, outside the field, an influence on both of us is the book by Belenky, Clinchy, Goldberger & Tarule (1997); the characterisation of 'connected-knowing' groups gets close to our approach:

> In connected-knowing groups people utter half-baked half-truths and ask others to nurture them … members of the group must learn to know and trust each other. In such an atmosphere members do engage in

> criticism, but the criticism is "connected" … because members of the group shared a similar experience. This is the only sort of expertise connected knowers recognize, the only sort of criticism they easily accept (Belenky et al., 1997, p. 118).

We too place importance on discussion always being focused on something 'knowers recognise' and we deliberately begin any meeting with a task designed to create a shared experience (e.g., a lesson observation, or a reading, or a video viewing). By invoking the notion of a connected-knowing group, we are recognising the importance of the affective dimension in describing any method of professional development. One implication of recognising the biological basis of being is that the affective cannot be separated from the cognitive.

In the next section, we show how our approach to teacher development, informed by enactivism, translates into specific working practices.

4 Mathematics education courses at the University of Bristol

In an enactivist frame, '*instruction*, in the strict sense of the term, is radically impossible' (Stewart, 2011, p. 9), where instruction is 'a process of information transfer from teacher to pupil' (p. 9). The design, therefore, of professional development courses at the University of Bristol follows a different path to that of lectures covering content related to a syllabus that needs to be covered, given that the design principles are enactivist. These principles are applied to courses for prospective teachers, teachers in their first year of teaching and those studying for a taught (as opposed to research based) Masters degree.

Before illustrating the design of one of the core course units, we give some background context through the words of an external examiner for the Masters Pathway in Mathematics Education. Firstly:

> The mathematics education team at Bristol are, for the most part, well established and very many teachers in the area have studied at various times with individuals in the team. This has led to the establishment of a network of supportive and knowledgeable teachers who seem very happy to support initiatives in the university and host students in their schools as part of their MEd studies.

We are not in the position of needing to establish a way of working with resistance from the community. In many local schools, the majority of the mathematics teachers studied for their teaching qualification at the University of Bristol, in a similar way to how they will work on the Masters courses and experienced teachers get involved in supporting colleagues with their studies.

Secondly (continuing the words of the external examiner):

> The standards set and achieved in the Mathematics Education strand of the MEd have remained impressively high during my period as external examiner. Much of the work could have formed the basis for PhD study. I suspect this is amongst the very best Masters Level courses currently being offered in the UK.
> The pathway is strong and coherent. Students are introduced to a wide and rich range of ideas in mathematics education and move towards expertise in the field.
> The nurturing of individual students—and their subsequent development over the year—has been impressive.

Such accolades support us in our belief in the effectiveness of the ways of working on the courses, and, consequently, we are able to focus our attention on supporting and challenging individuals rather than on delivery of content. Our interventions are contingent upon the learning of the students, the 'things that were *already possible* at that stage of development' (Stewart, 2011, p. 9). We cannot know what those interests and concerns will be independently of the students.

What does a syllabus look like for our Master's courses? We have copied parts below:

> Professional Development through Collaborative Working on an Issue in Mathematics Education (20 credit points)
>
> 10 meetings of two hours over 2 terms, 5 in each term, with the rest of the work being school-based with some support from the group leader

Unit Aims

- Carry out an individual school-based action research project on an issue in mathematical education.
- To work as part of a collaborative group on the range of issues within the group.
- To reflect on effective ways of working in such a group.

Statement of Learning Outcomes
After taking this unit, the student should demonstrate that they:

- have knowledge of a range of important issues related to action research in mathematics education
- have carried out an investigation over two terms into an issue developing skills in action research
- have developed as a reflective practitioner.

In addition to the aims and outcomes outlined above, the methods of teaching are specified as:

> Participants will be part-time students and mathematics teachers in local schools. Activities are given such as the keeping of a research diary to support participants in identifying their issue to be worked on as part of the process of action research, with discussion in early sessions of how to approach, monitor and evaluate stages of the action research process.
> In university-based sessions members of the group will:

- negotiate a programme of inputs targeted to their needs
- share their current thinking with the wider group for critical comments as their individual work develops.

> There is also a school-based visit by the group leader acting as a critical friend.
> The assignment for assessment can be presented as a portfolio or as a 4000-word report of the research undertaken.

The important points to note here are that structures are specified without content. The action research process is also not specified, there are many different alternatives, and students can use the action research process as best suits their purposes.

To summarise, our enactivist stance commits us to the following principles to support collaborative groups of teachers, both on Master's programmes and small-scale research projects:

(a) the group size should be less than or equal to 10,
(b) meetings should be spread out over an extended period of time,
(c) the teachers should come from a range of schools and be volunteers rather than conscripts,
(d) the leader of the group sets up a loose structure for meetings and time is given to each participant to discuss their emerging thoughts about their issue,
(e) the leader of the group gives individual readings in between meetings to support participants thinking about their issue, or there could be tutorials for participants between meetings linked to their Masters study,
(f) the leader(s) of the group will make one or more visits to each teacher's school to further support thinking about the issue and/or data collection, and, in some cases, the teachers visit each other's classrooms.

Alongside these principles is the need to establish norms for discussion in meetings. The leader of a group will usually be explicit in the first meeting about participants trying to be comfortable with silence, paying attention to others who may be about to speak, and talking through the detail of experience. It is common for the leader, especially in early meetings, to question the meaning of a phrase or word someone else uses, or to question an underlying assumption or judgment behind something being said. Taking such a questioning stance serves both to model a way of interpreting what others say that stays aware to assumptions we bring, and also to undermine expert-novice boundaries by disrupting any sense of the "infallible leader". There is of course a lot more that could be said about the ways discussions are facilitated and chaired. Our focus in this article, however, is on more structural aspects of the way of working.

In the next section, we analyse the workings of one particular collaborative group that ran via the University of Bristol, supported through a grant from the UK's National Council for Excellence in Teaching Mathematics (NCETM). We give further evidence for the effectiveness of our way of working and illustrate in more detail the principles articulated above. In this group, we also set out with a mixed group of experienced teachers and novices.

5 Evidence of effectiveness within one collaborative group

The NCETM project that is the focus of this section comprised a group of three newly qualified teachers, who had all just finished a year's teacher training course at the University of Bristol, two experienced teachers (one of whom was Alf) and three teacher educators (one of whom was Laurinda). These teachers wanted to work with Laurinda and each other on the work of Gattegno. There was a deliberate mix of 'novice' and 'expert' teachers, with the assumption however that everyone would be learning from each other.

The written aim of the project was:

> establishing a collaborative cross-school research group of teachers responding to […] student teachers' interest in the work of Gattegno, to investigate the effective use of current resources to support the learning of mathematics and to develop new ideas. We aim to support these new teachers to the profession in continuing their reflective and research practices (https://www.ncetm.org.uk/enquiry/5207, accessed 8 Feb 2011).

The practical applications of Gattegno's ideas are well documented in his writings (e.g., 1971, 1987) and we offer a summary below since some familiarity with his ideas is needed to make sense of the evidence we later present of teacher development.

5.1 Gattegno's approach

Learning is an active process and learning mathematics is *mathematisation* rather than memorisation of skills. We possess *functionings*, i.e., '[n]one of us remembers [our native language], we function in it' (1971, p. 12). Gattegno talks about having to pay the price for using memory, which is needed for arbitrary (Hewitt, 1999) things such as the link between the symbol, '9', and the word, 'nine'. Having paid the price to learn the first nine symbols and a few extra sounds such as '-ty' and 'hundred', the rest of the number system can be worked on directly through perception, action and functionings, working with Cuisenaire rods or for the structure, through a chart that Gattegno devised (Brown, Hewitt & Tahta, 1989, p. 57) (Fig. 1).

These functionings, the mental powers of children that can be brought into use by teachers, are: (1) the power of *extraction*, finding 'what is common among so large a range of variations'; (2) the power to make *transformations*, based on the early use of language '*This* is *my* pen' to '*That* is *your* pen'; (3) handling *abstractions*, evidenced by learning the meanings attached to words; and (4) *stressing and ignoring*, without which 'we can not see anything' (functionings paraphrased from Gattegno, 1971, pp. 9–11). Gattegno also has suggestions for how these powers can be 'used in the process of education' (p. 16): (1) students can notice differences and assimilate similarities so ask them to tell you what distinctions they are making—what is the same? what is different?; (2) students can use their power of imagery, so ask them to shut their eyes and respond with mental images to verbal statements; (3) students can generalise given that algebra is a fundamental power of the mind' (linked to abstraction) (distilled from Gattegno, 1971, pp. 22–29).

As an example, another resource used by Gattegno, in seeking to make mathematics visible and tangible to learners, is the geoboard, where learners explore placing elastic bands on a grid of nails (or applet equivalents). One powerful activity on a nine-point grid is asking learners to make all the triangles they can (Fig. 2).

The challenge leads to learners using their powers of discrimination as they make the task manageable in some way through classifying, deciding which triangles are the same, which are different and explore congruence and similarity without naming these concepts until they can be perceived through abstraction.

The term 'awareness' is a technical term used frequently by Gattegno in such phrases as 'Only awareness is educable' and 'awareness of awareness'. Gattegno wants students to use their mental powers, and the teacher subordinates teaching to their learning. For example, in offering two examples and asking what is the same or different, the teacher works contingently with the student responses.

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Fig. 1 A Gattegno tens chart

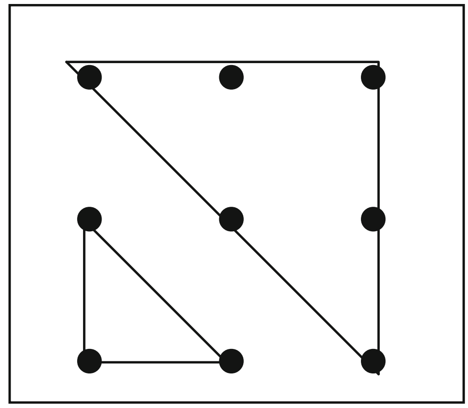


Fig. 2 Are these triangles the same or different?

Through our own work on this project, we have become convinced that Gattegno's ideas are consistent with enactivist principles, e.g., his focus on awareness and its link to action. We plan to go into these connections more deeply in a future publication.

Having given this context—on the NCETM project all the participants wanted to work on the practical applications of Gattegno's ideas—we now move to analyzing the outcomes of the project.

5.2 Methods of data analysis

Enactivist thinking leads to a privileging of final pieces of data in the process of analysis, for reasons linked to the concept of 'equifinality'. 'Equifinality' is associated with early cybernetic, or general systems, thinking (von Bertalanffy, 1969). At this time, it was assumed that systems move from one equilibrium position to another and that a system not in equilibrium was in danger of collapse, and hence would resolve itself as quickly as possible into an equilibrium position, before being triggered out of it by some other event, only to return again to equilibrium. The concept of 'equifinality' described how such stability seeking systems would often appear to reach the same equilibrium position from a wide variety of initial conditions.

Later systems thinking, influenced by the mathematics of chaos and complexity, has come to recognise the incredible sensitivity on initial conditions that certain systems display (famously, the weather). It is now understood that systems are able to exist and operate far from equilibrium (Juarrero, 2002). One insight here is that complex systems are finite. Although existing far from equilibrium is inherently unstable, systems (such as humans) are able to maintain such instability for a finite time before collapsing.

It might seem as though collaborative groups, with their myriad complexities of individuals and relations, would be complex systems existing far from equilibrium. Yet a surprising observation is that, although individuals differ and particular routines and language vary, there is something identifiably the same about, e.g., collaborative groups at the University of Bristol (or more generally any class of an experienced teacher). One aspect of what stays the same is the way (if not the content) people talk to each other. We believe, therefore, that the patterns of language in a group or classroom do not form a far from equilibrium system, but rather consist of a simpler, equilibrium seeking system; and therefore there can be some sense of an equilibrium that is achieved in each group, independent of the varying initial conditions. In other words, our experience suggests that equifinality operates across patterns of language in groups of experienced teachers. Plausibility is added to this suggestion by noting that equifinality is a concept still used today in some branches of family therapy to describe how family patterns can become constrained (Stroh Becvar & Becvar, 2000).

On the assumption of equifinality, the final piece of data should exhibit the most stable patterns, or perhaps the patterns that are most fundamental. Enactivist analysis proceeds by identifying patterns in the final data item, and then tracing these patterns back through the rest of the dataset. The intention in the tracing back is not to tell any story of causality, but rather to trace the emergence of pattern.

We collected audio recordings of all the group meetings and, as part of the final meeting, we asked the teachers to discuss what they found valuable about their participation in the project being particularly interested in aspects that they all thought were important. There was also time in the final meeting for the teachers to work on distilling their learning into a common framework for dissemination, including writing describing the impact of the project on their teaching. We were then able to look back at the data we collected from earlier meetings and where, for instance, presentations to the rest of the group were highlighted as being important, we track back in the data and look at those events. In our analysis for this article, we were concerned with patterns in teacher development and the role of purposes.

In the next section, we present three types of evidence for the effectiveness of the group. The first (in Sect. 5.3) is from the discussion in the final meeting of the group. The second source of evidence (Sects. 5.4–5.7) is transcripts of audio data of sessions run in the group by each of the three newly qualified teachers, and the third source of evidence (in Sect. 5.7) is from writing done by each of the teachers at the end of the project.

5.3 Evidence from final discussion

We have evidence of development and learning in all the participants. We present here evidence of development and change in the three newly qualified teachers: Alistair, Barry and Louise (actual names, used with consent).

In a final discussion about what the new qualified teachers would take away from their involvement in this project, there were three types of experience all three of them identified as being important in their professional development. We list these categories below, giving one quotation and a short commentary on each.

> Transcription notation: We indicate whose session it was by putting an A, B or L before the timing). (.) indicates a pause of less than a second, (2) indicates a pause of 2 seconds. [*text*] indicates an editor comment. [text] indicates unclear speech and our best guess at the words. Three dots indicate some speech missed out of the transcription, generally to make the reading easier. *Italics* indicate that the words are spoken as if in the voice of another.

5.3.1 Thinking about their own practice:

> a lot of it's been a time for me to think about (.) what do I actually want to achieve in my classroom (.) I spent a long time thinking about what I thought mathematical behavior was (.) or is

In saying 'I spent a long time thinking about what I thought mathematical behavior was (.) or is' this teacher is articulating what we recognize as a purpose; since the teacher's view of mathematical behavior was directly linked to his actions in the classroom as he worked on ways of provoking in his students those behaviours he valued.

5.3.2 Working with more experienced others:

> as well as feeling more of an equal (.) it's good to know the experience is there (.) at the same time (.) so you can bring your questions and you know that there'll be people who've met these issues before or who can talk about them (.) so you know you'll go away with much more in depth understanding of what the issues are … all the group were working together … I think maybe they're learning too

This teacher is recognising an important aspect in the setup of the group that we described earlier in this article, that there was not a strong demarcation of roles between those working in school and university. We would agree with the final statement quoted above (referring to the teacher educators): ‘I think maybe they’re learning too’. The mix of experienced and novice teachers can be seen to support the development of the novices.

5.3.3 Leading a session in the group:

> it helped me quite a lot in the bit where we were asked to lead a bit ourselves (.) helped a lot in that because what we offered led to quite a rich discussion within the group and it had come from us and not from one of the more experienced people

We have taken this category (‘lead a bit ourselves’) to trace back in our data and explore what happened in the session where the newly qualified teachers led. We now move to the second source of evidence for effectiveness in which we analyse interactions between participants in each of the sessions run by Louise, Alistair and Barry. The selection of sections of the transcript for analysis was made by looking for sections that gave evidence of effective behaviours on the part of the teachers, in relation to a purpose.

5.4 Evidence of change: Louise, finding a purpose

In Louise’s session she began by playing a mathematical game with the group. Having finished playing, Louise set out the theoretical ideas that informed her choice. She is looking for games that ‘teach a skill’. The following exchange occurs between D and Louise:

L09.56	D	you’re saying using a task to teach a skill (.) so I can imagine someone coming back and arguing um (.) I don’t think you’re teaching a skill you’re practising the skill (.) I wonder what your response would be
L10.12	Louise	yes, in this one, in this game, yeah
L10.15	D	so you would agree that it’s practising a skill
L10.16	Louise	it seems like it (.) because it doesn’t seem like there can be a development

We interpret this last line as a re-reading of her frame that a task must ‘teach a skill’. Part of ‘teaching’ a skill, she now sees, is that there must be some ‘development’ in it. It is, of course, unclear what this means except that she does not think the game she offered now satisfies the criteria she is wanting. We see this as evidence that Louise has a new purpose (development of a skill) to bring to her use of games in the classroom. This kind of purpose is a pre-cursor to acting; it is the kind of idea around which complexes of behaviour can accumulate. In the learning of experts, we see the raising to consciousness of such purposes in planning and teaching as critical to future development. Not only do such ideas provide a principle for planning, they also provide a tool to aid reflection on experience and hence support further learning. A purpose such as finding ‘games that develop a skill’ fits the criteria for purposes, of being in a middle position between a philosophical level (e.g., ‘getting kids enjoying lessons’) and a specific behaviour (e.g., a particular game). In this case, the example of a philosophical level idea is likely to lead to blanket judgements of lessons or activities without supporting the identification of specific behaviours that were, or were not, effective. If Louise had ended with a specific game to try, again, this is not likely to help her find other games in different contexts. By identifying a specific feature of a game, she knows what she is looking for, both in planning and in the classroom, and therefore has a tool to use in her own future development.

5.5 Evidence of change: Alistair, using purposes

Alistair’s session with the group began with him handing out a transcript he had created from a lesson of his that took place several months before the meeting. He wanted the group members to try and analyse how he responded to students in discussion. He offers a comment of his own, after Laurinda pointed to a specific section of the transcript that interested her. Alistair comments on a teaching strategy he sees himself as using at that moment (which was his first turn in the transcript of the lesson):

A05.22	Alistair	I was intrigued that the first teaching strategy was [1] [seemed] [.] completely different to where I am at now
A05.26	Laurinda	and that’s how (.) try to capture that in some writing (.) even if it’s only a paragraph

Alistair is articulating his sense of change, and Laurinda, perhaps with an eye on the Master’s unit he was doing, encourages him to capture evidence of this change while he can. Later on in his session, Alistair muses on how someone in the group had described him, from watching the video of the lesson in question, as ‘not ever giving anything away’.

A17.05	Alistair	I’m intrigued by the [description of] not ever giving anything away (.) and actually maths-wise it feels like (.) maybe I’m not (.) giving anything away (.) having read this closely on the maths (.) but (1) I’d started to think that actually in every response (1) how do I guide where it goes then if I don’t (.) there must be times when I do give things away

We read Alistair here as grappling with a deep question. He seems to be saying that the evidence on the transcript is that he does not ‘guide’ the mathematics in his lessons—

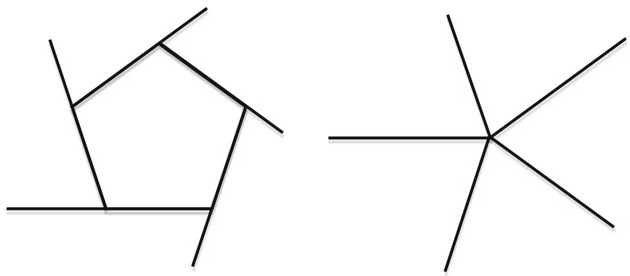


Fig. 3 Barry's image of a pentagon, and then with side lengths reduced to zero

and yet he is aware this cannot actually be the case. His focus on transcripts and how he responds seems to be driven by a desire to find out where and how he *does* influence and direct class discussions. He wants to hear and respond contingently on what students in his classroom say; yet how is this to be balanced with his need, as the mathematician and teacher in the room, to take responsibility for facilitating discussions such that students are supported in learning the things they need to learn: 'how do I guide where it goes then'? This phrase we recognise as a purpose; it is an idea or question that Alistair has clearly been grappling with. It is informing his collection of data for analysis of his own practice and we see evidence in Alistair's words that he has begun a process that is leading to new actions in the classroom. He has a conscious purpose and this purpose is accruing new actions, as can be inferred from Alistair's contribution at A05.22, where he articulates a sense of his own movement. His purpose is supporting him seeing his classroom in a different way, and acting differently within it.

5.6 Evidence of change: Barry, acting from a purpose

In the session Barry led, he showed the group a dynamic geometry file he had created, which displays regular polygons from 3 to 30 sides in an animation, with exterior angles shown by extending each side in one direction. A slider controlled the size of the polygon. At one point in his session D asked if he could see the image with the length of the sides of the polygons set to zero, this effectively altered the image to a set of half lines emanating from a point, equally spaced (because the exterior angles were still shown) (Fig. 3).

B29.36 Barry this end discussion has helped me a bit (.) because I'd never thought about (.) dragging it in to make a point (.) that's not what I'd (.) even thought about as (.) as something it could do (.) but what I want my sketches to be able to do is if someone has a query or just (.) *what happens if you do this* (.) I want it to be able to be possible to answer that (.) so it wasn't a problem for me just to drag the point into the centre because it has capability (.) and even if I've never thought of that before it doesn't matter because it can do that

We interpret Barry's words as indicating two movements. There is clearly a new action about the sketch he presented to the group—and an appreciation of the power of reducing the polygon side length to zero, so that you could directly observe that the sum of exterior angles was a complete turn. However, Barry also articulates wanting sketches to be able to answer 'what happens if?' questions from those who use them. There is clarity and strength in this description of what Barry wants sketches to do in general. Such an articulation of action linked to purpose, even if this is something he may have recognised before, is powerful in terms of the chances of these criteria being present for him as he next sits down to create a dynamic geometry image.

The statement 'I want it to be able to answer that … even if I've never thought of it' we recognise as a purpose—it guides planning and reflection. He has already created sketches (including the one shown in this session) that fit this criteria, and in the session Barry's use of the sketch was effective in relation to that purpose; he is becoming expert in relation to creating and using these sketches.

5.7 Similarities and differences across the three sessions

There had been few detailed instructions to the teachers about how to organise their 'session' with the group. It is striking that all three of them chose to do something active with the group, and then invite discussion (Louise got everyone playing her game, Alistair had a transcript for us to read, and Barry had a sketch for us to interact with). We see evidence here that the teachers had taken on one of the disciplines of our way of working, of beginning with a shared experience.

Without looking for this in the data, the three examples provide evidence of what we take as a progression of stages of development. In order to begin learning like an expert there must be some identification of a purpose—as exemplified with Louise. We have evidence that this purpose continues to be relevant for Louise, as she continues to research the use of games in her practice, and the topic is the starting point for a Masters' dissertation. Having identified a purpose, using 'intelligent awareness' provokes awareness—and we see this clearly in the data from Alistair's session in his articulation that what he saw himself do on a video from a few weeks before seemed 'completely different to where I am at now' (A05.22). The purpose that we read into Barry's words indicates a further development—of arriving at principles (in awareness) that will be on-going guides to effective behaviour, in this case in relation to developing dynamic geometry sketches.

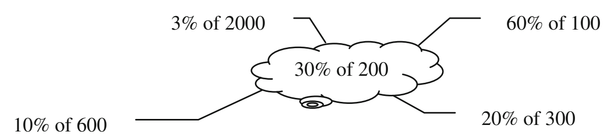
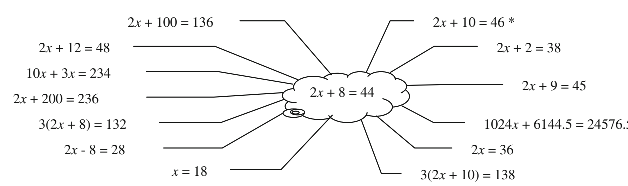
To conclude the analysis of evidence for teacher development, we move to our third source of data, which is writing done by each of the teachers at the end of the project.

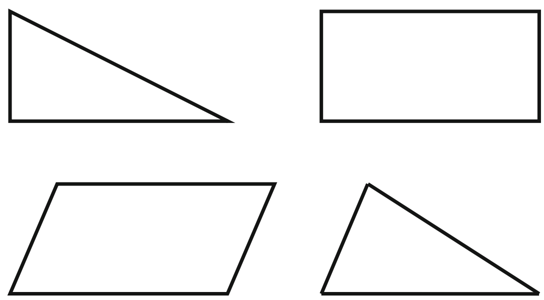
5.8 Evidence from teacher writing

All the new qualified teachers created a distilled report on their focus issue in a standard format that was suggested to them at the end of the project (see https://www.ncetm.org.uk/enquiry/5207, accessed 8 Feb 2011.) This writing provides evidence of, among other aspects, what they saw as the positive impact of the project on their students, e.g.:

> Pupils were motivated within the mathematics when their attention was to some other goal rather than the mathematics itself and this energized pupils (Louise)
> Pupils were engaged, interested and enthusiastic about the lesson.
> Pupils were happy to follow the construction methods and developed a deeper understanding than a traditional method.
> Pupils were engaged and enjoyed setting each other challenges. (Barry)

We have chosen to include Alistair's writing in more detail, as we see in it striking evidence of development and insight, for someone (at the time of writing) in their second year of teaching. We aim to set up situations (common experiences) from which teachers can make distinctions, in the same way that Alistair now does this with his students. We read in his writing that he has arrived at some answers to the question of what, for him, it means for students to be working mathematically, while at the same time, raising many new questions. Excerpts from Alistair's writing are below.

> **The Economy of Teaching Mathematics: noticing differences and assimilating similarities**
>
> *Background*
>
> The teacher's role is in focusing students' attention and supporting mathematical discussion. As teachers we have relied too much upon training students' memories and not enough on their 'automatic unconscious functions', 'The Powers of Children'.
>
> When I joined this working group I had many issues and questions to which I wanted to find definitive answers that would contribute to my aim of discovering the most effective way to teach mathematics. Many of my initial questions centred around what it means to work mathematically, and through being a member of this group I realised that making transformations and noticing similarities and differences are two behaviours I consider to be mathematical. Over the last two years I have in particular explored strategies that take advantage of students' ability to notice differences and assimilate similarities.
>
> *Results: Examples of the power of making distinctions in mathematics lessons*
>
> 1. Transformation webs
>
> The task given to the students is to 'find as many equivalent statements' as they can. I have varied the wording with which I tell this to the students: 'Find as many questions as you can that have the same answer', 'Find as many questions as you can that are the same as this one', 'If this statement is true, find as many other statements as you can that you know must also be true', 'Find as many equivalent statements to this as you can'. I always tell students to try to find ones that they think no one else in the room will have, in order to let them know that there is scope for creativity.
>
> 1.2 Percentages
>
> 
>
>
> 1.3 Transforming equations
>
> 
>
>
> * $2x + 10 = 46$ was given as an example
> Some of the more able students worked out the value of x and then used this to create *new* true statements involving x, instead of transforming the original statement. I also became aware that my example was too leading as students were only writing down equations of that form. One student had written $2x + 0 = 36$ which demonstrates the reluctance to change form.
>
> 2. Images
>
> The arrangement of shapes (below) was shown on the board and the pupils were asked to write down any similarities and any differences between any of the shapes. These could be similarities/differences between all four shapes, or between just two of the shapes.



Pupils seemed to find it harder to think of differences in the shapes. If a pupil told me they couldn't find any differences then I would choose two of the shapes at random and ask the pupil "Are these two shapes the same?". When the pupil replied "no", I would then ask "What is it that makes these two shapes different?". This always seemed to enable the pupil to see more, even though I hadn't given any suggestions as to what the differences might be. I've found that asking if the two triangles have the same area is an effective question for focusing students' attention and getting them to talk to each other about what they see.

Conclusions

The powers of makings transformations and making distinctions are constantly intertwined; in order to make transformations successfully you need to know which properties need to stay the same and which ones can be adjusted.

The use of images with 'the right question', and stimulus that encourages pattern-spotting can be effective ways of focusing students' attention on surprising similarities and differences, and can provide motivation for students to discuss and listen to each other.

Students gain the most from their powers of noticing differences and similarities when the culture of the classroom allows students to explore their mathematical ideas, be prepared to discuss and listen to each other, and not be concerned whether their offers are 'right' or 'wrong'. What students notice depends on what *they* see and this is not for the teacher to judge; the role of the teacher is to focus students' attention such that students can interact with each other discuss what they see with others.

Impact on my professional development

Throughout the year I've found myself writing down lists of questions and current issues for me in my teaching, only to find that this list keeps growing. I now have many more questions to investigate than at the beginning of the year but despite this I feel much better equipped to make decisions in my classroom. I've realised that through studying my classroom carefully I discover deeper complexity in my questions that always leads to further questions. Despite not finding definitive conclusions I gain awareness of the complexity of the questions and the underlying issues involved, helping me to discover effective teaching strategies and be more aware of the effects my actions may have in the classroom.

In this writing, Alistair has given evidence of his own learning. A particular aspect of this learning is moving away from absolutes, the need for perfection to a sense of 'effective' practice as being adequate or 'good enough' to the task of supporting the learning of his students in the classroom. To do this, he studies his classroom carefully, employing the use of video to focus on the detail of what happens to be aware of his actions and their effects. His decision-making is supported through purposes linked to actions (e.g., planning for students to be able to notice differences), and as he studies his classroom more questions are raised. He literally is able to see more and in turn, those new questions lead to new awarenesses and purposes. Through staying with the detail he sees more, his decision making is more complex and he is acting with expertise.

Alistair chose his own focus, yet his conclusions mirror the principles we set out above for working with teachers. We see evidence of the biological basis of being in the issues Alistair tackles—as he is grappling with what to do with the students so that they make distinctions.

6 Conclusions

The framework for our way of working in collaborative groups to support the development of expertise is informed by our commitment to the enactivist sense of the biological basis of being and the links between perception and action. To summarise our approach to teacher development, we have evidence for the effectiveness of the following principles that are linked to our enactivist commitments:

- Establishing ways of working from the first meeting of a group so that no one is set up as 'expert', through participants working on shared experiences, such as reading the same text and discussing issues arising, sharing writing in relation to a task, observing teaching together or focusing on an extract from a videotape of teaching in a non-judgmental yet critically questioning way (as you act, so you see the world—expectations must be established from the first moment).
- Focusing discussions on experiences of communal activities (knowing is doing).
- The teachers finding their own issues, their desires, with a central place for 'purposes'; discussions in the

> group are not at a philosophical level but rather a working through of experiences to identify common issues, leading to finding purposes that can accrue a range of possible actions, supporting flexible and adaptable behaviours in the classroom.

It could be argued that we are talking about the setting up of a generic collaborative group and in some ways this is true. Where is mathematics in all of this? We are ourselves mathematics teachers and educators and so what we notice and work on links to our desires, we learn about mathematics teaching because that, within these communities, is what we are interested in.

We do not believe that we have a recipe for the running of successful groups. However, in running this group, we wanted to research a way of working that is distinct from other models in the literature, and that has been effective in the local community and supported generations of prospective mathematics teachers to be seen as experts in a relatively short space of time. Members clearly did value this group and grew and changed over the course of its lifetime. It can be seen as a risk to begin work with others (teachers or students) knowing that, within a given structure, the actual issues to be worked on will come from the participants. We have grown to trust that issues always will arise to support participants in learning, particularly from a visible or tangible starting point; the risk is in having faith in our own capacities as conveners to impose enough of a discipline on the group to keep conversation focused on evidence, and in staying vulnerable enough to what is said that we are open ourselves to change, sharing our own learning in the group.

Earlier in this article we quoted Liljedahl (within Brown & Coles, 2010, p. 377) framing the professional development of mathematics teachers into three strands: content, method and effectiveness. While we placed this article within the strand of methods, our analysis of the workings of one collaborative group implies a re-framing. We believe it is essential, when investigating professional development, to look at features of the wider community within which courses or projects take place. From an enactive standpoint, the effectiveness of the way of working we have analysed in this article is impossible to separate from the rich history of interaction between generations of participants on courses at the University. We do not offer our practice as a method that could be adopted simply in other contexts; but more in the hope of supporting others make new distinctions in their own practice.

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