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ORIGINAL ARTICLE

Representation of the notion "learning-as-participation" in everyday situations of mathematics classes

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Abstract It will be argued that the recent discussion about "learning-as-participation" is primarily led rather on an abstract level and therefore needs specific mathematics classroom research in order to substantiate how these general theoretical perspectives are represented in social practices at school. Blumer's differentiation between "sensitizing concepts" and "definite concepts" in theories of social science that had been adapted to everyday classroom situations of teaching and learning mathematics will be applied in order to develop an empirically definitive concept for the description of forms of participation that facilitate the learning of mathematics.

Keywords Representation · Everyday mathematics classroom situations · Social interaction · Multi-age · Primary classes · Production design · Recipient design · Participationism · Learning-as-participation

1 Meta-theoretical and methodological considerations about representation and learning-as-participation

In the recent discussion in mathematics education, learning is often considered to be a form of participation in specific social practices:

> …human mental activity is neither solo nor conducted unassisted, even when it goes on 'inside the head.'… Mental life is lived with others, is shaped to be communicated, and unfolds with the aid of cultural codes, traditions, and the like (Bruner 1996, p. XI; see also Lave & Wenger 1991; Cobb & Bauersfeld 1995a, b; Cobb 2007; Krummheuer 2007; Sfard 2008).

There are several approaches that adhere to this general position, which Sfard (2008) characterizes as "participationism" (p. 76). From this stance, she claims, the metaphor of "learning-as-acquisition" has to be replaced by one of "learning-as-participation" (p. 92). Combining the words "cognitive" and "communicational", she introduces the neologism of "commognitive" (p. 83) in order to stress the insight that "cognitive processes and interpersonal communication are … different manifestations of basically the same phenomenon" (Sfard 2008, p. 83). In considering this conceptual shift, one must as well allow for a concept of teaching that is also an integral feature of the process of social interaction in the classroom. Teaching has, so to speak, a "systemic effect" on situations that enable certain members to participate as learners.

Sfard (2008) further develops her argument clarifying that mathematics and mathematics learning emerge in specific forms of discourse, and that a membership is won through "participation in communicational activities of any collective that practices this discourse" (Sfard 2008, p. 91). Becoming a member of such a discourse involves the learning of the rules and routines of these discourses.

As convincing as this general approach might sound, on an empirical level, it seems to me to be less elaborated with regard to how these forms of participation are shaped. According to Blumer (1954), any empirical theory in social

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science is characterized by a combination of "sensitizing concepts" and "definitive concepts". Sensitizing concepts offer rather a perspective and a framework in which way this theory approaches the empirical domains of interest. They are necessary elements of such theories. Definitive concepts in contrast contain empirical elements, which clearly help to describe what a certain phenomenon looks like.

> "Whereas definitive concepts provide prescriptions of what to see, sensitizing concepts merely suggest certain directions along which to look" (Blumer 1954, p. 7).

Sensitizing concepts can hardly be proved empirically. They function rather like axioms in a mathematical theory. Empirically related research questions ask less *whether* these concepts come to view in "reality", but rather, *how* they orient one's perspective in order to interpret this "reality". For the definitive concepts, however, one can ask *whether* they come to view in the chosen strip of reality and *how* they help to understand collected empirical material.

In my view, in most of these mentioned theoretical considerations of participationism, the notion of "learning-as-participation" has the status of a sensitizing concept. Sfard (2008) usually speaks of a "metaphor" (e.g., p. 92). Thus, there is a need to transpose this concept toward a definitive status.

Methodologically, one can address this need by referring to the notion of "representativeness of concepts" within the approach of grounded theory (Strauss & Corbin 1990, p. 190): The aim of qualitative research would be to find the representativeness of the developed theoretical concepts within the *interpretations* of the selected parts of reality. This would be unlike quantitative research, which aims at representativeness on the level of the *sampling* (p. 190 f). Typical for the qualitative research according to grounded theory is also that this representativeness is bound to specific empirical domains and is not understood in the sense of a universal generalization.

> "The purpose of a grounded theory is to specify the conditions that give rise to specific sets of action/interaction pertaining to a phenomenon and the resulting consequences. It is generalizable to those specific situations only" (Strauss & Corbin 1990, p. 251).

The specific situations of interest here are the everyday situations of (primary) mathematics classes (see below).

With regard to Blumer's differentiation between sensitizing and definitive concepts, it is clear that this claim for the representativeness of concepts holds only for definitive concepts. Thus, transposing the notion of "learning-as-participation" into a definitive status necessitates the creation of new concepts of a higher representativeness. In short, I name this endeavor the "representation of the notion of 'learning-as-participation' in everyday mathematics classroom situations".

Besides these meta-theoretical and methodological foundations, one also needs an initial understanding of how a process of learning-as-participation begins, continues and ends. Therefore, often it is referred to Lave and Wenger's approach. They initially developed the concept of "legitimate peripheral participation":

> "Legitimate peripheral participation is proposed as a descriptor of engagement in social practice that entails learning as an integral constituent" (Lave & Wenger 1991, p. 35).

Learning is then to be understood as the successful process of becoming a "full participant" (Lave & Wenger 1991, p. 37) in a certain discourse, step by step. However, the two authors deliberately do not apply their concepts to learning processes in schools (Lave & Wenger 1991, p. 39), although they underline the necessity of additionally focusing on these forms of social practices as well (p. 41). As sensitizing concepts, they offer a perspective on how to look at learning, namely as participation. However, it remains unclear in which forms of participation a learning individual is involved and how, e.g., these forms can be described with definitive concepts on an empirical level in everyday mathematics classroom situations.

In order to gain insights useful for the development of a definitive concept, it is still to be questioned whether learning mathematics is *represented* (taught) by some specific kind of participation in everyday mathematical classroom discourse and furthermore by which "cultural codes, traditions and the like" (s. Bruner), it unfolds in regular classroom situations?

From an interactionist and ethnomethodological perspective, *everyday mathematics classroom situations* are accomplished in the course of interaction by the participants of a class (Cobb & Bauersfeld 1995a, b). In a certain sense, the emergence of these situations is "conducted" by their own rules and routines, which resembles that which Bruner calls "cultural codes, traditions and the like". The participants accomplish through the turn-by-turn organization of their interactional moves an "interaction order" (Goffman 1983) that has its own status and dynamics.

Within this theoretical framework, Birgit Brandt and I developed several empirically definitive concepts in order to describe different roles of participation in everyday mathematics classroom situations (Krummheuer & Brandt 2001; Krummheuer 2007; Brandt & Tatsis 2009). In this paper, I will outlines these concepts and relate them to the participationist view of "learning-as-participation". Rephrasing Blumer's idea of sensitizing and definitive

concepts, the question of this paper is how is the sensitizing idea of learning-as-participation represented in the definitive roles of participation that emerge in everyday teaching–learning situations in mathematics classrooms?

2 An example: the analysis of the production and reception of mathematical statements in students' group work

I will introduce some *definitive* concepts that represent the sensitizing conception of learning as participation by illustrating them in a primary everyday mathematics classroom situation. Furthermore, this illustration serves also as a source for reflection on specific features of such everyday mathematics classroom situations and their implication on a participationist view on teaching and learning mathematics at school.

Methodologically, this research is based on micro-ethnographic approaches, such as conversation analysis (Sacks 1998; Ten Have 1999), analysis of argumentation (Toulmin 1969; Krummheuer 1995), analysis of production and reception, and the principles of comparison according to grounded theory (Strauss & Corbin 1990). These analyses demand a relatively highly resolved form of transcription of video documents of mathematics classroom interaction that include non-verbal and paralinguistic information. The limited space of this article does not allow further details. In the following, only the result of these intertwined methods of analyses can be presented.

The interactional processes in classes involve relatively complex structures of conversation that are not appropriately conceptualized by the dyadic structure of face-to-face interaction. If more than two people are participating in a conversation, one speaks then usually of a "multi-party interaction" or "multi-party conversation" (Sacks 1998). We use the term of "polyadic interaction". In such interactions, the use of the terms "speaker" and "hearer", which is relatively uncomplicated in dyadic interactions, appears here to cause fundamental difficulties (Goffman 1981), making amendments and modifications necessary. Instead of the concept of the speaker, the term "production design" will be introduced that designates any person who is involved in the production of an utterance and that person's role as he/she participates in this production. In a similar way, the concept of the hearer will be replaced by the term "recipient design", which indicates the different status that is apparent when the participant is addressed by another member in the interaction.

In the following, these concepts will be clarified in an example of students' individual work in a primary mathematics class. In order to show the applicability of the definitive concepts of the recipient design and the production design, I choose a relatively uncommon scene of a spontaneously emerging cooperation between two third graders in a multi-age mathematics class. Thekla (third grade), Jakob (first grade), Lenni and Salih (both second graders) are sitting together with the assignment to work on their weekly schedule. At such age-mixed tables, they are allowed to help each other or to look for help at another table as well (Fig. 1).

	Thekla (3rd grade)	Jakob (1st grade)
	Salih (2nd grade)	Lenni (2nd grade)

Fig. 1 Sitting arrangement at the table

Thekla is working on her mathematics assignment and has already finished two of her tasks, when Steven (third grade) comes over to the table. The transcript shown in Fig. 2 depicts only the appropriate pieces of interaction at the table that belong to the conversation of Steven and Thekla. For this reason, there are jumps in the line numbering. The children have been given three tasks: $360 \div 90$, $90 \div 30$, and $210 \div 30$. The transcript only deals with the work on the first task.

2.1 The recipient design

2.1.1 Concepts of the recipient design

Considering the complex structure of a polyadic interaction in general, one has to first take into account that a person who delivers an utterance does not necessarily address it to everybody in the acoustic reach of his/her voice. As in the example in line 67, Thekla addresses her words "I don't know" seemingly to Steven. Obviously, her three partners at her table also hear these words and Thekla does not undertake any effort to prevent that. She seems to tolerate their indirect participation. In general, in a polyadic interaction, there are participants who are directly "addressed" by a speaker and other recipients who are not equally involved in the developing conversation thus making the differentiation between *direct participation* and *not direct participation* (Goffman 1981). The listener to whom the speaker allocates the right to take over the next turn, we will characterize as the "*conversation partner*". Usually, this recipient status is associated with the obligation of a high level of attentiveness. Those who are also addressed but are not assumed to be the next speaker but may be later involved, we will call the "*co-hearers*". Those who are tolerated by the speaker, but are not considered in the same manner to take part in the conversation, are the so-called "*over-hearers*". Listeners excluded deliberately from the

Fig. 2 A piece of transcript from the conversation

67	Thekla	*looking up to Steven* I don't know (unintelligible)
68		say again how often does the four go into the thirty-six\
79	Steven	into the **thirty**-four/
80	Thekla	into the thirty-**six**\ .. *with her fingers* **four\eight\twelve\sixteen**
83	<Steven	two n sevn five times ty twenty-four
84	<Thekla	twenty twenty-four twenty-eight
85	>Steven	thirty-two thirty-six\ nine
86	>Thekla	thirty-two **thirty-six** \ *takes the top of her pen, writes*
89	Thekla	*while writing to Steven* what's up (unintelligible)
90	Thekla	*leans back for a second, counts two fingers and then writes again*
97	Steven	three into the nine/
98	<Thekla	*with her fingers* three six nine \ three *is writing*
99	<Steven	nine three times\ but it's easy
103	Steven	*to Thekla* but it's right or / .. but these are easy tasks

utterance we will call the "eavesdroppers". The four recipient roles can be presented in a table in such a manner as in Fig. 3.

2.1.2 Recipient analysis of the scene

Thekla is working on the task 360 ÷ 90. She first asks Steven a question with "I don't know (unintelligible) say again how often does the four go into the thirty-six\" <67, 68> and asks him with this statement for the solution of a division task that is actually connected in a more complex solution process than the actual task at hand. She selects Steven only through eye contact <67-68> as a *conversation partner*. Steven confirms this selection in <79>; he himself selects Thekla as a *conversation partner* and shows with his reaction (both thematically and socially) that he has noticed that Thekla has addressed him and he has accepted. Thekla then again selects Steven as a conversation partner for the solving process of subtask of 36 ÷ 4.

Thekla makes a note of the result of this subtask and then she counts quietly with her fingers for the solution of the next task, without involving Steven directly in the writing process or in her calculations. He can then be referred to in this status as an *over-hearer*. In this assigned status, he follows Thekla's working process attentively and extracts from this process enough information in order to shift back to Thekla as a *conversation partner* for the next task with "three into the nine/" <97>.

Steven's careful following of the conversation is accepted by Thekla and is subsequently used to involve him in the solution process for the next tasks. In the processing of the following tasks (not contained in the transcript extract), the calculation of the partial results is almost completely taken over by Steven. He then names almost immediately the shortened division task and the respective partial result, and is able in this way to actually revert back to recollected multiplicative number facts. Thekla receives the partial results as a *conversation partner* and incorporates them into the overall solution.

Meanwhile, the two second graders Lenni and Salih are working on a different painting task and are accomplishing a conversation that is apparently relatively independent of

Fig. 3 Recipient design

Accessibility to an utterance			
direct participation of the recipient to the utterance		**not direct participation** of the recipient to the utterance	
addressed by the speaker	**unaddressed** by the speaker	**tolerated** by the speaker	**excluded** by the speaker
conversation partner	**co-hearer**	**over-hearer**	**eavesdropper**

the conversation between Thekla and Steven. There is no further information about the activities of Jakob, a first grader. He seems to work independently on his worksheet. From the perspective of the conversation between Thekla and Steven, the second graders Lenni and Salih and the first grader Jakob are tolerated *over-hearers*. Neither Thekla nor Steven directly addresses these three classmates at their table, although Lenni, Salih and Jakob are able to acoustically follow the utterances produced by Thekla or Steven. The transcript delivers no indication that they pick up any idea of this conversation. However, they would have the opportunity to learn or at least to recapitulate a part of the row of fours (4 – 8 - 12 – 16 and 24 - 28), a strategic approach for a division problem (how often does 4 go into 36), and, additionally on the social level, they could learn how to involve a classmate in ones own attempts at problem solving.

2.2 The production design

With the help of the analysis of production design, it should be possible to more precisely ascertain which type of participation is taking place in the polyadic interaction.

2.2.1 Concepts of the production design

The fundamental idea involves the assessment of the responsibility or originality of the utterance of a speaker. An example might help to introduce this method of analysis (Fig. 4).

How original is Thekla's contribution in <86>, which responsibility can one ascribe to her reiterating the numbers 32 and 36 as part of the row of fours?

The responsibility or the originality that one inherits for what one is saying is related in great part to two components of an utterance. One can inherit responsibility or claim originality for:

- the syntactic structure with a specific word choice and form (formulation function) and/or
- the content-related (semantic) contribution (content function).

If a speaker is responsible in both components (syntactically and semantically) for his or her utterance, then we call him/her the "author" of the utterance. Speakers who neither take responsibility nor have originality for the semantic content of their utterances, we call "relayer" of an utterance, which might be the case for Thekla in the introductory example above. If a speaker takes over the identical formulation of the parts of a preceding utterance and with them attempts to then express his own new idea, we then name the speaker a "ghostee". If a speaker takes over the idea of a preceding utterance and then tries to express this idea with his/her own new formulation, we call these speakers "spokesman" (Fig. 5).

2.2.2 The analysis of the production design of the scene

Now, we will move on to dealing with the allocation of responsibility for the contributions of Thekla and Steven in the introduced scene. Thekla and Steven do not communicate the idea of breaking down the total problem into smaller problems by canceling the zeros. This is only brought into the interaction through Thekla's initial question. This idea is initiated by Thekla in her role as *author* through the formulation of the first task of "four into the thirty-six" <80>. This idea is then picked up anew for the following tasks, so that the other children can be seen accordingly as *spokesmen*, with Thekla as the *initiator*. Alternatively, Steven must have picked up her idea since

Fig. 4 The transcript

80	Thekla	into the thirty-**six**\ .. *with her fingers* **four\eight\twelve\sixteen**
83	<Steven	two n sevn five times ty twenty-four
84	<Thekla	twenty twenty-four twenty-eight
85	>Steven	thirty-two thirty-six\ nine
86	>Thekla	thirty-two **thirty-six** \ *takes the top of her pen, writes*

Fig. 5 The production design

	Responsibility for the **Content** of an utterance	Responsibility for the **formulation** of an utterance
author	+	+
relayer	-	-
ghostee	+	-
spokesman	-	+

he was able to reconstruct for himself, where Thekla was. He must close the gap from "four into the thirty-six" <80> to the outright task of 360 ÷ 40, in order to be able to find the next task in the book. For this inference, he can be seen as a *ghostee* of an idea.

The children figure the results of the shortened quotients through the listing of the times table as a further step in the calculation. Although they list the results partially synchronically and asynchronically, they appear not to be imitating each other, but rather to be expressing the idea of the times table by mentioning the number facts. The listing of the times table also fits the interpretation of the division problem as 'how often does (the divisor) fit into (the dividend)' that is first formulated by Thekla in <68>. Steven even later picks up this interpretation. For both of them, it is self-evident that the tasks can be solved in this manner, expressed at least in Thekla's action of noting the solution in some form in her notebook. The table of the production design shows additionally a column that refers to the mathematical idea that can be assumed to be expressed by the corresponding utterance (Fig. 6).

In summary, this collective work can be described with a view to production design as follows:

Thekla can be characterized as the initiator of the working schema of "canceling and listing of the rows", a schema that she adheres to throughout the assignment. Both children separately and as a pair produce in a "duet" the listing of the rows as a calculating strategy of the determination of the quotients. For third graders, they are accomplishing a profoundly standardized and routinized solution approach.

If one looks at the calculating model that stabilizes itself between Thekla and Steven, it becomes clear that almost all of Thekla's steps are activated in the function as an *author* and that Steven picks up the ideas executively as a *spokesman*. The calculating model is provided by Thekla and is no longer challenged in its appropriateness. Thus, no new ideas are picked up by Thekla, which could have offered her a substantial possibility for learning. This might be different with Steven since he finally (not included in this transcript; see Krummheuer & Brandt 2001, p. 135 ff.) refines the accomplishing strategy by referring to the multiplicative number facts of the division problems in a canceled form. For example, to his question "how often fits 3 into 21" in the last problem 210 ÷ 30 she suggests:

128	Steven	you just have to do three times seven \

Here, he acts in the status of an *author*. One can assume that Steven finally found out that a division task can be reformulated as a multiplication task and that one can then (relatively) easily apply the known facts of the multiplication table.

3 Conclusions and commentary on the analyses

A short scene from a mathematics lesson is introduced and reconstructed with reference to the concept of learning-as-participation. It was initially argued that the theoretical position of participationism that lies behind this notion is unbalanced toward sensitizing concepts and is lacking definitive concepts that apply to everyday mathematics classroom situations. From the interactionistic research tradition (in mathematics education), one conceptualizes these everyday classroom situations as interactive processes that develop according to their own routines, rituals, and order. Insofar, the question arises, in which ways the sensitizing concept of "learning-as-participation" is represented in the definitive interaction order of everyday classroom situations.

Referring to everyday classroom situations, one first must consider that the structure of the interaction is polyadic, which implies that the adaption of the structure referring to a face-to-face interaction is inappropriate with respect to both the role of the speaker and the role of the listener. In his/her utterance, a speaker can rely on formulations (syntax) and/or the content (semantics) of previously produced utterances, which might stem from a variety of (more or less) actively participating interlocutors. The concept of the production design attempts to clarify the specific relationship between these utterances. Similarly, the concept of the recipient design considers the fact that a person producing an utterance does not necessarily want to address it to all members involved in the social event.

From the analyses of the presented classroom scene of a multi-age class in a phase of individual paper work, it becomes apparent that:

- Students participating in a *production design* in the role of an *author* are not necessarily in a position of a learner. They either know already what is to be learned or they are at a stage of learning, where they do not recognize the weakness of their applied understanding of the problem situation and do not offer their interpretations. Only in the first case could it be called, in the sense of Lave & Wenger (1991), a "full participation" (p. 37) and it represents an aspect of the idea of learning-as-participation.
- Students participating in a *production design* in the role of a *relayer* can be taken to be at the very beginning of a cooperative learning process. They try to imitate what they hear and see and possibly this initiates reflections

Fig. 6 Analysis of the production design

speaker and function	utterance	idea of the utterance
	Other people responsible	
Thekla: author	Say again how often does four go into thirty-six\ <68>	Calculating the task as shortened division Division as 'fitting into the'
Steven: relayer	Into **thirty**-four/ <79>	Division as „fitting into the“
	Thekla	
Thekla: author:	*with her fingers* **four \ eight \ twelve \ sixteen** \ <80>	Listing of the row
Steven: spokesman	twenty-four <83> thirty-two thirty-six\ nine <85>	Listing of the row Quotient as the result
	Thekla	
Thekla relayer	thirty-two **thirty-six**\<86>	Listing of the row
	Steven	
Stevens ghostee	*orients himself in Thekla's notebook (recognizable from the subsequent linguistic exchanges)*	Tracing back to the unreduced task
	(Thekla)	
Steven spokesman	three into nine / <97>	Calculating the task as reduced division Division as „fitting into the“
	Thekla	
Thekla: spokesman	*with her fingers* three six nine \ <98>	Listing of the row
	Thekla	
Steven: spokesman	nine times three \ <99>	Listing of the row Quotient as the result
	Thekla	
Steven: spokesman	but it is easy <99> .. but these are easy tasks <103>	Working schema delivers the end result
	Thekla (Writing down the results.)	

inside their cognitive system that might lead to a restructuring of their definitions of the situation. In a definitive sense, this role in a production design could be associated with Lave & Wenger's concept of the "legitimate peripheral participation" (see below under recipient design).

- With respect to their learning-as-participating, students cooperating in a *production design* in the role of a *spokesman* or *ghostee* are somehow positioned between the role of an author and a relayer. They paraphrase a given idea in its presented formulation or they bring in a new idea not yet able to formulate it in a novel way. From my understanding of the recent discussion in the literature, these two forms of participation in a production design are *novel* representations of the sensitizing concept of learning-as-participation in a definitive form that suggest stages in a learning process developing from imitation to autonomy.
- Students participating as recipients of an utterance might also be in an attitude of learning. Generally, one has to consider that a student in any status of an emerging *recipient design* might be able to pick up some ideas that change his/her interpretation of the problem situation. Considering the dynamics of the interactional turn-taking process, the commitment of a recipient to listen and pursue the ongoing actions increases when the speaking person addresses him/her directly. In this case, the obligation arises for the recipient to possibly take over the next turn. That implies a certain degree of attentiveness, which can be seen as a positive condition of initiating a learning process. Considering the term of "legitimate peripheral participation", one would possibly only see it represented in the indirectly addressed statuses of the overhearer and the eavesdropper and would not identify this concept with the relayer in the production design.
- In the context of everyday mathematics classroom situations, bringing these results together on the definitive level of a theory, one can conclude for the notion *of* "*learning-as*-participation" that the described evolution by Lave & Wenger (1991) from "legitimate peripheral participation" to "full participation" can be represented on the level of definitive concepts as in Fig. 7.

With regard to everyday mathematics classroom situations, one has to take into account that the legitimate peripheral participation is not only bound to different statuses of the recipient design, but also that already within this sensitizing metaphor the change of a role within the production design might take place. The repeating or reiterating of someone else's utterance does not involve a substantial contribution to the solution of a mathematical problem but might initiate a restructuring in the cognition the student. This is even more likely if he/she acts in the role of a spokesman or ghostee. As a spokesman, a student paraphrases a mathematical idea that has been previously introduced by someone else and by putting this idea in ones own words might count as an indicator of an ongoing learning process. Replying as a ghostee can be taken to be a situation, in which a student wants to express his/her own new mathematical idea, though he/she still is not able to put this in his/her own words. In both cases, I see a form of participation that is more than just peripheral. These two forms are characterized by an active contribution to a joint mathematical solving process, which these students could not entirely accomplish by themselves. This would be the case if they could participate in the role of an author, which represents the conception of a full participation.

In closing, one might ask whether these differentiations on the level of definitive theoretical concepts about the notion of "learning-as-participation" are specific only to *mathematics* learning processes. From the participation-bound view, one hardly can think about learning without taken into account the specificities of the social situation in which it takes place. In this paper, I restricted the reflections to everyday mathematics classroom situations, where discourse about mathematics emerges. Looking at definitive forms of participation in these situations, of course, the learning of mathematics is not the principle concern. Mathematical meanings emerge in the interactions between teacher and students in different ways, and from this theoretical perspective mathematics never appears in its pure form but only always as a "local production" (Garfinkel 1972). Further research in mathematics education would be needed in order to clarify in which way the outcome of such local productions impacts the form of participation and also the learning of mathematics of the students.

Fig. 7 Summary

definitive concepts represent ⇒	**sensitizing concepts**
eavesdroppers, over-hearer	legitimate peripheral participation
co-hearer	
relayer	
spokesman, ghostee	not existing
author	full participation

4 Transcription legend

4.1 First column

Here, the line numbering is noted. The numbering refers back to the lines in the original transcript. During the working process, from time to time an extension of the numbering was used, for example 29.1 is an addition to the twenty-ninth line in the first draft. To some extent, there were also lines that were cut out, for example when the commentary or the description of non-verbal activities contained interpretations. For this reason, there are jumps in the line numbering.

4.2 Second column

Here, the (altered) names of active participants of the interaction are listed, in as far as the video recordings were able to extract.

4.3 Third column

It contains the verbal utterances (normal) without attention to punctuation; these utterances are amended through paraverbal information, such as stress and prosody (see below). Utterances that are not without some doubt are set in brackets; utterances that are completely incomprehensible are specified through (*unintelligible*) the non-verbal activities of the participants (*in italics*). The end of such an activity is then noted +, e.g., Wayne points repeatedly during the following utterance to the boxes on the worksheet until seventeen:

452	Wayne	look here \ here it is *points to the boxes* twelve thirteen fourteen fifteen sixteen seventeen + and here / in this box always tens

4.4 Paralinguistic special characters

Paraverbal information is identified by the following special characters:

.	pause (max. 1 sec.)
..	pause (max. 2 sec.)
…	pause (max. 3 sec.)
\	falling of the voice
-	voice stays in abeyance
/	lifting of the voice
then	bold for strong emphasis
y e s	earmarked for stretched out pronunciation

If an utterance directly connects with the antecedent, then it will be marked with #, as for example:

40	Sabrina	Aja \ Aja \#
41		*Aja is still standing behind the teacher, who is busy with another child.*
42		*As Sabrina calls, she turns and comes back to the table.*
42	Patrick	# (Sabrina) seventeen doesn't come out of this \ *erases*

In a crossover in speaking, the utterances in their written form are similar to scores in music; the lines to be read parallel are marked by brackets in from of the names ("<"), as for example:

454	<Efrem	and this box is always three four five six
455	<Wayne	yes \ yes *knocks with his pen on the*

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