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ORIGINAL ARTICLE

Modeling empowered by information and communication technologies

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Abstract In this article, we describe our work with mathematical modeling (MM) at different educational levels and discuss how the use of information and communication technologies (ICTs) empowered such work. Characteristics of two trends in research which have influenced our work are presented: one is a Brazilian perspective of MM, and the other is the use of ICTs in mathematics classrooms seen through the lens of the theoretical construct "humans-with-media". We introduce some key questions regarding the notion of mathematical model and the phases of the modeling process that were paramount for us. Finally, we describe and analyze two experiences using modeling in different educational contexts, and present some evidence of the empowering role of ICTs in such contexts.

Keywords Mathematical modeling · Information and communication technologies

Abbreviations
ICTs Information and communication technologies
MM Mathematical modeling

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1 Introduction

The aim of this article is to present our work with mathematical modeling (MM)[1] at different educational levels and discuss how the use of information and communication technologies (ICTs) empowered such work.

Our search for new teaching models centered on the students' work, and our interest in emphasizing their engagement in the educational process led us to the study of proposals based on two strong trends in Brazil. One is related to a particular Brazilian MM perspective we will describe later, and the other is the use of ICTs in mathematics classrooms considering the theoretical construct "humans-with-media" as developed by Borba and Villarreal (2005).

With these trends in mind, we decided to carry out two didactical projects at different times and educational levels, one at the secondary school level and the other at the university level. In both projects, our main concern was with the implementation of modeling in mathematics classrooms; however, during the projects design and their realization in classroom, the use of ICTs as empowering partners in these scenarios arose almost naturally.

The experiences that we report in this paper were developed with educational goals, and now we aim to look at them with an analytical purpose: provide evidence and discuss the ways that modeling is empowered by ICTs. In our analysis, we will consider, on the one hand, some specific characteristics of the modeling approach we adopted for our work, and on the other hand, the epistemological perspective of "humans-with-media" which emphasizes the role of media in the production of knowledge.

[1] In order to avoid repetitions, sometimes we will just use the word modeling instead of mathematical modeling.

In the next section, we depict the international debate about MM and the influence of work carried out in Brazil in this debate. Within this framework, we present some principles about modeling that we adopt in our work. In Sect. 3, we describe the epistemological perspective about ICTs as resonant partners. A review of how ICTs can empower modeling closes the section.

2 Mathematical modeling: defining a position

Although the first inspiration of our work with modeling has a Brazilian flavor, we would like to situate our decisions, discussions and positions within the international debate.

The international debate about MM and applications in mathematics education has been increasing over the last several decades. Many authors (Blum et al. 2007) recognize that the field is complex and difficult to survey due to the considerable work done during this period. Niss et al. (2007) present a vast number of issues and challenges or questions raised by this work. They also point out that, since ICME 3-1976, the programs of such congresses show evidence of the emergence of a research perspective considering applications and modeling in mathematics education. The international movement of MM gained momentum due to two significant events: the realization of the *International Conferences on the Teaching of Mathematical Modeling and Applications* (ICTMAs) since 1983, and the establishment of the so-called *Community of Teachers of Mathematical Modeling and Applications*, which, since 2004, is an ICMI study group.

MM is an old research process commonly used among applied mathematicians and specialists from different fields of knowledge involved in the creation of models to describe a given phenomenon and predict future behaviors. Nearly four decades ago, modeling also began to be associated with the teaching and learning of mathematics. According to Kaiser-Messmer (1991), "*The consideration of applications and modelling examples in mathematics teaching has once again become more prominent since the beginning of the seventies*" (p. 83).[2] This quotation seemed to point to a prominent role for applications and modeling in the mathematics curricula at that time. Many mathematics educators believed students should be given the opportunity to experience what mathematicians experience, including MM activities. But the mere "*consideration of applications and modelling examples in mathematics teaching*", as mentioned by Kaiser-Messmer (1991), seems to fall far short of providing the opportunity for students to engage in the complete MM process in school that mathematicians or other scientists do.

According to Blum et al. (2003), the term 'applications and modeling' has been used to "*denote all kinds of relationships whatsoever between the real world and mathematics*" (p. 153) where 'real world' is considered as "*everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics*" (p. 152). The author points out that, while modeling focuses on the direction going from reality to mathematics and emphasizes the processes involved, the applications focus on the opposite direction.

In the educational context, there are different ways of bringing applications and modeling into the mathematics classroom. Without the intention of presenting an exhaustive list, we will just mention some of them based on Borba and Villarreal (2005) and on our own experience:

1. Apply mathematical knowledge that has already been taught to solve a problem that can be real or artificial.
2. Show a real problem to motivate students to study a mathematical content that can be used to solve such a problem.
3. Work with projects in which the teacher chooses a theme from the real world and also poses the problems.
4. Work with themes from the real world chosen by the students, who also design projects and solve problems that they themselves have posed, with the help of the teacher.

Perspectives (1) and (2) are associated with applications of mathematics to the real world. In both cases, the main concern is with mathematics, and the main educational goal is to show the usefulness of mathematics through applications to the real world. According to Muller and Burkhardt (2007), these activities are *illustrative applications* and "*the student has no doubt as to the mathematics to be used—it is the topic just taught*" (p. 269).

Perspectives (3) and (4) favor the establishment of an educational scenario to develop modeling activities. In both cases, the main concern is with the real world, emphasizing an interdisciplinary dimension of modeling and promoting a non-internalist view of mathematics. The educational goals go beyond the mere application of mathematical contents, stressing, especially in the last case, the significant participation of the students in their mathematics classrooms as problem posers and project designers, according to their own interests. In line with Muller and Burkhardt (2007), such activities promote *active modeling* and "*a variety of mathematical tools will be useful for different aspects of the analysis… Choosing and using tools appropriately is the major part of the challenge to the student*" (p. 269).

[2] The paper shows different spellings for some words (modeling/modelling, emphasize/emphasise, etc.) since, in the quotations, we decided to respect the original spelling from the authors.

The fourth perspective, which emphasizes students' choice of the problems, as well as interdisciplinarity, has a strong presence in Brazil, and was the one that attracted our attention. Such a perspective introduces an emphasis on socio-political values and implies critical curricular challenges.

In summary, we can say that the ways of understanding applications and modeling in educational contexts are varied and may lead to different didactical proposals. In the research context, this heterogeneity is also present. In that sense, Kaiser et al. (2007) point out that:

> …various approaches promoting applications and modelling in school or university teaching come from very different theoretical perspectives spanning the debate from ethno-mathematics to problem solving (p. 2041).

In this context of diversity, Kaiser and Sriraman (2006) proposed a classification of international perspectives on modeling in mathematics education, and they observed that "*there does not exist a homogeneous understanding of modelling and its epistemological backgrounds within the international discussion on modelling*" (p. 302). According to Kaiser et al. (2007), these perspectives are characterized by different views about goals and intentions, the role of the context, and perceptions of the modeling process.

Among the perspectives developed in Kaiser and Sriraman (2006), it is worth noting the inclusion of the so-called socio-critical modeling which, according to the authors "*can be characterized as a continuation of the emancipatory approach*" (p. 306) associated with socio-critical endeavors of mathematics teaching. They assert that this perspective refers to socio-cultural aspects of mathematics education and is closely related to ethnomathematics. The perspective emphasizes the role of mathematics in society, and a main teaching goal within this perspective is the promotion of critical thinking and reflexive discussions among the students.

The socio-critical perspective is also rooted in a well-established modeling trend in Brazil. In fact, the only example of this perspective quoted in Kaiser and Sriraman (2006) article is a study by the Brazilian researcher Barbosa (2006).

In the Brazilian context, the educational concern with modeling is closely related to the ethnomathematics movement, launched by Ubiratan D'Ambrosio, and the use of MM as a teaching and learning strategy pioneered by the Brazilian mathematician Rodney Bassanezi (Borba and Villarreal 2005).

We came across this Brazilian perspective for the first time in an article published by Bassanezi (1994). Since the 1980s, he has been disseminating his teaching experience with modeling in different Brazilian contexts. Over the last several decades, this research area has undergone considerable development in Brazil. For example, Barbosa (2001a), Borba and Bovo (2002), Araújo (2002), Malheiros (2004), and Oliveira (2007), among others, have worked at different educational levels. Many of these authors have drawn on Bassanezi (1994) guidelines, of which we would like to highlight the following:

- To take into account the specific realities of every region and the students' interests, aiming at increased motivation and at an effective participation of the students in their communities or in a larger context in which they take part […]
- To appreciate the human resources, explore and develop teachers' and students' skills, making them feel able to give the community their contribution and form socially active individuals.
- To keep interdisciplinarity in mind. (p. 31)

These guidelines emphasize, on the one hand, the interdisciplinary character of MM, and on the other hand, the importance attributed to the participation of the students. Analyzing the first two guidelines, Borba and Villarreal (2005) consider that they "*are strongly related to our responsibility as educators to form democratic and participatory citizens in their communities*" (p. 46). In this way, such guidelines call socio-political values into play.

According to Barbosa (2001b), the Brazilian experiences with modeling have a strong anthropological, political and socio-cultural inclination since they are concerned with the students' interest and their socio-cultural contexts. "*This can be considered a hallmark of the Brazilian work with modeling…*" (pp. 1–2).

As in the international context, over the last several decades, there has been a growing movement of Brazilian mathematics educators interested in modeling. Barbosa (2007) reports an increasing production of theses and dissertations with modeling as their main focus. He also writes about the constitution of the *Working Group on Mathematical Modeling* inside the *Brazilian Society of Mathematics Education* and the organization, every 2 years, of the *National Conference on Modeling in Mathematics Education* since 1999. These facts, together with the existence of research groups at universities (such as GPIMEM[3] and NPMM[4]) engaged in inquiries about modeling in different educational contexts, and the recent launch of the *Journal of Mathematical Modelling and*

[3] GPIMEM: *Grupo de Pesquisa em Informática, outras mídias e Educação Matemática* (Technology, other media and Mathematics Education Research Group) from the State University of São Paulo.

[4] NPMM: *Núcleo de Pesquisas em Modelagem Matemática* (Group of Research in Mathematical Modeling) from the State University of Feira de Santana, Brazil.

Application in Brazil, are clear indications of the growth and importance of MM in the Brazilian context.

Finally, it is worth noting that the particular Brazilian perspective of modeling we have described above is closely related with *project work* as presented in Skovsmose (1994) within the theoretical framework of Critical Mathematics Education. The work of Greer et al. (2007) addresses some themes related to the socio-critical perspective. Among other topics, this paper discusses modeling social issues and how modeling can be used as a tool for critical analysis of various unjust and discriminatory practices in the USA. Julie and Mudaly (2007) also focus on learners' and teachers' engagement with the MM of social issues in South Africa.

In our country (Argentina), many local curricular documents suggest the development of modeling activities in mathematics classrooms at secondary school level. Particularly, they emphasize the relationships between the real world and mathematics through modeling processes, but most of the activities are *illustrative applications* to solve semi-real problems as defined by Skovsmose (2001). Although there are some publications suggesting the use of MM in classrooms, the engagement of teachers and students in *active modeling* is rare.

In view of this situation, and considering the particular Brazilian modeling perspective we have described above, we decided to design and implement some modeling projects in our educational contexts, adopting a modeling perspective characterized by the following principles:

1. Open nature of the activities posed to the students, due to the free choice of a non-mathematical theme of interest to study and make questions, and the absence of a pre-determined mathematical content to be taught during the class.
2. Interdisciplinary nature of the work.
3. Promotion of reflections about mathematics itself, the models created, and the social role of mathematics and MM.
4. Creation of a learning environment in which the students are invited to participate.
5. Domain of the whole modeling process considering all the phases as will be described in Sect. 4.1.

These principles framed our didactical projects. We consider this perspective to be radical, since it represents a challenge for the classical curricular structure in our mathematics classes.

3 Modeling and ICTs: resonant partners

In this section, we will first introduce our position regarding the role of ICTs in the production of knowledge. Then, we will focus on the relationship between modeling and ICTs. A brief review of research from the international and the Brazilian contexts closes the section.

3.1 Humans-with-media: our epistemological position regarding ICTs

The use of technology in mathematics classrooms is another important trend among the Brazilian trends in mathematics education research. As in the case of modeling, there are many perspectives regarding the use of technology in the school. Dichotomous positions considering the inclusion or not of computers, or the mere use of calculators in mathematics classrooms, are always present in educational discussions. The usual roles that have been assigned to technological media are auxiliary or motivational. Such roles fail to consider the presence of the technology as media to "think with" and produce knowledge.

In recent years, some researchers have reported on the central role of the media in the production of knowledge (Kenski 2007; Borba and Villarreal 2005; Noss and Hoyles 1996). Particularly, we have adopted the idea that knowledge is produced by collectives of humans-with-media. The notion humans-with-media[5] was presented in Borba and Villarreal (2005) and is associated with two main ideas. One is that cognition is not an individual enterprise, but a social one, which is why the construct explicitly includes humans, in the plural. The other key idea is that cognition includes tools, media with which the knowledge is produced, and this component of the epistemic subject is not auxiliary or complementary, but essential. Media are constitutive of the knowledge: different media produce different knowledge. The media define the practices, the contents and the ways of knowing. In this way, the presence of computers, calculators or any other media reorganizes thinking and alters the production of knowledge.

The above brief discussion about the use of ICTs in mathematics education suffices to clarify our position regarding their role in the construction of knowledge.

3.2 Modeling with ICTs

"*Technological impacts*" was one of the issues raised by Blum et al. (2003) in the *Discussion Document* prior to ICMI Study 14: *Applications and modeling in mathematics education*. They recognize that the availability of many technological devices, such as calculators, computers,

[5] The notion of humans-with-media has its roots in the *theory of reorganization* as posed by the Russian psychologist Tikhomirov (1981) and the ideas of collective thinking and technologies of the intelligence presented by the French philosopher Lévy (1993).

Internet, and computational or graphical software, as well as all kinds of instruments for measuring, for performing experiments or for solving daily life problem, are highly relevant for applications and modeling. The authors consider that:

> These devices provide not only increased computational power but broaden the range of possibilities for approaches to teaching, learning and assessment. Moreover, the use of technology is in itself a key knowledge in today's society. On the other hand, the use of calculators and computers may also bring inherent problems and risks (p. 167).

The final publication of the contributions to the 14th ICMI Study offers various papers focusing on technology that we will review later.

The relevance of modeling and ICTs in mathematics education has also been stressed by Kaiser et al. (2006), who asserted that:

> Together with the use of information technology, the introduction of mathematical modelling and applications is a prominent general feature of the recent developments in the practice of mathematics teaching, especially with regard to secondary level teaching (p. 82).

This quotation emphasizes the relevance of both trends, but nothing is said about the possible synergy between them.

The amount of research produced by the international community of mathematics educators regarding problems related to MM in different educational contexts has been increasing in recent years, but there are still few studies considering the conjunction of modeling and technology. Authors such as Pead et al. (2007) have pointed out that modeling becomes more powerful with the use of ICTs. Their study focuses on the different uses of technologies, such as spreadsheets, graphing tools, dynamic geometry, applets, programming, etc., in learning mathematics through modeling. They show and analyze examples of activities for secondary and university levels. They conclude that technologies "… *allow students to work with mathematical concepts which are traditionally seen as too difficult for them*" (p. 318) and "*challenge the view that applications and modeling can only be introduced after the student has developed all the required mathematical knowledge*" (p. 318). These conclusions refer to the empowering role of technologies in the construction of mathematical knowledge. It seems to us that the authors' epistemological position regarding the use of technology is compatible with the notion of humans-with-media.

In her paper for the 14th ICMI Study, Stillman (2007) raises several issues related to perspectives in applications and modeling at the secondary level. One of them is related to the use of technology. On the one hand, she asserts that "*Technology allows more authentic modelling situations*" (p. 467), and on the other hand, she warns about the possibility of "*hiding the mathematics into "black boxes" as commonly happens with sophisticated workplace instruments*" (p. 467). Lingerfjärd (2007) refers to the rare inclusion of modeling in teacher education programs. Among some possible obstacles, he points out that MM often requires the use of technology, and many mathematicians or mathematics educators who work in teacher preparation may reject such approach. Both articles make reference to different kinds of problems associated with ICTs.

In our analysis of the Abstracts of ICTMA 14, we were also able to find some presentations that make reference to the use of technology in modeling activities from different points of view. Some of them mention only the auxiliary role of technology to make long or tedious computations more quickly, or even solve some calculations without knowing an algorithm yet (e.g., multiplication of matrixes). Other authors mention concerns regarding how the use of ICTs can promote or hinder the development of modeling competence. Others refer to the possibilities of visualization and simulation that particular dynamic software allow (e.g., Ndlovu et al. 2009). Weitendorf (2009) emphasizes that, with the use of technology, some problems become trivial while others can just be solved with it. This author proposes analyzing the role of technology in the modeling cycle. Haapasalo (2009) presents examples of geometric modeling and emphasizes the fact that the preparation of meaningful situations using technology allows the students to be "designers of their own learning". Geiger (2009) investigates the factors that may affect teachers' adoption of technology with the aim of improving students' modeling activities at the secondary school level.

Now we turn our attention to Brazilian work. In this context, we will concentrate on the work of the research group GPIMEM. Many of their (present and past) members have produced numerous studies focused on the interplay between modeling and ICTs considering the notion of humans-with-media as the theoretical framework to analyze this interplay. Examples of such studies are Borba and Penteado (2001), Araújo (2002), Malheiros (2004), Diniz (2007), and Borba et al. (2007).

Borba and Penteado (2001) present several examples showing uses of ICTs in different educational contexts. They emphasize the possibility of experimentation allowed by the ICTs within modeling projects. Araújo (2002) followed the development of students' modeling projects in a calculus course for chemical engineering majors, observing how the students used the computer to build graphs and study some functions. Malheiros (2004) studied the way that biology majors used the Internet as a source to develop modeling projects in the context of their mathematical course. Diniz (2007) developed his work in this same educational context. He investigated how students use ICTs in MM projects, including the Internet and the use of e-mail in activities of simulation, predictions, inquiry and communication of modeling experiences. From another educational context, extensive experiences and research results about online education for teachers are presented in Borba et al. (2007). According to these authors, the use of Internet transforms modeling projects and, consequently, the modeling process itself, by allowing participants to collaborate in inquiry activities, discuss, and communicate about some topics.

Summarizing, Borba and Villarreal (2005) emphasized that the particular Brazilian modeling approach we have described in Sect. 2 "*becomes even more powerful with the use of new technology and can bring substantial change to curricula developed inside and outside the classroom*" (p. 29). In this way, they recognized that modeling and ICTs are resonant partners in mathematics education. This is also the way we understand the empowering role of ICTs in modeling scenarios.

In the next section, we first introduce a brief discussion concerning two key questions related to modeling that were relevant to us. Then, we present two examples from two didactical projects we carried out, one at the secondary level and the other with a small group of university students. The examples we present come from the contexts of these two projects. In all the examples, we look for evidence of the ways in which ICTs empowered modeling processes.

4 Modeling with ICTs in action

4.1 Two preliminary key questions related to modeling

Two key questions were paramount in our work:

- What is, or what can be considered to be, a model?
- Which are the phases or sub-processes of a MM process?

Although we recognize that many authors in the international context have answered these questions (see, e.g., Blomhøj and Højgaard Jensen 2003; Blomhøj 2004), we will refer to answers provided by some Brazilian authors because they were the ones who initially inspired our work with modeling. These authors' notions about mathematical model and the phases of modeling were also discussed with our students during the implementations of our proposals.

According to Bassanezi (1994)

> … a mathematical model is almost always a system of equations or algebraic inequalities, differentials, integrals, etc., obtained through establishing relations among variables considered essential to the phenomenon under analysis (p. 31).

Since the mathematical contents mentioned in this definition seem too advanced for young students, we expanded it with the characterization posed by Biembengut and Hein (2000) in order to have an adequate definition for our younger students:

> …a set of symbols and mathematical relations that intend to translate, in some way, a given phenomenon or real problem situation, is denominated mathematical model… A model can be formulated in familiar terms, using numerical expressions or formulas, diagrams, graphs or geometrical representations, algebraic equations, tables, computational programs, etc… (p. 12).

This characterization opens up the possibility of bringing MM closer to young children, and encourages us to propose modeling experiences with students from different educational levels, as we will show further.

Considering our second question related to the phases of MM, we found very illustrative and useful the diagram in Fig. 1 that shows the phases described in Bassanezi (2002).

This diagram was particularly important for our work, since we used it as didactical tool while we were planning the activities for the students. At the same time, it was a useful tool for the students to reflect about their own modeling processes while working on modeling activities.

As illustrated in the diagram, once the theme of interest is selected, the MM process begins. After choosing the theme, one can pose problems or questions associated with the theme and start searching for data or maybe designing an experiment to obtain them (experimentation). The abstraction is the phase in which variables are selected and hypotheses or conjectures are posed. When the questions or problems posed in natural language are translated into the mathematical language, a mathematical solution is

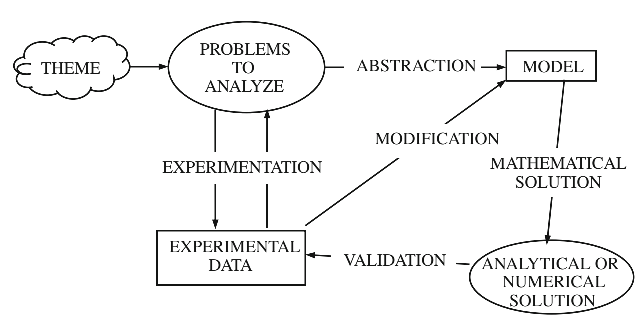

Fig. 1 The modeling process

obtained. The process of acceptance or rejection of the model is the validation. In this phase, the model is checked against the data, followed by analysis of whether it represents the best solution for the problem according to the original goals. If the model is rejected, a phase of modification may start, and a new cycle is initiated again.

This short description of the MM process shows its complexity as a research procedure. At the same time, it also shows the complexity of implementing didactical projects according to the principles of the modeling we decided to adopt.

4.2 Secondary students and their modeling process with ICTs

4.2.1 Experience 2004: a non-traditional context

During 2004, a group of three secondary mathematics teachers (Mina among them) and a university teacher (Esteley) participated in an innovative experience (*Experience 2004*) in mathematics education developed with the aim of implementing MM projects in three secondary schools in Córdoba (Argentina). A more detailed description of the *Experience 2004* can be found in Cristante et al. (2007), Esteley et al. (2007), Marguet et al. (2007) and Mina et al. (2007). An important challenge for the teachers was to produce classroom activities based on research findings related to modeling and according to the principles we have adopted and described at the end of Sect. 2.[6] The teachers coped with this challenge by creating didactical projects adapted to the traditions and culture of each of the three schools as well as to the characteristics of each teacher.

Two of the teachers decided to begin the school year in a traditional way and then move on to modeling. They worked with 72 pupils aged 15–16 years old. Forty-four pupils were with one teacher in just one class in one school, and 28 with the other teacher in another. The third teacher designed and implemented a modeling approach from the first day of classes with her 120 students aged 11–12 years, divided into three classes. In spite of some differences at the beginning of the experience, at different times, spaces, and environments, the three teachers followed the same pattern in their classes with respect to the organization of the modeling activities.

In all the courses, the MM process and the definition of model were objects of teaching. In this process, each teacher discussed the phases of MM with their students using the diagram shown in Fig. 1 as a means for understanding and reflecting about mathematics, problem solving and the MM process. At that moment, problem posing activities were developed with different emphases in each course. In all of them, teachers opened discussions for students to reflect on many mathematical tools and concepts that were emerging around those problems. These activities satisfy mainly principles 2, 3, and 4 outlined at the end of Sect. 2.

The next step in the *Experience 2004* was the time for the students to work in groups and develop a complete process of MM. With the teacher's guidance, most of the groups of students were able to select a theme of interest, pose a problem, select variables and construct a model. These activities are related to principles 1 and 5. In this way, particular thinking collectives of students and teachers were constituted. Some groups made use of analytical representations, others Cartesian graphs, tables or geometrical figures, drawings or diagrams.

When the groups of students finished their modeling process, they had to present a written report on the complete process using a template created by the teachers with a word processor. The final step was sharing the knowledge generated. At the end of the school year, a meeting was organized that took place in one of the schools. All the students from the five courses, some principals, other teachers, and some parents participated in the meeting. Each course selected, by vote, one project to be presented orally in the meeting, and the final presentations of each of the five studies were made using computer slides prepared according to a template provided by the teachers. One of those projects will be presented in the next section.

During their work, the students were supported by the teachers with the help of "others". Those "others" were human actors: members of the team, parents, professionals or any person who was knowledgeable about the theme selected by the students; and non-human actors: books, the Internet and mathematical software. It is important to note that among the three secondary school teachers only one of them had extensive experience with ICTs, but the interactions with such "non-human" actors became a new challenge for all three of them. They accepted the challenge, and during the MM process in their classrooms, the three teachers decided to use a particular user-friendly non-human actor, *Graphmatica*, a freeware equation plotter with numerical and calculus features.

Related to this choice, and without elaborating, we want to briefly portray a particular empowering relationship between modeling and ICTs. It seems that when ICTs are used in a modeling scenario (*Graphmatica* in this case) the teachers are also able to create new meaning for both, modeling and ICTs. One of the teachers said:

> Now, when I try to suggest to other teachers that they use Graphmatica in the way we did [she means, in an exploratory way and without knowing a model

[6] From now on when we mention the word principles we will be referring to those stated in Sect. 2.

previously], it seems to me that it is not easy to understand if you are not doing mathematical modeling.

We recognize in her words the establishment of a relationship between modeling and ICTs. Modeling seems to be offering the space for the teacher to construct a new meaning for the use of ICTs, and ICTs are the media to think with and produce MM processes.

4.2.2 *Timber transportation*

In this section, we analyze some outcomes of a modeling process related to timber transportation, carried out by a group of 11–12-year-old students, and how ICTs empowered their work. These students' project was one among others developed in the didactical context of *Experience 2004*. In this context, we decided to face the challenge of working with these young students, and many problematical concerns arose: What type of modeling process could these children carry out if we expected some level of autonomy? What degree of mathematical sophistication in their models could be expected? Which sources of relevant information related to the students' selected topic of interest could be used? How could students' original work be recorded and communicated? Would students be capable of doing a critical evaluation of their work and processes, and even appreciating the social role of mathematics? It is not unusual that these concerns appear in modeling contexts as reported in the literature about modeling with young students (Biembengut 2007; English and Watters 2004; Greer et al. 2007).

Now we analyze the students' modeling project searching for evidence of how ICTs could overcome the problematic issues stated above and enhance students' MM process.

It is worth noting that previous to the realization of students' modeling projects, the notion of mathematical model was negotiated in class discussions between students and the teacher. The notion was collectively developed, grounding it in the classroom experiences and it was stated as follows:

> A mathematical model is any mathematical structure [diagram, formula, table, algebraic equation, etc.] designed from relationships established among variables considered relevant, to match some entity; and it [the model] allows us to make predictions, to make decisions, to explain, and to understand.[7]

This notion and the diagram presented in Fig. 1 were used as "thinking tools" in every modeling process, giving explicit guidance and instruction throughout the process. Since all these ideas were new and did not appear in the textbooks used by the students, a web page was designed in order to allow students to retrieve this information. ICTs served as a tool to communicate novel information.

To start with their modeling projects, students were invited to select a theme of interest to them to study using mathematics. A group of five boys selected the topic of timber transportation. The students wrote in their final report that:

> … we were working with wood in handicraft classes, and we liked it because we thought that a real situation, such as timber transportation, might have to do with mathematics, in the sense that the transportation costs would be related in some way with the bulk transported.

Students sought information from several carriers through telephone contacts, and decided to use lists provided by fax by one of those companies. The quantity of wood, in kilograms, and the corresponding cost in Argentinean pesos, was listed for different distances between two cities in Argentina. The students selected two distances, Buenos Aires–Córdoba and Buenos Aires–La Rioja because

> … these fixed distances are among those that appeared on the carrier's lists, and because they seemed interesting to us, because they join inhabited places, and due to people's needs, transportation is in high demand …

Students discarded some previously considered variables such as fuel requirement, type of timber, and the size of timber, because "*they were included in the cost of transportation*".

After these modeling phases (definition of some aspect of the real world to be investigated, looking for pertinent information, selection of relevant variables, etc.), students were able to pose the problem that guided their project:

> Knowing a distance, what is the relationship between the cost of transportation and kilograms of transported wood?

According to our analysis, communication media, such as telephone and fax, gave the students the opportunity to gather real data from a carrier. It would be almost impossible for them to obtain such data if they appeared in person at the company. In fact, other groups of students reported that some shops or industries did not give any information for their projects when they asked for it in person because of their young age. Without an image of the receptor, fax and telephone allowed the students of the timber transportation project to be treated as "serious" costumers.

[7] This notion of mathematical model is consonant with that given by Biembengut and Hein (2000).

The fax provided abundant written information about transportation costs between different pairs of cities. In this context, students were able to make a selection using some kind of “social” criteria like “*they* [cities’ distances] *join inhabited places, and due to people’s need, transportation is in high demand …*”.

The two-column format of data provided by fax permitted students to realize that previously considered variables such as fuel requirement, type and size of timber could be discarded, obtaining a fine delimited problem with two variables (distance vs. cost). The two-column list surely brought into play previously learned notions and processes and, as a result, students also recognized that timber transportation “*might have to do with mathematics*” and that this science might have to do with, “*in some way*”, relating variables, as earlier quotations showed.

Following the modeling process and based on the notion of model previously constructed, the students analyzed and completed the data provided by the carrier, organizing them into a two-column table (kilograms of wood transported vs. cost of transportation) for a fixed distance of 1,168 km (the distance between Buenos Aires and La Rioja) as shown in Fig. 2. By analyzing information such as “up to 5 kg, the cost is $10.19” or “up to 10 kg, the cost is $10.72” (as appeared in table format in the carrier’s lists), students decided to incorporate some data into their own table; for instance, the corresponding cost for 2.5 kg, 7.5 kg, etc. Such work was evidence of their correct interpretation of the data and, at this point, students used the word processor as a tool to register completed information in a desired way.

Later, students translated the table’s data into a Cartesian graph (see Fig. 3) using *Graphmatica*. This software was used intensively during the classes and allowed students to translate a two-entry table of corresponding variables into its related graph, or vice versa.

Additionally, *Graphmatica*’s curve-fitting features allowed students to find the curve and the analytical expression which best fit their data. The plotted line in Fig. 3 shows that students selected a first-order polynomial equation to fit their data. On the one hand, this choice seemed visually acceptable, but on the other hand, they were probably looking for a known curve like the straight line studied in previous classes. *Graphmatica*, as a user-friendly and versatile tool for representing relations among variables, allowed students to confidently select a graphical model that they were able to understand at that moment.

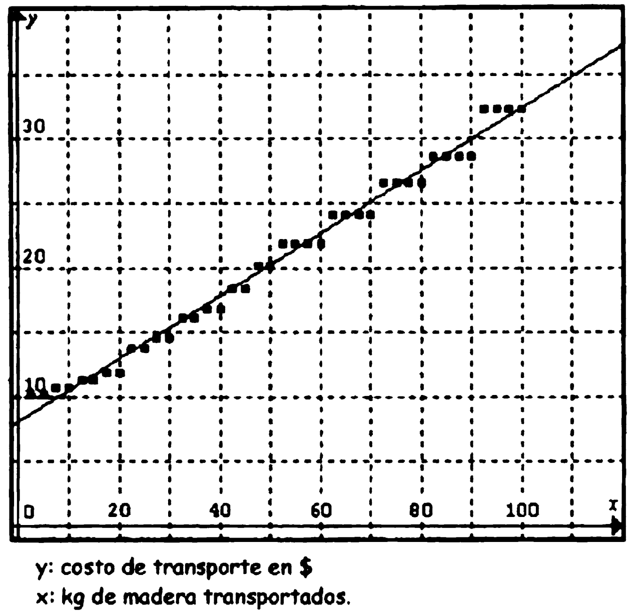

Fig. 3 Distance × cost graph. *x* represents kilograms of wood, and *y* cost of transportation in pesos

From the table in Fig. 2 and the graph in Fig. 3, we can observe that a step function appropriately describes the phenomenon. But piecewise functions were unknown to these children at that moment, and were beyond the scope of a mathematical course for 11–12-year-old students. However, this graph obtained by the students could be used later as a springboard for learning this topic, since now they would have a familiar context to work from.

At the end of their written report, students provided a mathematical expression for the line in Fig. 3 obtained using *Graphmatica* again, and it was recognized by them as the analytical model of the phenomenon (see Fig. 4). The children recognized the usefulness of *Graphmatica* in their

Tabla N°1

Distancia de Bs. As a La Rioja. (1168 Km): dato fijo.

Kg de madera transportados	Costo de transporte
2.5 kg	$10.19
5 kg	$10.19
7.5 kg	$10.72
10 kg	$10.72
12.5 kg	$11.28
15 kg	$11.28
17.5 kg	$11.87
20 kg	$11.87
22.5 kg	$13.76
25 kg	$13.76
…	…

Fig. 2 An excerpt of the table made by students relating kilograms of wood transported versus their costs (in Argentinean pesos) for a fixed distance (1,168 km)

0.2424.x + 8.11= C

C: costo de transporte.
x: kg de madera transportados.

Fig. 4 Analytical model of the phenomenon studied by students, where *C* represents the cost of transportation, *x* the kilograms of wood transported process when they asserted that it was "*an useful software for making graphs and obtaining mathematical models*".

Lastly, using this analytical model and calculators, students evaluated the validity and prediction features of the model for different values of x by comparison with its respective data in the carrier's lists. Therefore, they concluded that:

> … for $x = 300$ kg, from the model it is obtained $C = 80.83\$$, and from the list $C = 75.15\$$. For $x = 400$ kg, $C = 105.07\$$ from the mathematical model, and from the real data $C = 92.58\$$. For $x = 600$ kg, $C = 153.55\$$ from the mathematical model, and from the real data $C = 119.74\$$

With these calculations and analyses in mind, students inferred that "*the model is useful to predict the cost of transportation up to 400 kg, approximately*", since they realized that for values of $x = 600$ kg and above, the analytical model did not provide a cost which fitted the real data. In their written report, students concluded that:

> … the real data provided by the carrier COTIL CARG S.A. could be represented by the mathematical models obtained. To do that, we had to make tables with the data in an experimental phase, analyze them, and graph them using GRAPHMATICA, which we have already mentioned, and using this software, we get the mathematical model.

The above excerpts show the diversity of representations generated by students using technology: table, graphs, and analytical expressions. The students recognized these representations as models for their problem and they were able to translate one representation into another one. The children were able to obtain the curve that fit the data and the analytical expression pictured in Figs. 3 and 4, respectively, owing to the use of *Graphmatica*, since these first year secondary school students did not have the algebraic expertise needed to deal with these mathematical objects at that moment. In this case, media were constitutive of the knowledge generated in classrooms.

This experience provides evidence for asserting that these young students have been able to deal autonomously with an entire modeling process, according to our modeling principles. The students obtained interesting and, to some extent, sophisticated mathematical models. They use the definition of model, constructed collaboratively in classroom, and the diagram of Fig. 1 as important "thinking tools" to reflect on their modeling process. In the same way, the ICTs acted as essential partners for the students in their MM process as well as in the communication process (use of webpage and the fax). Without the media, this sound modeling process would have been even impossible for these students. Media empowered the students' modeling process and the teaching processes. Teacher, students and ICTs constituted a powerful thinking collective of humans-with-media.

4.3 University students and modeling

4.3.1 Description of the experience

The experience we present here was carried out with three agronomy students at the University of Córdoba, Argentina. The students were attending a course entitled *Mathematical modeling: applications for solving agronomical problems*, a special course for teaching assistants helping with the first year mathematics course for agronomy majors. The aim of the course was to generate a learning environment in which MM transcends the mere application of mathematical models, and where students' engagement in problem solving, problem posing and creation of models is fundamental. The specific teaching goals of the course were to: (1) relate mathematics with other sciences, (2) strengthen and consolidate the students' mathematical and agronomical web of knowledge through the creation of models associated with agronomical phenomena, (3) develop research abilities such as: problem posing; data collection, analysis and interpretation; formulation of hypotheses; creation and validation of models; elaboration and presentation of research reports and (4) support the pertinent use of software that can be used in the MM process. These goals are in synergy with principles 2, 4, and 5 of our modeling perspective.

In order to achieve these goals, the work was developed in two phases: modeling as a teaching object and modeling in action. During the first phase, the students were familiarized with concepts associated with modeling through dialogues with the teacher, short lectures, readings and problem solving activities. First, the teacher asked the students to explain what they associated with the word "model", to mention contexts in which the word was used, and to give examples applied to different sciences. Then, different definitions of mathematical model were considered (including those that we presented in Sect. 4.1), the phases of the modeling process were characterized, and the diagram shown in Fig. 1 was analyzed. Secondly, the students worked with a collection of problems to be solved and discussed with the aim of recognizing different types of models and identifying the phases of the modeling process. Thirdly, the students studied, in detail, examples of mathematical models associated with real biological and agronomical situations. As a complementary activity in this first phase, a researcher from the area of Edaphology, which is concerned with the influence of soils on living things, particularly plants, was invited to talk about her research and the kind of mathematical model she was using in her study. During this phase, the software *Graphmatica*, mentioned in the previous section, was introduced and used to visualize data, make graphs, or validate conjectures.

During the second phase, a complete MM process was implemented by the students, who developed a project that aimed to pose and solve an agronomical problem of interest to them in the field of genetics.

The three students who participated in this experience had previously taken a traditional Calculus I course, which was the only university-level mathematics course they had ever taken. The course was structured around the sequence teacher lecture–examples–exercises. The types of problems in which mathematics was applied were, according to Skovsmose (2001), semi-real problems. Their main goal was to apply a mathematical content that had just been taught. In this learning environment, the students did not have the opportunity to develop a complete modeling cycle or to criticize the kind of problems they have to solve or their relevance in the real world.

4.3.2 An example of the students' production

As we described before, during the first phase of the experience, the students solve some problems posed by the teacher. In this section, we show students' production while solving the following traditional problem:

> If a plant measures, at the beginning of an experiment, 30 cm and every month its height increases 50% of the height of the previous month,

a) Create a mathematical model that makes it possible to predict the height of the plant for any time t
b) Criticize the statement of the problem and indicate the phases or sub-processes of the modeling process which are present in the model generated in item a)

This problem presents an un-real situation in which a plant grows indefinitely and the mathematical model that represents it is $h(x) = 30 \times (1.5)^x$ where x represents months and $h(x)$ the height in month x measured in centimeters.

We chose this particular problem to show the diverse activities and approaches that the students developed. Although the problem was simple and presented a fictitious situation, it was possible for the students to develop modeling activities and critical attitudes in a technologically enriched teaching environment.

In the context of the traditional calculus course that our three students had attended previously, this problem could be posed with the aim of applying exponential functions after the content had been taught. In the context of our modeling course, in which we did not teach any particular mathematical content, the students did not relate the problem to exponential functions at the beginning of their solving process. First, they proposed a function that they wrote as $h(x) = 30+\ldots$ pointing out that it could have logarithmic behavior and drawing a graph like the one sketched in Fig. 5.

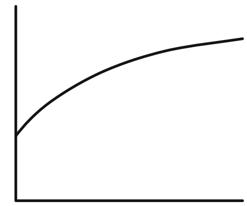

Fig. 5 Students' graph

After making these conjectures, they indicated that "*A logarithmic* [function] *would not cross the axis y; it has logarithmic behavior but it won't be purely logarithmic*". They were referring to the fact that the formula of the function that they were looking for could not be $y = \log_a x$, since that function does not cross the y-axis. Although the *log* function they had in mind was an increasing function, the shape of the curve they had drawn suggests the existence of an upper limit to its growth. The students knew that a plant does not grow indefinitely, and their graphical conjecture is maybe related to that biological knowledge. This fact can be considered as evidence of the connections the students tried to establish between their knowledge about a life phenomenon and their knowledge about functions. They were trying to translate a biological model into a mathematical model. They were going into a MM process. Meanwhile, this real biological knowledge would not help them to solve this un-real problem. They had to concentrate just on the data provided by the statement of the problem. After this graphical analysis, they embarked on an algebraic analysis, trying to find an expression for $h(x)$, and they said: "*In fact, if I put x plus 50% of x, x would vary for the next time*". In this particular claim, when the students said x, they were referring to the height of the plant. It shows that the students were thinking in a recursive way to determine the height of the plant. After some discussions, they computed the height of the plant for the first months and concluded that a general formula to calculate it would be:

$$h(x) = h_{(x-1)} + \frac{h_{(x-1)}}{2}$$

$$h(x) = 1.5\, h_{(x-1)}$$

where $h(x-1)$ represents the height of the plant in the month prior to x, and $h_{(x-1)}/2$ represents 50% of the height in the month prior to x. At this moment, the students decided to use the *Graphmatica* software to plot the points they had previously computed (see black points in Fig. 6). While visualizing those points, one of the students recognized that it seemed to represent an exponential function.

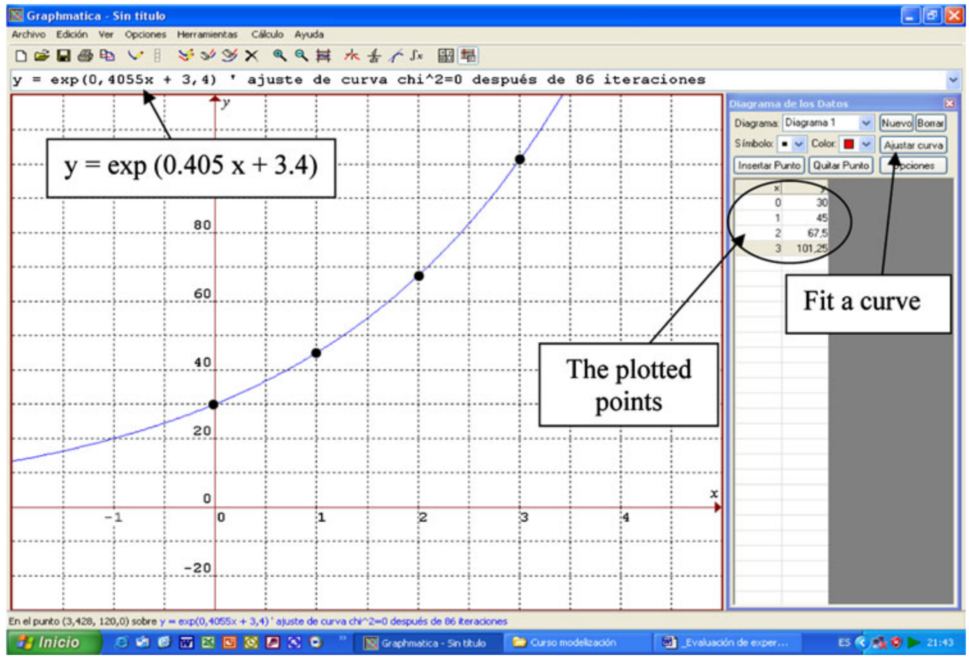

Fig. 6 Graph and analytical expression of the fitted curve on the computer

Another student said that "*to compute values for the height of the plant in successive months, using the calculator, one only needs to always push times 1.5*", referring to the fact that the height of the plant for each month is obtained multiplying the height in the previous month by 1.5. This pattern was perceived with the use of the calculator, and allowed the students to arrive at the formula that they were looking for: $h(x) = (1.5)^x 30$.

After that discussion, the students used *Graphmatica* to fit a curve using the points they had entered. The computer showed the graph and its analytical expression (Fig. 6).

The students made the graph of $h(x) = (1.5)^x 30$ using *Graphmatica*, and they verified visually that this graph coincided with the one provided by the software through the curve-fitting feature, $y = e^{(0.405x+3.4)}$. Then, they made some calculations with the calculator to verify that both expressions were equivalent.

The activities that the students developed during the solution of the problem showed the diversity of approaches they used: visual, algebraic, numerical and technological. At the same time, the students developed activities related to different phases of modeling. For example, they formulated conjectures about the type of functions they were looking for; and they used the computer to validate the algebraic model they had obtained. The use of technology accompanied the students' processes and was a tool with which they were thinking about the problem. In this case, the calculator had a main role when the students realized that, to calculate the height of the plant for each month, it was sufficient to "push × 1.5". For us, this was a nice example of a thinking collective of students-with-calculator trying to solve a problem. We consider that in this case, the technology empowered the students' engagement with modeling activities.

The kind of problem that the student solved with calculator and computer as partners can also be solved just with paper and pencil. But the use of those technologies made possible the coordination of different types of mathematical representations of the growth of the plant considered in the problem. The students went beyond the mere application of a known model; they constructed a particular model that fits the conditions established in the problem even though they did not go through all the phases of a MM process as shown in Fig. 1.

Finally, the students discussed the viability of this model and concluded[8]:

> There are many factors that you should take into account; you don't know which plant it is.
> The model doesn't fit the reality because a plant doesn't grow indefinitely.
> If it [the model] were an exponential function, it [the plant] would increase indefinitely. Maybe this [model] works for a given period of time, but here [in the problem] it is not limited to a given period of time.

We can observe that the students contrasted the model with reality. They observed that a plant may grow exponentially during a certain interval of time after which such growth is no longer possible. All these expressions show

[8] The information included between brackets aims to clarify some of the students' expressions.

that the teaching environment encouraged the students to criticize the given problem and the model obtained.

We recognize that this example does not fit all the principles of the modeling approach we have assumed. The problem did not represent a real phenomenon. The biological context seems to be a disguise to hide the application of exponential functions. Meanwhile, we think the example shows that the use of a non-real problem in a teaching environment where modeling is at the center can provoke the engagement of the students in the solution, and even more importantly, in the critique of it. This example enables us to think that even with non-real trivial problems, issues related to modeling emerge if we create an appropriate environment. Considering Kaiser and Sriraman (2006) classification of modeling perspectives, we think that the plant problem is compatible with the characteristics of task within the educational modeling perspective,[9] since a subject-learning goal related to it may be to explore exponential functions.

5 Conclusions

We have made an extensive review of the international literature focused on modeling and modeling empowered by ICTs. Those works depict important and diverse issues about these topics of study. We have also pointed out that, in our country, MM is proposed in many curricular documents, and it is associated with mathematical applications to other sciences and with problems coming from real contexts. Our experiences show different ways of setting up a modeling scenario according to the modeling principles we have adopted. Such principles were originally inspired by some Brazilian trends. We brought some evidence of how ICTs appeared as empowering partners when collectives of teachers-and-students-with-media engaged in MM processes.

Modeling becomes more powerful with the use of ICTs, but after our experience in the secondary school, we can be even more radical and say that, within the constraints of our educational institutions, the absence of ICTs would have made our work more difficult, less rich, or even impossible. The broad and rapid access to information through the Internet encouraged the teachers to face the challenge of inviting students to become problem posers and project designers, choosing a theme of their interest. The use of the Internet as a communication tool was another relevant role of ICTs. The web page and the templates mentioned in Sect. 4.2 facilitated the broadcast of important and original material created by both students and teachers. It seems that, as stated by Blum et al. (2003), the availability of many technological devices is highly relevant to modeling and broadens the range of possibilities for teaching and learning.

Considering the use of mathematical software, the availability of *Graphmatica*, and the teachers' decision to use it, allowed secondary students to construct and understand a sophisticated mathematical model, and for university students to validate their first analytical model developed with the help of a calculator. It is worth noting that, for secondary students as well as for university students, the calculator was a tool that also accompanied the MM process, to validate their linear model in the first case, or to construct their exponential model in the second one.

It is worth noting that sometimes it may happen that the students are not able to recognize the mathematics that is "behind the model" provided by *Graphmatica* using the curve-fitting feature. This issue, related to the notion of *black boxes*, led us to ask: Should the teachers make such mathematics visible to the students? If the answer is yes, how should they do that? The way the agronomy students used the calculator to compare the exponential model they had created and the one provided by *Graphmatica* is one possible way. A discussion related to these questions can also be found in Esteley et al. (2007).

The software did not only act as a partner for the students but also for the teachers. *Graphmatica* let the teachers open spaces for their students in order to visualize and think about mathematical representations that most of the time were beyond the scope of the official curriculum. In that sense, in the projects and specific examples that we have described and analyzed, we have offered evidence regarding the intervention of students and teachers as curriculum developers at different educational levels.

The creation of modeling scenarios according to the modeling principles we have adopted implies a challenge to the traditional curricular structure. When we start a modeling process in which students choose a theme of interest, we cannot guarantee that all the mathematical contents of the official curriculum will appear during the process. Moreover, "*such a possibility breaks free from a 'sacred rule' in which students have little or no say in curricula*" (Borba and Villarreal 2005, p. 56).

Finally, in this paper, we analyzed and provided evidence of how ICTs empower modeling processes and, even more, how the modeling scenario may imply curricular changes. However, if we go further, we may ask: if we had not been in a modeling environment, would the actors immersed in such context have been able to recognize the relevance of technological devices? The answer to this

[9] It is worth noting that this task may also be connected to the contextual modeling approach. Kaiser et al. (2007) inform that (at CERME 5) "*it was difficult for some researchers to see the differences between educational and contextual modelling*" (p. 2038).

question implies a discussion about the interplay between modeling and ICTs.

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