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ORIGINAL ARTICLE

Understanding kindergarten teachers' perspectives of teaching basic geometric shapes: a phenomenographic research

Hatice Zeynep Inan · Ozlem Dogan-Temur

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Abstract Geometry is one of the disciplines children involve within early years of their lives. However, there is not much information about geometry education in Turkish kindergarten classes. The current study aims to examine teachers' perspectives on teaching geometry in kindergarten classes. The researchers inquired about teachers' in-class experiences in geometry and asked a series of questions such as "what are the benchmarks in your kindergarten class?"; "what kind of tools and materials you use to teach geometry in your class?"; "what shape do you teach first in your kindergarten class?"; "what do you expect to hear when you asked your students 'what is square'?"; "how do you teach rectangular?". The study utilized one of the qualitative research methods, namely phenomenography, to collect the data and analyze the data. The study involved with eight kindergarten teachers who work in different schools in central Kutahya, Turkey. The researchers collected data by conducting face-to-face half-structured interviews. The findings of this phenomenographic research showed that kindergarten teachers have some difficulties in teaching geometry and have lack of knowledge and skills in teaching geometry in kindergarten classes.

Keywords Geometry · Kindergarten · Teacher perspectives · Geometric shapes · Phenomenography

H. Z. Inan (✉) · O. Dogan-Temur
Egitim Fakultesi, Dumlupinar Universitesi, Merkez Kampus, Tavsanlı Yolu, 10.km, Kutahya, Turkey
e-mail: haticezeynep@hotmail.com

1 Introduction

One of the indispensible elements of kindergarten education is to equip the child with the basic geometric concepts during kindergarten period and to teach him/her how, where and when to use these concepts by associating them with real life. Achievements of kindergarten students in Turkey in terms of geometric concepts are included in the 36–72 Months Program of Turkish Ministry of National Education (2009). When a child starts primary school without being equipped with the necessary achievements he/she is supposed to have had during kindergarten period, this may cause him/her to lag behind all through his/her education life, to go through difficulties, to lose his/her self-confidence in the relevant subjects and as a result to fail. Many relevant researches have shown that if a child is equipped with the necessary attainments in geometry during kindergarten period through accurate methods and techniques, this will contribute to that individual's future academic life and even raise his/her IQ (e.g. Clements and Sarama 2006).

Since the times when kindergarten education started, geometry has occupied an important place in programs (e.g. Frobel's 1826/1905-blocks, cited by Brunkalla 2009), but kindergarten classes still have difficulties in geometry. For example, relevant researches have shown that these geometric concepts, which constitute the base of mathematics, and spatial reasoning, have not been presented sufficiently during kindergarten period and that teachers have not paid sufficient effort on these attainments (Clements 1998). Therefore, it is necessary that in certain periods children be provided with such geometric attainments that are consistent with their developmental processes in such a manner that is consistent with their developmental processes and individual differences; that necessary physical hardware be provided in the classroom and school environment; and that sufficient counseling, guidance and time be allowed for the student to make use of the opportunities.

This research aimed to reveal the experience and thoughts of kindergarten teachers about the attainments of children during kindergarten period—especially during 5–7 years old—about geometric concepts. In this study, Phenomenography (Marton 1994)—a qualitative research methodology—was used in order to reveal the experience of teachers in line with the aim of the research. Phenomenography is an empiric study focusing on the experience and differences in the perceptions of people about the phenomenon around us (Marton 1994) and it aims "what is in people's mind" (Webb 1997). Accordingly, phenomenography helped the researchers in the current study in terms of their aim, which was to understand teachers' view of the teaching and learning process in geometry education and their own experiences related to the interest topic. The data acquired through face-to-face interviews with a group of kindergarten teachers in the urban of were analyzed and interpreted through phenomenography method.

2 Geometry in kindergarten

2.1 Geometry attainments in the world

Geometry continues to claim its place in kindergarten education as a sub-title of mathematics in many parts of the world. Clements (1998) states that geometry involves concepts and competence about figures and space. New studies started on relevant regulations and relevant attainments upon the spread of kindergarten education during the last century, through new evaluations of education perspective in the light of new theories and theorists, the paradigms changing on the basis of discipline and upon the acceptance of kindergarten education as a compulsory part of general education. Countries which have been going through this process of change and advancement including Turkey have revealed the geometric attainments during kindergarten education. As the standard scope of educational attainments expanded, kindergarten geometry attainments became included into the discipline-based educational paradigm.

When we examine geometry attainments for kindergartners on the world, related standards took attention. Early Learning Standards (ODE 2004) can be given as an example for this. Geometry attainments within mathematics standards include the characteristics of objects and their spatial relations. ODE, referring to geometry and spatial reasoning, implies recognizing the figures and structures around and spatial perception and awareness. In short, the attainments referred in the context of specific geometric attainments in ODE are as follows:

1. Matching of two- or three-dimensional objects or figures around and in playgrounds (e.g. two squares of the same size, two "stop" signs).
2. Classification of two- or three-dimensional objects or figures around and in playgrounds (e.g. paper figures, two balls of different dimensions).
3. Recognizing, naming, creation and definition of two-dimensional objects or figures around and in playgrounds (e.g. a square, triangle, rectangle, and a circle).
4. Recognizing, naming, creation and definition of three-dimensional objects in child language (e.g. circle: a ball, cube: a box, cone: ice-cream cornet).
5. Starting to use a language for the objects around and in playgrounds (e.g. down, up, under, on, in, out, in front of, behind, in the middle, next to, ahead, upside down; ODE 2004).

Moreover, National Council of Teachers of Mathematics [NCTM] (2009) stresses importance of geometry in our lives and states that geometry and spatial sense offer us ways to interpret and reflect on the physical environment. NCTM indicates that there are four basic standards as follows:

Instructional programs from prekindergarten through grade 12 should enable all students to

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems (p. 1).

ODE (2004) states that attainments in children about geometry and spatial competence can be achieved in suitable environments through such methods and techniques that are parallel to the developmental characteristics of children. It also adds that children can acquire these attainments by doing, living and trying themselves; therefore, only providing the required opportunity and environment (e.g. children can prepare different geometric figure to use in artistic activities, they can design figures from blocks or classify similar blocks, they can personally apply spatial relations in the scope of geometry such as climbing up or coming down) can give them experience. In short, being an advocator of Piaget view, ODE emphasizes that geometry should be given through "hands-on" activities. A child should be absorbed in geometry activities not as a duty or under pressure but as fun (Clements 1999).

2.2 Geometry attainments in the Turkish Ministry of National Education Program

In order to understand geometry attainments for kindergartners in Turkey, it is essential to examine Turkish standards. The following are the basic attainments and important points in geometry and spatial competence presented in the kindergarten education program for children of 36–72 months old prepared and revised by Turkish Ministry of National Education (2009) General Directorate of Pre-school in 2002–2003 education period.

2.2.1 Aim: recognizing geometric figures

The child can

1. recognize that every object has a shape.
2. distinguish the circle, square, triangular and rectangular objects.
3. form different models by using circles, triangles, squares and rectangles.

   - During the activities prepared for these purposes, teachers pay attention to using certain terminology like a circle and ring. The ring is hollow inside but only covers a circle area; however, a circle defines a full circle area. Accurate usage of these two geometric terms is crucial for preparation for primary school.

2.2.2 Aim: application of instructions of location

The child can

1. say the location of an object in a given environment correctly.
2. locate himself/herself correctly according to the given instruction.

   - For this purpose, such instructions as "Stand by the chair; Line up under the window; Wait behind the puppet stage" should be given to the child and enough time should be allotted for him/her to perform them.

3. put an object correctly to the instructed location.

   - For this purpose, the instructions should involve such prepositions as under, on, in front of, behind or next to.

The attainments of geometric and spatial relations mentioned above are valid for children of 36–72 months old attending the private and public schools of Turkish Ministry of National Education. Educators should adjust their aims, programs and activities parallel to these attainments.

2.3 Teaching of geometry in the light of theories

2.3.1 Piaget

Inhelder and Piaget (1958) stressed that children in the preoperational stage learn from concrete evidence while adults learn in abstract way. They stated, "The reactions of this stage are simple; in all his explanations the child is limited to describing what he sees. As neither classifications nor organized operations of serial ordering are yet available, he fills in his observations with precausal linking (finalism, animism, moral causality, etc." (p. 48). Accordingly, it is essential to provide children opportunities to work on physical objects instead of imaginary things related to geometry.

Analyzing the education of kindergarten geometry in the light of Piaget's cognitive development theory, it can be seen that he focused on concepts intertwined with geometry such as three mountains experiment which is about the (preoperational, 18 months to 6 years) kindergarten children' forming a perspective. However, it must be noted that as stated by Malaguzzi (1998), Piaget used these concepts not to work artificially on the matter but to point out the cognitive stages the child goes through while constructing knowledge. After all, the child comes across these concepts in real life situations.

Mooney (2000) emphasizes that Piaget focuses on the meaningfulness of the matter for the child in his/her learning process. That is, a child can construct subjects that have been set in a certain context and thus having a certain meaning for the child more easily. Therefore, in the teaching of such disciplines as geometry which disguise in an absolute abstractness unless studies have been carried out on them, the subject should be made meaningful for the child and also be correlated with the child's life and experiences.

According to Piaget, such characteristics that can be observed instantly (NRC 2001) should be studied in order to bring about geometry attainments concordant to the developmental level of children. Many researchers (e.g. Casey, Erkut, Ceder, Young 2008; Clements 1998; Malaguzzi 1998) based on Piaget and the constructivist approach claim that during geometry teaching in kindergarten classes, such methods and techniques that allow the child to participate in the process actively will be more efficient than direct instruction method. For example, it was seen that kindergarten students could complete visualization (what arises from the merging of the pieces) and fictive rotation (the image when objects are rotated in a two- or three-dimensional environment) activities successfully through story telling (Casey et al. 2008). These activities can set geometry teaching into a meaningful and enjoyable context because they remain in the child's memory and they can appeal to different skill groups.

Studying on children's acquisition of spatial reasoning competence and geometry teaching, Clements (1998) claims that children can perceive the concept of perceptual space from their childhood, but they can learn geometric concepts only by touching, drawing their shapes and discovering. In his studies, Clements states that children of earlier ages benefit from pictures, things that can be manipulated, relevant computer programs/games in geometry teaching and in the development of their visual language.

On the other hand, according to Piaget's theory of cognitive development, it is impossible for the child to grasp certain matters without reaching a certain age/developmental level. Therefore, a certain passage of time is required. Contrary to Piaget's constructivist approach, Vygotsky defends social-constructivist view. In Vygotsky's social-constructivist theory, it is stated that the learning process is not like the one in Piaget's developmental view and that environment is dominant because social environment is crucial.

2.3.2 *Vygotsky*

Vygotsky (1978) stresses that a child can construct more information when he/she interacts with a more knowledgeable peer or with an adult than when he learns alone. Mooney (2000) adds that while a child gradually constructs his/her knowledge, he/she can advance and develop his/her competence through external aid and support.

Emphasizing the training of teachers about kindergarten geometry education, Chard, Baker and Clarke (2008) state that children suffer during their future educational lives if they have not had a solid mathematical background (e.g. geometry, measuring, mathematical language, and numbers) in their kindergarten period. Chard et al. add that students who have difficulties in mathematics can succeed through well-managed student–teacher relations and that researches of high quality are required for this purpose. Vygotsky's social-constructivist view, getting ahead of Piaget's constructivist approach, contributes to the matter in the role of environment during the child's efforts on geometry and spatial concepts.

2.3.3 *van Hiele*

van Hiele (1986) analyzes the development of geometric thinking in five periods, namely, visual, analytic, informal deduction, formal deduction, and rigor.

1. Visual period: Students at this level can perceive geometric figures as a whole. They determine, name and compare figure according to their appearances (e.g. A square is a square arbitrarily).
2. Analytic period: At this level, students scrap figures into parts and compare and describe them according to their properties. They discover the characteristics of the figure through different activities. In order for them to pass to the following stage, it is helpful to arrange the data they have gathered about the geometric figures into tables and to make deductions from those tables.
3. Informal deduction: Students at this level can recognize the relations between figures and between the characteristics of figures. They can also grasp the role of figures and classify them according to their properties (e.g. they can say that a rectangle is a special kind of parallel edges).
4. Formal deduction: At this level, students can grasp the meaning and importance of a proof based on axioms, theorems and definitions. Making use of previously proved theorems and axioms, they can prove other theorems through deduction.
5. Rigor: At this level, distinguishing different axiomatic systems, students can come up with theorems in these systems, can analyze these systems and can make comparisons between them (Olkun and Uçar 2004, pp. 174–176).

Being able to think at other levels by thinking at basic thinking levels shows a hierarchic structure. It is impossible to short cut these levels (van Hiele 1986, p. 51). However, many relevant researches have shown that teachers cannot adapt their geometry teaching to their students' levels (e.g. Mason 1997; Gutierrez and Jaime 1998).

In his studies based on van Hiele's model, Clements (1998) determined that children first consider geometric figures as a whole and at the next stage interpret in terms of their details and their location in the space. He showed by means of the misconceptions of the students that figures and spatial reasoning are closely intertwined. Hence, he concluded that spatial reasoning, visualization, gaining a perception should be emphasized in the teaching of figures.

There are grave differences between Piaget's and van Hiele's theories. While Piaget's researches are about developmental psychology and they emphasize that geometric thinking can advance through development, van Hiele discusses the advancement of geometric thinking in the learning process. The phases put forward by Piaget for concept development cannot fully explain the learning process in geometric thinking (van Hiele 1986, p. 101).

3 Method

Since it is aimed to conduct more focused research and to obtain rich data examining a few cases instead of making generalizations, the study utilized a qualitative research method. Sandberg (2005) states, "because truth is always something unfinished within the interpretive tradition, the criteria proposed do not enable researchers to generate absolute truth claims. Instead, they give researchers the opportunity to produce more informed and thorough knowledge claims in relation to their ontological and epistemological assumptions." (p. 62). As indicated by Sandberg, interpretivist studies do not aim one absolute truth for everyone, but aim to define and describe the situation, which is rich in in-depth knowledge, unique to its owner but advisable and inspiration to others.

The study utilized the one of interpretivist method, phenomenography, for some reasons. The study aimed to examine experiences and thoughts of teachers on kindergartners' learning of geometric concepts. Consistent with this aim, phenomenography enables researchers to facilitate the thematization of aspects of the interviewee's experience (Marton, 1994). Marton (1994) states, "The experiences, understandings, are jointly constituted by interviewer and interviewee" (p. 2). Accordingly, it was meaningful for the current study to utilize the method of phenomenography in order to enable researchers to focus on the experiences of the kindergarten teachers, which is the basic focus of this research. Therefore, the research made use of a qualitative data gathering technique, individual interview, which is common in phenomenographic research. Phenomenography was also used to analyze the data and generate the themes occurred from teachers' experiences and thoughts.

3.1 Sample

The sample was composed of eight kindergarten teachers employed in the kindergarten classes of four public primary schools, two private independent preschools and one private school all in the urban of Kutahya. Moreover, all the teachers participated in the study hold an undergraduate degree in early childhood education programs. To choose the research sites, the researcher relied on what Patton (1990) calls the "Purposeful Sampling" method, which involves selecting information-rich cases for in-depth research. The sites were information-rich for the researchers to pursue their inquiries and also convenient places to access and conduct the current research.

3.2 Data collection

The data were gathered via interview technique of qualitative research method. For this purpose, an interview form composed of seven questions was developed (see Appendix 1). These questions were first applied to two kindergarten teachers and then were revised. Opinions of two experts were sought for the validity of the interview form. Those experts were colleagues who studied phenomenography and experienced in conducting research utilizing this method. Moreover, they both were in the field of early childhood education so that their expert knowledge in both the field and the research method was helpful in development of the interview questions. It is essential to state that the procedure of developing the interview questions was not linear but spiral process, because the researchers went back and forth among the related literature, two kindergarten teachers' opinions, and experts' opinions while developing the questions.

3.3 Phenomenography

Phenomenography is an empiric study focused on the experience and differences in the perceptions of people about the phenomenon around us. Experience, perception and grasping can be used interchangeably in phenomenography. Since everyone's perceptions, grasping and experience will be different, so will their ways of defining phenomena. The meaning in the text forms the exterior structure of the phenomena, while the meaning in the parts of the text forms the interior structure of the phenomena. These exterior and interior structures exist together in the structure of experience (Marton 1994).

Webb (1997) argues the value of phenomenography and explains it based on one of the pioneer of such method, Marton as follows:

> Phenomenography… considers only the 'second order' or conceptual thoughts of people. Phenomenography attempts to aggregate 'modes of experience… forms of thought' (Marton 1981, p. 181) into a limited number of categories. Phenomenographers do not claim to study 'what is there' in the world (reality) but they do claim to study 'what is there' in people's conceptions of the world. (… words are skipped; pp. 199–200).

Since phenomenography aims "what is in people's mind," it helped the researchers in the current study understand teachers' view of the teaching and learning process in geometry education and their own experiences related to the interest topic.

3.4 Data analysis

The data of this research were gathered in compliance with phenomenography method and during data analysis phenomenographic analysis was used. The interview recordings were first listed three times and then the meaning categories in the last text were analyzed. The main meanings were labeled, the concepts were grouped and categorized, and the categories were named. The eight participant teachers were given numbers from one to eight regardless of their names or working places and these numbers were shown next to the categories and sample conversation sentences in the tables.

Marton and Pong (2005) describe their data analysis in their phenomenographic research as follows: state, "All the interviews were transcribed to yield data for a two-stage analysis. The first stage focused on identifying and describing the conceptions in terms of their overall meanings. This was done by marking and segmenting the transcripts according to the themes addressed. A unit was formed whenever there was sufficient evidence that a particular overall meaning had been expressed. The second stage of analysis focused on identifying the structural aspect of each conception expressed. The units, now denoted by the various overall meanings, were studied in detail, to identify within each unit the elements of the phenomenon that were focused upon, and to devise a description of each conception's structural aspect. In doing so, we paid attention to the explicit variations that the student brought in as they focused on a particular element, as well as the variations that were implied by that element." (p. 337).

4 Findings

The participant teachers in this research were asked seven questions during face-to-face interviews. Below are the findings and interpretations of the findings for each question reflecting the categories of the teachers' answers and sample conversation sentences. The eight participant teachers were given numbers from one to eight.

When kindergarten teachers were asked about geometry attainments in the Turkish Ministry of National Education program, it was seen that they mentioned about geometric figures both in the program and not (see Appendix 2). To the question "What are the kindergarten geometry attainments?", the categories emerged from the data are as follows: Triangle, square, circle, rectangle (1, 2, 3, 4, 5, 6, 7, 8), which exist in the program and oval (1, 2, 5, 7), polygon (2, 4, 7, 8), and prism (5, 8) which do not exist in the program. They also stated that they expect children to know figure characteristics (7), and similarities and differences (7) among figures. Some of the excerpts taken from the interviews with the teachers are as follows:

> Four main shapes were emphasized. We take these four main figures into account. (8)
> Triangle, circle, ellipse, square, rectangle. Generally we also use similes. I also give pentagon as well. (2)
> Figure concepts, numbers, triangle, circle, rectangle, oval, prisms, angular figures. (5)
> Recognizing the similarities and differences between figures. (7)
> I pay attention to whether my students can use geometric concepts in daily life. I give the concept to make him/her familiar. (8)

Whereas the Turkish Ministry of National Education program involved triangle, circle, square and rectangle, during the interviews categories such as oval, polygon, prism, figure characteristics, similarities and differences were also emerged. Again, four of the participant teachers found the attainments sufficient, while the other four maintained that their expectations were higher. Some of excerpts from the interviews with the kindergarten teacher are as follows:

> What matters is not the attainment but the activity. Activity should be emphasized rather than attainment. (4)
> Not sufficient for me, but what students get matters. I give as many as possible: as much as the student gets. (2)
> No, geometry attainment exits only in cognitive area, but it should also be in language and social area. In fact, I think it's about language development. (8)

Another question asked teachers during the interview was "What kind of equipment do you use in geometry teaching?"; the study shows that kindergarten teachers do not make use of materials for geometry education effectively (see Appendix 3). It is seen that teachers benefit from materials around but not much technology and playful staff in teaching geometry. The categories emerged from the data are as follows: Furniture (1, 6, 7), Body (2, 8), Books (1, 2, 8), Model formation (2), Games (2), Toys (3, 5, 7, 8), Blocks (3, 4, 7), Tangram (7), Cards (2, 7), Puppets (8), Cut & Paste (5, 8). Some of excerpts from the interviews with the kindergarten teacher are as follows:

> There aren't many alternatives. We use books. We use cards. They use their bodies. (2)
> Furniture at home, in class. Studying by using their bodies. Books. Homework. (1)
> There are toys, handmade materials, boards. We give examples like house furniture, balls, and circles. Classroom furniture. (5)
> I use puppets. This lets them to introduce figures and themselves. (8)

During the interviews with kindergarten teachers, it was remarkable to observe that the teachers use similar equipment during their activities about geometric figure concepts. The mostly used equipments are classroom furniture, books and toys. The teacher number 2 stated that she sees the game as a tool and emphasized that students at this level could only learn while playing. The teacher number 7, on the other hand, stated that she sees tangram as an efficient tool in geometry teaching to kindergarten students.

The findings of the study showed that the teachers used to teach circle first but they do not do that because the program requires but they do this just because of their earlier experiences in geometry teaching (see Appendix 4). Upon the question "In what order you teach figures in geometry teaching? Why?", the teachers stated the following categories: Circle, triangle, square, rectangle (1, 2, 8), Circle, square, triangle, rectangle (3), Circle, square, rectangle, triangle (4, 6, 5), Square, rectangle, circle, triangle (7) and Easy (1, 6, 7, 8), Agonic (2), Suitable for muscle skills (2, 4, 5), A lot in the vicinity (3, 6, 8). The teachers said that they generally start geometry teaching with a circle (1, 2, 3, 4, 5, 6, 8) because drawing it is easy, it has no corners, it is suitable for muscle skills of the children and most of the objects around them are round. Some of excerpts from interviews with teacher are as follows:

> With a circle, because it is more suitable for their muscle skills. Shapes start with them. A circle is what they draw most easily on dashed lines. (4)
> I start with a circle because it is the easiest shape for the child to grasp. (6)
> Circle, square, triangle, rectangle. Rectangle is the last because it is complicated as it is composed of two different dimensions. Three sides with the same size in the triangle, while four sides with the same size in the square, but only one stroke in the circle. (3)

It is also remarkable that the square is among the lead in teaching, which is because it has four sides of the same size and this makes it easier for the students to learn it since it does not have a different characteristic. The teacher number 3 stated, *The rectangle is the last because it is the most difficult since it is composed of two different dimensions while the triangle has three, the square has four equal sides and a circle can be drawn at one stroke and its continuous*.

The study shows that the teachers are cognizant of developmentally appropriate practices for kindergartners but hold higher expectation for them in geometry teaching. The teachers' answers to the question "How do you expect from your students to answer when you ask what a square is?" can be categorized as follows: 4 sides–4 corners (1, 2, 7), 4 equal sides (4, 5), What does it look like (1, 3, 4, 5, 6, 8), and Different from a rectangle (4). It is interesting that the participant teachers expected from kindergarten students to know the characteristics of square concept. The characteristics they wanted them to know are four sides, four corners, showing square shapes around and knowing its difference from a rectangle. Some of the example excerpts from the interviews conducted with teachers are as follows:

> It is enough for me when he/she says how many sides and corners it has. (7)
> He can tell me about a daily object.

Only the teachers numbered 1, 3, 4, 5, 6, 8 emphasized that it was sufficient for their students who had just met the teaching of geometric figures only to tell them what the figure looked like.

The study shows that the teachers focused on characteristics of figures, similarities and differences among them but they did not use technology and play at all. The teachers gave different answers to the question "How do you teach a rectangle?" The categories emerged from the data were as follows: Foreknowledge is a square (3, 4, 6, 7), Showing examples from vicinity (2, 3, 4, 7, 8), Drawing (2, 7), Cutting (7), Playing games (2, 8), Pointing out characteristics (2, 3, 5, 7), Taking attention to similarities and differences (4, 5, 2, 6, 7), and Playing puppet shows (8).The teachers numbered 3, 4, 6 and 7 said that they could teach the properties of a rectangle by emphasizing the characteristics and figure of a square. They also added that they would give examples from their environment and that showing the similarities and difficulties, they would teach the shape. Only two teachers stated that they would use games. Some of the excerpts from the interviews are as follows:

> After teaching the square, I try to explain and exemplify its difference from the square in various activities. (6)
> I give examples from the class. I point out the differences in the matchings in worksheets. I give examples. (4)
> I first teach how to draw. I explain the figure rectangle. Two long and tow shorter sides. With guidelines. Then I ask them to find rectangular things in the class and then we play games. I ask them to find that shape in class and them make them draw. (2)

The study shows that the teachers relate geometry attainments with some other attainments in the programs. Those are: Cognitive attainments (2, 3), Numbers (2, 5), Order (4, 6), Matching (4, 6, 7), Pattern (6), Colours (5, 7), Classification (4), Hand skills (2), Language development (8), Art (8), Social development (8), and Differences (7). It was determined that kindergarten teachers related geometry attainments to many other attainments which are concepts such as numbers, order, matching, pattern, colors, classification, hand skills, language development, arts, social development and differences. Some of the excerpts from the interviews are as follows:

> With drawing, stories and poem. We make formations with toy blocks. We design houses, railways. (1)
> Matching, differences. (7)
> I relate to concepts. Figures, colours and numbers. How many circles are there here? (5)
> I relate to cognitive development attainments. (3)

With order, matching, classification and attainments involving these. (4)
I relate it to language development. I relate it to arts. (8)

Last question was "How do you use three-dimension objects in geometry teaching?" The categories emerged from the interview data are as follows: Looking at the shape of the object (1, 2, 6, 7), Touching (3, 6), Using models (4, 7), Playing games (8), Paying attention to differences between figures (6), and Forming other figures from the given ones (7). During the interviews with the teachers, they revealed that while teaching geometric figures, they use three-dimensional objects, they make models and play games, stress the differences between figures, making the students touch objects and feel what they look like. Some of the excerpts from the data are as follows:

I start with a three-dimension cube and cylinder. I use them in the activities. I make children form models with blocks. We teach geometric concepts via projects. (4)
I enjoy as much as the students when we go through games. There is no pressure in games. (8)
I use the frontal view of a bookcase. (7)
By asking what it looks like or by using furniture. (1)

5 Discussion and recommendations

Clements, Wilson, and Sarama (2004), who worked on forming compositions from geometric figures, stated parallel to the research conducted by Piaget and Inheilder in 1967 that the developments in children in terms of geometry went through first physical shapes (i.e. figurative knowledge) and then mental constructs (i.e. in mental imagery and eventually as explicit mathematical entities, or operative knowledge). They stated in their studies that the kindergarten students coincidentally succeed in rotating the geometric figures in their compositions (picture maker level), while the students of second grade consciously and knowingly complete their compositions thanks to their cognitive competence (shape composer level). Parallel with the literature, this study shows that the teachers helped children understand and learn geometric figures using real life objects. It is seen that teachers benefit from materials around, such as furniture, books, and toys, but not much technology and playful staff in teaching geometry.

On the other hand, young children's understanding of the geometric figures and grasping their characteristics on the mental level are not consistent with their developmental level (Clements et al. 2004). In this study, however, many teachers expect their kindergarten students to analyze geometric figures on the mental level. For example, it is interesting that the participant teachers expected from kindergarten students to know the characteristics of square concept, such as four sides and four corners although it is not developmental appropriate to expect them to understand geometric shapes at mental level.

According to Sherard (1981), geometry is a fundamental skill with its characteristics such as forming interaction, developing people's spatial perception power, developing one's problem solving skills through brain exercise, and finding solutions for real life problems. Parallel to the literature, it was observed in the study that teachers could make their students intertwine geometry with real life situations. In particular, the findings of the study show that teachers encourage their students to learn about figure through real life objects. However, it was observed by the researchers that these real life objects were limited to those in the classroom environment.

Driskell and Olivia (2004) cite the research conducted by Clements, Swaminathan, Hannibal, and Sarama as follows: Clements, Swaminathan, Hannibal, and Sarama (1999) conducted individual interviews with 97 children between 3 and 6 years of ages on description of figures. Their results showed that the children had a little difficulty in defining a circle and only the children at the age of four frequently categorized rings and curved figures as circles. The children at the age of 6 had less difficulty in defining a circle. Most of the children could define a square, but the children at the ages of 4 and 5 misdescribed a rectangle as a square. What is interesting is that, the children at the age of 4 classified a square as a rectangle. On the other hand, school-aged children did not point out a rectangle as a square when they were asked to choose a rectangle among the same-sized-sided figure and they stated that a rectangle is composed of two long and two short sides. As a result, the children made fewer mistakes in defining a circle and a square than in defining a rectangle and a triangle (cited by Driskell and Olivia 2004). In this study, what the teachers said depending on their experience is in line with this result. Parallel to the literature, seven out of eight teachers revealed that they start teaching geometry with a circle. Again, a rectangle and a triangle lag behind in teaching activities of the teachers. Only the teacher numbered 3 showed that she has misinformation by saying "*The triangle has three equal sides, while the square has four and the circle is composed of just one stroke*" (3), which is thought to lead to misinformation. It is seen that this teacher places the triangle to the second in the teaching of figures because of his misinformation.

Children in their earlier stages can easily distinguish a circle from cornered figures but they mostly confuse it with ellipse, which is caused by the fact that they cannot grasp concepts such as diameter, radius and ring. A teacher, therefore, can use concrete materials while teaching a circle to let the students compare them with an ellipse and recognize the differences (Arnas 2006). During the interviews, four teachers stated that they use ellipse in their teaching techniques; however, they did not state that students confuse it with a circle and they did not state that they therefore focus on the differences between the two figures.

In the literature, it was also stated that relevant computer games and programs are also effective in geometry teaching and to the development of visual languages in children of earlier ages (e.g. Clements 1998; Sarama and Clements 2004). However, in this study, none of the teachers mentioned about computer-supported geometry attempts. However, it is seen that there are internet accessed computers available for the kindergarten students in the sample of the study.

A game is such an activity that supports the social, emotional, motor and cognitive developments of children; that is conducted with spontaneously, pleasure and willingly; and that provides options (Garvey 1990). The opinions of students about the three-dimensional figures start with games as in many other mathematical concepts. Children who play with blocks at early ages are in fact forming the base of a more systematic study to be carried out in the future (Olkun & Uçar, 2004). In this study, it is interesting that only one of the teachers considers three-dimensional objects as game tools while they generally stated that they use them while teaching three-dimensional objects and their characteristics.

One of the frequently used methods and techniques in geometry teaching is story telling which might be putting geometric concepts into interesting characters in a story. Casey et al. (2008) found in one of his experimental studies that story telling in kindergarten is more effective than direct method in geography teaching. Using geometry jigsaws, this technique analyzes the component, the whole relation, and thus enables students to visualize and imagine a concept. In this study, just one of the teachers stated that she puts the figures into characters and presents its material in such a manner to her students.

When the teachers were asked "which attainments in our program do you relate geometry attainments?", it was observed that they relate geometry attainment with various attainments. While doing the relations, attainments will be repeated and will also be dealt with many different activities. It is thought that this will contribute to the learning of such a young child of geometric figures and gaining spatial skills too.

Clements and Sarama (2000) stated that misconceptions at early ages persist during the further periods such as thinking that there is only an equilateral triangle but no scalene triangle. If teachers over simplify the subject at early ages and meanwhile lead to misconceptions, this might continue all through the education life. Clements and Sarama state, 'They [young children] are likely to accept triangular forms with curved sides and reject triangles that are too "long," "bent over," or "point not at the top." Some three-year-olds accept any shape with a "point" as being a triangle' (p. 483). Accordingly, it is essential to give young children correct information related to geometry. In this study, one teacher stated that she defines a triangle as being composed of three equal sides.

One of the most important findings of the study is that the teachers do not look up on spatial relations and skills in the context of geometry. In many kindergarten programs all over the world, it is seen that figure and the location of figures are the indispensable parts of geometry (e.g. 2006 the 36–72 Months Program of Turkish Ministry of National Education 2009; ODE 2004). However, the current study showed that teachers focus mostly on teaching of geometric shapes. Further research can be done on teaching of spatial relations in the kindergarten geometry education.

Suggestions are:

1. Teachers should be informed about geometry attainments of kindergarten students that are parallel to the students' development level. They should not force students to grasp concepts that are not suitable for their developmental level; otherwise, the students might lose their self-confidence in maths.
2. Teachers should be urged to prepare such geometry teaching activities and schedules that students will enjoy and be interested in. Games should be emphasized.
3. Teachers should be informed about location, spatial relations and these subjects should also be included into the program rather than just figure in geometry teaching.
4. Teachers should be made aware of misconceptions and personal development programs should be prepared for this purpose.
5. Teachers should be trained about different methods and techniques used in geometry.
6. Computer programs used in geometry teaching should be presented to teachers and they should be encouraged to this end.

This research has put forward some opinions and thoughts of kindergarten teachers about geometry teaching. Teachers should be trained about the problems faced in the evaluation conducted in the light of modern theories, applications, and the 36–72 Months Program of Turkish Ministry of National Education. Further studies can be conducted in the future to answer the questions arising from this study such as "What are the misconceptions of teachers about geometric concepts, are the teacher sufficiently informed about the cognitive development of children in the context of geometry, what are the responses of students to different methods and techniques used in geometry teaching, what are the attitudes of kindergarten students towards computer-supported geometry education in the sample of Turkey, and how can we conduct studies on spatial relations?"

Appendix 1

Interview questions:

1. What are the kindergarten geometry attainments?
   - 1.a. Are these attainments sufficient?
   - 1.b. If not, which other geometry attainments can be added?
2. What kind of equipment do you use in geometry teaching?
3. In what order you teach figures in geometry teaching? Why?
4. How do you expect from your students to answer when you ask what a square is?
5. How do you teach a rectangle? Tell us about the process.
6. Which other attainments in the program do you relate geometry attainments with?
7. How do you use three-dimension objects in geometry teaching?

Appendix 2

See Table 1.

Table 1 The kindergarten geometry attainments

1. What are the kindergarten geometry attainments?
1.a. Are these attainments sufficient?
1.b. If not, which other geometry attainments can be added?

Intended	Expressed
Triangle, square, circle, rectangle (1, 2, 3, 4, 5, 6, 7, 8)	Four main shapes were emphasized. We take these four main figures into account. (8)
Oval (1, 2, 5, 7)	Triangle, circle, ellipse, square, rectangle
Polygon (2, 4, 7, 8)	Generally we also use similes. I also give pentagon as well. (2)
Prism (5, 8)	Figure concepts, numbers, triangle, circle, rectangle, oval, prisms, angular figures. (5)
Figure characteristics (7)	Recognizing the similarities and differences between figures. (7)
Similarities and differences (7)	I pay attention to whether my students can use geometric concepts in daily life. I give the concept to make him/her familiar. (8)
Sufficient (1, 3, 4, 5)	What matters is not the attainment but the activity. Activity should be emphasized rather than attainment. (4)
Insufficient (2, 6, 7, 8)	Not sufficient for me, but what students get matters. I give as many as possible: as much as the student gets. (2)
More figures (6, 7)	No, attainment only in cognitive area, but it should also be in language and social area. In fact, I think it's about language development. (8)
Different developmental areas (8)	

Appendix 3

See Table 2.

Table 2 Kinds of equipment used in geometry teaching

2. What kind of equipment do you use in geometry teaching?

Intended	Expressed
Furniture (1, 6, 7)	There aren't many alternatives. We use books. We use cards. They use their bodies. (2)
Body (2, 8)	Furniture at home, in class. Studying by using their bodies. Books. Homework. (1)
Books (1, 2, 8)	There are toys, handmade materials, boards. We give examples like house furniture, balls, and circles. Classroom furniture. (5)
Model formation (2)	I use puppets. This lets them to introduce figures and themselves. (8)
Games (2)	
Toys (3, 5, 7, 8)	
Blocks (3, 4, 7)	
Tangram (7)	
Cards (2, 7)	
Puppets (8)	
Cut & Paste (5, 8)	

Appendix 4

See Table 3.

Table 3 Approaches in geometry teaching

3. In what order you teach figures in geometry teaching? Why?

Intended	Expressed
Circle (1, 2, 3, 4, 5, 6, 8)	With a circle, because it is more suitable for their muscle skills. Shapes start with them. A circle is what they draw most easily on dashed lines. (4)
Square (7)	I start with a circle because it is the easiest shape for the child to grasp. (6)
Easy (1, 6, 7, 8)	Circle, square, triangle, rectangle. Rectangle is the last because it is complicated as it is composed of two different dimensions. Three sides with the same size in the triangle, while four sides with the same size in the square, but only one stroke in the circle. (3)
Agonic (2)	
Suitable for muscle skills (2, 4, 5)	
A lot in the vicinity (3, 6, 8)	
Circle, triangle, square, rectangle (1, 2, 8)	
Circle, square, triangle, rectangle (3)	
Circle, square, rectangle, triangle (4, 6, 5)	
Square, rectangle, circle, triangle (7)	

Appendix 5

See Table 4.

Table 4 What a square is

4. How do you expect from your students to answer when you ask what a square is?	
Intended	Expressed
4 sides–4 corners (1, 2, 7) 4 equal sides (4, 5) What does it look like (1, 3, 4, 5, 6, 8) Different from a rectangle (4)	It is enough for me when he/she says how many sides and corners it has. (7) He can tell me about a daily object

Appendix 6

See Table 5.

Table 5 How to teach a rectangle

5. How do you teach a rectangle? Tell us about the process.	
Intended	Expressed
Foreknowledge is a square (3, 4, 6, 7) Examples from vicinity (2, 3, 4, 7, 8) Drawing (2, 7) Cutting (7) Games (2, 8) Characteristics (2, 3, 5, 7) Similarities and differences (4, 5, 2, 6, 7) Puppet (8)	After teaching the square, I try to explain and exemplify its difference from the square in various activities. (6) I give examples from the class. I point out the differences in the matchings in worksheets. I give examples. (4) I first teach how to draw. I explain the figure rectangle. Two long and tow shorter sides. With guide lines. Then I ask them to find rectangular things in the class and then we play games. I ask them to find that shape in class and them make them draw. (2)

Appendix 7

See Table 6.

Table 6 Other attainments in the program related to geometry attainments

6. Which other attainments in the program do you relate geometry attainments with?	
Intended	Expressed
Cognitive attainments (2, 3) Numbers (2, 5) Order (4, 6) Matching (4, 6, 7) Pattern (6) Colours (5, 7) Classification (4) Hand skills (2) Language development (8) Art (8) Social development (8) Differences (7)	With drawing, stories and poem. We make formations with toy blocks. We design houses, railways. (1) Matching, differences. (7) I relate to concepts. Figures, colours and numbers. How many circles are there here? (5) I relate to cognitive development attainments. (3) With order, matching, classification and attainments involving these. (4) I relate it to language development. I relate it to arts. (8)

Appendix 8

See Table 7.

Table 7 How to use three-dimension objects in geometry education

7. How do you use three-dimension objects in geometry teaching?	
Intended	Expressed
The shape of the object (1, 2, 6, 7) Touching (3, 6) Models (4,7) Games (8) Differences between figures (6) Forming other figures from the given ones (7)	I start with a three-dimension cube and cylinder. I use them in the activities. I make children form models with blocks. We teach geometric concepts via projects. (4) I enjoy as much as the students when we go through games. There is no pressure in games. (8) I use the frontal view of a bookcase. (7) By asking what it looks like or by using furniture. (1)

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