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ORIGINAL ARTICLE

Online distance mathematics education in Brazil: research, practice and policy

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Abstract In this article, we address online distance mathematics education research and practice in Brazil, which are relative newcomers to the educational scene. We present the national context of education in Brazil, highlighting the organization of the educational system, and also a summary of national legislation on distance education and an overview of digital inclusion in the country. We outline the potential and relevance of distance education for the Brazilian educational system and show how it could intervene in the system. With respect to research and practice in online mathematics education, we present support for research, examples of studies and highlight different aspects being addressed, including its essential components. In addition, we discuss the synergy between distance education and teacher education, and mathematics distance education and modeling, as well as other initiatives in the national scenario.

Keywords Distance education · Mathematics education · Internet · Teacher education · Modeling

Abbreviations

VLE	Virtual learning environment
ICTs	Information and communication technologies
VCM	Virtual center for modeling

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1 Introduction

In this article, we address distance education in Brazil, conducted over the Internet in particular, with a focus on mathematics education, portraying various initiatives and research trends. We consider distance education to be an educational modality that takes place partially or totally at different times and/or in different spaces. It can take place by means of television, correspondence, Internet, or other media, and we refer to that conducted via the Internet as online distance education.

In Brazil, public access to the Internet began in 1995; however, online distance education is recent, and the number of studies that focus on online mathematics education is still small and restricted to a few groups. Nevertheless, due to various factors, including the lack of educational institutions in certain regions of the country, growing (although still insufficient) digital inclusion among the population, political and governmental incentives, and because it has still not been fully exploited, there has been an intense increase in recent years in participation in distance courses in Brazil. This increase has provided incentives for research in online distance education, including in the field of mathematics.

The quality of distance courses has been the subject of frequent debates in Brazil. For many years, distance education was considered to be second class education, a form of mass education. Various initiatives, however, have shown that there are alternatives that privilege interaction and dialogue between teachers and students and among students (Engelbrecht and Harding 2005). This distance education format limits the number of students a teacher can work with, but opens up many possibilities for education to occur, filling spaces that face-to-face education cannot. Maltempi (2008), e.g., argues that the demand for

in-service education, together with the continental dimensions of Brazil and the shortage of teachers/researchers in certain regions of the country, stimulates the adoption of distance education, which is at times the only viable alternative.

Still considering the quality of distance courses, and in spite of all the controversy surrounding testing to evaluate knowledge, we nonetheless note that the results of the 2005 and 2006 Enade[1] showed that distance education students obtained better scores than students enrolled in face-to-face courses in 7 out of 13 fields of knowledge evaluated. Among students nearing the end of their studies, the best performance for distance education students was observed for administration and mathematics students, whereas those who studied teacher education and pedagogy via distance education obtained the lowest scores. Also, in 2008, the Brazilian Ministry of Education reduced the size of four distance education institutions due to problems in the quality of the teaching. We believe that these results and actions indicate that distance education is being taken seriously at this point in time.

The discussion regarding the quality of distance education courses is unquestionably relevant; however, we have not progressed further on this topic due to the lack of data and research in this field, especially in mathematics. Our focus is to show the progress in research about online mathematics distance education in Brazil, portraying its various initiatives and trends. We note a strong link between online distance education and teacher education, and a movement toward associating it with mathematical modeling.

We begin by presenting the national context in education in Brazil, focusing on its organization, legislation specific to distance education, and aspects of digital inclusion. We then discuss research in online mathematics education, which is strongly associated with the practice of distance education. Finally, we conclude with analyses and trends in mathematics distance education in Brazil.

2 The national context in education

In order to provide the reader with information we believe to be important for understanding trends in mathematics education in Brazil, and distance education using the Internet in particular, we present data in this section that portray the organization of education in Brazil and its current demands and trends. We also describe national legislation establishing guidelines for distance education and present data characterizing digital inclusion of the Brazilian population. Thus, in this section, we do not provide specific data about online distance mathematics education, as there is no existing data on the topics we discuss.

2.1 Organization, data, and demands of the educational system

Responsibility for the educational system in Brazil is shared among the Federal, state, and municipal levels, with the Federal Government having the main responsibility for macro-policies through the Ministry of Education. Basic-level education includes pre-school (up to 5 years of age), elementary education (the only stage that is obligatory, consisting of 9 years of schooling), and high school (only upon completion of elementary school, and consisting of 3 years of schooling). Higher education includes undergraduate and graduate programs.

There is a common basic curriculum in Brazil that is to be guaranteed by all elementary and high schools in Brazil, although they are also allowed to complement the curriculum with contents and activities of local interest.

In 2008, approximately 30 million students were enrolled in elementary education, representing almost all children of this school age, and eight million in high school.[2] At the higher education (college and university) level, data from 2007 show slightly more than 4.8 million enrolled. These numbers indicate that only a small portion of the population make it to college, pointing to considerable room for growth at this educational level. In recent years, the role of distance courses at this level has been increasing in importance.

Data from the INEP[3] show that the number of college undergraduate distance courses in Brazil grew from 46 (20,700 students) in 2002 to 408 (302,500 students) in 2007, representing 7% of college-level enrollment. The number of graduates from distance programs increased from 131 in 2001 to 25,000 in 2006. The majority of these programs are designed to prepare Portuguese, mathematics, biology, history, and geography teachers for the public elementary and high schools in towns located far from urban centers and in rural areas. Data from INEP also show that students enrolled in college-level distance courses

[1] The objective of the National Examination of Student Performance (*Exame Nacional de Desempenho do Estudante*) is to assess student productivity in undergraduate courses in relation to programmatic contents and their abilities and competencies, in face-to-face as well as distance courses.

[2] Data from the 2008 School Census (available at http://www.in.gov.br/imprensa/visualiza/index.jsp?jornal=1&pagina=8&data=29/10/2008).

[3] The Anísio Teixeira National Institute of Educational Study and Research (*Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira*, INEP) promotes studies, research, and evaluations of the Brazilian Educational System with the objective of informing public policy in education.

differ from students enrolled in traditional face-to-face programs in that they are more likely to be non-white, older (average 34 years old), poorer, married, already employed, and their parents have lower educational levels.

It can thus be said that Brazil is currently experiencing a rapid expansion of college-level distance education programs, serving mainly older students, probably due to disadvantages accumulated in elementary and high school, to which access was made more difficult by their economic condition.

These programs also serve students who require greater flexibility to study, in terms of time as well as location, either because they live far from educational institutions[4] or work different shifts, for example. This is the case for some elementary and high school teachers who have no college degree. In Brazil, it is estimated that only 15% of public school teachers have college degrees. In addition, according to the Ministry of Education, there is a shortage of 246 thousand teachers in Brazil, and 300 thousand do not have degrees in the subjects they teach (teachers with degrees in mathematics teaching physics, for example).

When we associate these numbers with the scarcity of higher education institutions in certain regions of Brazil, we understand why distance education has become an increasingly popular option in recent years for updating and addressing the deficit of elementary teachers in Brazil. In college-level programs in pedagogy, specifically, designed to prepare teachers to teach at the pre-school level and the first 5 years of elementary school, there has been a 4% decrease in enrollment in traditional face-to-face programs, while enrollment in distance courses increased 183%. Thus, for every three enrollments in face-to-face programs, there is already one in a distance program (data from the 2006 census). In the specific case of mathematics teacher education, there were no such courses in 2000, and by 2006, there were 24 programs with 26,421 students enrolled.

2.2 Legislation regarding distance education in Brazil

Distance education was only officially recognized as a teaching modality in Brazil in 1996 with the passage of the Law of Directives and Bases for National Education, by the Ministry of Education. In 1998, a decree regularized the article of this law that addressed distance education and established the following definition for distance education in Brazil:

> Form of teaching that enables self-learning mediated by systematically organized teaching materials, presented in different informational supports, used alone or in combination, and transmitted via diverse means of communication (Decree N° 2.494, February 10, 1998).

With this decree, undergraduate and graduate programs acquired the right to be ministered totally in the form of distance education, with the presence required only for the practical laboratory activities and examinations. The teaching plan is expected to be based on the same amount of study hours as face-to-face programs.

In 2001, the Ministry of Education established the National Education Plan (*Plano Nacional de Educação*, PNE) in which "the Brazilian government commits itself to broadening educational services at various teaching levels" (Giusta 2003, p. 20). With this, "distance education gains importance as a means to promote access to learning environments by a significant portion of the population" (Giusta 2003, p. 20), such as youth and adults, professionals who need or seek complementary education, etc. In the PNE, distance education appears as an important theme, with a chapter dedicated exclusively to it. In the case of higher education, the PNE established three objectives yet to be achieved:

> 1. Promote, by the end of the decade [2010], the provision of higher education for at least 30% of the population aged 18 to 24 years. 2. (vetoed) 3. Establish a policy of expansion that decreases inequalities in the provision of services existing among different regions of the country. 4. Establish a broad interactive system of distance education, which can even be used to increase possibilities for providing services in face-to-face programs, regular programs as well as continuing education programs.

Thus, we see that the government of Brazil has been committed to increasing higher education by means of distance education since 2001, which has in fact been occurring, as pointed out in Sect. 2.1. One of the first moves toward this end was, still in 2001, the authorization of the provision of non-face-to-face courses within face-to-face undergraduate programs. It was established that the courses could be conducted in part or totally by distance, up to a maximum of 20% of total hours required for completion of the degree. Similarly, but at the high school level, the state of São Paulo, in October of 2008, began to allow schools to offer up to 20% of the total class hours required via distance education.

With respect to item 3 in the above quotation, distance education is also seen by the Brazilian government as a way to decrease sharp regional inequalities in Brazil.

[4] According to the Brazilian Association of Distance Education (*Associação Brasileira de Educação a Distância*, ABED), close to 70% of Brazilian cities have no institutions of higher education.

Progress in this respect can be noted, but as will be seen in the following subsection, there is still a long way to go before regions and sectors that have historically suffered from discrimination reach the same level as the more privileged.

A new regulation regarding distance education emerged in 2005 in which:

> distance education is characterized as an educational modality in which the didactic-pedagogical mediation in the teaching and learning processes occur with the utilization of information and communication means and technologies, with students and teachers developing educational activities in diverse places and times (Decree N° 5.622, December 19, 2005).

In the years that followed, new decrees were established aiming to regulate, supervise, and evaluate institutions of higher learning that offer distance courses as well evaluate the programs themselves.

Although there is a federal legislation to guide distance education in Brazil, some states, such as São Paulo, have opted to create their own legislation based on the norms of the Ministry of Education.

2.3 Digital inclusion

For distance education to be possible, access to computers connected to the Internet is essential. Data on Internet access in Brazil, however, reflect the reproduction and reinforcement of pre-existing social differences and exclusion, with possible changes in this situation in the last 3 years.

According to Waiselfisz (2007), the number of people with computers in their homes grew from 12.5% in 2001 to 18.5% in 2005, and the number with Internet connections at home rose from 8.3 to 13.6% (a 63.9% increase). More recent data indicate that, in the first trimester of 2008, the number of Internet users in Brazil passed the 40 million mark (23 million homes), corresponding to more than 21% of the population.

If the gap that separates Brazil from the more advanced countries is large—in some countries, close to 80% of the population has Internet access—the differences within Brazil itself are even greater. Data from 2005 show that the level of Internet access in the state of Alagoas (7.6%) is 5.4 times below that of the Federal District (41.2%), where Brasilia, the city with the highest income per capita, is located. The gap between the lowest income group (0.5% with access) and the highest income group (77% of whom have access) in the country is greater still: 154 times. It is evident that the internal differences in Brazil—by income, race, and geographical region—are much greater and deeper than those between Brazil and other nations. Castells (2003) made it clear that the largest concentration of Internet access can be found in those countries and/or regions with the highest per capita income, and emphasized that the centrality of the Internet can be considered equivalent to marginalization for those who do not have access. He, thus, considers it to be yet another factor contributing to social exclusion, from a global point of view, as it has not yet fulfilled the expectations held by many that it would democratize knowledge.

In recent years, since 2005, the Brazilian government initiated various programs to increase the population's access to computers and information technology. In particular, there is a goal that all of the 130 thousand public schools will have computers for teaching by 2010, 50% of them with broadband Internet connections. Special financing opportunities are being provided by the government for teachers at various levels to purchase personal computers. Notwithstanding, according to data from the ABED, the postal service was still the main means used for distance education in Brazil in 2007, surpassing the Internet.

We, nevertheless, believe that the process of digital inclusion has been progressing sufficiently to allow various initiatives in online distance education, which is expected to increase in the years ahead. More privileged groups of the population will undoubtedly continue to benefit more, and serious problems resulting from digital exclusion will emerge and have to be faced in Brazil.

Another relevant factor for online distance education in Brazil is the language in which contents are presented on the Internet, especially when we consider that online distance education may (and we believe this to be a trend) be based on open contents. Despite representing millions of web pages, it is estimated that only 1.5% of contents are in Portuguese, which is less than the percentage the world population that is lusophone.

3 Research and practice

There is no established tradition of research on online distance education (in general) in Brazil (Litto 2009). Within mathematics education, relatively few studies related to online distance education have been carried out as yet in comparison to research that focuses, e.g., on other trends in mathematics education. However, research on different aspects of mathematics distance education has been conducted in association with practice, i.e., based on online courses proffered by researchers interested in the theme or undergraduate distance courses offered by institutions of higher learning.

It can be noted that research on online mathematics education began with the availability of short courses

generally oriented toward continuing education for mathematics teachers. These courses were designed not only for research purposes but also with the aim of contributing to teacher education, and are characterized by intense interaction between teachers and students and among students themselves, with a student/teacher ratio of around 20:1. During the courses, theories and ideas that the researcher/teachers had been investigating using different software programs, technologies, and approaches were investigated online in the context of distance education. This is the case of research seeking to contribute to understanding about the way the production of mathematical knowledge is modified when it occurs in a distance education environment, which is one of the focuses of the Research Group in Computers and other Media in Mathematics Education (GPIMEM), one of the pioneers in research on mathematics distance education in Brazil. We understand mathematical production, as the process of exploration of mathematical concepts and testing of properties, and validation and development of conjectures, with the aim of generalizing them.

Before we analyze these courses and research, we provide an overview of the ways research is supported and encouraged in Brazil, and finally, we discuss some of the essential components of mathematical distance education.

3.1 Organization of support for research in Brazil

In general, public agencies are the main source of financial support for research in Brazil, principally at the Federal level, through Capes (*Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*) and CNPq (*Conselho Nacional de Desenvolvimento Científico e Tecnológico*). Some states have scientific research foundations, with the most active being FAPESP (*Fundação de Amparo à Pesquisa do Estado de São Paulo)*, of the state of São Paulo. These agencies, through calls for proposals, or together with the Federal Ministry of Education or State Secretaries of Education, encourage the development of research on themes they judge to be lacking in information or pertinent according to the national context at the moment.

Although Litto (2009, p. 14) states that "we are far from seeing Distance Education as a field of study that can be considered 'scientific' in the classical sense of the word, [since] we observe that rigorous scientific practices, such as the establishment of precise definitions of phenomena in this area, are nonexistent".

Research related to the application of digital technologies in education, and distance education in particular, has been receiving some stimulus in Brazil in recent years. Since 2005, e.g., Capes has offered scholarships to students interested in developing research on distance education through their Program to Support Research in Distance Education (*Programa de Apoio à Pesquisa em Educação a Distância*, PAPED). Such research recently received considerable incentive with the creation of the Open University of Brazil (*Universidade Aberta do Brasil*, UAB) in 2006, a federal university, and the Virtual University of the State of São Paulo (*Universidade Virtual do Estado de São Paulo*, Univesp) in 2008, whose immediate objective is to broaden the training of teachers at the basic education level in fields such as mathematics.

Such research is developed mainly in public universities, generally by research groups composed of professors and students from various levels who submit projects that are judged based on the merit and experience of the members of the research group. Virtual learning environments (VLE) have resulted from this research that continue to be improved and that, because they are free, favor their adoption and the consequent expansion of distance education in Brazil. Among these, we highlight TelEduc (http://www.teleduc.org.br) and TIDIA-Ae (http://tidia-ae.incubadora.fapesp.br/portal).

It should be pointed out that the increased demand and availability of college-level distance courses in recent years has been accompanied by an increase in the amount of research on online distance education, particularly in mathematics education, as will be discussed in the following subsections. This increase in distance education courses was made possible, and was shaped by, specific legislation, which we briefly presented in Sect. 2.2.

3.2 Aspects of the nature of online distance mathematics education

Gracias (2003) conducted an investigation about the nature of the reorganization of thinking in an online distance course attended by mathematics teachers. The course, entitled "Trends in Mathematics Education", rather than using VLEs, which were rare at the time and were not free, used synchronous (chat) and asynchronous (web page and e-mail) communication tools. Gracias' study emphasized the notion of the reorganization of thinking proposed by Tikhomirov (1981) in which the computer is seen as a mediator of human activities, leading to the reorganization of processes of searching, creating, and storing information as well as of human relations. Based on Lévy's (1993) notion of thinking collectives, which postulates that thinking is a collective act of human and non-human actors, Gracias states that the pedagogical proposal of the course and the technologies, the chat tool in particular, enabled a group of people to experience collective intelligence, to the degree that the social and cognitive potential of each person could be developed and broadened in a reciprocal manner. According to Lévy (2003), the basis and objective of collective intelligence is the mutual

recognition and enrichment of people, "intelligence distributed all around, incessantly valued, coordinated in real time, that results in the effective mobilization of competencies" (Lévy, 2003, p. 28, translated from the Portuguese).

In a new version of the same online course, this time administered in the TelEduc environment, Santos (2006) investigated mathematical production in distance education, focusing mainly on spatial geometry. Course participants were sent activities whose solutions were to be explored using a dynamic geometry software and discussed afterward by everyone via chat. According to the author, the results obtained over the course of the investigation show that information and communication technologies (ICTs) conditioned the discussions and the conjectures elaborated during the geometrical constructions and transformed the mathematical production.

The nature of mathematical learning in continuing education courses for mathematics teachers was investigated by Zulatto (2007) in courses that were conducted mainly via videoconference available in a VLE. The proposal of these courses included the collective production of knowledge in geometry, discussion of the use of dynamic geometry software in the classroom, and training of teachers in technological issues related to the use of this software. She reported that the videoconference tool made it uniquely possible to visually share geometrical constructions and for everyone to become involved, encouraging interaction and active participation. The results led her to conclude that, in this context, the mathematical learning was characterized by: collaboration, in virtue of the discussions woven from the contributions of all the participants; collectivity, in that the mathematical production was conditioned by the thinking collective composed of human actors and media; and argumentation, as the mathematical conjectures and justifications developed intensely during the process, facilitated by the technologies present in the constant, collaborative interaction.

An example of this collaboration in one of the courses proffered via videoconference is presented in Borba and Zulatto (2006). A file was given to the teachers with the figure MNOPQ (presented in Fig. 1). Teachers were asked to find the symmetric figure, in relation to axis "q". Teachers were reminded in the text that the symmetric figure had to remain symmetric, even after being dragged.

According to the authors, when the teachers worked by themselves, most of them used a "count dot approach" in which they counted how many dots a given vertex was from "q" (see Fig. 2). During the synchronous meeting, the teachers worked together to solve the problem and observed that, when the vertex was dragged, the symmetry between the figures was lost (Fig. 3).

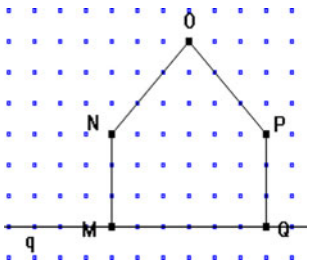

Fig. 1 Starting the activity

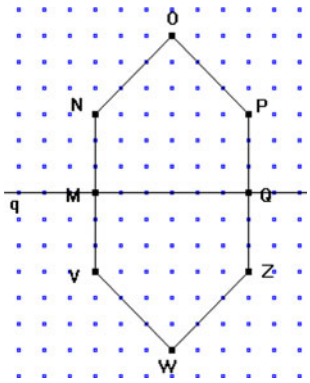

Fig. 2 First solution

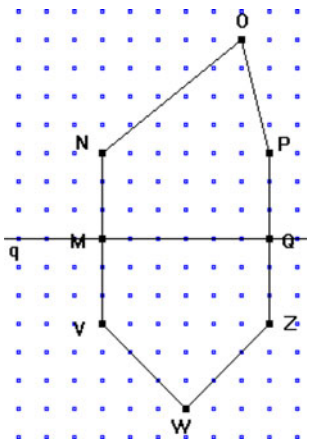

Fig. 3 Dragging problem

Borba and Zulatto (2006) reported that few teachers noticed that the statement itself called attention to the issue of "dragging", so they asked the teachers how a figure could be constructed that would resist dragging. Various suggestions were made, characterizing collaborative work resulting in different constructions that solved the problem, one of which is presented in Fig. 4.

These studies, while they have different focuses, were all based on short courses designed for mathematics teachers and all used the notion of humans-with-media

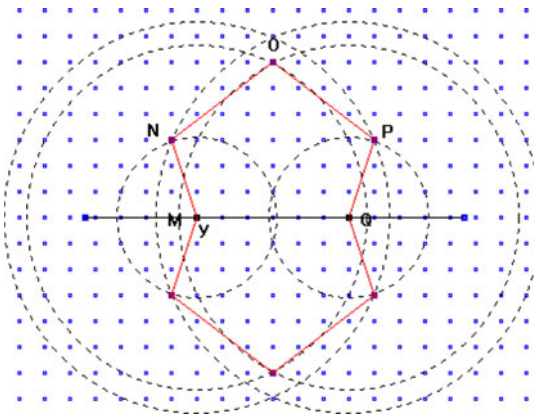

Fig. 4 A final solution

(Borba and Villarreal 2005), which postulates that knowledge is produced by a collective formed of humans and media, as a reference.

Bello (2004) also addressed the possibilities of the production of mathematical knowledge in a VLE based on a course that dealt with geometry contents, emphasizing the collaboration between participants and the interventions of the teachers and tutors.

3.3 Virtual learning environments

As a result of the initial incursions into mathematics distance education, analyses of the tools used in teacher–student and student–student communication, synchronous as well as asynchronous, began to emerge. These analyses point to dialogical differences that are only possible in VLEs, which often restructure the relations normally established in face-to-face education. The chat tool, because of its singular characteristic of establishing a "written–spoken word", was the most explored. Thus, Gracias (2003) and Borba and Villarreal (2005) described and analyzed the experience of discussing mathematical contents via chat. The analysis of discourse using chat was carried out by Bairral (2004) based on his experience with online continuing education courses for teachers. The principal objective of these courses was to investigate the importance of mediation in continuing education in geometry via distance courses. Interactions took place synchronously and asynchronously using the resources provided by the VLE and others, such as MSN (instant messaging), for example. The author presented some contributions of tele-interactions for teacher development in the context of mathematics education.

Based on courses administered online, Maltempi (2007) identified aspects of distance education that are conditioned by the forms of interaction available in the VLEs. He showed evidence of how communication via chat, which is used often in online courses, provides a unique symbolic experience; space, time, acceptance, visibility and identity are different from that of everyday experience because communication is based on written text via the Internet. Based on this premise, he argues that this fact has implications for the way in which school relations become established (crystallized), and thus, for the dynamic of teaching and learning. For example, the teacher has more control over conversations that take place in the classroom compared to online chats, where the student's presence (the student) is represented by the written word, which is continuous, and parallel conversations are central, since all that is said is "heard".

To a lesser degree, the videoconference has also been analyzed (Borba and Zulatto 2006; Zulatto 2007). These authors point out that the videoconference can facilitate a collaborative process in mathematical production; since depending on the VLE used, it is possible that a geometrical construction, e.g., can be carried out by various people in a synchronous manner.

Another aspect that should be emphasized, according to the authors, is the role of visualization in the process of learning mathematics. With the environment formed by videoconference and other software, visualization is favored, encouraging interaction and dialogue as activities are discussed.

These experiences in online mathematics education allowed the researchers to experience the process of teaching and learning mathematics via distance education, whose principal form of communication is textual. Difficulties with the symbolic notation characteristic of mathematics were highlighted by Engelbrecht and Harding (2005) and Borba et al. (2005). The former claims that such difficulties prevented mathematics from being a pioneer in distance education. In addition to the symbolic notation, Borba et al. (2005) point to difficulties with the discussion of geometry in distance classes and present important demands for software developers to strengthen online mathematics education, among them the need to facilitate the use of mathematical symbols in all the text-based communication tools.

Although text communication in VLEs can be a limiting factor, it made research possible that would not have made sense under other conditions. This is the case of the study of Rosa (2008) who investigated how the construction of online identities in an online distance education course contributes to the teaching and learning of the concept of definite integral. In addition to the factors of time and space normally associated with distance education, this author included the dimension of identity (the representation of multiple identities) as a preponderant factor in a distance course (Rosa and Maltempi 2008). The context of the course was an online Role-Playing Game in which future mathematics teachers assumed other roles and simultaneously explored mathematics concepts. Rosa (2008) state

that the construction of online identities takes place through a process characterized by transformation, immersion and individual agency as students learn the concept of definite integral; students produce this concept from the perspective of their online identity, which they constructed at the same time they constructed worlds where they experienced hyperreality and mathematical knowledge.

3.4 Online qualitative research methodology

Borba et al. (2007), Rosa (2008) and Malheiros (2008) discuss the implications of applying qualitative research methodology in a distance education environment, shedding more light on a dimension of mathematics distance education presented by Borba (2004).

Based on Lincoln and Guba (1985) and Denzin and Lincoln (2000), they discuss and provide examples of how methods such as interviews, observation, and others used within a qualitative research paradigm are transformed when applied in an online environment. Issues such as data collection and analysis, triangulation, and what constitutes a "natural environment" are discussed using examples from their research.

Borba et al. (2007) believe that online environments can be considered natural environments. They claim that life today is impregnated with the Internet, as are parks, schools, and other "natural" environments where research studies are carried out seeking to link researchers' understandings with people's experiences.

3.5 Online evaluation

Another focus of research on distance education in Brazil has been student evaluation. Lopes (2004) discusses the viability of implementing evaluation processes, with emphasis on formative evaluation and self-evaluation, among others, within a VLE based on experiences teaching geometry to high school students. Rosa and Maltempi (2006) draw on constructionist theory (Papert 1980) to discuss the evaluation process in the context of an online distance course. Given the frequent teacher–student interaction that becomes established, they argue that evaluation should be understood as a formative process in distance education. In another study, Bairral (2007b) discusses issues related to the evaluation process during continuing education in distance courses proffered to mathematics teachers.

3.6 Teacher education

In a study that addresses discourse, interaction, and mathematical learning in VLEs, Bairral (2007a) presents experiences and analyses courses conducted with in-service as well as future mathematics teachers. He provides examples of situations in which the subjects reflect on their practices using synchronous and asynchronous communication. One of the conclusions he draws is that online distance education presents epistemological challenges that merit further study.

Although it is often not the main focus, many of the studies mentioned in this paper refer to continuing education of mathematics teachers via distance education. This is the main focus of a study by Richit and Maltempi (2009) who discuss pedagogical–technological education in mathematics as a process of appropriation of knowledge regarding the pedagogical use of technologies to address curricular contents. In their discussion, which highlights issues related to distance education from the perspective of the dialectical theory of knowledge, they draw on data from a short semi-distance course that they taught to elementary-level mathematics teachers.

Another study, which is in its initial phase, aims to investigate how the use of hypertext (Lévy 1993), characterized mainly by a lack of linearity, can contribute to the learning of differential and integral calculus during a distance course (Oliveira 2008). The author also seeks to contribute to understanding regarding how learning takes place via hypertexts and what changes in approach, by teachers as well as students, might be required by their use in distance courses.

Another study currently underway aims to analyze how mathematics teachers with no previous formal mathematics education training, and who enrolled in an online undergraduate mathematics teaching program, relate issues originating from their practice to the educational process they are engaged in (Silva and Araújo 2008).

There are also studies involving questions of online education in the face-to-face education of future mathematics teachers. Gonçalvez (2007) described an experience in which students were exposed to online mathematics distance education in a face-to-face course as a way to prepare them for this educational modality.

We note fewer research findings regarding undergraduate education of mathematics teachers administered via distance education compared to research on continuing education. As mentioned, Federal and state (in the case of São Paulo) governments in Brazil are striving to provide an undergraduate education to teachers who are already working through the creation of online courses in public educational institutions. Private teaching institutions are also offering distance courses (in greater numbers than the public institutions, and representing the majority of enrollments in this modality), in an attempt to profit from this fast-growing market. It is thus our belief that the number of studies on undergraduate distance education of mathematics teachers is likely to increase.

3.7 Online distance mathematics education and mathematical modeling

Some research has been carried out in online distance education contexts seeking to understand how other trends in mathematics education, as well as teacher education, may be applied in distance education. This is the case of mathematical modeling, which has been associated with digital technologies for some time. We understand modeling to be a pedagogical strategy that privileges students' selection of the themes to be investigated with the aid of mathematics. In this process, searching, collaboration, and communication are emphasized, and students' contact with mathematical contents is contextualized, making it easier to relate them to everyday issues. The teacher acts as a mediator in the investigative process and production of knowledge. Various authors have developed and reported experiences using this pedagogical approach, including Bassanezi (2002), Mousoulides and English (2008), and Borba (2009).

Much of the research on online distance education cited in this article encourages and values interaction, searching, collaboration, and investigation, the same characteristics emphasized in the mathematical modeling approach. Thus, we believe a synergy that exists between mathematical modeling and ICT, particularly in the context of online distance education. Examples from research described in Borba and Malheiros (2007), Malheiros (2008), and Borba (2009) illustrate this synergy. They show how the Internet, VLEs, and different software contribute to modeling in various ways, ranging from the search for and representation of data (elaboration of texts, tables, and graphs, researching themes, etc.) to the analysis of this data (investigation, reorganization, validation, etc.) and synthesis of knowledge (which can be communicated via the Internet in different ways). Throughout this process, communication and collaboration can be broadened using the Internet, contributing as well to the processes of modeling and production of knowledge.

Some research has been carried out on the use of mathematical modeling in online distance education in Brazil. Jacobini (2002) presents a proposal for using online distance education as a way to support face-to-face teaching of statistics, with modeling being the main teaching and learning method.

Another study involving online distance education and mathematical modeling composes part of a study conducted within the sphere of the virtual center for modeling (VCM) (Borba and Malheiros 2007). This center is an online environment for the investigation of issues related to modeling, characterized by the exchange of information and experiences based on the collective participation of teachers and researchers. It can be considered a "virtual locus" for exchange and intervention in everyday classroom activities, based on the use of the Internet to develop and execute modeling projects. Borba and Malheiros (2007) consider the VCM to have a dual nature, as many people meet there seeking a space for learning and sharing experiences in mathematical modeling as well as developing research. Currently, issues involving videos and modeling are also being researched in the VCM. The center aims, among other things, to contribute to understanding regarding some of the possibilities of the Internet to transform teaching practices related to modeling.

The elaboration of modeling projects by mathematics teachers during an online course was the focus of a study by Malheiros (2008). In this context, the author described how collaboration took place through dialogue and interaction facilitated by different media over the course of the development of the projects. She also outlined considerations regarding the role of interest, of ICT, of collective intelligence, of the mathematical contents, among others, related to the elaboration of modeling projects.

One group of students chose the theme of landline telephone rates for her project, as she wanted to know which company offered the best rates. In this case, it was clear that the student's interest in the theme motivated her to develop the project. Working with another student, they began to collect data from the websites of telephone companies, and based on analysis of the information they found, they presented their doubts about one of the company's billing system using VLE tools.

Interacting with the professors and with each other, they made progress on their modeling project. The dialogue and collaboration were quite intense, as they sought answers to the difficulties that arose. The final version of the project was presented to the entire group in the second-to-last synchronous meeting via chat, after the project had been posted beforehand using one of the tools available in the virtual environment for all participants to read and discuss.

In this example, the synergy between modeling and ICT was present throughout the process, as the modeling project was elaborated via the Internet using different ICTs without any face-to-face contact between the two authors of the project or between them and the rest of the group involved in the course. For example, to sketch a mathematical representation of a given situation, one of the authors photographed a graph she had drawn using pencil and paper and posted it in the VLE to share her mathematical ideas and continue the mathematical modeling process. Thus, with the theme they had chosen as their starting point, the two authors sought solutions to the problems they had posed using computers and other media, in an investigative process characteristic of modeling.

In relation to undergraduate education of mathematics teachers via distance education, Luz (2003) used modeling

as the methodological approach to construct a virtual space for the discussion of mathematics teaching and learning processes.

Thus, modeling is a trend in mathematics education that can be used in online distance education, and research on this theme is ongoing in Brazil.

3.8 Online distance mathematics education: essential components

As mentioned above, distance education in Brazil is defined and guided in general terms by legislation. Both face-to-face and distance education are required to have the same number of hours as well as face-to-face evaluations. But what would be the essential components of online distance education, particularly with respect to mathematics? Such components would make explicit the fundamental differences between face-to-face and distance education.

Our analysis of the research and practice in distance education in Brazil indicates that such components have not yet been established; however, we can make conjectures. To do so, it is important to say that we believe learning takes place in the interaction between the subjects, students and teachers, the media, and the object of knowledge. Thus, communication is fundamental to learning, and when it takes place via the Internet, this different mode of communication may be the principle component of online education that distinguishes it from face-to-face education. Communication via the Internet takes place in spatial–temporal dimensions and through tools that emerge and condition new ways of teaching and learning. In this regard, some of the studies we have cited include considerations about the use of mathematical symbols, communication tools such as chat and videoconference, and tools for studying geometry.

The impossibility, in the majority of known VLEs, of easily typing mathematical symbols in the text communication is undoubtedly one component that distinguishes online mathematics education from offline. In addition, given the increased importance of text communication in distance education, it is reasonable to believe that those who type faster may have an advantage over the others; in the same way, speaking more slowly could be important in a videoconference, depending on the speed of the connection.

Studies involving the use of dynamic geometry software show us that there are differences in the way they are used in distance education depending on the VLE utilized. For example, in Santos (2006), the geometric constructions were made offline and individually with a software, and the files containing the partial and final constructions were posted afterward in the VLE and discussed via chat. In the study carried out by Borba and Zulatto (2006), the geometric constructions could be made online in a collaborative manner, as the VLE provided tools that made this possible. Examples of this type of software use in mathematics teaching are not usually found in face-to-face teaching contexts, where the model adopted tends to be presentation of contents followed by the use of software to carry out activities, individually or in group, in a computer laboratory.

We have tried to provide evidence of some specific components of online mathematics education gleaned from the studies discussed above. We believe that the VLE, particularly the communication resources they provide, together with the teachers view of knowledge and technological fluency, conditions the essential components that characterize online mathematics education.

4 Conclusions and trends

Although recent, Brazil already has a history in online distance education which is the fruit of governmental policies and incentives together with private interests and social demands. In mathematics education specifically, our search showed that there is still relatively little research being done; despite our efforts to give national breadth to the theme, most of the studies we presented have their origins in our research group (GPIMEM), which has members in various regions of Brazil.

Brazil is still experiencing a large expansion of online distance education, to the extent that we may someday see the end of the strong dichotomy between distance and face-to-face education. This dichotomy may give way to an education that we can call *contemporary* that will know how to use ICT to educate people, and will involve face-to-face as well as distance activities. This is a symptom of digitalization in our everyday lives which generates technological habits that students and teachers end up bringing with them into the school. Certainly, this phenomenon is the fruit of digital inclusion in society and, as we presented, still has a long way to go in Brazil. Thus, the tendency is that we will have contemporary education in the more privileged regions of the country while others continue to suffer from basic problems such as shortages of schools and teachers.

Preliminary data from the 2007 Census on Higher Education show a worrisome decline in the number of students earning degrees in specific teaching programs, such as biology, mathematics, and chemistry. From 2006 to 2007, there was a 4.5% decrease in the number of graduates earning teaching degrees. These numbers, together with the fact that nearly 300 thousand teachers lack the appropriate qualifications for the classes they teach, point

to one of the main deficiencies in Brazilian education: the devaluation of teachers. This devaluation is the result of low wages and the lack of social respect, which can only be changed through policies that attribute greater value and status to the profession. Online distance education can address the urgent need to prepare more and better teachers in diverse fields of knowledge, mathematics among them, and the data we have presented show this, as long as it is done well.

The use of distance education to educate teachers can also result in a body of teachers prepared to engage in distance education in the future, since having experienced the process of distance education, teachers may seek to become qualified to use this modality themselves. With the expansion of this educational modality, the demand for professionals who know how to act in this new environment, which requires a logic that is inherently different from face-to-face education in many respects, is likely to grow. We believe that online mathematics education currently reflects face-to-face mathematics education, in the sense that the experiences, beliefs, and teaching models of the latter are reproduced in the former. In other words, if the teacher bases his/her practice on the paradigm of *definition, example, exercise*, he/she is likely to reproduce this paradigm in the context of distance education, even though the model of communication is different. As mentioned earlier, the profile of the student differs, being older and more autonomous, as does the profile of the teacher who in addition have a deep knowledge of the contents to be taught, must also have a command of the technology mediating the teaching and learning process. This technology, the VLE in the case of online distance education, conditions the process through the tools it makes available to the teachers and students. Both need to have a command of the tools and learn to express themselves through them. In this sense, new tools can facilitate teaching and, especially in mathematics, much still needs to be done so that algebra, analysis, and geometry, e.g., can be expressed in ways similar to how they are expressed in face-to-face education, whether synchronously or asynchronously.

It is worth pointing out that in online distance mathematics education, as in face-to-face education, visualization, manipulation, and the utilization of different methodological resources are important for the production of mathematical knowledge. VLEs should anticipate these issues and provide possibilities for students and teachers to interact and relate in different ways and by different means using distinct resources.

Although we found no published findings in Brazil, we would like to note the existence of some initiatives by mathematics education researchers, mainly those who work with ICT, with respect to using VLEs to support face-to-face classes. For example, the teacher may use a VLE to: communicate with students outside classroom hours; distribute exercises, other activities, and test results; receive student work assignments; and occasionally to give a class via discussion forum or chat. When using a VLE, all communication is recorded, and the teacher can use these records to assess students. This is the case in a study reported by Bueno-Ravel and Gueudet (2009) who present and discuss different uses of a VLE by a professor to complement the teaching process in the context of a face-to-face trigonometry course.

Considering research currently being developed, we note one ongoing study involving performance art theory: the Digital Mathematical Performance project. The study of what makes a performance relevant in the world of cinema, theater, music, and poetry has been used in the elaboration of digital mathematical performances (Gadanidis and Borba 2008). These researchers are studying possibilities for synergy between mathematics education and theories originating from the arts found in the virtual world and analyze how these issues can also be used in the context of online distance education.

We also found initial studies about the use of WebQuests in the process of teaching and learning algebra in a virtual community (Barros 2008). In this study, the author will investigate algebra conceptions that can be identified in the WebQuests produced by the participants of that community.

Finally, we call attention to the fact that, in the process of reviewing research on online distance mathematics education in Brazil, we noted a lack of research in Brazil (Litto and Formiga 2009) as well as internationally. We searched in recent issues of Psychology of Mathematics Education (PME) and the journals ZDM Mathematics Education, Educational Studies in Mathematics and International Journal of Computers for Mathematical Learning and found little about online mathematics education. Those show that it is an evolving field and that additional studies and experiences are needed in order to better understand its essential components.

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