

Place and use of new technology in the teaching of mathematics: ICMI activities in the past 25 years

Colette Laborde · Rudolf Sträßer

Accepted: 3 October 2009 / Published online: 20 October 2009
© FIZ Karlsruhe 2009

Abstract Looking back at the place of technology in the past ICMEs during the last decades and on the two ICMI studies devoted to technology, it is obvious that the role and use of technology has given rise to a diversity of points of view and attitudes across the world. The ICMEs and the two studies are places where researchers, teacher educators and practitioners meet. To what extent do they reflect the evolution of the trends of research and/or of integration of technology into real practice? The study will develop a general analysis of the theoretical frameworks, issues and wishes related to the use of technology in mathematics teaching from the proceedings of past ICMEs and of the two ICMI studies. Both a quantitative and a qualitative point of view will be adopted. From the great diversity of questions and approaches, the study attempts to formulate the main trends and their evolution over time within the ICMI community, as well as some pertinent issues for the coming years.

Keywords Computer software · ICT · ICME · ICMI studies · Integration of technology into mathematics teaching · Curriculum · Teachers · Instrumentation · Evolution of research focus

1 Mathematics and its teaching and learning: a history of tools?

The history of mathematics is full of examples showing that the availability of certain tools definitely influences, if not decides, the course of the conceptual development of mathematics as a scientific discipline. As an illustration, let us mention the Arabic notation of numbers in a positional system for arithmetic and the use of ruler and (Euclidean) compass for geometry (with the classical problems of trisection of angles and doubling a cube as prototypic tasks).

With the history and development of the scientific discipline being most influential in the teaching and learning of a related subject [for a certain overstatement see the concept of scholarly knowledge introduced by Chevallard (1985, 1992)], the use and development of tools are also most important for the teaching and learning of a certain subject. When “Geo-Dreieck” was introduced to German geometry teaching after World War II, it took some time to accept that this tool could be used to draw parallels easily without the tedious ruler and compass procedure from Euclid. The conceptually correct and easy trisection of angles using “Geo-Dreieck” was not accepted as a topic in secondary geometry teaching (and learning). It would be an interesting exercise to rewrite the history of geometry teaching and learning, or even of mathematics as a history of available tools. As stressed by Bartolini Bussi et al. in this issue, from the end of the nineteenth century, it became clear, in particular for F. Klein, that teaching methods had to be supplemented by new tools to help students advance in mathematics. Such a historical analysis of a tool in mathematics education was developed by Brock and Price (1980), who reported on the introduction of the use of squared paper in the teaching of mathematics. As the use of squared paper was not really adopted at the beginning of

C. Laborde
Université Joseph Fourier, Grenoble, France
e-mail: Colette.Laborde@imag.fr

R. Sträßer (✉)
Justus-Liebig-Universität, Giessen, Germany
e-mail: rudolf.straesser@uni-giessen.de

the twentieth century, the journal “L’Enseignement Mathématique”, which became the voice of ICMI from 1909 on, published in 1910 suggestions for the use of squared paper in an article by Sainte Laguë (1910). This is another example of the early concern of ICMI for the use of tools in the teaching of mathematics.

The most prominent, recent and modern tool nowadays is the so-called new technology, which we prefer to discuss under the catchwords of “computer, software and communication technology”. With the computer fundamentally being a mathematical machine, it does not come as a surprise that the new technology is also influential, discussed and even researched for mathematics and its teaching/learning. Many countries support this integration at the institutional level, but the everyday practice of a large part of teachers generally does not follow this institutional demand.

There is obviously a tension between, on the one hand, the enthusiasm of pioneers advocating the use of ICT in the teaching and learning of mathematics and, on the other hand, the reality of its use in the mathematics classrooms. Twenty years ago, and in a rather naïve way, it might have been proposed that one could reduce the teaching and learning of mathematics with the advent of new technology, because computers and appropriate software could take over most of the mathematical tasks a person was bound to learn in general education, at least in industrialized countries. This claim would never be stated as such nowadays. The development of research on the use of ICT in mathematics teaching has shown the complexity of the processes and has raised the importance of certain aspects (such as the mediation of mathematical content through technology, the changes in the ways of solving problems using technology that students must learn, conceptual and practical differences when teaching and learning with new technology, the necessary change of tasks), relevant to learning mathematics using ICT.

2 Investigating two kinds of ICMI activities

The role of ICMI is twofold: supporting or even stimulating the development of research on teaching and learning mathematics, and influencing the teaching and learning of mathematics in the world. Apart from other activities, ICMI organizes two major types of activities to fulfill this role:

- ICMI studies gather a small number of selected experts (in most cases less than 100) to synthesize and exchange research results or prospect new questions;
- ICME congresses, open to all mathematics teachers, teacher educators, researchers in mathematics education

and practitioners from across the world, gather a very large number of participants.

The International Commission on Mathematical Instruction (ICMI) following its reconstruction after World War II soon realized the importance of the technological development of computers and mathematical software and its impact on teaching. The first ICMI study took up the issue under the title of “The Influence of Computers and Informatics on Mathematics and its Teaching” (for the discussion document see Churchhouse et al. 1984).

The issue was taken up again in study no. 17, entitled “Digital technologies and mathematics teaching and learning: Rethinking the terrain” (short title: “Technology Revisited”, for the discussion document see <http://www.math.msu.edu/~mathsinc/ICMI>). At the same time and with the International Congresses (abbreviated “ICMEs”), ICMI created forums where participants could address the use of technology in mathematics teaching and learning.

An analysis of the documents of both types of meetings will allow us to follow the integration, addressed by ICMI, of ICT over time, the issues that were put to the fore, the points of view that were adopted and how they evolved. In a final section, a comparison of both activities may reveal whether there were differences between the two types of meetings, probably due to the differences in their audience and their objectives. In addition to this qualitative aspect, the proceedings of ICMEs allow us to determine more quantitatively the importance of technology with respect to the whole range of topics addressed in the congresses. Section 3 is devoted to the analysis of the proceedings of ICMEs, while Sect. 4 is devoted to ICMI studies. Section 5 contrasts the findings of both analyses and Sect. 6 concludes by opening new perspectives.

3 The place of ICT and issues discussed in the ICMEs over the past decade

We analyzed the proceedings and programs of the last ICMEs over the past decade, starting with ICME 8, the conference held in 1996. The proceedings may not reflect the opinion and work of all participants in ICMEs, as there is a limited number of participants presenting in groups or offering lectures. The small number of pages allotted to group reports may also not do justice to all points of view and interventions expressed during the group work. This is a weakness of the methodology adopted here, but this procedure takes advantage of the selection adopted by the proceedings. We start with the assumption that our inquiry into the proceedings of ICMEs offers a representative image of the major views presented within the framework and responsibility of ICMI, because the leaders of the

Table 1 Frequency of the activities devoted to technology in the ICMEs (from ICME 8, 1996, onwards)

ICME	Working groups (ICMEs 8 and 9), Discussion groups (ICMEs 10 and 11)		Topic (study) groups		Lectures		Others
	On technology	Total number	On technology	Total number	On technology	Total number	
8 (1996)	2	26	3	26	1 plenary 6 regular lectures	4 57	
9 (2000)	1	13	1 ^a	23	4 regular lectures	52	
10 (2004)	0	24	1	29	1 subplenary 7 regular lectures	5 80	1 Thematic afternoon (4 themes)
11 (2008)	1	28	1	38	1 plenary lecture 5 regular lectures	8 57	Workshops 12/45

^a The title of the topic study group is “The use of Multimedia in Mathematics Education” (TSG7)

groups, as well as the lecturers, are chosen by an International Programme Committee (IPC) nominated by ICMI, which is responsible for the text of the proceedings. IPCs appointed by ICMI for each ICME conference are chosen in a way that geographical and gender balances are achieved. There is also concern that different trends within the mathematics education community be represented. These IPCs have guidelines to follow the geographical and gender balance in their choice of WG leaders and plenary speakers, as well as a diversity of themes. This led the authors of this study to believe that most perspectives in mathematics education were represented.

3.1 The place of ICT in the ICMEs over the past decade

A first rough indicator of the place of technology in the ICMEs is given by the number of lectures and groups devoted to this topic. In Table 1 we take into account only activities that mention technology in their title.

The importance of activities related in an explicit way to technology seems to fluctuate. ICME 8 was a congress that exhibited on the surface level a peak in the interest for technology with:

- two working groups entitled “The impact of technology on the mathematics curriculum” and “The role of technology in the mathematics classroom”;
- and three Topic Groups entitled “Role of calculators in the classroom”, “Computer based interactive learning” and “Technology for visual representation”;
- a plenary lecture “Information technology and mathematics education: enthusiasm, possibilities and realities” (Tall 1998, pp. 65–82).

The denomination for groups of the following ICMEs was more general. ICME 10 did not propose a large

number of working groups and topic study groups, but had a thematic afternoon devoted to Technology and Mathematics Education. In the continuation of ICME 10, ICME 11 organized one discussion group and one topic study group on technology, and also a plenary lecture devoted to technology and entitled “Technology and mathematics education”. It was delivered by C. Hoyles.¹

A possible interpretation of the decrease in the number of working, topic and discussion groups since ICME 8 could be related to the fact that integration of technology into mathematics teaching makes its use less exotic and decreases the need for specific groups devoted to the topic. In that case, a larger place must be devoted to technology in the groups focusing on other topics, such as those devoted to teaching specific mathematical content areas (such as algebra, geometry or calculus) or teaching at specific educational levels (secondary or tertiary, for example). A review of the reports of these groups from ICME 8 to ICME 10 was carried out to test this hypothesis.

3.2 References to technology in ICME 8 to ICME 10 groups not devoted to technology

Reports from groups, the main focus of which was not technology, showed the diversity of attitudes toward technology among these groups. Several categories can be distinguished among the groups as shown in Table 2. The category “Technology scarcely mentioned” refers to the fact that it is part of one or few presentations and it is not quoted as an issue in the discussion. “Technology

¹ C. Hoyles was awarded the first Freudenthal medal by ICMI in 2004, in particular for her work on technology in mathematics teaching, recognized by ICMI as a contribution, both in terms of theoretical advances and through the development and piloting of national and international projects.

Table 2 Distribution of the groups not devoted to technology in relation to technology according to categories of treatment of technology

Technology	Not mentioned	Scarcely mentioned	Questioned	As a future issue to be addressed	As a tool (and not discussed)	As an issue	Largely discussed
ICME 8					1 WG		
26WG	14 WG		2 WG	4 WG	2 TG (nos. 17 and 23)	5 WG	1 WG
26 TG	12 TG	4 TG	2 TG	1 TG (17)		2 TG	1 TG
ICME 9							
13WGA	4 WGA	2 WGA	2 WGA	5 WGA			1 WGA
23 TSG	11 TSG	4 TSG			2 TSG	4 TSG	1 TSG
ICME 10						0 TSG	13 TSG
24 DG		2 TSG	3 TSG	1 DG	2 DG	3 DG	
29 TSG		8 DG					

A group may belong to several categories (for ICME 10: “DG” means “discussion group”)

questioned” refers to reports that cite possible dangers and question the appropriateness of using technology.

From Table 2, it appears that technology was not taken more often as an issue at ICME 9 than at ICME 8 in the groups not specifically devoted to technology. Globally, on both ICMEs, a minority of groups tackled the issue of technology (less than 20%). More than half of the groups did not mention technology or did not make it an explicit issue to be discussed. In some of these groups, technology was mentioned as an issue for the future (see for example, proceedings of ICME 9, WGA5, pp. 245–246, WGA 10, p. 272).

The picture changes when looking into the proceedings of ICME 10: The word “technology” is used 265 times in the complete proceedings book, with more than half of the appearances in the reports on topic study groups (see proceedings of ICME 10).² Over the last few years, technology seems to be taken into consideration in the review of the state of art done by TSGs, even in those groups that do not have technology as the main focus of study. In addition to this, there was a subplenary “on behalf of Survey Team 5: Information and communication technology in mathematics education” (see Proceedings of ICME 10, p. 228) and a “thematic afternoon” on “Technology in mathematics education” (see Proceedings of ICME 10, p. 281). Discussion groups seem not to be as devoted to technology as topic study groups are, where an increasing presence of technology seems to reflect a growing impact of technology on the teaching and learning of mathematics. If this is the beginning of an evolution, it may be judged by a more careful analysis of ICME 11 as soon as the proceedings are available.

3.3 Issues and points of discussion on ICT

3.3.1 Issues in groups not specifically devoted to ICT

When technology is questioned, “danger or pitfalls of technology” are evoked, sometimes under the more balanced expression “promises”. Judging from and the focus of the discussion as mentioned in the reports, technology was an issue for three main reasons:

- Technology is a catalyst for change in the curriculum or in the teaching practice (WG11 to WG13, WG 15 to 16 of ICME 8, see proceedings of ICME 8, pp. 145–161, 171–175; see the topic study groups nos. 2, 3, 7, 9, 11, 12, 13, 26, 27 of ICME 10, proceedings of ICME 10, pp. 298).
- Technology is a tool that deeply changes mathematical activity, such as modeling or processing data in statistics, experimenting in algebra, geometry or statistics (spreadsheet, dynamic geometry), visualizing in geometry (e.g., TG12 at ICME 8, see proceedings of ICME 8, p. 299; TSG1 at ICME 9, proceedings of ICME 9, p. 293; the topic study groups nos. 2, 3, 7, 9, 11, 12, 13, 26, 27 of ICME 10, proceedings of ICME 10 pp. 298).
- Technology may help students construct a better understanding: technology offers an intermediate level between the physical reality and the formal mathematical model (for ICME 8, see WG12, WG14 and TG17 in proceedings of ICME 8, pp. 145, 163, 341, respectively; TSG4 of ICME 9 in proceedings of ICME 9, p. 303, topic study groups nos. 18, 19, 20, 22 of ICME 10, proceedings of ICME 10, pp. 368). With technology, mathematics becomes more experimental and allows students to change the conditions of the problem, check strategies and receive feedback.

The emphasis is mainly on the content taught with technology and the interactions between technology and the student, the changes brought about by technology on

² The information in Table 2 about ICME 10 was condensed from a search of the word “technology” in the CD shipped with the proceedings, restricted to the contents of the “complete book”.

the students' strategies and interpretations of representations. Sometimes, reports state that teachers must learn how to use technology, e.g., in WG19 of ICME 8: "Teachers need to be guided and supported in using IT" (in Proceedings of ICME 8, p. 189). Groves in WG11 of ICME 8 stated "It is not a trivial matter to use technology effectively in classrooms. Teachers need to rethink math and children learning of math as well as develop new and substantially different skills for teaching and assessment" (see Proceedings of ICME 10, p. 149).

More specifically for ICME 10, technology was one of the major topics addressed by topic study groups devoted to the teaching and learning of a specific mathematical domain (such as algebra, geometry or calculus) or to a specific educational level (secondary, tertiary). The proportion of contributions reporting on experiments with students investigating how technology impacts the solving processes, students' thinking and cognitive processes is increasing, while simultaneously speculations about the potential of technology are decreasing. As formulated by Drijvers in TSG 2 (p. 299), there is more communication among research, development and practical conditions of teaching. Thomas (TSG12, p. 345) also confirmed the convergence of theory and practice. As a consequence, the role of teacher education with regard to the integration of technology was recognized to be critical (TSG 2, DG 20). Technology was also discussed in the topic study group about mathematics education in and for work (TSG 7), as technological environments widespread at the workplace may hide the mathematics underlying the models used by these environments (Sträßer, p. 320). Hoyles and Noss (p. 320) claim that the models are no longer symbolic, but "situated" in these "techno-mathematical artifacts". Finally, technology linked to the use of Internet is also addressed. In particular, a classification of courses relying on Internet is proposed by Engelbrecht and Harding (p. 305) in TSG3. DG 24 about distance teaching addresses the issue of e-learning materials, e-educational standards and e-environments fostering a distributed communication among remote students. The discussion about the dialectical link between tools and teaching (tools shape the teaching and learning, and teaching shapes the use of tools) emerged in some groups. A research report in TSG 10 about geometry (Lopez Real and Leung, p. 333) introduced the notion of "conceptual tool" when analyzing the role of dragging in problem-solving processes of students using a dynamic geometry environment.

To conclude on the topic study and discussion groups at ICME 10 not devoted to technology, technology was cited by most of the groups and in a more specific way than in the earlier ICMEs. Reports no longer dream about all the possibilities technology may bring, but analyze with a more critical eye the use of technology by students. The

theoretical reflection seems to move on and to be in tune with the reality of mathematics teaching.

3.3.2 Focus of the groups on ICT, from ICME 8 to ICME 10

The reports of the groups on technology of past ICMEs confirm that the focus was more on the impact of technology on the curriculum and on the learning than on the teachers themselves. However, from the information available in the reports and in some additional texts of presentations within the groups (in particular, in Borba et al. 1997), one can identify a change in the approach of the issue of integrating technology into teaching.

At ICME 8 (more than 10 years ago), the perspective of technology as a "catalyst for change" in the teacher's role was generally adopted. Some reports on long-term experiences (Dugdale in WG15 of ICME 8; Olive in WG16 of ICME 8 and Borba et al. 1997) indicated significant changes in the teacher's role when calculator or computer was used extensively in the mathematics classroom: the teacher becomes more a stimulant, a manager for learning, an orchestrator of the interactions between technology and students... The theoretical frameworks of these projects were often situated in a socio-cultural approach, in which interactions among students and between teacher and students were critical. In those long-term projects (several years), the teachers were very much supported by summer institutes and worked in strong interaction with researchers, who could impact on the nature of activities given to students. Nevertheless, some difficulties of teachers are expressed in those presentations: a teacher experiences difficulty in engaging in a discussion with students in which she had missed the main conflict (Hershkowitz & Schwarz 1997); a teacher has difficulty in matching the intentions of students in their reactions (Olive 1997). The presentation of Valero (1997) about teachers' beliefs about calculators stressed the length of the process of change and the importance for teachers being part of a research team in the curriculum design. We found agreement that teachers needed to be provided with resources as well as assessment activities to limit their uncertainty in using technology. Allen (in TG19 of ICME 8) insisted that the principal vehicle through which teachers could reconstruct their pedagogies was the writing and use of teaching scenarios.

Less information is available on ICME 9 as there was only one working group and, therefore, one report. The enthusiasm of the ICME 8 presentations on the impact of technology on teaching is far less apparent. Technology is no longer presented as a catalyst for change. The need for more work on several aspects of the integration of technology is expressed in several places. Activities making use of technology at ICME 8 seemed to be open ended, rich and long. The nature of the activities given to students was discussed

at ICME 9: simple tasks supporting learners versus giving access to more advanced topics for bright students. The subgroup devoted to conceptual and professional development of students and teachers expressed the concern of professional development of teachers without any other precisions on the research studies and innovative projects on the topic. Briefly speaking, it seemed that at ICME 9 more questions arose about the use of technology, which was less innovative but more in tune with the reality of the classroom: “It was felt that more work was needed [...] to make the many creative ideas still more adapted to practical conditions and to know more how pupils and teachers make use of these ideas” (Proceedings of ICME 9, p. 276).

The ICME 10 topic study group about technology and mathematics education 15 expressed the move of the research focus “from the individual doing mathematics with software to research attempting to recognize the role of the teacher and curriculum demands on the learner”. One unanimous point was the need for more research that takes the teacher as the central focus, and in particular the relationship between the teacher and technology. The presence of a powerful feature in a software program does not guarantee that teachers will use it. For example, dragging for testing is not really used by teachers who have a history of mediating learning through static representations (Abigail Lins TSG15 of ICME 10, p. 356). It is recognized that the theoretical approach of instrumentation in which the artifact becomes an instrument for the user was not really applied to the appropriation of technology into teachers’ pedagogical practices (for the instrumental genesis approach see Rabardel 1995, for a short description related to CAS: Artigue 2002). It must be mentioned that this growing interest in teachers is not specific to research on the use of technology, but is part of a more general tendency of research in mathematics education (Ponte and Chapman 2006, p. 462, confirmed by a sequence of eras in Sfard 2005, p. 409).

ICME 10 was the first ICME congress to offer a thematic afternoon on five different themes. The choice of technology among the five themes reflects the concern of the scientific program committee of ICME 10 for the issue of the use of technology in mathematics education. This afternoon was very rich and gave rise to a diversity of presentations in a total of 17 sessions involving 52 speakers. It was meant for a broad range of participants, from novice users to experts, and for all levels of education: elementary, secondary and tertiary. Through a range of lectures, panels and hands-on sessions, four main themes were addressed:

- new developments in ICT for mathematics education;
- advantages and pitfalls concerning technology in mathematics education;

- Internet: accessibility, use and misuse;
- technology in distance teaching and learning.

The first two themes gave rise to presentations reporting on experiences from around the world on productive ways to introduce technology in school systems, including developing countries. In particular, algebra was the focus of four sessions and three workshops in which both positive aspects and difficulties were addressed. It was mentioned that despite the promising use of CAS, difficulties appeared that led curricula to privilege approaches to functions with non-symbolic software. There is increasing complexity not only for students, but also for teachers who have to change their way of teaching when using CAS.

The two latter themes reflect the growing importance of technology as a medium for communicating at distance, and storing huge amounts of information and easily accessing it. Although they are not linked to specific contents, these new functionalities of technology may deeply affect teaching and learning processes, especially the work of teachers. The report concentrates on the positive impact of these new possibilities: Resources from various countries are available for teachers, supporting their daily practice. Collaboration between teachers can be supported by e-mail, teacher education may combine the best of traditional and online teaching, and a positive experiment of students from different countries solving collaboratively mathematical problems via e-mail was reported. A similar phenomenon is occurring for these new possibilities brought by technology as it did for software: the focus is on pioneering and successful experience. Limits and difficulties are not yet identified.

3.3.3 Lectures at ICMEs

The proportion of lectures (plenary and “regular”, i.e., paralleling 2–12 lectures in the congress program) devoted to technology was quite stable from 1996 to 2008, as seen in Table 1 (around 10%). The theme of technology was less represented in lectures than in groups. However, the intention of the organizers of ICMEs to bring technology to the forefront was apparent as, with the exception of ICME 9, all ICMEs offered a plenary or subplenary lecture on technology.

We could classify the lectures into several categories:

- speculations on the possibilities and potential of computer technology in mathematics and in mathematics curricula;
- reports on teaching at the secondary or tertiary level integrating technology;
- theoretical issues about the changes produced by new technologies into mathematics, mathematics learning

and teaching, discussed on the basis of experiments or school observations.

The same tendency as in groups emerges from the set of lectures over time. The first category of lectures mainly offered positive speculations on the potential of technologies and these were held mainly in the earlier ICME 8. Later on, lectures on the actual teaching with technology was replaced in the two other ones.

Theoretical issues seem to emerge, mainly since ICME 10. Several lectures share a common issue of mediation of mathematics through technology or of new kinds of representations offered by technologies (such as at ICME 9, Osta, Laborde; at ICME 10, Arzarello, although his plenary lecture was not specifically devoted to technology, Noss, Biehler, Thomas; at ICME 11 Hoyles). Osta (ICME 9) pointed out very clearly the issue of how the conceptualization of the same geometric object varies when represented with different tools, by giving the example of the circle. By referring to semiotic approaches and in particular to Peirce, Thomas (ICME 10) stresses the role of the representational versatility in learning mathematics, how important it is for the learner to be able to establish links between representations and to translate meaning from one representation to another one. He gives examples from research on the use of CAS by students. Noss (ICME 10) claims that “the difficulty of a mathematical idea is often as much in the “representational infrastructure” with which it is expressed as in the idea itself” and that the challenge brought by computer technology is for researchers and teachers to find new ways to represent mathematics that are “learnable and rigorous”. Hoyles in her plenary at ICME 11 also considers that ICT can transform the teaching and learning of mathematics through the new representational infrastructures it may offer. The instrumentation approach was quoted by the lecture of Noss at ICME 10 as focusing on the dialectical link between the notation or the representational infrastructure and the learner, each of them being shaped by the other one.

ICME 11 is probably the first ICME in which some lectures strongly focus on the role of the teacher for the integration of technology into the usual practice of mathematics teaching (plenary lecture by Hoyles, regular lectures by Kieran and Lagrange). Lagrange advocates the use of a plurality of theoretical frameworks to address the complexity and uncertainty of teachers’ activity involving classroom use of technology, such as:

- Saxe’s cultural perspective and the notion of emergent goals used in particular by Monaghan (2005) for analyzing the activity of the teacher in technology-based lessons;
- the theory of didactical situations and the attached notion of “milieu”: how teachers deal with feedback

provided by the computer, which often is not sufficient for the students to produce new knowledge on their own;

- the practitioner model of teachers’ aspirations about the use of technology to support mathematics teaching and learning (introduced by Ruthven and Hennessy 2002). In lessons observed by Lagrange, teachers’ aspirations seemed to play a critical role in the way they deal with emergent goals;
- the instrumental genesis: how teachers construct themselves an instrument from the artifact for two purposes, for doing mathematics with technology and for using technology as supporting the learning of their students.

The design of tasks with technology is also an issue theorized in Lagrange’s and Kieran’s lectures on the use of CAS by referring to the triad: task, technique and theory. Technology impacts the mathematics teaching system by providing new techniques for performing tasks and thus calling for different theorizations. This perspective embraces the issue of designing adequate tasks in a more comprehensive framework taking into account the mutual relationships between the knowledge to be taught, the tools and the ways to use this knowledge.

A change in the lectures (similar to the change in the groups) seems to have taken place starting from ICME 10. There is a move from a focus on the learner to a focus on the teacher and toward referring to more theoretical approaches or even recognizing the necessity of referring to more comprehensive frameworks or a plurality of theoretical frameworks.

4 Issues discussed at two ICMI studies on ICT

ICMI studies are initiated to have a description of the state-of-the-art and possible future developments on an important issue in mathematics education. The institutional backbone of an ICMI study is a study conference organized by an international program committee (appointed by the ICMI executive committee) and a follow-up “study book” documenting the conference and sometimes more of the expertise on the issue at stake. The very first ICMI study was on the role of technology with the conference in 1985. ICMI study no. 17 was deliberately initiated to rethink the role of technology in mathematics education, more than 20 years later, with the study conference held in the end of 2006).

4.1 ICMI study no. 1

In the discussion document, the ICMI study no. 1 was clearly structured around three questions:

“1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science? 2. How can new curricula be designed to meet the needs and possibilities? 3. How can the use of computers help the teaching of mathematics?” (see Churchhouse et al. 1984, p. 161).

Two features of this structure stand out. Question 1 is a clear indication of the importance of the discipline of mathematics for didactics of mathematics (or mathematics education research, we do not want to enter into this ongoing terminological debate at this time). The developments in the scientific discipline mathematics were so important at that time that most of the plenary sessions and one of the three working groups in the study conference were devoted to questions clearly linked to developments inside the discipline or defined by mathematical topics (such as the four-color-theorem, discrete and continuous mathematics, also linked to the curriculum question no. 2, computer algebra and logic; for a more detailed account see the rather comprehensive report on the study conference by Biehler et al. 1986).

The underlying problem in these discussions was how to cope with the changing relation between experiments, explorations and proof in the discipline mathematics and within its teaching. Is a “brute force” approach like the computer-based proof of the four-color theorem a mathematical proof, even if it can never be completely controlled by an individual mathematician? Are activities in applied mathematics and statistics with a focus on “How can it be (best) done?” an acceptable and prototypic piece of mathematics, maybe even to be mirrored in a teaching process more oriented to exploration than to formal proof?

Question 2 somehow stands for the second important feature of the conference. In line with the then widespread focus on curricular issues, question 2 concentrated on curriculum design as a consequence of the advent of the computer and software and was subdivided into ten questions on specific curricular issues (see Churchhouse et al. 1984, pp. 166–168). During the conference, much time was used to discuss the inclusion of computer-related content material into the teaching of mathematics and “problems of implementation” of curricula (the title of one plenary session in the conference). In this respect, the conference was an excellent example of “the era of the curriculum” (as Sfard states in her plenary presentation at the ICME-10-conference; see the ICME 10 proceedings, p. 90, or Sfard 2005, p. 409 for Sfard’s classification of “eras”).

Compared to these more or less detailed questions 1 and 2, question 3 on the teaching of mathematics with the help of computers is rather vague. In Sect. 3 (loc.cit, 168–172), the discussion document has five subsections (“general effects of computers”, “objectives and modes of operation”, “treatment of particular areas” as the longest

subsection, “assessment and recording”, “training of teachers”). Curricular issues are given special attention; again, the user/learner is absent at least from the headlines, while the teacher is given at least some attention as a person to be trained for appropriate use of new technology. ICMI study no. 1 had not yet entered the “era of the learner” nor the “era of the teacher” (according to Sfard 2005, p. 409).

The proceedings of the conference (see Churchhouse et al. 1986) very clearly mirror the foci of the conference. The “Report of the... Meeting” covers 12 pages for the “Effect on Mathematics” with detailed comments on mathematical activities (such as “proof” and “experimentation”) and subject areas, followed by 11 pages on the “Impact of Computers and Computer Science on the Mathematics Curriculum”, especially mentioning the impact on discrete mathematics and calculus and the role of “Exploration and Discovery in Mathematics”. Another 11 pages deal with “Computers as an Aid for Teaching and Learning Mathematics”, together with other issues mentioning the “relation between teacher and student”, which is changed by the introduction of the computer. The “provision of software” and “cultural, social and economic factors” are also discussed. Eleven “Selected Papers” from the conference are reprinted, which more or less concentrate on changes implied by using computers within mathematics and curricular issues related to computer use (mainly for upper secondary and university mathematics).

With the focus on curricular issues, it is worth mentioning that the curriculum area was not really the one where success was available after the study. In 1992, UNESCO edited a revised edition of the conference proceedings (Cornu and Ralston 1992), where Burkhardt and Fraser give a quite discouraging report on the consequences for mathematics curricula and mathematics teaching: “The lack of progress in Domain C (the teaching and learning of mathematics, inserts from CL and RS) is the major mismatch between intentions and outcomes over the last 7 years. It is notable that even the use of simple calculators has not been fully integrated into the curriculum in any country in a way that realizes their known potential for enhancing mathematical performance (even on traditional skills!). The reasons are less clear than is sometimes thought by those who ascribe it simply to teacher inertia and/or parental opposition” (Burkhardt and Fraser, p. 6). Further down in the document, Burkhardt and Fraser suggest that the “work on large scale implementation should become a priority over the next decade... However, the difficulty of achieving large scale change of any kind is often underrated, or at least neglected. It clearly needs empirical study of the dynamics of change in the education system as a whole, with all the factors this brings in. We already know far more about the benefits that could flow

from the use of technology... than is realized in practice. Without attention to Domain C, this mismatch will simply get worse" (loc.cit., p. 8). Even if official curricula prescribe the use of new technology, implementation in the classroom seems far from obvious.

A comparison of the original proceedings (edited by Churchhouse et al.) and the version edited by Cornu and Ralston (1992) is quite interesting. The 1992 version offers the "quick pace of change of computers, mathematics and its teaching" as reason of the new version (see the "Preface" from UNESCO), while the editors characterize the changes: "For this edition the report of the Strasbourg meeting itself has been brought up to date by the leaders of the three workshops held at that meeting and five of the articles in the first edition have been updated for this edition. In addition, the editors have solicited four new articles written just for this Edition" (from the "Editors' Foreword to the Second Edition"). One of the new papers (by Stephen B. Maurer) is on algorithms and algorithmics, a curriculum topic high on the agenda in the early 1990s, while the other three additionally confirm the focus on curricular issues and teachers (see the paper by Cornu).

4.2 ICMI study no. 17

More than 20 years later, the discussion document for the ICMI study no. 17 (see "Digital technologies..." 2006) shows a different structure. After explicitly linking the new effort to the first ICMI study, it tries a new approach: "While we noted the first Study was largely focused on modeling mathematics, more recently work has focussed much more generally on the multitude of ways technology can shape teaching and learning mathematics, while reciprocally being shaped by its use.... New robust paradigms for thinking about tool use in the context of mathematics education are beginning to emerge and ICMI Study 17 aims to take a further step forward in this direction" (see discussion document, p. 5). To follow this brief introduction, the discussion document identified seven "themes" to "provide complementary perspectives on the use of digital technologies in mathematics teaching and learning", which were the following: "Mathematics and mathematical practices; Learning and assessing mathematics with and through digital technologies; Teachers and teaching; Designing learning environments and curricula; Implementation in curricula and in classrooms; Access, equity and socio-cultural issues; Connected and networked classrooms" (from the discussion document, p. 6).

From this citation, two changes are obvious: developments inside the discipline mathematics are less important than in the ICMI study no. 1, whereas ICMI study no. 17 starts from the assumption that there are theoretical approaches and paradigms, which help with a detailed

analysis of the teaching process and teachers. We deliberately played down the learning aspect, because the "era of the learner" was somehow less present in the discussion document of ICMI study 17. The "student" is mentioned only twice in the document, with the first instance talking about problems of student assessment (discussion document, p. 8) and asking for the potential of the new technology for "students with special needs" (discussion document, p. 12). The word "learner" does not show up in the discussion document.

According to the discussion document, the seven themes should be tackled within five "approaches" (discussion document, p. 13f), which somehow confirm the description given so far: With the approaches "impact on mathematics", "roles of different technologies", "contribution to learning mathematics", "the role of the teacher" and "theoretical frameworks", the idea of the existence and importance of theoretical frameworks is confirmed, the teacher is clearly identified, while the learner shows up only in the respective activity. The four instances of "learn" bring us to the two places dealing with "students" mentioned above. The other two places deal with "learning environments" and learning from teachers. The "contribution to learning mathematics ... could be addressed in terms of cognition or affect, with regard to mathematical fields, activities and contexts at different school levels, or in contexts in and out of school" (discussion document, p. 13).

If one looks into the plenary activities of the study conference held in Hanoi in December 2006, the general tendencies described above are confirmed. In the conference presented by Seymour Papert, the "keynotes" at the beginning and end of the conference reflected on the difficulties in implementing the use of new technologies in the classrooms all over the world, with the suggestion of avoiding the mistakes of the introduction of the "New Math" reform and a technical solution: the \$100 laptop (a project offering "one laptop per child", see <http://laptop.org/>). In the second keynote address, Michèle Artigue outlined three "perspectives... to reflect on the potential and limitation of what has been achieved so far for thinking about the future: the theoretical perspective, the teacher perspective, the institutional and curricular perspective" (from the abstract of Michèle Artigue's keynote).

Some of the papers reacting to the discussion document were presented in "parallel sessions" (in most cases four parallel presentations with 25 min for each individual paper), so one can have only a rather global idea about these contributions. Judging from the titles of these presentations (to be found in the info on the study conference on the respective Web site), two areas of mathematical contents were especially analyzed (namely algebra and, more often, geometry). The question of a sustainable development is still open, but there are some theoretical

perspectives on the use of computers, software and communications technologies, which are now available in a way that there is even an opportunity to start comparing them (see the “TELMA”-approach and its description in TELMA 2006, contribution “c54” of the CD of the study conference).

At the time of writing the final version of this text (i.e., May 2009), the “study book” on ICMI study no. 17 was not yet in the bookstores. Following a hint from one of the anonymous reviewers, we (CL and RS) could analyze the document, which was sent to the publisher of the book (Hoyles and Lagrange 2009). After a foreword, the book is structured into five sections with the following headlines: “Design of learning environments and curricula”, “Learning and assessing mathematics with and through digital technologies”, “Teachers and technology”, “Implementation of curricula: issues of access and equity” and “Future directions”. Judging from these headlines, the foci on curricular issues and teachers are confirmed, while a closer look into the “learning” and “future directions” section seems appropriate. Following the description of the “learning” section in the foreword, two features stand out: in a whole chapter (no. 7), special attention is given to theoretical frameworks for analyzing the introduction of new technology into learning mathematics, which is taken up later with a discussion of different frameworks to analyze “Technology, communication, and collaboration” (headline of chapter no. 11). The “future directions” section was derived from the plenary sessions at the study conference, namely... from the panel about design for transformative practices... (with) creators and designers of well-established and widely used software environments,... (and another one with) panelists who were asked to present their views and experience of the role of connectivity and virtual networks for learning mathematics” (inserts from CL and RS). The plenary talk by Artigue is also documented (see Artigue 2009), which also prominently took up the issue of theoretical frameworks.

The theoretical framework, which is most often used, namely the “instrumental genesis approach”, points to the other outstanding feature of the section on “learning”. With its distinction between artifacts and instruments and the attention paid to “utilization schemes”, it clearly indicates the importance of the human factor in the introduction and use of new technology. For education, these actors are mainly in a teaching or learning role, and the section on “Teachers and technology” pays due attention to one actor in the game. The discussion document of the study is quite revealing for the other actor, the learner. A mere linguistic analysis of the respective part of the discussion document shows that the part on “Learning and assessing mathematics with and through digital technologies” of the discussion document was written in a more

prescriptive than descriptive attitude, it often asks “should” questions. Directly dealing with assessment problems confirms this interpretation. On the other hand, prioritizing the instrumental genesis approach as theoretical background somehow should act in the opposite direction, privileging empirical analyses. We (CL and RS) interpret this as: the ICMI community has fully embraced the teacher as the most important actor in the introduction and use of technology, while the student learner has not been taken into account in the same way as the teacher. To check this perspective, we did a word search for utilization scheme in the proceedings file and found nine places of use of these two words. This concept is somehow especially apt to practice an empirical option of the instrumental genesis approach. Four occurrences serve as introduction and definition of the concept, while another four of them describe utilization schemes found in empirical studies (all four in the section on “Learning”). The last occurrence of utilization scheme is in the register (wrongly giving only four page numbers). Looking only for “utilization” produces one more place in the document: Here the authors describe a “utilization schema” and immediately link it to instrumental genesis.

5 Contrasting the analyses of ICMEs and of ICMI studies

Not surprisingly, the issues addressed in the first ICMI study seem to deeply differ from the contents of the ICME congresses: ICME 8 took place 11 years after the ICMI study n 1. The ICMI study devoted a large part of the discussion on the impact of ICT on mathematics and its development, whereas teaching was the focus of ICMEs. However, one can see a link between the emphasis placed by the ICMI study on the change brought by technology on the mathematical activity (in the discipline itself or in the teaching) and technology considered as a catalyst of change on the curriculum in ICME 8 or as contributing to introduce an experimental dimension into the mathematics classroom.

The second ICMI study reflects far more on the influence of research in mathematics education on the use of ICT in teaching processes and on the role of the teacher. In this, it is much closer to issues discussed at ICME 10 and in the topic study group about technology at ICME11. A specific feature of the study book of ICMI study no. 17 is the explicit discussion of theoretical frameworks, with a visible stress on the instrumental genesis approach. ICMI study 17, ICME 10 and lectures at ICME 11 converge in recognizing the complexity of integration of technology under various aspects: the role of the teacher, the social dimension of the classroom, the curricular perspectives and

stress on the importance of the development of several theoretical frameworks taking into account this complexity. A study of the proceedings of the ICMEs seems to show that theoretical frameworks are more visible in lectures than in group work. Two reasons may explain this difference:

- the format of the group reports is less adequate for giving account of a theoretical approach, since the page number is rather limited;
- the lecturers are chosen by the program committee for their expertise, whereas the participants of groups come from various backgrounds not necessarily linked to research. One of the original features of ICMEs is precisely to organize the meeting of researchers and practitioners.

An interesting issue is the shallow picture of the empirical, actual learner in the recent ICMI study, whereas the first ICMEs (ICME 8, in particular) seemed to take into account interactions between learners and technology. A survey of research shows that the focus of research in the last 20 years was more on the learner than on the teacher or on the class (see Laborde et al. 2006). The paradigm of micro-world inspired many studies, which carefully analyzed the solution processes of students faced with computer tasks as well as their learning processes (several research reports were presented in PME conferences). The second ICMI study took place 22 years after the first one and during this long period of time, the paradigm of learning with and through technology was largely investigated. The second ICMI study may have preferred to address questions of the moment rather than synthesizing results. This raises a general question on how the frequency of ICMI studies on a specific topic may affect the content of the study and give a biased view of research results. A different interpretation may be that only a more careful analysis of the study book would offer a different picture of what was going on in the study conference and what was documented in the study book, which would be in the bookstores soon.

6 Conclusion

We would like to conclude this investigation on the ICMEs and ICMI studies' proceedings and documents by pointing out to some issues that arise from our study and suggesting orientations for debate or future research in the communities of teachers, educators and researchers.

- (1) The developments in the discipline of mathematics have become less important for the educational use of computers, software and communication technology.

The relative autonomy of the educational system, of research in didactics of mathematics and of classroom practice, creates uses and rejection of new technology, which is not fully controlled by developments inside the discipline of mathematics.

- (2) Problems of implementation of pieces of (educational) software, learning environments and use of communication technology are far from being solved. As was already mentioned in the intermediate report by Burkhardt and Fraser, the discrepancy between intentions, suggestions and potentials to use new technology and the actual use of it is still wide. The "royal road" to the educational use of computers, software and communication technology within mathematics teaching and learning is still to be discovered, if it ever exists. Technology develops at high pace and new facilities are systematically penetrating mathematics teaching. The example of the recent distance possibilities shows the same process as for the integration of software: the first reports mainly focus on the successful aspects while not identifying the limits or difficulties. After the first pioneering phase, when the technology is more widely used, a more complete picture of its use is reflected by the reports.
- (3) It seems obvious that a mere analysis of the artifacts (computers, software, communication technology) is not sufficient to make this technology to be used in teaching and learning mathematics. "User studies" (often referred to in informatics) are an unavoidable prerequisite for the implementation of new technology in the mathematics classroom. To state it in the terminology of one of the theoretical frameworks widely used in didactics of mathematics (see Rabardel 1995): the analysis of the artifact is an insufficient presupposition to introduce and understand its use. Only an analysis of the instrument, i.e., the interaction of the artifact and the utilization schemes of its users (teachers and students), the analysis of its "instrumental genesis" will help in the implementation of computers, software and communication technology in the mathematics classroom.
- (4) Apart from a lot of most challenging, well-designed software and suggestions to use new technology in the classroom, the most important innovation seems to be for research: different theoretical frameworks have been developed and used within research on (the use of) new technology in teaching and learning mathematics as reflected by the second ICMI study. Theoretical frameworks are needed because of the fast pace of change of computers and software. For longitudinal studies, only appropriate theoretical frameworks allow for comparisons between the actual

uses of new technology. The time is ripe to start testing not only the domain of functioning of these frameworks to know more about the strengths and weaknesses of the frameworks, but also of (the use of) computers, software and communication technology in teaching and learning mathematics.

Literature outside the ICMEs and ICMI study proceedings provides for individual pieces of mathematical domains, a wide range of ideas, artifacts and suggestions for their use. This is especially true for geometry (for a research overview see Laborde et al. 2006). Software and suggestions for using it are also available and well analyzed in algebra (see for instance the overview in Artigue 2002 or more recently Ferrara et al. 2006). The format of ICMI studies and of ICMEs is perhaps not the most appropriate one to give a valid picture for those ideas, as either they offer surveys or discussions on vivid general questions. The role of ICMI studies and ICMEs is not to repeat what is addressed in specific research conferences, but to discuss farther reaching questions by taking into account both the perspectives of research and of the reality of teaching.

References

- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal for Computers in Mathematical Learning*, 7(3), 245–274.
- Artigue, M. (2009). The future of teaching and learning mathematics with digital technologies. In C. Hoyles, & J.-B. Lagrange (Eds.), *Mathematics education and technology—rethinking the terrain. The 17th ICMI study* (pp. 497–510). Heidelberg: Springer.
- Biehler, R., Sträßer, R., & Winkelmann, B. (1986). Report on the ICMI Symposium on “The Influence of computers and informatics on mathematics and its teaching”. *Zentralblatt für Didaktik der Mathematik*, 18, 26–36.
- Borba, M., Souza, T., Hudson, B. & Fey, J. (Eds.). (1997). The role of technology in the mathematics classroom. In *Proceedings of working group 16 at ICME 8*. Rio Claro: UNESP-State University São Paulo.
- Brock, W. H., & Price, M. H. (1980). Squared paper in the nineteenth century: instrument of science and engineering, and symbol reform in mathematical education. *Educational Studies in Mathematics*, 11, 365–381.
- Burkhardt, H., & Fraser, R. (1992). An overview. In B. Cornu & A. Ralston (Eds.), *The influence of computers and informatics on mathematics and its teaching* (Vol. 4, pp. 1–10). Paris: UNESCO.
- Chevallard, Y. (1985). *La transposition didactique (2nd enlarged ed. 1991)*. Grenoble: Pensée sauvage.
- Chevallard, Y. (1992). A theoretical approach to curricula. *Journal für Mathematikdidaktik*, 13(2/3), 215–230.
- Churchhouse, R. F., Cornu, B., Ershov, A. P., Howson, G., Kahane, J.-P., van Lint, J. H., et al. (1984). The influence of computers and informatics on mathematics and its teaching. *L'Enseignement Mathématique*, 30, 161–172.
- Churchhouse, R. F., et al. (Eds.). (1986). *The influence of computers and informatics on mathematics and its teaching* (Vol. 1). Cambridge: Cambridge University Press.
- Cornu, B., & Ralston, A. (Eds.). (1992). *The influence of computers and informatics on mathematics and its teaching* (Vol. 44). Paris: UNESCO.
- Digital technologies and mathematics teaching and learning: Rethinking the terrain. The 17th ICMI Study “Technology Revisited”*. (2006). Retrieved September 29, 2007, from <http://www.math.msu.edu/~mathsinc/ICMI>. (also in: *L'enseignement mathématique*, 51 (2005), 3–4, 351–363,
- Drijvers, P., Kieran, C., & Mariotti, M.-A. (2009). Integrating technology into mathematics education: theoretical perspectives. In C. Hoyles, & J.-B. Lagrange (Eds.), *Mathematics education and technology—rethinking the terrain. The 17th ICMI study* (pp. 105–150). Heidelberg: Springer.
- Ferrara, F., Pratt, D., & Robutti, O. (2006). The role and uses of technologies for the teaching of algebra and calculus. In A. Gutierrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education* (pp. 237–273). Rotterdam: Sense.
- Healy, L., & Kaput, J. (2008). *TSG 15: The role and the use of technology in the teaching and learning of mathematics*. In M. Niss, & E. Emborg (Eds.), (2008). Proceedings of the 10th international congress on mathematical education, 4–11 July, 2004. (pp. 355–358) Roskilde.
- Hoyles, C., & Lagrange, J.-B. (Eds.). (2009). *Mathematics education and technology—rethinking the terrain. The 17th ICMI study*. Heidelberg, New York: Springer. (available in a preprint version for inspection).
- IMFUFA, Dpt. of Science, Systems and Models, Roskilde University. Hershkowitz, R. & Schwarz, B. (1997). The technology and the development of sociomathematical norms in classroom. In Borba, M., Souza, T., Hudson, B. & Fey, J. (Eds.), *The role of technology in the mathematics classroom, Proceedings of Working Group 16 at ICME 8* (pp. 15–35). Rio Claro: UNESP-State University São Paulo.
- Laborde, C., Kynigos, C., Hollebrands, K., & Strässer, R. (2006). Teaching and learning geometry with technology. In A. Gutierrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education* (pp. 275–304). Rotterdam: Sense Publishers.
- Olive, J. (1997). Technology as a catalyst for change in the teaching of mathematics. In M. Borba, T. Souza, B. Hudson, & J. Fey (Eds.), *The Role of Technology in the Mathematics Classroom, Proceedings of Working Group 16 at ICME 8* (pp. 144–153). Rio Claro: UNESP-State University São Paulo.
- Ponte, J., & Chapman, O. (2006). Mathematics Teachers’ knowledges and practices. In A. Gutierrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education* (pp. 461–494). Rotterdam: Sense Publishers.
- Proceedings of ICME 10. (2008). In M. Niss, & E. Emborg (Eds.), *Proceedings of the 10th international congress on mathematical education, 4–11 July, 2004*. Roskilde: IMFUFA, Dpt. of Science, Systems and Models, Roskilde University.
- Proceedings of ICME 8. (1998). In C. Alsina, J. M. Alvarez, M. Niss, A. Pérez, L. Rico, & A. Sfard (Eds.), *Proceedings of the 8th international congress on mathematics education, Seville 14–21 July 1996*. Seville: S.A.E.M. ‘Thales’.
- Proceedings of ICME 9. (2004) In H. Fujita, Y. Hashimoto, B. R. Hodgson, P. Y. Lee, S. Lerman, & T. Sawada (Eds.), *Proceedings of the Ninth International Congress on Mathematical Education*. Dordrecht: Kluwer.

- Rabardel, P. (1995). *Les hommes et la technologie*. Paris: Armand Colin. (translated into English: “People and Technology—a cognitive approach to contemporary instruments”, available on the Internet in 2006, could not be retrieved on Oct. 4, 2007).
- Sainte Laguë, A. (1910). Note sur les usages du papier quadrillé. *L'Enseignement Mathématique*, 12, 5–17.
- Sfard, A. (2005). What could be more practical than a good research? On mutual relations between research and practice of mathematics education (Survey Team 1 Report on Relations between research and Practice of Mathematics Education). *Educational Studies in Mathematics*, 58, 393–413.
- TELMA (KALEIDOSCOPE) European Research Team (Artigue, M., Bottino, R., Cerulli, M. et al.) (2006). *Developing a joint methodology for comparing the influence of different theoretical frameworks in technology enhanced learning in mathematics: the TELMA approach*, “c54” on the CD of ICMI study 17.
- Valero, P. (1997). Teachers’ Beliefs and Technology in the Classroom. In Borba, M., Souza, T., Hudson, B. & Fey, J. (Eds.), *The Role of Technology in the Mathematics Classroom, Proceedings of Working Group 16 at ICME 8* (pp. 185–192). Rio Claro: UNESP-State University São Paulo.