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ORIGINAL ARTICLE

Characterizing exemplary mathematics instruction in Japanese classrooms from the learner's perspective

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Abstract This paper aims to examine key characteristics of exemplary mathematics instruction in Japanese classrooms. The selected findings of large-scale international studies of classroom practices in mathematics are reviewed for discussing the uniqueness of how Japanese teachers structure and deliver their lessons and what Japanese teachers value in their instruction from a teacher's perspective. Then an analysis of post-lesson video-stimulated interviews with 60 students in three "well-taught" eighth-grade mathematics classrooms in Tokyo is reported to explore the learners' views on what constitutes a "good" mathematics lesson. The co-constructed nature of quality mathematics instruction that focus on the role of students' thinking in the classroom is discussed by recasting the characteristics of how lessons are structured and delivered and what experienced teachers tend to value in their instruction from the learner's perspective. Valuing students' thinking as necessary elements to be incorporated into the development of a lesson is the key to the approach taken by Japanese teachers to develop and maintain quality mathematics instruction.

Keywords Mathematics classroom · Japanese lessons · Cultural activity · Learner's perspective · Lesson study

1 Introduction

The findings of large-scale international studies of classroom practices in mathematics include aspects of instruction as identified with a resemblance among participating countries while instruction in Japan seemingly unique (Clarke, Emanuelsson, Jablonka & Mok, 2006; Hiebert, et al., 2003; Stigler & Hiebert, 1999; Stigler, Gonzales, Kawanaka, Knoll & Serrano, 1999). Japanese mathematics teachers, for example, appeared to spend more time on the same task in one lesson than their counterparts in other countries by having students work on a challenging problem and discuss alternative solutions to it (Hiebert et al., 2003). Also, experienced teachers in Japan typically highlighted and summarized the main points at particular phases of lessons to have their students reflect on what they have learned (Shimizu, 2006b). These striking characteristics can be regarded as indicating some indispensable elements of mathematics classroom instruction that are valued and emphasized by Japanese teachers.

The current paper aims to examine those aspects of mathematics classroom instruction that appear to make Japanese lessons different from the other countries and to explore key characteristics of exemplary mathematics instruction in Japanese classrooms.

For this aim, selected findings of large-scale international studies of classroom practices in mathematics are examined for discussing the uniqueness of Japanese mathematics lessons and what Japanese teachers value in their classroom instruction. Particular attention is given to the ways lessons are structured and delivered with an emphasis on presenting and discussing students' thinking on alternative solutions to the problem. Cultural values attached to the characteristics of quality mathematics instruction by Japanese teachers are also discussed. Then, an analysis of post-lesson video-stimulated interviews with 60 students in three "well-taught" eighth-grade mathematics classrooms in Tokyo is reported to discuss participants' views on a "good" mathematics lesson. The data are those from the Learner's

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Perspective Study (LPS), an international study of the practices and associated meanings in mathematics classrooms taught by competent teachers in participating countries (Clarke, Keitel & Shimizu, 2006). Central to the LPS is the notion of the competent teacher as recognized within the local culture (Clarke, 2006). In this paper, the characteristic of exemplary mathematics instruction is examined by referring to those lessons taught by experienced teachers selected for the study as the competent teachers in the Japanese context. The results of the analysis are discussed with a focus on the co-constructed nature of classroom practice. Japanese teachers approach to maintaining quality mathematics classroom instruction through the particular form of activity called "lesson study" (Fernandez & Yoshida, 2004; Shimizu, 2002) is also discussed with an emphasis on the importance of incorporating students' thinking into the development of lessons.

In this paper, the following questions will be addressed to examine characteristics of exemplary mathematics instruction in Japan: (a) What do Japanese mathematics teachers value in structuring and delivering their instruction from a teacher's perspective? (b) What are the characteristics of a "good" mathematics lesson from the learner's perspective? (c) How can we incorporate these two perspectives into our characterization of exemplary mathematics instruction in Japanese classrooms?

2 Aspects of Japanese mathematics instruction

2.1 Mathematics lessons as "problem solving"

Japanese teachers, in elementary and junior high schools, in particular, often organize an entire mathematics lesson around the multiple solutions to a single problem in a whole-class instructional mode (Shimizu, 1999). This organization is particularly useful when a new concept or a new procedure is going to be introduced during the initial phase of a teaching unit. Even during the middle or final phases of the teaching unit, teachers often organize lessons by posing a few problems with a focus on the various solutions students come up with.

A typical mathematics lesson in Japan, which lasts 45 min in the elementary schools and 50 min in the lower secondary schools, has been observed as divided into several segments (e.g., Becker, Silver, Kantowski, Travers, & Wilson 1990; Stigler & Hiebert 1999). These segments serve as the "steps" or "stages" in both the teachers' planning and delivering actual teaching–learning processes in the classroom (Shimizu 1999):

- Posing a problem
- Students' problem solving on their own
- Whole-class discussion
- Summing up
- Exercises or extension (optional depending on time and how well students are able to solve the original problem)

Lessons usually begin with a word problem in the textbook or a practical problem that is posed on the chalkboard by the teacher. After the problem is presented and read by the students, the teacher determines whether the students understand the problem well. If it appears that some students do not understand some aspect of the problem, the teacher may ask these students to read it again, or the teacher may ask questions to help clarify the problem. Also, in some cases, he or she may ask a few students to show their initial ideas of how to approach the problem or to make a guess at the answer. The intent of this initial stage is to help the students develop a clear understanding of what the problem is about and what certain unclear words or terms mean.

A certain amount of time (usually about 10–15 min) is assigned for the students to solve the problem on their own. Teachers often encourage their students to work together with classmates in pairs or in small groups. While students are working on the problem, the teacher moves about the classroom to observe the students as they work. The teacher gives suggestions or helps individually those students who are having difficulty in approaching the problem. He or she also looks for the students who have good ideas, with the intention of calling on them in a certain order during the subsequent whole-class discussion. If time allows, the students who have already gotten a solution are encouraged by the teacher to find an alternative method for solving to the problem.

When a whole-class discussion begins, students spend the majority of this time listening to the solutions that have been proposed by their classmates as well as presenting their own ideas.

Finally, the teacher reviews and sums up the lesson and, if necessary and time allows, that he or she poses an exercise or an extension task that will apply what the students have just learned in the current lesson.

In sum, from a teacher's perspective, Japanese lessons can be characterized as being structured with a set of segments that includes students' problem solving and a whole discussion as major parts. In this sense, Japanese lessons can be characterized as "structured problem solving."

2.2 The Japanese lesson pattern

The video component of the Third International Mathematics and Science Study (TIMSS) was the first attempt

ever made to collect and analyze videotapes from the classrooms of national probability samples of teacher at work (Stigler & Hiebert, 1999). Focusing on the actions of teachers, it has provided a rich source of information regarding what goes on inside eighth-grade mathematics classes in Germany, Japan, and the United States with certain contrasts among three countries. One of the sharp contrasts between the lessons in Japan and those in the other two countries relates to how lessons were structured and delivered by the teacher. The structure of Japanese lessons was characterized as "structured problem solving", here again, while a focus was on procedures in the characterizations of lessons in the other two countries.

The following sequence of five activities was described as the "Japanese pattern" (Table 1): reviewing the previous lesson; presenting the problems for the day; students working individually or in groups; discussing solution methods; and highlighting and summarizing the main point.

In this lesson pattern, the discussion stage, in particular, depends on the solution methods that the students actually use. In order for making this lesson pattern to work effectively and naturally, teachers have to have not only a deep understanding of the mathematics content, but also a keen awareness of the possible solution methods their students will use. Having a very clear sense of the ways students are likely to think about and solve a problem prior to the start of a lesson makes it easier for teachers to know what to look for when they are observing students work on the problem. The pattern seems to be consistent with the description of mathematics lessons as problem solving in the previous section, though there are some differences between them as "reviewing the previous lessons" above and "exercises or extension" in the previous section.

Characterization of the practices of a nation's or a culture's mathematics classrooms with a single lesson pattern was, however, problematized by the results of the LPS (Clarke, Mesiti, O'Keefe, Jablonka, Mok & Shimizu, 2007). The analysis suggested that, in particular, the process of mathematics teaching and learning in Japanese classrooms could not be adequately represented by a single lesson pattern by, at least, the following two reasons. First, lesson pattern differs considerably within one teaching unit, which can be a topic or a series of topics, depending on the teacher's intentions through out the sequence of lessons. Second, elements in the pattern themselves can have different meanings and functions in the sequence of multiple lessons. Needless to say, it is an important aspect of teacher's work not only to implement a single lesson but also to weave multiple lessons that can stretch out over several days, or even a few weeks, into a coherent body of the unit. It would not be possible for us to capture the dynamic nature of activities in teaching and learning process if each lesson was analyzed as isolated.

Table 1 The Japanese lesson pattern (Stigler & Hiebert, 1999, pp. 79–80)

Reviewing the previous lesson
Presenting the problems for the day
Students working individually or in groups
Discussing solution methods
Highlighting and summarizing the main point

An alternative approach was proposed to the international comparisons of lessons by the researchers in LPS team. That is, a postulated "lesson event" would be regarded to serve as the basis for comparisons of classroom practice internationally. In LPS, an analytical approach was taken to explore the form and functions of the particular lesson events such as "between desk instruction", "students at the front", and "highlighting and summarizing the main point" (Clarke, Emanuelsson, Jablonka & Mok, 2006).

In particular, the form and functions of the particular lesson event "highlighting and summarizing the main point", or "Matome" in Japanese, were analyzed in eighth-grade "well-taught" mathematics classrooms in Australia, Germany, Hong Kong, Japan, Mainland China (Shanghai), and the USA (Shimizu, 2006b). For the Japanese teachers, the event "Matome" appeared to have the following principal functions: (1) highlighting and summarizing the main point, (2) promote students' reflection on what they have done, (3) setting the context for introducing a new mathematical concept or term based on the previous experiences, and (4) making connections between the current topic and previous one. For the teachers to be successful in maintaining these functions, the goals of lesson should be very clear to themselves, activities in the lesson as a whole need to be coherent, and students need to be involved deeply in the process of teaching and learning. The results suggest that clear goals of the lesson, a coherence of activities in the entire lesson, active students' involvement into the lesson, are all to be noted for the quality instruction in Japanese classrooms.

2.3 A story or a drama as a metaphor for an excellent lesson

Associated with the descriptions of "structured problem solving" approach to mathematics instruction discussed above, several key pedagogical terms are shared by Japanese teachers. These terms reflect what Japanese teacher value in planning and implementing lesson within Japanese culture.

"Hatsumon", for example, means asking a key question to provoke and facilitate students' thinking at a particular point of the lesson. The teacher may ask a question for probing students' understanding of the topic at the

beginning of the lesson or for facilitating students' thinking on the specific aspect of the problem. "Yamaba", on the other hand, means a highlight or climax of a lesson. Japanese teachers think that any lesson should include at least one "Yamaba". This climax usually appears as a highlight during the whole-class discussion. The point here is that all the activities, or some variations of them, constitute a coherent system called as a lesson that hopefully include a climax. Further, among Japanese teachers, a lesson is often regarded as a drama, which has a beginning, leads to a climax, and then invites a conclusion. The idea of "KI-SHO-TEN-KETSU" which was originated in the Chinese poem, is often referred by Japanese teachers in their planning and implementation of a lesson. It is suggested that Japanese lessons has a particular structure of a flow moving from the beginning ("KI", a starting point) toward the end ("KETSU", summary of the whole story).

If we take a story or a drama as a metaphor for considering an excellent lesson, a lesson needs to have a highlight or climax based on the active role of students guided by the teacher in a coherent way. Stigler and Perry (1988) found *reflectivity* in Japanese mathematics classroom. They pointed out that the Japanese teachers stress the process by which a problem is worked and exhort students to carry out procedure patiently, with care and precision. Given the fact that the schools are part of the larger society, it is worthwhile to look at how they fit into the society as a whole. The reflectivity seems to rest on a tacit set of core beliefs about what should be valued and esteemed in the classroom. As Lewis noted, within Japanese schools, as within the larger Japanese culture, Hanseiself—critical reflection is emphasized and esteemed (Lewis 1995).

In sum, the selected findings of large-scale international studies of classroom practices in mathematics examined above suggest that "structured problem solving" in the classroom with an emphasis students' alternative solutions to the problem can be a characterization of Japanese classroom instruction from a teacher's perspective. Also, a coherence of the entire lesson composed of several segments, students' involvement in each part of the lesson, and the reflection of what they did are all to be noted for the quality instruction in Japanese classrooms. For understanding what Japanese teacher value in their instruction with cultural influence on them, a story or a drama can be a metaphor for characterizing an excellent lesson in Japan.

3 What constitutes a "Good" mathematics lesson: the learner's perspective

Given the fact that teaching and learning are interdependent activities within a common setting, classroom practices should be studies as such (Carpenter & Peterson, 1988). Also, if we consider teaching as *cultural activity* (Stigler & Hiebert, 1999), we need to look into what participants, both the teacher and students, value in the classroom and how they perceive the lesson with associated values embedded in cultural activities in classroom. In the following part of this paper, associated values attached to a "good" lesson are explored from the learner's perspective.

Exploring students' views on mathematics lessons will open a window through which we can examine values held by students in the context of teaching (Bishop, Seah & Chin, 2003). Previous international studies of mathematics classroom have identified coherent sets of actions, and associated attitudes, values, beliefs, and knowledge, that appear to constitute culturally-specific teacher practices (Stigler & Hiebert, 1999). The larger international study, of which this study is a part, hypothesizes that there is also a set of actions and associated attitudes, values, beliefs, and knowledge of students that constitute a culturally-specific coherent body of learner practices (Clarke, Keitel & Shimizu, 2006).

3.1 Data and methodology

Data collection for the current paper was conducted at three public junior high schools in Tokyo. The technique for undertaking this study involved the development of complex "integrated data sets" that combined split-screen video records of teacher and students with transcripts of post-lesson interviews and copies of relevant printed or written material (Clarke, 2006).

The teachers, one female and two males, roughly represented the population balance of mathematics teachers at the school level in Japan. All of the three mathematics teachers were very experienced with the experience of teaching mathematics more than 20 years. Two of them were writers of mathematics textbooks that are widely and commercially available in Japan. The criteria for identifying them reflected a locally-defined "teaching competence". Namely, the three mathematics teachers were identified for their visibility in presenting at teacher conferences, active roles in the study groups of teachers in Tokyo, and the recognition in the community of mathematics teachers as a teacher who teach mathematics in excellent ways.

The topic taught in each school corresponded to three different content areas prescribed in the National Curriculum Guidelines; linear functions, plane geometry, and simultaneous linear equations.

Semi-structured post-lesson video-stimulated interviews with the students occurred on the same day as the relevant lesson. In each lesson two students sitting next to each other were selected as "focus students" for that particular lesson. These students were interviewed individually after the

Table 2 Selected prompts in post-lesson video-stimulated interviews

Prompt four: Here is the remote controller for the video-player. Do you understand how it works? (Allow time for a short familiarization with the control.) I would like you to comment on the videotape for me. You do not need to comment on all of the lessons. Fast-forward the videotape until you find sections of the lesson that you think were important. Play these sections at normal speed and describe for me what you were doing, thinking and feeling during each of these videotape sequences. You can comment while the videotape is playing, but pause the tape if there is something that you want to talk about in detail.

Prompt seven: Would you describe that lesson as a good one for you? What has to happen for you to feel that a lesson was a "good" lesson? Did you achieve your goals? What are the important things you should learn in a mathematics lesson?

Table 3 The description and examples of categories for coding

Code	Description	Example
Understanding/ thinking	Those responses that refer to their understanding and thinking in the classroom	*I can understand the topics to be learned*. (J2-03M)
Presentation	Those responses that refer to presenting their ideas in the classroom	*I can present my solution on the blackboard*. (J3-07I)
Classmates	Those responses that refer to other students' presentations and explanations	*There is an opportunity of listening to classmates*. (J1-09S)
Whole class discussion	Those responses that refer to the whole class discussion	*We all in the classroom exchange ideas actively*. (J1-06U)
Teacher	Those responses that refer to teacher's explanation	*I listen to teacher's final talk. I always take a note and check a point*. (J3-06S)
Other	Other responses	*By preview the topic at home, I attend the lesson with a preparation*. (J3-09K)

lesson. Among three of them, two teachers were interviewed three times, roughly once a week, during the period of videotaping and one teacher was interviewed twice.

The methodology employed in this study offered both the teachers and the students the opportunity in post-lesson video-stimulated interviews to identify for the interviewer those events in the lesson that the participant felt to be significant. The teacher and the students were given control of the video replay and asked to identify and comment upon classroom events of personal importance. Semi-structured post-lesson video-stimulated interviews, which occurred on the same day as the relevant lesson, included such prompts as follows (Table 2).

Interviewers were supposed to be explicit during the interviews in specifying the point to which the teachers and the students referred. It is clearly possible that students identify significant classroom events quite differently from those intended by the teachers. The analysis that focused on Prompt four was reported elsewhere (Shimizu, 2006a) and this paper reports on the analysis of the students' response to Prompt seven.

The post-lesson interviews with 60 students, 20 students from each of three schools, were transcribed and subjected to the analysis. For the analysis of the interview data, a coding system was developed. Table 3 shows the description of each coding category with an illuminating example of students' response to Prompt seven. The first five categories and one additional category, "other", had appeared from the initial analysis of transcriptions from one of the three schools (labeled as "J1"). Then all the students' response to the prompt were classified into six categories for coding by the author and research assistant. When discrepancies in coding between coders appeared, they were resolved by discussions. It should be noted that these codes do not constitute a mutually exclusive coding system.

3.2 Results

Table 4 shows the result of the analysis as a whole of students' response to the Prompt seven in video-stimulated interview. It is noted that the percentages do not add up to 100, because the coding system is not mutually exclusive.

As Table 4 shows, nearly half of the students interviewed (45.0%) described "understanding" or "thinking" to be happened in a "good" lesson. As the example in the Table 3, "I can understand the topics to be learned", illustrates, the students in this category regarded a lesson as "good" one if he can have a clear understand of mathematical topic taught in the lesson. Those students who mentioned to "understanding/thinking" seemed to attach values directly to the importance of their own thinking and understanding in the lesson. Some students in this category also referred to other activities in the classroom, such as the case of MANA from J2 who mentioned to teacher's

Table 4 Students' response to the Prompt seven in video-stimulated interview

Codes	Responses
Understanding/thinking	27 (45.0%)
Presentation	10 (16.7%)
Classmates	4 (6.7%)
Whole class discussion	16 (26.7%)
Teacher	10 (16.7%)
Other	10 (16.7%)

explanation as the object of understanding: "*Even if your answer is wrong…to be able to understand what the teacher explained. If that happens, I think//that it was a good lesson*." Roughly a quarter of the students (26.7%) identified "whole class discussion" as the "component" of a "good" lesson. Then, two categories "presentation" and "teacher" follow the "whole class discussion". Only four students (6.7%) explicitly described the activities related to their "classmates" in mathematics classroom.

There is a difference between the first four categories and "teacher" category in terms of types of the activities referred by the students. That is, first four categories are directly related to students' own learning activity, while "teacher" category is related to both students' learning and teacher's instructional activities. The example of "teacher" category in Table 3, for instance, is the one that referred to the teacher's final talk (highlighting and summarizing the main point), taking a note, and checking the key point of lesson. This example illustrates that teacher's instructional activities can also be a component of a "good" lesson to the students.

Table 5 shows the same result by schools. Table 5 shows that students in each school described what constitutes a "good" lesson by referring to learning activities that related to "understanding/thinking" frequently. There are some differences in students' response among three schools. Ten students interviewed at school J1, for example, described activities related to "whole class discussion" as the component of a "good" lesson, while only one student at school J2 did that. Also, nearly half of the students at J2 mentioned to "understanding/thinking" in the interviews. There was no outstanding "peak" in the number of students' responses at J3.

Table 5 Students' response to the Prompt seven by schools

Code	J1	J2	J3	Total
Understanding/thinking	8	13	6	27
Presentation	2	4	4	10
Classmates	1	1	2	4
Whole class discussion	10	1	5	16
Teacher	2	6	2	10
Other	2	2	6	10

3.3 Relating teacher and learner perspectives

To understand the characteristics of a "good" mathematics lesson, a detailed analysis was also conducted with an eye of relating students' responses to Prompt seven to those by the teacher who taught each classroom.

Suzu, a student from the school J3, for example, responded to the questions, "When you think it's a good class?" and "What should happen in the class?", as follows.

01. INT: When you think it's a good class,
02. SUZU: Yes.
03. INT: What should happen in the class?
….
04. INT: Do you have anything that you think is a good class?
05. SUZU: I can present my answer, and then listen to my friend's way as well,
06. INT: Yeah?
07. SUZU: The teacher's final comment, or answer,
08. INT: Yeah?
09. SUZU: Listen to it carefully, and to make a good note from it.

The student clearly mentioned to the importance of presenting his answer to the problem to the class and of listing to his classmates' way to solve the same problem. He also referred to listening to "*The teacher's final comment, or answer*" carefully and of "*making a good note from it*". These comments suggest that, students' views on a "good" lesson are shaped through the classroom practices co-constructed by the teacher and the students. If the teacher keeps summarizing and highlighting the main points of the lesson as a daily routine, for instance, the students may become aware of the importance of the particular lesson event which tends to come on the final phase of lesson in the form of teacher's public talk together with time for note-taking, and then he or she will "listen to it carefully" and try to "make a good note from it". The teacher's summarizing and highlighting, in turn, have to rely upon students' understanding of the mathematical topic taught which is to be summarized and highlighted.

Teachers' comments on what constitutes a "good" lesson also suggest the co-constructed nature of a "good" mathematics lesson. Mr K, the teacher of JP3, in the second interview, for example, mentioned to the importance of students thinking on alternative solutions and their understanding in a "good" lesson as follows: "*practically, what I think is that the students think in many ways…and they understand it well…The students can ask me or each other where they can't understand.*" Here, Mr K expressed that he valued to have his students think in many ways and understand the topic well through the interaction with him and classmates. On the other hand, Mr N, the teacher of JP2, in

the first interview referred to a shared goal between the students and himself in a "good" lesson: "*the best lesson is where the teacher and the students can both agree that today's lesson was a good lesson. Um, not just one side but, well this is for the students so of course it's good if the students say it's good. Of course that's good but from a teacher's perspective, if the students assessed something else as good and ignored what the teacher wanted them to understand the most, that's. That sort of class really doesn't have much meaning so I think if the goal of the teacher and the students are the same, that's the best thing*".

These comments suggest that in a "good" lesson teacher and student practices can be conceived as being in a mutually supportive relationship. This is not to presume that teacher and students have the same goals or values, or even that they perceive the importance of particular classroom activities in the same way. The analysis suggests that a "good" lesson is a co-constructed classroom practices by the teacher and the students.

4 Discussion

4.1 The co-constructed nature of classroom practice in quality mathematics instruction

The selected findings of the large-scale international studies of classroom practices reveal the uniqueness of the way Japanese mathematics teachers structure and deliver their lessons and what Japanese teachers value in their instruction. Students' views on what constitutes a "good" mathematics lesson are related to what their teacher values in structuring and delivering their instruction from a teacher's perspective. The two perspectives can be incorporated into the characterization of exemplary mathematics instruction in Japanese classrooms.

The results of the analysis showed that students in each school participated to this study referred to the activities related to the category "understanding/thinking" frequently, though there were some differences in students' response among three schools. Although those students seemed to attach values directly to the importance of their own thinking and understanding in the lesson, students also referred to other activities in the classroom. In fact, the students in the post-lesson interviews interviewed described learning and teaching activities related to the lesson events within "structured problem solving" as the "components" of a "good" lesson. Two categories "presentation" and "whole class discussion", in particular, can be regarded as directly related to the part of the "structured problem solving" approach to teaching mathematics in the classrooms. Further, there were students who referred to teacher's highlighting and summarizing the main point as well as their own learning activities as the case of student SUZU.

It is possible that these valued outcomes may have little connection to "knowing", "learning" or "understanding", and that students may have very localized or personal ways to describe lesson outcomes (Clarke, 2006, p. 33). This was not the case with most students interviewed in the study. Their descriptions had strong connections to "understanding" and "thinking" and other classroom activities. The results of the current study suggest that values held by Japanese students in the context of a "good" lesson directly related to the way in which mathematics lessons are structured and delivered with an emphasis on students' thinking as intended by teachers. Not only the teacher who teach the lesson but also student who behave in response to the teacher's instructional activities play key role in the co-constructed practice in an exemplary mathematics instruction.

4.2 Maintaining the focus on students' thinking in planning and implementing a lesson

There are opportunities for Japanese teachers to learn with and from their experienced colleagues to pursue an excellent lesson with a focus on students' thinking in classroom. "Lesson study" is an approach to develop and maintain quality mathematics instruction through a particular form of activity (Fernandez & Yoshida, 2004; Shimizu, 2002). Valuing students' thinking as necessary elements to be incorporated into the development of a lesson is a key to the approach taken by Japanese teachers.

Generally a lesson study consists of the following events: the actual classes taught to pupils, observation by others, followed by intensive discussion called as the study discussion. Designing, enacting, and analyzing are the three stages of lesson study that evolve before, during, and after the lesson. There is extensive preparation to made before the class, and there will be extensive work to be done after the lesson study as well, which will be used as a follow up and as a preparation for the next lesson studied. These events form a cycle or iterative process. In the process of a lesson study, lesson plans are used as "vehicles" with which teachers can learn and communicate about the topic to be taught, anticipated students' approaches to the problem presented, and important teachers' roles at various phases of lessons (Fig. 1).

Describing anticipated students' responses, among others, is a key to lesson planning because the whole-class discussion depends on the solution methods the students actually come up with. Teachers need to have not only a deep understanding of the mathematics content, but also a keen awareness of the possible solution methods their students will use. Having a very clear sense of the ways students are likely to think about and solve a problem prior to the start of a

Steps	Main Learning Activities	Anticipated Students' Responses	Remarks on Teaching
Posing a problem			
Students' problem solving on their own			
Whole-class Discussion			
Summing Up			
(Exercise/Extension)			

Fig. 1 A common framework for writing lesson plans (Shimizu, 1999, p.113)

lesson makes it easier for teachers to know what to look for when they are observing their students work on the problem.

In sum, key features of the approach by Japanese teachers to develop and maintain quality mathematics instruction include teachers' deep understanding of the relationship between mathematics content to be taught and students' thinking about the problem to be posed. Anticipating students' responses to the problem is the crucial aspect of lesson planning in the Japanese approach to teaching mathematics through problem solving.

5 Conclusions

This paper aimed to examine key characteristics of exemplary mathematics classroom instruction in Japan. The uniqueness of the way Japanese mathematics teachers structure and deliver their lessons was discussed in relation to what Japanese teachers value in their instruction by reviewing selected findings of large-scale international studies of classroom practices. The structured problem-solving approach to teach mathematics in the classroom was related to the analysis of post-lesson video-stimulated interviews with 60 students in three "well-taught" eighth-grade mathematics classrooms. The results revealed that students' conceptions of what constitutes a "good" lesson can be characterized in accordance with what Japanese teachers tend to value in their instruction. Valuing students' thinking to be incorporated into the development of a lesson is discussed as key aspect of the approach taken by Japanese teachers to develop and maintain quality mathematics instruction.

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