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ORIGINAL ARTICLE

How a Chinese teacher improved classroom teaching in Teaching Research Group: a case study on Pythagoras theorem teaching in Shanghai

Yudong Yang

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Abstract In China, a school-based teaching research system was built since 1952 and Teaching Research Group (TRG) exists in every school. In the paper, a teacher's three lessons and the changes in each lesson were described, which might show a track of how lessons were continuously developed in TRG. The Mathematical Tasks Framework, The Task Analysis Guide, and Factors Associated with the Maintenance and the Decline of High-level Cognitive Demands developed in the *Quantitative Understanding: Amplifying Student Achievement and Reasoning* project (Stein and Smith in Math Teach Middle School 3(4):268–275, 1998; Stein et al. in Implementing stardards-based mathematics instruction. Teachers College Press, NY, pp. 1–33, 2000), were employed in this study. Based on the perspective of Mathematical Task Analysis, changes of three lessons were described and the author provided a snapshot for understanding how a Chinese teacher gradually improved his/her lessons in TRG activities.

Keywords Case study · Mathematical lessons · Pythagoras theorem · Mathematical tasks · Teaching research activities

1 Introduction

Could Pythagoras Theorem be explored out by students? was a first prize videocase that was honored by the China National Teacher Education Union, in Beijing, 2005. In fact, the excellent lesson, which won first prize, experienced an improving process in teaching research activities conducted by the Teaching Research Group (TRG) in the research lesson teacher's school. Compared to the teachers from developed countries, Chinese mathematics teachers did not have high records of formal schooling. But some studies had found that Chinese mathematics teachers had profound understanding on elementary mathematics and they had better pedagogical content knowledge on mathematics (Ma 1999). One possible reason was that all mathematics teachers were involved in various teaching research activities conducted in the school-based teaching research network.

Different from western culture, the classroom teaching of Chinese mathematics teachers is open for colleagues' observation, studies and discussion. Mathematical TRG exists in each school in mainland of China because of the educational system. The three-level teaching research network (province-level Teaching Research Office (TRO), county-level TRO, and school-level TRG has more than 50 years' tradition.

TRG is the basic unit in the network and its main responsibility is carrying out studies on teaching to solve practical problems of teachers. Early in 1952, the Ministry of Education stipulated in *Provisional Regulation for Secondary School (draft)* that the "Teaching research groups should be set up in all subjects in secondary schools". It is formed by teachers who teach the same subject and usually a teacher always teaches one subject for 2–3 classes of the same age group in a school in mainland of China. The duty of TRG is "to study and improve the way of teaching" (Ministry of Education 1952). In *Secondary School Teaching Research Group Rulebook (draft)* issued by Ministry of Education in 1957, the duty of study was further emphasized: "A Teaching Research Group is an organization to study teaching. It is not an administrative department. Its task is to organize teachers to do

Y. Yang (✉)
Shanghai Academy of Educational Sciences, Shanghai, China
e-mail: mathedu@163.com

teaching research in order to improve the quality of education, but not to deal with administrative affairs" (Ministry of Education 1957). In 2001, the *Decision on Basic Education Reform and Development* pointed out that "all level teaching research offices should actively participate into editing textbooks and teaching experiments of basic education reform; to learn from other nation's advanced experience; to promote the excellent experience on teaching in basic education reform" (The State Council 2001).

What changes happened when a teacher developed his/her lesson in TRG activities? How did the TRG activities influence a teacher's lesson? In the paper, a teacher's three lessons and the changes in each lesson were described, which might show a track of how lessons were continuously developed in TRG. By this case study in Shanghai, the author wanted to develop some ideas on how a Chinese teacher improved his/her lessons in collaborative team work of TRG and got professional progress in their teaching research activities. Considering the unique educational background in mainland China, though what the lessons looked like and what happened in the improving process in the Shanghai case were very representative, the author wanted to point out that the discussion and conclusions in the paper only fit to the concrete Shanghai teacher and Shanghai lessons and it could not be arbitrarily generalized in the all lessons or teachers in mainland of China.

2 Background for Chinese classrooms

Mathematical teaching in China emphasized the basic knowledge and basic skills (Zhang, Li, & Tang, 2004), which resulted in students' high achievements in some large-scale international comparisons (Fan & Zhu, 2004), e.g., the International Assessment of Education Process (IAEP) (Lapointe, Mead, & Askew, 1992) and the International Mathematics Olympiad (Wong 2004). And some scholars mentioned (Lopez-Real, Mok, Leung, & Marton, 2004), Chinese classroom was very similar with most of other Asian countries', i.e. large classes, whole class teaching, examination driven teaching, content rather than process oriented, emphasis on memorization, etc. So some scholars criticized that the over-exercising and over-drilling ignored the learning of mathematical essence (Tsatsaroni & Evans, 1994; Partners in Change Project 1997; Romberg & Kaput, 1999; Uhl & Davis, 1999). Some experts in China advocated that the teaching of mathematics should de-emphasize its appearance, reinforce its substance (Song & Chen, 1996) and distinguish the educational mathematics from the academic mathematics (Zhang 2001; Zhang & Wang, 2002).

The exploring styles of learning and teaching were advocated by Teaching Research Officers since the National Mathematics Curriculum Standards for Compulsory Education (Ministry of Education 2001) was issued. Bao (2004) had compared the composite difficulty between the new standard-based textbooks used in all schools in the experimental districts and the old teaching-syllabus-based textbooks used in the schools in the non-experimental districts, and found that the new standard-based textbooks emphasized more on investigation and context.

Based on the current situation in China, mathematics teaching has emphasized more on mathematics essence (Yang & Li, 2005). That is, taking the point of view of dynamic quasi-empiricism mathematics philosophy, it is proposed not to over-emphasize the acquisition of mathematics skills. Instead, teaching should root in students' common sense and experience, and go deep into problems' mathematical substance. In this way, teaching should let students experience mathematics activities similarly as mathematicians, such as mathematical conjecturing, plausible reasoning, exploring, validating, and justifying etc., then reorganize their new common sense and experience progressively. For what mathematics students experience in school would influence their recognition of mathematics in their future life (Dossey 1992).

3 Methodology

3.1 A theoretical perspective for mathematical tasks analysis

Stein and Smith (1998) used the Mathematical Tasks Framework (Fig. 1) to analyze hundreds of teaching cases in the *Quantitative Understanding: Amplifying Student Achievement and Reasoning* (QUASAR) project from 1990 to 1995, which showed how a mathematical task was changed when it was carried out in three stages: (1) when a task appeared in curricular or instructional materials, it was an ideal task set up by a curriculum expert or textbook editor; (2) when a task appeared in classroom teaching, it

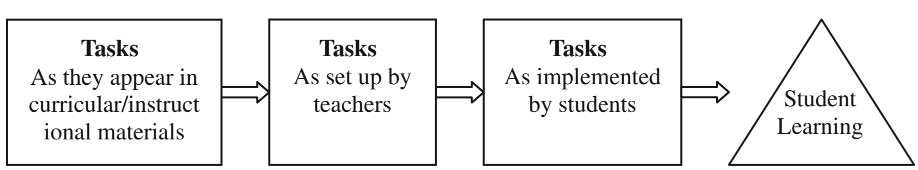

Fig. 1 The mathematical tasks framework (Stein & Smith, 1998)

was an operational task set up by teachers; (3) when a task appeared in students' learning, it was an implemented task worked by students. From stage 1 to 3, the mathematical task would not necessarily keep up on a same level and some changes happened in the continuous process. What students achieved relied on the three stages.

In their analysis of hundreds of teaching cases, Stein and Smith found that a higher cognitive task was always translated as a lower cognitive task by teachers in classroom teaching. And only when a task was set up by teachers in a high cognitive level did it have the possibility to be implemented in a high cognitive level by students. Furthermore, they defined four types of tasks in two levels (Table 1): the first two tasks were low-level demands in cognition (memorization task and procedures without connection tasks), and the other two tasks (procedure with connection tasks and doing mathematics tasks) were high-level demands in cognition.

Stein and Smith (1998; Stein, Smith, Henningsen, & Silver, 2000) found, a high-level demands task might be kept in a same level or declined to a lower level when implemented by students, but a lower level task had no possibility to be implemented in a higher level by students. For example, doing mathematics task, which is a high-level demands task, after it was set up by a teacher in classroom teaching, there might be four kinds of results. It might be implemented by students in a same level as a doing mathematics task, and it also might be implemented by students as a procedures with connection task, or a procedures without connection task, or a memorization task. But if a teacher set up a low-level demands task, e.g. a procedure without connection task, it had no possibility to be implemented as a procedure with connection task or a doing mathematics task. What resulted in students' implementation kept or declined in some level? They concluded the factors associated with maintenance and decline of high-level cognitive demands (Table 2).

The theoretical framework of mathematical task analysis constructed by Stein and Smith (1998; Stein, Smith, Henningsen, & Silver, 2000) would be used in this paper to

Table 1 The Task Analysis Guide (Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2000)

Cognitive level	Type of tasks	Features
Low-level demands	Memorization tasks	Reproducing previously learned facts, rules, formulae, or definitions
		Being solved without using procedures or the time is not enough to use a procedure
		Involving exact reproduction of previously seen material which is clearly and directly stated
		No connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced
	Procedures without connections tasks	Being algorithmic because use of procedure is either specially called for or its use is evident based on prior instruction
		Requiring limited cognitive demand for successful completion
		Without connection to the concepts or meaning that underlie the procedure being used
		Being focused on producing correct answers rather than developing mathematical understanding
		Requiring no explanations, or explanations that focus solely on describing the procedure that was used
High-level demands	Procedures with connections tasks	Focusing students' attention on understanding of mathematical concepts and ideas
		Suggesting pathways to follow (explicitly or implicitly) that are broad general procedures that have close connection to underlying conceptual idea
		Making connections among multiple representations helps to develop meaning in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations)
		Requiring some degree of cognitive effort to understand the conceptual ideas that underlie the procedures
	Doing mathematics tasks	Requiring complex and non-algorithmic thinking
		Requiring students to explore and understand the nature of mathematical concepts, process, or relationships
		Demanding self-monitoring or self-regulation of one's own cognitive process
		Requiring students' relevant knowledge and experiences and making appropriate use of them to solve the task
		Requiring students to examine task constrains that may limit possible solution strategies and solutions
		Requiring considerable cognitive effort and may involve some level of anxiety for the student

Table 2 Factors associated with maintenance and decline of high-level cognitive demands (Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2000)

Factors of maintenance or decline	Explanations
Factors associated with the maintenance of high-level cognitive demands	
M1	Scaffolding of students' thinking and reasoning is provided
M2	Students are given the means to monitor their own progress
M3	Teacher or capable students model high-level performance
M4	Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback
M5	Tasks build on students' prior knowledge
M6	Teacher draws frequent conceptual connections
M7	Sufficient time is allowed for exploration-not too little, not too much
Factors associated with the decline of high-level cognitive demands	
D1	Problematic aspects of the task become routinized
D2	The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
D3	Not enough time is provided to wrestle with the demanding aspects of the task
D4	Classroom-management problems prevent sustained engagement in high-level cognitive activities
D5	Task is inappropriate for a given group of students
D6	Students are not held accountable for high-level products or processes

show what had been changed in each lesson and how mathematical tasks were set up by the teacher and how they were implemented by students.

3.2 Source of data

3.2.1 The school and the teacher

The school was an ordinary junior high school in Qingpu District, which is located in west suburb of Shanghai city and with an economy developed at a middle level in Shanghai. The Mathematics Teaching Research Group consisted of seven full time mathematics teachers in the school. Two of them had more than 15 years' mathematics teaching experience (T1 and T2 are used to substitute their names), three of them had 5–10 years' mathematics teaching experience (T3, T4 and T5 are used to substitute their names) and two of them had less than 5 years' teaching experience (T6 is used to substitute one of them). Ms N, was the youngest teacher in the mathematics TRG who had 2 years and 3 months' mathematics teaching experience and graduated from Shanghai Teachers' University. In this paper, we will study her three lessons on Pythagoras theorem teaching.

In Shanghai, even in mainland China, mathematics TRG exists in every primary and high school. For mathematics, Chinese language and English language, they are the main subjects in every school and the teachers in these three TRG usually teach 2 or 3 classes of the same age group and every teacher in these three TRG only teaches one subject, mathematics, Chinese language, or English language. Ms N, after her 4 years' study of Mathematics Education in Shanghai Teachers' University, taught mathematics in three parallel classes from grade 6 to 8. When the lessons data were collected, Ms N just lived through 3 months of her third year's teaching journey.

3.2.2 The lessons

Three lessons on Pythagoras Theorem which taught by Ms N in three grade-8 classes were videotaped, and the related TRG activities were videotaped, too. In China, the TRG activities were mainly pre-lesson discussion on lesson plans, post-lesson discussion on teaching contents and teaching methods.

The first lesson was prepared by Ms N solely, so there was no pre-lesson discussion for the first lesson. After the first lesson, all members of TRG discussed the problems of the lesson first. And then, the members of TRG discussed how to improve the lesson next time, so new lesson plan ideas came out and Ms N made a new lesson plan after the post-lesson discussion. Actually, the discussion after the first lesson actually consisted of two parts: the first part was post-lesson discussion aiming at finding problems of the lesson just taught and the second part was pre-lesson discussion aiming at making a new lesson plan for the next coming lesson. After the second and third lesson, the same discussions happened though there was no chance for Ms N to practice new ideas in her three classes after the third lesson. The improving process was a typical procedure in TRG in Shanghai, mainland of China.

3.3 Data analysis

3.3.1 Analysis of the lessons

In Shanghai, most of the lessons were teacher-centered and most of the mathematics teaching kept a basic model because of historical reasons. Review of old knowledge was always the first part of a lesson and summarization of what had been learned was always the last part of a lesson. For a theorem-teaching lesson, the middle part often had three steps: producing the theorem, justifying the theorem and applying the theorem. But different teachers would emphasize different steps in the middle part which showed their different aims in teaching the same topic. So in the analysis of lessons, firstly the structure of the lesson would be identified to show a whole picture of the lesson. And then two key tasks were focused in the paper, producing the theorem and justifying the theorem. For applying the theorem, it often involved doing exercises, which will not be analyzed in the paper.

The *Mathematical Tasks Framework* (Fig. 1) was used to distinguish the transferring tasks appearing in textbooks, set up by teachers and implemented by students in lessons. By comparison of the three levels of tasks, what had been changed in tasks would be revealed. In the paper, tasks set up by teachers and implemented by students would be checked in detail.

The *Tasks Analysis Guide* (Table 1) was used to define the cognitive level of the tasks set up by teachers and the tasks actually implemented by students. *Memorization Tasks* and *Procedures Without Connections Tasks* were defined as Low-level Demands and *Procedures With Connections Tasks* and *Doing Mathematics Tasks* were defined as High-level Demands. By this analysis guide, the tasks which really happened in classrooms would be judged if they were high cognitive demands tasks or not and the result would show us how the tasks to be changed among textbook, teacher and students.

The *Factors Associated with the Maintenance and the Decline of High-level Cognitive Demands* (Table 2) was used to recognize the main factors that influenced the teacher's implementing tasks in her teaching. In Table 2, seven common factors associated with the maintenance of high-level cognitive demands and six common factors associated with the decline of high-level cognitive demands were described. In this study, the maintenance factors were coded as M1–M7 and the decline factors were coded as D1–D6 in the key tasks analysis of the lessons.

3.3.2 Analysis of the interview of the teacher

After each of Ms N's lessons and the post-lesson discussion, an interview with the teacher was carried out. So there were three interviews to collect the teacher's opinions on the lesson and on the discussion in the TRG activity. There were two focuses in analyzing the three interviews: (1) what did the teacher see as important in the lesson? (2) what did the teacher feel about others' opinions in the discussion of the lesson in TRG activity?

3.3.3 Analysis of the TRG discussion

After each of Ms N's lessons, the TRG talked about the problems of the lesson and the possible improvement in the next lesson. All the discussions were videotaped and transcribed. The transcripts analysis focused on two questions: (1) what the other teachers in the TRG see as important in the lesson? (2) what kind of opinions influenced the next lesson of Ms N?

4 Results

The results of three lessons' analysis were presented as three parts, and then three lessons were analyzed by comparison in summarization. In each analysis of the lesson, lesson structure, two key tasks and discussion were presented.

4.1 The first lesson

The first lesson was taught in one of Ms N's three classes, which was scheduled for 45 min and actually lasted about 45 min.

4.1.1 The lesson structure

The durations for different activity segments could be approximately described in this sequence:

1. Producing the proposition by questions (8 min and 15 s). By asking the question (if we know the two edges of a right-angle in a triangle, how can we get the bevel edge?), Ms N introduced a history material which was from an ancient Chinese book ZHOU BI SUAN JING: Gou 3, Gu 4 and Xian 5 (which means if one right-angled edge is 3 and the other right-angled edge is 4 in an triangle, then the bevel edge is 5). After asking students to draw two right-angled triangles and measure their sides, Ms N gave out the proposition: $a^2 + b^2 = c^2$.
2. Justifying the proposition by explaining and asking students to read the related content in the textbook (15 min and 51 s). After writing down the algebraic formula $[a^2 + b^2 = (a + b)^2 - 2ab = (a + b)^2 - 4 \times 1/2ab]$ on the blackboard, Ms N asked students to

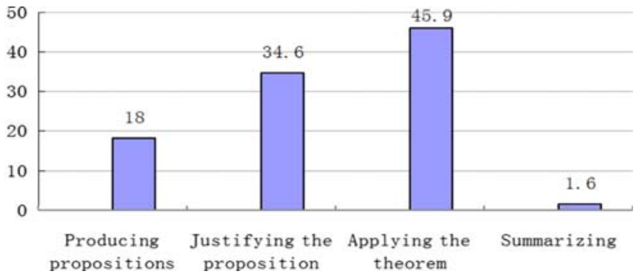

Fig. 2 Time percentage of each segment in the 1st lesson

read the proving process in the textbook for several minutes and then gave some explanations to facilitating students' understanding.

3. Applying the Pythagoras theorem to solve four questions in the textbook (21 min and 2 s).
4. Summarizing what was learned in this lesson briefly (43 s) Fig. 2.

4.1.2 The two key tasks

4.1.2.1 Producing the proposition *How did Ms N set up the task?* First, Ms N asked a question to inspire students: "if we know the two edges of a right-angle in a triangle, how can we get the bevel edge?" After introduced a history material about Gou 3, Gu 4 and Xian 5, Ms N let students to draw the triangle. Then she let students draw another triangle (5, 12, 13) and calculate three edges' square. By these two sets of data, she asked: "what is the relationship among three sides of the triangle?" From the Tasks Analysis Guide, it was a very open question for students. It needed complexly non-arithmetic thinking to guess a proposition and there was not an exact way to achieve. It also needed students' hard cognitive efforts to answer the question which had the characteristic of a Doing-Mathematics Task. So the task was set up by teacher as a high-cognitive demands task.

How did students implement the task? In Table 3, seven factors which influenced students' high-cognitively implementation of the task.

From the lesson segment in Table 3, the whole process of producing propositions had very limited cognitive demands and it even seemed to be quite easy to find $a^2 + b^2 = c^2$ for students. Ms N had transferred the high-cognitive demands task into several computational questions by asking students to compute numbers' square. When students get two set of data in the form of $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$, there was no cognitive challenge for them to produce the proposition: $a^2 + b^2 = c^2$. According to the characteristic of mathematical tasks described in the Task Analysis Guide, the task of producing propositions implemented by students was a Procedures-without-connections task.

4.1.2.2 Justifying the proposition *How did Ms N set up the task?* Ms N did not give students the chance to try to justify the proposition by themselves but directly wrote down the algebraic formula $[a^2 + b^2 = (a + b)^2 - 2ab = (a + b)^2 - 4 \times 1/2ab]$ on the blackboard. Then Ms N asked students to read the proving process in the textbook for several minutes and gave some explanations on how the proposition was justified in the textbook. So the task of justifying the proposition was set up in "telling" way, which had the characteristic of a Procedures-without-connections task:

- The procedures of justifying the proposition had been arranged by the teacher (directly told students how to operate the algebraic formula)
- The cognitive challenge for students was much less for the textbook had told everything about the justifying process
- The teacher emphasized on the outcome of justifying a correct proposition but did not care of developing students' idea on how to justify the proposition.

How did students implement the task? As Stein and Smith's study pointed out (1998; 2000), only if the teacher set up a task in a high-cognitive demands level, was there possibility for students to implement it in a high-cognitive demands level. That was to say, there was no possibility for students to implement a task in a high-cognitive demands level if a task was set up by the teacher in a low-cognitive demands level. It was obvious that the task set up by Ms N in the lesson was a low-cognitive demands task and so it was needless to discuss if students implemented the task in a low or high cognitive-demands level.

4.1.3 The discussion in TRG

The post-lesson discussion lasted about 1 h and 35 min and all the discussions on the lesson were concentrated on three topics.

4.1.3.1 What should be taught in the lesson? From Ms N's lesson structure, applying the theorem was seen as the most important part in the teaching but some teachers questioned about it.

> Ms N: In my lesson plan, the key segment is applying the theorem. In my opinion, justifying the theorem is too difficult for students. Well, I think there are difficulties for students to find a way to justify it, so I let them to read the justifying process in the textbook and I gave some explanations…

Table 3 The associated factors influenced students' implementing

Segments of the lesson	Factors
T: In a triangle, if the two right-angled edges are 3 and 4 cm, then the bevel edge is 5. Right? This is a conclusion. Well, now use your ruler to measure them to check. Ok, use your own paper… Draw a triangle, its two right-angled edges are 3 and 4 cm. Well, let us see if the bevel edge is 5 cm right?	D1 (Ms N "took over" the thinking and told students how to validate the figure)
(Students began to draw pictures)	
T: Ok, one edge is 3 cm, the other one is 4 cm, then you can see how long the bevel edge is	D2 (Ms N only took care of the correctness of the expected answer)
S (choral): 5 cm	
T: Well, next triangle, please draw it. One of the right-angled edge is 5 cm (stop 5 s to wait for students' drawing), the other right-angled edge is 12 cm, which is much longer. How long is the bevel edge?	D1 (Ms N "took over" the thinking and told students how to validate the figure)
(students drew the triangle and measured the bevel edge)	
T: How long?	
S1: 13	
T: Ok, we got two sets of data just now: one is 3, 4 and 5; one is 5, 12 and 13. Let us guess, 3 and 4 are right-angled edges, and 5 is bevel edge, right? Here 5 and 12 are right-angled edges, and 13 is bevel edge. Then what is the relationship among three sides of the triangle? Please discuss the question in your group and guess what kind of conclusion that we may get	D5 (only 2 set of data, students' learning without scaffold) D3 (not enough time to finish a high-cognitive demands task)
(Every 4 students as a group talked for 2 min, and Ms N patrolled)	
T: If the two right-angled edges are 3 and 4, correspondently, we can find that 3^2 plus 4^2 is equal to 5^2, isn't it?	
S(choral): Yes	
T: Well, Let us see the second set of data. Is 5^2 plus 12^2 equal to 13^2? (no answer, in silence)	D1 (the high-cognitive demands task has been transferred as a computational question)
T: $5^2 = ?$	
S(choral): 25	
T: $12^2 = ?$	
S(choral): 144	
T: The sum?	
S(choral): 169	
T: Right? It is $5^2 + 12^2 = 13^2$	
S(choral): Yes	
T: Ok, now I noticed someone had found something. Well, who would like to tell what you have found?	D1 (by the computational tasks above, the thinking process of producing a proposition had been suggested by teacher's two set of data: $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$)
S2: The first right-edge's square plus the second right-edge's square is equal to the bevel-edge's square	
T: Then we have such a conclusion. If we generalized the conclusion, what would we get? (Stopped for about 6 s, Ms N pointed the blackboard) If we know the three edges in the right-angled triangle, a, b and c, then what will we get according to the conclusion mentioned just now?	
S(choral): $a^2 + b^2 = c^2$	
T: Good, $a^2 + b^2 = c^2$. That is to say, the sum of the two right-edges' square is equal to the bevel-edge's square, isn't it? Ok, the proposition we have guessed out, now let us justify it	

T1: The Pythagoras Theorem seemed very simple and even some students knew it when they were in the primary school. But how such a theorem was found and how it was proved, few one can speak it out… So I think the key tasks should be reconsidered: what should students learn in the lesson?

T4: According to the new curriculum standard, mathematical teaching should pay attention to make students experience the mathematical activities as a mathematician. So the process of producing a proposition should be redesigned…

T2: As mathematical lessons, logical thinking is very important, which is the core of mathematics. Justifying the theorem should be finished by students in teacher's enlightening…

From the above discussion, we could see that how they gradually got consent to what should be taught.

4.1.3.2 How should the process of producing propositions be reasonable? When the TRG got consent to what should be taught in the lesson, their discussion transferred to how Ms N produced the Pythagoras Theorem.

T2: Just by two set of data (3, 4, 5) and (5, 12, 13), students smoothly "found" the theorem. That was because of Ms N's questions about these numbers' square. It was almost equal to telling students the outcome…

T3: We cannot ignore that maybe some students have read the textbook before Ms N's lesson and they have known the theorem…

T1: What is a reasonable supposal in mathematics? It should be based on enough supportive data. Mathematical supposal does not mean guess a riddle…

Ms N: Yes. After the lesson, a boy came to the dais and asked me, "Ms N, in the triangle of (3, 4, 5), may I get a relationship among three edges: $(3 + 5) \div 2 = 4$?" At that time I was thinking maybe it was too arbitrary to get the Pythagoras Theorem just by two special right-angled triangle…

Then the subsequent discussion moved ahead on how to redesign scaffolds to support students' producing propositions. They decided to use graph paper to build the connections between $a^2 + b^2 = c^2$ and corresponding squares' areas.

4.1.3.3 Should the thinking way of justifying the proposition be explored by students themselves?

T2: Ms N directly gave students the operation of algebraic formula was not appropriate. Why the algebraic formula was operated like that? The thinking way of justifying the proposition has not been revealed but just let students do formula operation without any reasons. Well, that is to say, the key algebraic formula $(a + b)^2 = 4 \times 1/2ab + c^2$ was just given, maybe students just knew HOW to prove the proposition but they did not know WHY it was proved like that. The thinking way of proving the proposition should not be took over by the teacher…

Ms N: Indeed, that was the difficulty for me… How can I make students naturally bethink of $a^2 + b^2 = (a + b)^2 - 4 \times 1/2ab$? In the reference book for teachers, it was designed to use four right-angled triangles to make up, but it was hard to be implemented by students. I do not know how to enlighten students, I do not know. Hope miracle happened? How to make students catch it? That's most difficult point for me in my preparation of the lesson…

The follow-up discussion moved ahead to how to overcome the difficult points in teaching. And the members of TRG gave out some suggestions on how to relate the justifying process with the meaning of figures of a^2, b^2 and c^2 in the graph paper.

4.2 The second lesson

In the post-lesson discussion after the first lesson, what was most valuable for students' learning had been talked about. Ms N took in most of the TRG's opinions and put her energy on two key tasks by designing the graph-paper worksheets for students: producing propositions and justifying the proposition. The second lesson was scheduled for 45 min and actually lasted 63 min and 53 s.

4.2.1 The lesson structure

The durations for different activity segments could be approximately described in this sequence:

1. Reviewing the method of area-calculating (12 min and 32 s). By asking students to calculate a catty-cornered square (Fig. 3), the method of replenishing or partitioning four right-angled triangles to calculate the area was clarified.

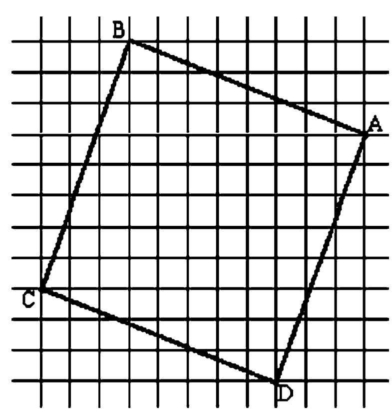

Fig. 3 A catty-cornered square in the graph paper

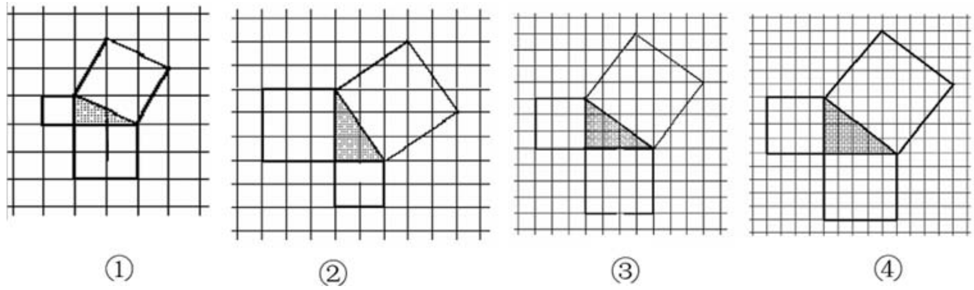

Fig. 4 Four right-angled triangles in the graph paper

2. Producing propositions by filling in a table (12 min and 57 s). After explaining the meaning of a^2, b^2 and c^2 in the graph paper, Ms N asked students to fill in values of four right-angled triangles (Fig. 4) in a datasheet (Fig. 5). When students finish filling in the datasheet, Ms N asked students to observe the datasheet and put forwards what they've found and students found the proposition $a^2 + b^2 = c^2$.
3. Justifying the proposition by students themselves on the worksheet (27 min and 34 s). Ms N asked students to justify the proposition by themselves and then asked students to explain their proving process.
4. Doing jigsaw games to verify the Pythagoras theorem visually (10 min and 13 s).
5. Summarizing what was learned in this lesson briefly (37 s) (Fig. 6).

4.2.2 The two key tasks

4.2.2.1 Producing the proposition How did Ms N set up the task? When Ms N set up the task, she first drew the squares of three edges of a right-angled triangle on the blackboard and gave some explanations on the geometrical meaning of a^2, b^2 and c^2. And then she asked students to calculate the value of a^2, b^2, ab, c^2 and fill in the datasheet on worksheet. Finally, she asked students to observe the datasheet and look for what rule might be there. Judging from the Tasks Analysis Guide, to find a rule needed students' complex and non-algorithmic thinking though the datasheet was a scaffold. This task needed students to observe the datasheet and to understand the relationship of several algebraic values. It was a Doing-Mathematics Task set up by Ms N.

How did students implement the task? In Table 4, the factors which influenced students' implementation were coded.

From Table 4, the task implemented by students was a high-cognitive demands one. First, the students' calculation and observation were built on the methods of calculation area and students' understanding of a^2, b^2 and c^2. Secondly, the datasheet was a scaffold for students to produce propositions which made the process of producing proposition was reasonable. Thirdly, Ms N gave students enough time for the process of producing propositions. Though sometimes Ms N took over students thinking and reasoning, the task was wholly implemented as a Doing-Mathematics Task.

4.2.2.2 Justifying the proposition How did Ms N set up the task? When Ms N set up the task of justifying the proposition, she clued students "to use the method of calculating area to justify the proposition" which suggested a pathway to be followed explicitly. So it was a typical Procedures-With-Connections Task.

How did students implement the task? In Table 5, the factors which influenced students' implementation were given (Figs. 7, 8).

From Table 5, analysis of the factors influenced students' implementation, the task implemented by students was a Procedures-Without-Connections one. For Ms N reduced the complexity of the task by telling "using the method of area calculation". So the process of justifying was declined as a computational task and students focused

	①	②	③	④	...
a^2					
b^2					
ab					
c^2					

Fig. 5 The datasheet

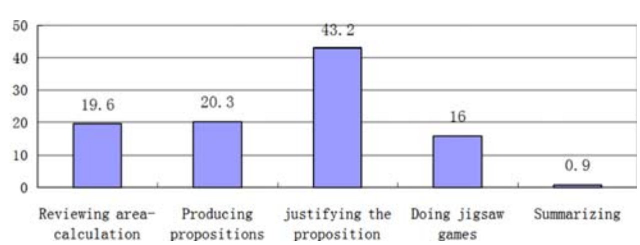

Fig. 6 Time percentage of each segment in the 2nd lesson

Table 4 The associated factors influenced students implementing

Segments of the lesson	Factors
T: Now there are four such figures on the No 2 worksheet, please calculate the value of a^2, b^2, c^2 and ab in each figure. Do not forget to fill the value in the data table	M1 (the teacher scaffold students' thinking and reasoning)
(Ms N walked around while students were doing the task)	
T: Well, when you finished the data table, please observe the data table and do some comparisons to look for some rules. If you find something, write them down on the worksheet	M7 (the teacher gave students proper exploration time)
(Lasted 4 min and 38 s)	
T: Ok. Now every four persons as a group to discuss what you have found. First, you should be sure that the values are correct. Then you may talk with your partners about the rules which you have found	M7 (the teacher gave students proper exploration time)
(Group discussion lasted 3 min and 26 s)	
T: Ok, just now you have done calculation. Now let us see what you have found by observation of the data table. Who would like to tell us? Yang Ming, please	M5 (the teacher built the task on students' prior work)
S1: $a^2 + b^2 = c^2$ (Ms N wrote it on the blackboard)	
T: $a^2 + b^2 = c^2$. Something else? Li Hua?	
S2: $\sqrt{(ab)^2} = ab$	
T: $\sqrt{(ab)^2} = ab$. Well, sit down please. Anything else?	
S3: when $a = 1$, $(ab)^2 = b^2$	
T: Ok, anything else?	
T: No more. Let us see the outcome here. When $a = 1$, $(ab)^2 = b^2$, this is a special outcome. From the figure, it seemed there is no meaning. Let us see the formula, $\sqrt{(ab)^2} = ab$. In fact, no matter what the values of a and b are, it is always right. It seemed no relationship with the figure which we are learning today. And then the formula $a^2 + b^2 = c^2$, what does a^2 mean?	D1 (without explanation from students, the teacher took over students' thinking and reasoning)
S(choral): One of right-edges' square	
T: a means a right-angled edge, so is b. And $a^2 + b^2$ means the sum of two right-angled edges' square. How about c^2?	
S(choral): The bevel-edge's square	
T: This is the very topic what we will learn today. In a right-angled triangle, the sum of two right-angled edges' square is equal to the bevel-edge's square. (while speaking, Ms N wrote it down on the blackboard)	

on correct calculation the area of c^2 rather than mathematical understanding of the whole process of justifying a proposition.

4.2.3 The discussion in TRG

The post-lesson discussion lasted about 1 h and 40 min. Except talking about Ms N's teaching behaviors in guiding students' justification of the theorem, there were two new topics produced and focused in the discussion by members of TRG.

4.2.3.1 Do students need to understand the necessity of justifying the proposition?

T2: When students got the proposition, $a^2 + b^2 = c^2$, Ms N directly came to justify the proposition. If I were a student, I would have such a question: why does it need be justified? The outcome is absolutely right according to the datasheet…

Ms N: Actually, I felt a little uncomfortable when I transferred to prove the theorem. But I did not recognize that was a problem at that time.

T4: I think, as a teacher, we should explain it to students: in mathematics a potentially correct proposition should be proved generally. But what I am thinking is that: can students understand it?

T1: It is necessary and important to create chances for students to understand the necessity of justifying a proposition. As you know, that's the very mathematical ideas in geometrical learning. You may ask students the question when they verified the proposition in the datasheet: how do you know it is always right in every right-angled triangle? Can you verify it by listing all examples of right-angled triangles?

Table 5 The associated factors influenced students implementing

Segments of the lesson	Factors
T: Now think it over. May we use the method of calculating area of c^2 to prove the proposition, $a^2 + b^2 = c^2$? That is to say the catty-cornered square. Try it on your worksheet	D1 (the teacher reduced the complexity of the task by telling how to do the problem)
(students did it for 10 min and 30 s)	
T: How did we calculate c^2? You may talk about it with your partners	M7 (Sufficient time is allowed for exploration)
T: Well, I noticed someone has finished it. Who would like to introduce your work? Li Yumin!	M3 (a capable student modeled high-level performance)
S1: Draw a big square in which CB is an edge	
T: Does it mean drawing a big square outside the right-angled triangle? Well, extend CB, and like this. (Ms N draw the figure) Then how did you justify the proposition?	D6 (the teacher accepted students' unclear explanation)
S1: Because of BC = AZ…	
T: Because of BC = AZ, so the four right-angled triangles are congruent, right? And then?	D2 (the teacher took over the student's reasoning)
S1: The area of quadrangle XYZC = $(\mathrm{AC} + \mathrm{AZ})^2$	
T: Using lowercase, it will be $(a + b)^2$ (Ms N wrote it down)	
S1: he area of square ABDE = $(a + b)^2 - 4 \times 1/2ab = (a + b)^2 - 2ab = a^2 + b^2$	
T: And then?	
S1: Because of the area of square ABDE = c^2	
T: Yes, because the edge of square ABCD is c, its area is c^2. So we get $a^2 + b^2 = c^2$. Sit down please	D2 (the teacher took over the student's reasoning)
T: Just now we justified the proposition by replenishing method. And the other method of calculating area, partitioning method, who would like to used it to justify the proposition. Zhang Wei!	D1 (the teacher reduced the complexity of the task by telling how to do the problem)
S2: Intercept four congruent triangles in the square ABDE and the area of each is equal to ΔBCA	M3 (a capable student modeled high-level performance)
T: Intercept? How to intercept? (drew figures while speaking) In the square ABDE, we partitioned it as four RtΔ and one small square. And then?	D2 (the teacher took over the student's reasoning)
S2: The area of square ABDE is equal to the area of square HIJK plus 4SΔBHA (while S2 speaking, Ms N wrote it down on the blackboard)	
T: Ok, and then?	
S2: It is $(b-a)^2 + 4 \times 1/2ab$	
T: Well, the edge is b, and this edge is a. So the area of the small square is $(b-a)^2$. And then?	D2 (the teacher took over the student's reasoning)
S2: = $b^2 - 2ab + a^2 + 2ab = b^2 + a^2$. And because the area of square ABDE is c^2, $c^2 = a^2 + b^2$. (while S2 speaking, Ms N wrote it down on the blackboard)	
T: $c^2 = a^2 + b^2$, right? Yes, by these two methods, may we justify the proposition?	
S(choral): Yes	
T: Ok, here we got a very important theorem called the Pythagoras theorem	

By discussion, the members of TRG began to talk more on how to redesign the lesson to guide students understanding the necessity of justifying propositions.

4.2.3.2 Should the logical proof be done strictly in reasoning?

T5: I have a doubt. Why did not Ms N take a tolerant way to admit student's reasoning in justifying the proposition? When the four congruent right-angled triangles were replenished or partitioned, why the big or small quadrangle was square has not been explained.

Ms N: I thought it was too difficult for students. That was the reason why in my first lesson I did not expect my students to prove the proposition. For example, when you replenished four congruent right-angled triangles on the catty-cornered square, you must explain why the three points are in the same line. That's too difficult for my students…

T2: As geometrical reasoning, the key steps should be explained by students.…

T1: I do not think so. When students came to replenish four triangles, it was natural for them to "see" a big

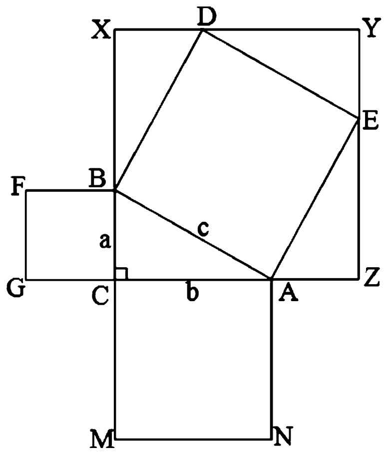

Fig. 7 Ms N's figure on the blackboard

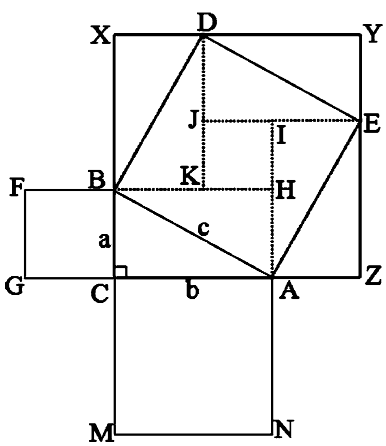

Fig. 8 Ms N's figure on the blackboard

square. If the teacher stopped here and asked them to do explanations, first, maybe it might block students' wholly reasoning process in justifying the proposition and secondly there is not enough time. The lesson overspent too much time…

T2: If it is too difficult for your students, I think, at least the teacher should do explanations…

T1: Maybe, but for most students, they just began to learn geometrical reasoning and the strict requirement may destroy their interest of further learning.

This topic was quite controversial and the six teachers in the TRG quarreled about it for a long time. At last, considering of the limited time in one lesson, most of teachers consented that the strictly reasoning should be required in the future learning. For the justifying the proposition, they thought the replenishing method should be communicated in the whole class for it was more understandable for most of students. For the other one, partitioning method, though several students might use it, would not be introduced in the whole class and the teacher should encourage more students to try it after the lesson.

4.3 The third lesson

In the post-lesson discussion after the second lesson, there were two main consents to improve the teaching. The first one was that the teacher should make students experience the necessity of justifying a proposition in the process of producing propositions. The second one was related with justifying the proposition. The teacher should make students get wholly mathematical understanding of the thinking way of the justifying process rather than very strictly doing logical justification. For centering at students' understanding and saving teaching time, the replenishing method would be the primary way to justify the proposition. Ms N still used the graph paper to design the worksheet for students and she did some modification. The third lesson was scheduled for 45 min and actually lasted 48 min and 57 s.

4.3.1 The lesson structure

The lesson structure was quite similar with the second lesson, but some adjustment was done by Ms N in some segments. The durations for different activity segments could be approximately described in this sequence:

1. Reviewing the method of area-calculating (6 min and 36 s). By asking students to calculate a catty-cornered square (same to Fig. 3), replenishing four right-angled triangles to calculate the area as the primary method was clarified.
2. Producing propositions by filling in a table (20 min and 16 s). After explaining the meaning of a^2, b^2 and c^2 in the graph paper, Ms N asked students to fill in values of four right-angled triangles (same to Fig. 4) into a datasheet (Fig. 9). When students finished the datasheet, Ms N asked students to observe the datasheet and put forwards some propositions.
3. Justifying the proposition by students themselves on the worksheet (10 min and 21 s). Ms N asked students to justify the proposition by themselves firstly and then asked students to communicate their method of justifying the proposition. Because of the hint of calculating a catty-cornered square in graph paper and

	I	II	III	IV	V
a					
b					
a^2					
b^2					
2ab					
c^2					

Fig. 9 The datasheet

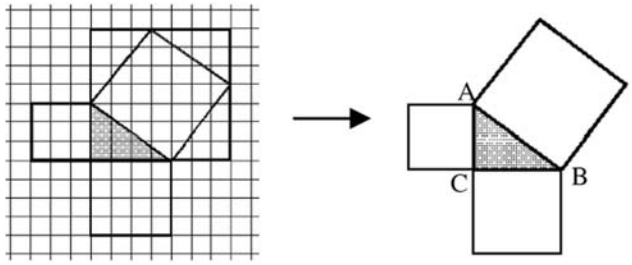

Fig. 10 Backout of the graph paper

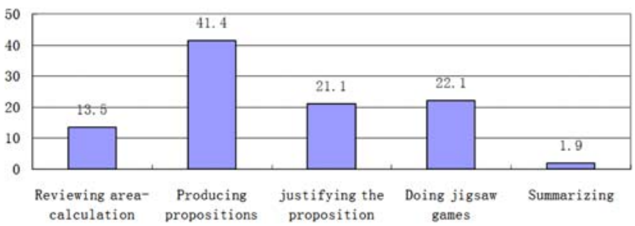

Fig. 11 Time percentage of each segment in the 3rd lesson

Ms N's emphasis on replenishing method, even if the backout of the graph paper (Fig. 10), students found the way to justify $c^2 = (a + b)^2 - 4 \times 1/2ab = (a + b)^2 - 2ab = a^2 + b^2$.

4. Doing jigsaw games to verify the Pythagoras Theorem visually (10 min and 49 s).
5. Summarizing what was learned in this lesson briefly (55 s) (Fig. 11).

4.3.2 The two key tasks

4.3.2.1 Producing the proposition *How did Ms N set up the task?* Ms N firstly drew the squares of three edges of the right-angled triangle on the blackboard and gave some explanations on the geometrical meaning of a^2, b^2 and c^2. And then she asked students to calculate the values of a^2, b^2, $2ab$, c^2 and fill them in the datasheet. Later, she asked students to observe the datasheet and look for what rules might be there. Judging from the Tasks Analysis Guide, the task needed students' complex and non-algorithmic thinking though the datasheet was a scaffold. This task needed students to observe the datasheet and to understand the relationship of several algebraic values. It was a Doing-Mathematics Task set up by Ms N.

How did students implement the task? In Table 6, the factors which influenced students' implementation were coded (Fig. 12).

From the process of students' producing propositions, it required students to access relevant knowledge and experiences, considerable cognitive effort to support or disprove what they have found. It was implemented by students as a typical Doing-Mathematics Task.

Table 6 The associated factors influenced students implementing

Segments of the lesson	Factors
T: Ok, no problem with the data? Then carefully observe the data in the table please, and think about what inference we might get from the data. Well, see it. If you find one, do not stop there and try to find more	M1 (scaffolding students' thinking and reasoning) M7 (appropriate exploration-time)
(Students observed and discussed the datasheet for about 1 and half a minutes)	
T: Then please tell me what inference you have found? Li Dan!	
S1: In my group, we found two conclusions: $2ab + 1 = c^2$ and $a^2 + b^2 = c^2$	
T: Oh? (S2 raised his hand) Well, Liu Yuyin, how do you think?	M4 (pressing explanation by questioning)
S2: Ms N, I just drew a right-angled triangle, $a = 2$, $b = 4$. $2ab = 16$ and $c^2 = 20$, so $c^2 \neq 2ab + 1$	
T: Pretty good! Liu Yuyin disproved it by a special example. It seemed that was very persuasive. So the proposition, $c^2 = 2ab + 1$, does not come into existence. Oh, do you want to speak something?	M4 (pressing for meaning by comments and feedback)
S1: We just found, when $a - b = 1$, $2ab + 1 = c^2$ could come into existence	
T: Sit down please. What you thought is reasonable and it seemed $c^2 = 2ab + 1$ was a conclusion with some conditions. Well, how about $c^2 = a^2 + b^2$? You, please	M4 (pressing for meaning by comments and feedback)
S3: It is always right judging by all the examples in the worksheet. But I am thinking, if I give more examples… Even if one hundred of examples are right, but the one hundred and first example does not match it, how can I do? So if I want to be sure of its correctness, I must know that all of its examples are right. If there is the only one example which does not match it, it would be still a conclusion with some conditions	
T: Sit down please. If we want to know if it is a theorem, judging by several examples is not enough. Then what we should do?	M4 (pressing explanation by questioning)
S(choral): Justification	

图号 量	I	II	III	IV	V
a	1	2	3	5	
b	2	3	4	6	
a^2	1	4	9	25	
b^2	4	9	16	36	
2ab	4	12	24	60	
C^2	5	13	25	61	

Fig. 12 One of the datasheets

4.3.2.2 Justifying the proposition *How did Ms N set up the task?* When Ms N set up the task of justifying the proposition, like in the second lesson, she clued students to "think of the method to calculate the area of cattycornered square" which suggested a pathway to follow explicitly. So it was a typical Procedures-With-Connections Task.

How did students implement the task? In Table 7, the factors which influenced students' implementation were coded (Fig. 13).

From the process of implementing the task by students in Table 7, Ms N kept questioning and building connections between calculation and figures. Though it was not justified very strictly but students got a whole understanding of the thinking way to justify the proposition. It was kept as a Procedures-With-Connections Task.

4.3.3 The discussion in TRG

The post-lesson discussion lasted about 1 h and 25 min. All the teachers in TRG positively reviewed the lesson though as a novice teacher Ms N still had some shortcomings in teaching technique. There were two focuses in the TRG activities.

4.3.3.1 How do the teacher deal with students' other propositions?

T5: When I observed the lesson in the classroom, I noticed that several students produced other propositions in their worksheet. Like these: $c^2 = (a + b)^2 - 2ab$, $c^2 = (a - b)^2 + 2ab$ and $a + b + a^2 = b^2$. Though the first two could be simplified as $a^2 + b^2 = c^2$, the last one was not understandable. So I think Ms N should give such students chances to speak them out.

T2: Yes, I noticed them, too. The last one was created by one set of special numbers. I suggest that Ms N should collect all the students' worksheets to analyze students' thinking.

Table 7 The associated factors influenced students implementing

Segments of the lesson	Factors
T: Think of the method to calculate the area of the catty-cornered square to justify $a^2 + b^2 = c^2$. What is c^2 equal to?	D1 (telling how to do the problem) M7 (appropriate exploration- time)
(Students did it independently for 4 min and 50 s)	
T: Well, let me ask somebody to tell us. Zhang Wen, try it	M3 (a capable student modeled high-level performance)
S1: Replenish three right-angled triangles around the catty-cornered square	
T: Replenish three right-angled triangles around the catty-cornered square. (Ms N drew them on the blackboard) Next step?	D6 (the teacher accepted students' unclear explanation)
S1: The area of the biggest square is $(a + b)^2$	
T: What does c^2 mean? … What is c^2 equal to?	M6 (keeping questioning the meaning) and M4 (building connection between figures and formula)
S1: So the biggest square subtracts four right-angled triangles. It is $(a + b)^2 - 4 \times 1/2ab$	
T: Subtract $4 \times 1/2ab$ and the area of each small right-angled triangle is $1/2ab$. Then we have got c^2, how do we justify the proposition?	
S1: Calculate out the square	
T: Well, let us calculate it	
S1: It is equal to $a^2 + 2ab + b^2 - 2ab = a^2 + b^2$	
(S1 said it and Ms N wrote it on the blackboard)	
T: We get $c^2 = a^2 + b^2$? May you explain the thinking way to justify it?	M4 (building connection between calculation and figures)
S: Yes. c^2 is the catty-cornered triangle and it is equal to that, the biggest square subtract four congruent right-angled triangles	
T: Good, sit down please. From the process of justification, we got the conclusion: the sum of two right-angled edges' square is equal to the bevel-edge's square. Now we verified its correctness and it is a true proposition, called Pythagoras theorem	

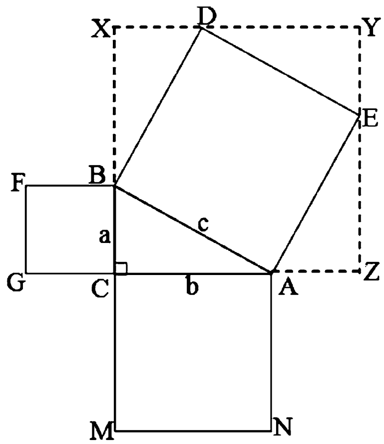

Fig. 13 Ms N's figure on the blackboard

T1: After Ms N dealt with $2ab + 1 = c^2$, she should have asked such a question: Anybody else has other findings? Such question may inspire students' more thoughts.

Ms N: Yes, I hurried up to move the lesson ahead. The second lesson overtimed too much, so I felt that I did not have enough time.

These teaching suggestions came from the teachers' classroom observation. Ms N could get some useful information which was not noticed by herself in the whole class teaching process.

4.3.3.2 Is the jigsaw game necessary for students to manipulate it? Though there was no possibility to have the fourth lesson, another new topic about redesign the lesson was brought out (Fig. 14).

T4: For the jigsaw game, I do not understand its value in the lesson. The theorem has been justified by logical reasoning. Is it necessary for students to manipulate it? To verify the theorem visually?

Ms N: I had the same doubt from the second lesson. Is the jigsaw game counted as another kind of justification? Students have proved the theorem before the game, so it is at most as a Verifying activity. If I had the fourth lesson, I would like use some problems to apply the theorem.

T2: The theorem has been justified. So the jigsaw game could be seen as an applying problem. Students needed to change places of the four right-angled triangles and presented that the area of the catty-cornered square was the sum of two small squares. I think it was an applying problem.

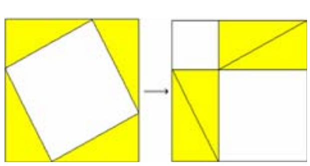

Fig. 14 Jigsaw game

In fact, the discussion on this topic reflected teachers' usual thinking way: in the limited lesson time, what was the important to be arranged in the lesson?

5 Summary and discussion

In this part, the analysis of the three lessons would be summarized to reveal what had been changed in Ms N's three lessons. About the study question, how Chinese teacher improved her/his teaching in TRG would be discussed.

5.1 What had been changed?

If each lesson was divided into five common parts: reviewing the method of area-calculating (segment A), producing propositions (segment B), justifying the proposition (segment C), applying the theorem/doing jigsaw games (segment D) and summarizing the learning (segment E), three lessons' segments could be gathered to do comparison. Figure 15 showed what the most important part was judging from the percentage of teaching time of each segment.

From Ms N's three lesson-structures (Fig. 15), it reflected the change of teaching behaviors. In the first lesson, applying the theorem was emphasized; in second lesson, justifying the proposition; and in the third lesson, producing propositions.

The three lesson plans were also checked, the change was listed in the Table 8. From Ms N's lesson plans done by herself, what the teacher saw as important in the lesson might be reflected, too.

In three lessons, two key tasks were focused and analyzed in this study: how the task was set up by the teacher and how it was implemented by students. The maintenance or decline of a high-cognitive-demands task in each lesson was summarized in Tables 9 and 10.

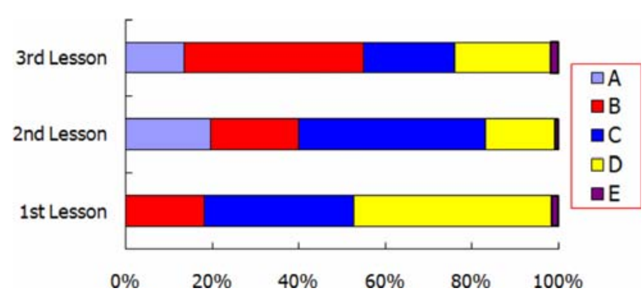

Fig. 15 Time percentage of segments in each lesson

Table 8 The tasks set up by Ms N in each lesson plan

	Tasks set up by Ms N in each lesson plan	Features
The 1st lesson	To justify the theorem	Emphasis on the content of the theorem and its application
	To apply it in exercises	
The 2nd lesson	To produce propositions	Emphasis on the process of producing propositions and the method of justifying it
	To justify it and become a theorem	
The 3rd lesson	To produce propositions	Emphasis on the whole process of producing propositions, its justifying way, and understand it visually
	To understand the way of justifying it	
	To verify the theorem visually by jigsaw games	

Table 9 Producing propositions in three lessons

	As set up by teacher	As implemented by students	Maintenance or decline	Factors associated with maintenance or decline
The 1st lesson	Doing-mathematics task	Procedures-without-connections task	Decline	Seven decline-factors (4 D1, D2, D3, D5)
The 2nd lesson	Doing-mathematics task	Doing-mathematics task	Maintenance	Four maintenance-factors (M1, M5, 2 M7) and 1 decline-factors (D1)
The 3rd lesson	Doing-mathematics task	Doing-mathematics task	Maintenance	Six maintenance-factors (M1, 4 M4, M7)

Table 10 Justifying the proposition in three lessons

	As set up by teacher	As implemented by students	Maintenance or decline	Factors associated with maintenance or decline
The 1st lesson	Procedures-without-connections task	No possibility to be implemented in a high cognitive level	–	–
The 2nd lesson	Procedures-with-connections task	Procedures-without-connections task	Decline	Three maintenance-factors (2 M3, M7) and seven decline-factors (2 D1, 4 D2, D6)
The 3rd lesson	Procedures-with-connections task	Procedures-with-connections task	Maintenance	Five maintenance-factors (M3, 2 M4, M6, M7) and two decline-factors (D1, D6)

Table 11 Discussion topics in TRG

	Main topics in TRG discussion	Features
The 1st lesson	What should be taught	Teaching aim-entered
	Reasonable process of producing propositions	
	Students' understanding of thinking way of justification	
The 2nd lesson	Students' understanding of the necessity in justifying	Focusing on key tasks
	Strictly logical proof in reasoning	
The 3rd lesson	Caring of students' other propositions	Caring of students' actual learning
	Jigsaw-game's necessity for manipulation	

Comparing the three lessons (Table 9), the task of producing proposition which was set up by the teacher and implemented by students declined in a low cognitive level in the 1st lesson, but maintained in a high cognitive level in the 2nd the 3rd lesson. And the maintenance-factors increased and decline-factors decreased gradually from the 1st lesson to the 3rd lesson.

Seeing from Table 10, the tasks of producing proposition in three lessons were differently set up by the teacher and implemented by students. In the 1st lesson, the task set up by the teacher was not a high cognitive one. In the 2nd lesson, the task set up by the teacher was a high cognitive one but it declined as a low cognitive one. In the 3rd lesson, the task was maintained in high cognitive demands. And the maintenance-factors increased and decline-factors decreased gradually from the 2nd lesson to the 3rd lesson.

The change that happened in the three lessons might be contributed to the TRG activities which influenced Ms N's teaching behavior. The main topics in three TRG activities were summarized in Table 11.

Looking back to the three post-lesson discussions, which were often started from the problems in former lesson and ended at the suggestions for the next lesson, some common features came out: the discussions were teaching aim-centered, focusing on key tasks and at last caring of students' actual learning in lessons.

5.2 How lessons were improved?

In the mainland of China, teachers have many opportunities to open their classroom to colleagues in school level, county level, and provincial level, which was seen as a high reputation by Chinese teachers. In fact, behind every exemplar lesson in different level, the lesson will be taught at least twice like the procedure described in this study and all the TRG members see it as the most important activity in school-based teaching research. So we may get a snapshot on how Chinese teachers improved their lessons from the Shanghai case.

5.2.1 Learn from other teachers and himself in TRG

Because of the school-based teaching research system, it was very common for Chinese teachers to learn others' experience in the TRG activities, which was called peer coaching and its value for teachers' professional development had been pointed out (Anderson & Pellicer, 2001; Lin, Yan, & Lin, 1999).

> *After the study of teaching, especially the discussion, I think the way of teaching is clearer than that in the textbooks. I have known it well. Where a question should be given to students and where an emphasis is arranged, and the teaching details guided by master teacher in discussion, are more useful compared to my own lesson design (from Ms N's interview).*

Except that, in Chinese school-based teaching research activities, teachers usually experienced his/her own routine lesson and then the improved lesson which absorbed suggestions in TRG activity. When the research lesson teacher had a chance to have three lessons repeatedly in parallel classrooms, he/she actually had a good opportunity to compare his/her own three lessons, which was actually a process to learn from himself/herself.

> *After the second lesson, I thought it over a lot. Though I thought a lot about the design before the lesson, I cannot help thinking it again and again. If I restarted the lesson, I would rethink the conjunction among the four worksheets, the language to express every question, the summarization after each activity. (From Ms N's interview).*

5.2.2 Construct profound understanding of mathematics in TRG

Chinese teachers did not have high-level educational certificates, but they had a more profound understanding of mathematics than their US counterparts in Ma's study (1999). In fact, the TRG members were teachers who taught the same subject and usually one teacher always taught one subject in 2–3 parallel classes in schools. So the discussion in TRG often related with their opinions on mathematics. When they talked about what should be taught and what should be learned, these kinds of questions were closely connected with the understanding of mathematics and teachers constructed their understanding of mathematics gradually in long period of teaching career.

> *A big idea about mathematics gave me deep impression. Let the students experience the process of justification and disproval. In my usual lesson I never thought about it. The mathematics examples, exercises, how to deal with them had been thought a lot before. From the discussion this time, I knew how to have such kind of lessons… (From Ms N's interview).*

5.2.3 Learn teaching theory in actions

Since Chinese curriculum reform was carried out in 1990's, teachers were facing more and more challenge from new ideas and teaching theory and they were required to attend many training courses. In training courses, teachers learned new information and ideas from experts' lectures but teaching theory was hard to practice in classrooms. Rather than that kind of "learning-in-listening", TRG activity was "learning-in-doing", in which teachers got grassroots professional development in their teaching practice (Paine & Fang, 2006). In the three post-lesson discussions, teachers tried to use graph paper as a scaffold to make students experience doing-mathematics: from producing propositions to disproving or justifying them.

> *In my first lesson, I put emphasis on applying theorem to answer questions for I thought the theorem was too difficult to be justified. Actually, after I introduced the justification in the textbook, I myself felt guilty. Is that counted as a justification-teaching? Now I knew the Scaffold Theory and understand how to use it in the teaching. I never thought of graph-paper, never expected it could be used in teaching a theorem… (From Ms N's interview).*

Though this was just a case from Shanghai, the three lessons' improving process was very representative because the TRG network generally exists in mainland of China. Of course, not every lesson developed in TRG could get honors in a national wide competition of lessons. Actually, when a teacher's lesson won prize in China, it often pooled a lot of collective wisdom and all the members of TRG saw it as their team's reputation. This paper just showed the process of how a lesson was changed step by step from a Shanghai Case. In many Chinese teachers' minds, a good lesson was always a process but not an outcome.

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