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ORIGINAL ARTICLE

Aesthetics as a liberating force in mathematics education?

Nathalie Sinclair

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Abstract This article investigates different meanings associated with contemporary scholarship on the aesthetic dimension of inquiry and experience, and uses them to suggest possibilities for challenging widely held beliefs about the elitist and/or frivolous nature of aesthetic concerns in mathematics education. By relating aesthetics to emerging areas of interest in mathematics education such as affect, embodiment and enculturation, as well as to issues of power and discourse, this article argues for aesthetic awareness as a liberating, and also connective force in mathematics education.

Keywords Aesthetics · Mathematics · Embodiment · Values · Inquiry · Affect

1 Introduction

According to the Ancient Greek divisions of philosophy, questions about aesthetics fall into the branch of axiology, which concerns itself with theories of values, including aesthetic values, and also ethical ones. Given the importance that its root term *axios*—which leads to *axioma*—plays in the discipline of mathematics. It may seem strange that axiology has been mostly ignored in the philosophy of mathematics, which has focused almost exclusively on the branches of ontology and epistemology. Axioms, for the Ancient Greeks, were the things that were taken to be self-evident, not needing proof, but used as starting points for a deductive system. *Axios* had the meaning of "being in balance", "having value", "worthy", and "proper". So aesthetics, far from being confined to more modern questions of artistic taste and style, involved theories about what is valued, how it is valued, and why it is valued. Which conception of aesthetics—the classical one, or the more modern one—should we choose to focus on in the context of mathematics education?

In his book on constructive postmodernism, Martin Schiralli (1999) describes the "fixed" view of meaning represented in the question "What do we mean by X?" (p. 57). He then argues for a view of meaning that attends to the genesis of concepts historically, the development of concepts in individuals, and the possibilities of meaning with regard to empirical and theoretical concerns. This view leads to a different question, namely: "What is there for us to mean by X?" (p. 57). This paper will be driven by the latter formulation in an attempt to help liberate aesthetics from its more modernist, fixed use, which has served both mathematics and mathematics education poorly. I base this claim on encounters I have had with a range of people—including teachers, researchers, parents, and others—who believe that it is either elitist or frivolous to focus on aesthetic concerns in mathematics education.

The elitist perspective derives in part from the fact that most contributors to discussions on aesthetics in mathematics (and in the sciences) are eminent mathematicians who talk about beauty, elegance and purity, which only very few people seem to be able to—or want to—appreciate. The frivolous perspective may be linked to common usages of the word aesthetic itself, either to describe hair salons and spas or to describe capricious, fashionable forms of taste. In the next section, I investigate where these understandings come from and what assumptions they lead to about the role of aesthetic awareness in mathematics education. I then consider some more contemporary views,

N. Sinclair (✉)
Faculty of Education, Simon Fraser University,
8888 University Drive, Burnaby, BC V5A 1S6, Canada
e-mail: nathalie_sinclair@sfu.ca; nathsinc@sfu.ca

which offer decidedly non-elitist and non-frivolous interpretations of the role of aesthetics in thinking and learning—and construct the aesthetics domain as essential to human meaning-making. Using these broader conceptions of aesthetics, I propose some relevant connections to current issues in mathematics education as well as questions for future study. I will also identify some pervading assumptions and practices, both in mathematics and in education, that will challenge the emergence of aesthetics as a liberating force in mathematics education.

2 Interpretations of the mathematical aesthetic

Conceptions of aesthetics draw on the multiple understandings of the aesthetic itself, as it has evolved over time. It is most often associated with the arts—aesthetics being seen as the philosophy of art—and is used to describe styles or tastes related to masterpieces of artistic products such as paintings, symphonies, and novels. This particular interpretation dates back to Alexander Baumgarten's (1739, 1758) use of the term aesthetics to mean "criticism of taste" and "the science of the beautiful". When applied in contexts outside of the arts, such as phenomena of the natural world, the word aesthetic is often used as a substitute for terms such as "beautiful," "pretty" or "attractive".

The etymology of the word suggests a somewhat different interpretation, relating the aesthetic to the senses and to sensory perception. As such, we can distinguish "the aesthetic", which relates to the nature of perceptually interesting artefacts, and "aesthetics", which relates to the science of human taste or sensory perception. While the former often appears adjectively, to describe artefacts, experiences, sensibilities, and judgements, the latter represents a more general or systematic theory of what might be considered beautiful, artful or tasteful.

In the domain of mathematics, no overall theories of aesthetics have been proposed. However, scholars have discussed varying ways in which aesthetic judgements arise in mathematics. There is a long tradition in mathematics of describing proofs and theorems in aesthetic terms, often using words such as "elegance" and "depth". Further, mathematicians have also argued that their subject is more akin to an art than it is to a science (see Hardy, 1967; Littlewood, 1986; Sullivan 1925/1956), and, like the arts, ascribe to mathematics aesthetic goals. For example, the mathematician W. Krull (1930/1987) writes: "the primary goals of the mathematician are aesthetic, and not epistemological" (p. 49). This statement seems contradictory with the oft-cited concern of mathematics with finding or discovering truths, but it emphasises the fact that the mathematician's interest is in expressing truth, and in doing so in clever, simple, succinct ways.

While Krull focusses on mathematical expression, the mathematician H. Poincaré (1908/1956) concerns himself with the psychology of mathematical invention, but he too underlines the aesthetic dimension of mathematics, arguing that the aesthetic is the defining characteristic of mathematics, not the logical. In Poincaré's theory, a large part of a mathematician's work is done at the subconscious level, where an aesthetic sensibility is responsible for alerting the mathematicians to the most fruitful and interesting of ideas. Other mathematicians have spoken of this special sensibility as well and also in terms of the way it guides mathematicians to choose certain problems. This choice is essential in mathematics given that there exists no external reality against which mathematicians can decide which problems or which branches of mathematics are important (see von Neumann, 1947): the choice involves human values and preferences—and, indeed, these change over time, as exemplified by the dismissal of geometry by some prominent mathematicians in the early 20th century (see Whiteley, 1999).

While many commentators have argued for the importance of aesthetic judgements in mathematics, they have differed in terms of the extent to which such judgements can be made objectively. As I describe in the following section, some consequences of the objective view include the following: aesthetic judgements are true and immutable; criteria can be established that will identify mathematical objects of aesthetic value; these criteria apply to the aesthetic objects themselves; mathematicians will agree on the aesthetic value of different mathematical objects. After elaborating the objectivist view, I will describe more subjective and contextual conceptions of the mathematical aesthetic.

2.1 Objective views of the mathematical aesthetic

Consider the following quotation by the textbook writers Holt and Marjoram (1973): "The truth of the matter is that, though mathematics truth may be beautiful, it can be only glimpsed after much hard thinking. Mathematics is difficult for many human minds to grasp because of its hierarchical structure […]". This statement not only suggests that mathematics object of aesthetic value can only be appreciated by a small elite, it also assumes that aesthetic values belong to the objects themselves, to "mathematical truth," and that mathematical beauty is permanent, like a Platonic ideal. Aristotle ascribes a similar objectivity, locating as he does the beauty within the mathematical object itself, and thus independent of the human mathematician: "The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful" (XIII, 3.107b). In other words, mathematical beauty is independent of time and culture.

The objective view of the mathematical aesthetic holds that mathematicians will agree on judgements of mathematical beauty (if they disagreed, that would imply possible subjectivity). The efforts of mathematicians such as G.H. Hardy (1967/1999) to identify the characteristics of mathematical beauty are dependent on this kind of agreement, and its underlying objectivist view. Hardy proposed "depth" and "significance" as primary features of mathematical beauty. Significance is related to the idea's fruitfulness, or its ability to lead to new ideas and its ability to connect different mathematical ideas. Depth, however, has "*something* to do with *difficulty*: the "deeper" ideas are usually the harder to grasp" (p. 109). As with the Holt and Marjoran, Hardy's conception of mathematical beauty severely restricts its accessibility to those who can understand difficult ideas. But this implies that mathematical practices that exploit more visual, accessible forms of representation and reasoning (as exemplified in many cultures, both historical and contemporary) may not produce beautiful or deep mathematical truths.

Several mathematics educators have followed suit in expressing the belief that the mathematical aesthetic is inaccessible to students and perhaps of less importance than acquiring "the basics". von Glasersfeld (1985) expresses the belief in his claim that children should not be expected to appreciate mathematics like they appreciate rainbows or sunsets. Dreyfus and Eisenberg (1986) argue that the aesthetic is important in mathematics, but that educators should focus their attention on addressing the problems of teaching students "the basics" before any attention should be paid to less pressing concerns. Silver and Metzger (1989) suggest that aesthetic responses in mathematical problem solving require an advanced level of metacognition, and they only gain momentum in advanced mathematical work.

Research on the mathematical aesthetic as conceived from an objectivist point of view focuses on such as: What criteria determine mathematical beauty? What mathematical proofs have aesthetic value? When do novices make the wrong aesthetic judgement? In the realm of education, research has sought to evaluate the extent to which students use the same criteria, or make the same judgements, as research mathematicians. For instance, Dreyfus and Eisenberg (1986) ask students to compare their own solutions to a problem to those preferred by mathematicians. They conclude that students' aesthetic judgements were poorly developed since the students did not tend to agree with the mathematicians. As I argue below, their finding suggests that students and mathematicians may not have the same aesthetic preferences, but this does not mean that students are incapable of aesthetic appreciation in mathematics.

In summary, the most prevalent view of the mathematical aesthetic conceptualises it as an objective judgement that only very few are capable of making, but on which the best mathematicians agree, and that apply to the prized objects of mathematics including theorems and proofs. Papert (1978) points out that for Poincaré, the aesthetic sense is innate: "Some people happen to be born with the faculty of developing an appreciation of mathematical beauty, and those are the ones who can become creative mathematicians" (p. 191). Papert stresses that the appreciation of mathematical beauty must be *developed*, but ascribes to Poincaré the belief that a sharp lines separates those who possess the aesthetic faculty from those who do not.

2.2 Challenges to the objective view

In his chapter of *Grands Courants de Mathematiques*, F. Le Lionnais (1948/1986) discusses the issue of beauty in mathematics, offering a broader conception of mathematical aesthetics than those described above. His discussion stretches the view along two different axes. First, he insists on the subjectivity of aesthetic responses that depend on personal taste. He contrasted mathematicians with Dionysian preferences to those with Apollonian one. Whereas Apollonian tastes privilege equilibrium, harmony and order, Dionysians tend to gravitate toward a lack of balance, form obliteration and pathology. Apollonians will look for structures and patterns, while Dionysians will focus on exceptions, counterexamples, and perhaps even strange or baroque concepts. By admitting a Dionysian proclivity, which contrasts with the more typical visions of mathematics expressed in the quotation from Aristotle, Le Lionnais suggests that mathematicians hold differing—and sometimes even opposing—aesthetic preferences. As a human being, the mathematician is bound to have personal inclinations and perspectives that will have an effect on what she values in mathematics.

Le Lionnais also expands the range of artefacts that are subject to aesthetic judgements by drawing attention to the "facts" and "methods" of mathematics that can be seen as either Dionysian or Apollonian in flavour. Facts include magic squares as well as imaginary numbers, while methods include proof by contradiction and inductive techniques. Le Lionnais' classification scheme of facts and methods is not as interesting as the sheer variety of mathematical ideas and objects that he offers as being capable of eliciting aesthetic responses.

The pluralising move of Le Lionnais challenges the intrinsic view of the mathematical aesthetic described above in several ways. In addition to recognising the subjectivity of aesthetic responses, which also challenges the assumption that mathematicians share a common aesthetic judgement, it recognises the way in which aesthetic responses are not confined to the ivory-tower mathematics of theorems and proofs—artefacts that are barely encountered in school mathematics.

Moreover, the mathematical aesthetic is driven not just by "beauty" and "elegance" but also, sometimes, by the "ugly" and the "vulgar". Interestingly, these latter judgements are often lost to history, as can be seen by looking at the way new ideas were initially seen by mathematicians. For example, Fénélon (1697/1845) warns others of the bewitching and diabolic attractions of geometry: "Défiez-vous des ensorcellements et des attraits diaboliques de la géométrie" (p. 493). Charles Hermite, recoils with "dread" and "horror" from non-differentiable but continuous functions, writing: "Je me détourne avec effroi et horreur de cette plaie lamentable des fonctions continues qui n'ont pas de derives" (Bailland and Bourget 1905, p. 318). Sometimes, the originally more Dionysian responses are retained in the mathematical words themselves, such as *irrational* numbers, *complex* numbers, the *monster* group, or *annihilators*. I will return to the theme of ugliness later in this article, in terms of the very narrow scope of aesthetics that has been studied in mathematics education.

The survey of mathematicians conducted by Wells (1990) provides a more empirically-based challenge to the intrinsic view of the mathematical aesthetic. Wells obtained responses from over 80 mathematicians, who were asked to identify the most beautiful theorem from a given set of twenty-four theorems. (These theorems were chosen because they were "famous," in the sense that Wells judged them to be well-known by most mathematicians, and of interest to the discipline in general, rather than to a particular sub-field). Wells finds that the mathematicians varied widely in their judgements. More interestingly, in explaining their choices, the mathematicians revealed a wide range of personal responses affecting their aesthetic responses to the theorems. Wells effectively puts to rest the belief that mathematicians have some kind of secret agreement on what counts as beautiful in mathematics.

Wells also sheds light on the changing values, over different times and cultures, which affect judgements of mathematical beauty. Rota (1997) also echoed this view in relating the mathematical aesthetic to different "schools" and eras: "…the beauty of a piece of mathematics is dependent upon schools and periods. A theorem that is in one context thought to be beautiful may in a different context appear trivial". (p. 126). Rota's work on umbral calculus provides a compelling example: this mathematical technique offers a notational device (treating subscripts as exponents) for proving similarities between polynomial equations. This technique first emerged as a sort of magic rule in the 19th century, but was later explained, and placed on a firmer foundation by the work of Rota and his students, which led to further applications and generalisations. While the Rota school found the umbral calculus aesthetically pleasing, its has since gone out of fashion. In sum, beauty is not only subjective; it is context-bound and inseparable from emotions and pleasure.

Moving away slightly from Wells's focus on the evaluation of finished products (theorems and proofs), Burton's (2004) work focuses on the practices of mathematicians and their understanding of those practices. Based on extensive interviews with a wide range of mathematicians, she proposes an epistemological model of "mathematician's coming to know," which includes the aesthetic as one of five categories (the others being: its recognition of different approaches, its person- and cultural/social relatedness, its nurturing of insight and intuition, and its connectivities). She points out that mathematicians range on a continuum from unimportant to crucial in terms of their positionings on the role of the aesthetic, with only 3 of the 43 mathematicians dismissing its importance. For example, one said, "Beauty doesn't matter. I have never seen a beautiful mathematical paper in my life" (p. 65). Another mathematician was initially dismissive about mathematical beauty but later, when speaking about the review process, said: "If it was a very elegant way of doing things, I would be inclined to forgive a lot of faults" (p. 65). While the first point of view arises from a question about defining mathematical beauty, the second statement relates to the way in which aesthetic responses affect decisions and judgements of mathematicians at work. The former view coincides with Schiralli's modernist interpretation of meaning (what is mathematical beauty?). In contrast, the latter, more pragmatic view draws on individual meanings in action and experience.

A more pragmatic approach to thinking about the mathematical aesthetic developed in my earlier work (Sinclair, 2004, 2006b) also draws on mathematical practice among research mathematicians, but focuses instead of the process on mathematical inquiry. Based on interviews with mathematicians, and also on an analysis of the structure of mathematical inquiry and the values that characterise the discipline, my tripartite model of the role of the aesthetic describes the way in which aesthetic values are involved in the selection of mathematical problems, in the generation of hypotheses and conjectures, and in the evaluation of mathematical solutions. In contrast with Burton, who relies solely on mathematicians' understandings of their own practices, the tripartite model also takes into account the way in which mathematical knowledge is produced and communicated in the mathematical community. So, for example, while mathematicians may not care about the beauty or elegance of their solutions or proofs, they do have to assess whether their solutions are "good" and "interesting" since these criteria are used to decide whether their work will be published. Similarly, mathematicians must adopt certain stylistic norms in writing their proofs and these norms are highly aesthetic in nature (see Csiszar, 2003). While mathematicians may not be explicitly aware of the values that guide their work, these values play a crucial role in mathematical practice.

This was the view adopted by Poincaré, who focused on the generative role of the aesthetic. However, for Poincaré, mathematicians were the only to possess the "special aesthetic sensibility" that was capable of generating productive ideas in the mathematician's unconscious mind. Papert (1978) challenges Poincaré on his elitist view, while fully endorsing the work of the aesthetic at the subconscious level. For Papert, mathematicians are not the only ones to possess the "special aesthetic sensibility;" instead, he shows how non-mathematicians can be guided toward correct mathematical ideas through appeal to aesthetic considerations. The aesthetic responses exhibited by Papert's non-mathematicians had little to do with Hardy's qualities of depth and significance, or even of surprise. Instead, they involved emotional reactions to an equation's form and structure—a desire to get rid of a square root sign, or to place the important variable in a prominent position. These responses do not provide solutions, now are they evaluated explicitly by the non-mathematicians; instead, they provide tacit guidance.

Sinclair's tripartite model is somewhat limited in the arena of mathematics education, if only because students rarely have the opportunity to engage in mathematical inquiry in the classroom (in particular, the selection of problems is usually made for them, and the evaluation of a solution is restricted to concerns with veracity). However, some empirical work has been conducted and has shown that, when provided with inquiry opportunities in rich environments, middle school children do indeed use aesthetic values in choosing problems, generating conjectures and evaluating their solutions (Lehrer, 2008; Sinclair, 2001; 2006a). These values sometimes, but not always, overlap with canonically mathematical aesthetic values such as fruitfulness, visual appeal, and surprise. Similar work, focusing on the problem-posing phase of inquiry has shown that prospective elementary teachers can use aesthetic values to pose more interesting mathematical problems (Crespo & Sinclair, 2008).

In working with university-level students, Brown (1973) reports on a "genealogical" tendency for students sometimes to prefer their own solutions to those of mathematicians—in this case, Gauss's solution to finding the sum of the first 100 whole numbers. Brown argues that while these solutions may be seen as "messy" they often encode the parts of the problem solving process that contribute to the student's understanding. These solutions are revelatory. It is interesting to compare Brown's approach to aesthetic preferences with Dreyfus and Eisenberg, who conclude that students lack aesthetic sensibility because they do not agree with mathematicians on the preferred solution. For Brown, a difference in aesthetic preference does not entail a lack of aesthetic sensibility.

Discussions of the mathematical aesthetic, even those of Le Lionnais and Wells, interpret the aesthetic as a mode of judgement that is neither epistemological nor ethical, but instead, related to what is considered good, significant or appealing. As a mode of judgement, the aesthetic is thus most commonly viewed as applying to finished products—such as theorems and proofs—but it can also arise in exploration. In addition, as a mode of judgement, the aesthetic is seen as operating distinctly from other modes of human behaviour, such as affect and cognition. In the next section, I describe very different conceptions of aesthetics that are not specific to the domain of mathematics, but that offer new and productive ways of reflecting on the role of aesthetic awareness in mathematical thinking and learning.

3 Contemporary views of the aesthetic

Over the past several decades, there has been a growing interest in the aesthetic,[1] and new conceptualisations of it that follow in part from contemporary views of the human mind as being inseparable from its body and the world around it. Instead of focusing on judgements, recent interpretations of the aesthetic have talked about human actions and meanings, and have sought to expand the range and deepen the influence of aesthetic responses and experiences. These interpretations draw on scholarly work in cognitive science, neuroscience, anthropology and philosophy, and can be categorise into four different themes related to experience, embodied cognition, inquiry, and evolutionary imperatives. In the sections below, I outline the distinguishing features of each theme and the connections to mathematics education research.

3.1 The aesthetic as a core component of being human

In his book *Art as Experience*, Dewey (1934) wants to reclaim the aesthetic from the narrow and elitist confines of "museum art" and place it as a theme of human

[1] Higginson (2006) documents some of this in relation to mathematics. Additionally, several books can now be found on the aesthetics of science *It Must be Beautiful: Great Equations of Modern Science* (Farmelo, 2002), *Beauty and the Beast: The Aesthetic Moment in Science* (Fischer, 1999) and of computing, *Aesthetic Computing* Fishwick (2006).

experience. For Dewey, the *aesthetic experience* is of central interest. Moreover, rarefied aesthetic experiences are simple an extreme form of what all humans experience in a wide variety of endeavours. Indeed, Dewey seeks, first and foremost, to situate the aesthetic squarely in more common, natural settings:

> in order to *understand* the esthetic in its ultimate and approved forms, one must begin with it in the raw; in the events and scenes that hold the attentive eye and ear of man, arousing his interest and affording him enjoyment as he looks and listens (p. 5).

While he traces the aesthetic to everyday human activities, he draws attention away from aesthetic judgements and instead focuses on the integration of thoughts and feelings that occur in experience. For Dewey, an experience has aesthetic quality whenever there is coalescence into an immediately enjoyed qualitative unity of meanings and values drawn from previous experience and present circumstances. In Dewey's conception, the aesthetic does not describe the qualities of perceptual artifacts; rather, it characterizes experiences that are satisfactory and consummatory. Aesthetic experiences can be had while appreciating art, while fixing a car, while having dinner, or while solving a mathematics problem. They are aesthetic in that they combine emotion, satisfaction and understanding. While previous philosophers focused on the form of perceptual objects (colour, structure, etc.), Dewey looks for integration with the human being in interaction with the world.

Dewey's aesthetic experiences can be had in mathematics, of course, and there have been several descriptions of the kind of overwhelming, satisfying and fulfilling experiences by mathematicians themselves, including the very moving testimony of Andrew Wiles (see Singh, 1997), but also claims that such experiences are the ultimate goal of mathematicians. Interestingly, the mathematician Gian-Carlo Rota (1997) has made a similar move to that of Dewey's in claiming that the notion of mathematical beauty or elegance is nothing more but a safe—and seemingly objective—way for mathematicians to communicate about their own emotionally charged experiences:

> Mathematical beauty is the expression mathematicians have invented in order to obliquely admit the phenomenon of enlightenment while avoiding acknowledgement of the fuzziness of this phenomenon […] (pp. 132–133).

The psychologist Csikszentmihalyi (1990) has also focused on qualities of human experience, and proposes the concept of "flow" to designate optimal experiences that are characterised by states of engagement, satisfaction, and goal-directedness, among others. There are some similarities to Dewey's notion of an aesthetic experience, even though Csikszentmihalyi does not talk explicitly about aesthetics. However, a central difference lies in Dewey's insistence on the location of aesthetic experience within the logic of inquiry, thus, integrating cognition with the emotional aspects of experience that "flow" describes.

In terms of mathematics education research, the notion of an aesthetic experience has rarely been used, though similar ideas have been expressed in different ways. For example, in relating the cognitive and motivational dimension of learning mathematics, von Glasersfeld (1985) writes "if students are to taste something of the mathematician's satisfaction in doing mathematics, they cannot be expected to find it in whatever rewards they might be given for their performances but only through becoming aware of the neatness of fit they have achieved in their own conceptual construction" (pp. 16–17). The notion of "neatness of fit" aligns closely with Dewey's qualitative unity; also, in speaking of satisfaction, and in integrating the cognitive and affective, von Glasersfeld describes something very close to Dewey's notion of aesthetic experience.

Neither Dewey nor von Glasersfled offer useful ways of describing what such experiences might look like for mathematical learners. This motivated Sinclair's (2002) work, which applied Beardsley's (1982) list of the defining features of aesthetic experience to the domain of mathematics. By analysing an example of mathematical problem solving, in terms of Beardsley's list (object directedness, felt freedom, detached affect, active discovery and wholeness), she finds that the features apply unevenly to the context of mathematics. However, the feature of object directedness seemed to be necessary to the aesthetic experience, and act as a precursor to active discovery and felt freedom. This feature refers to "a feeling that things are working or have worked themselves out fittingly" (p. 288) as one is fixed on the qualities or relations of a phenomenon. This finding might be useful in guiding the design of situations that can lead to aesthetic experiences.

While Dewey's notion of aesthetic experience might be challenging to operationalise in the mathematics learning context, it proposes two powerful, and distinct, commitments that have still not found sufficient expression in mathematics education research: (1) the refusal to separate emotion from cognition within the process of inquiry, (2) the view of the aesthetic as a continuous, unifying quality that underlies experience—not as a separate mode of judgment exercised after inquiry is complete.

Art as Experience was Dewey's final book. However, it has been argued that his intention was to publish a follow-up in which he linked his philosophical idea about aesthetics, experience and inquiry with his influential work on what constituted *educative experiences* (see Dewey, 1938).

Certainly, Dewey would have argued that aesthetic experiences are ones that promote growth—this goal being central to his conception of the goals of education. Jackson (1998) uses Dewey's work to argue for the increased role of arts learning in schools, drawing on Dewey's privileging of artistic experiences as highest expression of the aesthetic dimension of human experience. However, as Dewey himself argued, the arts are not the only enterprise in which aesthetic experience arise. This interpretation of aesthetic experiences in an educational context follows from a long-standing tradition in schooling in which the burden of a child's aesthetic development falls on the art or the music teacher, whereas the burden of that child's logical development gets conferred to the mathematics teacher. As I will argue later, this positioning of aesthetic development in the curriculum represents a common discourse that contributes to the relatively marginal role of aesthetics in mathematics education.

3.2 The aesthetic as a consequence of embodied cognition

More recently, Mark Johnson (2007) uses some of the Dewey's ideas around the notion of aesthetic experience, but adapts them more specifically to contemporary research in embodied cognition. Instead of using experience as the primary locus of the aesthetic, Johnson argues that human meaning-making itself is fundamentally aesthetic, using the word aesthetic now to describe all our physical encounters with the world. Johnson sees human understanding, including images, emotions and metaphors, as rooted in these bodily encounters. To make his argument, Johnson links recent theories of embodied cognition—namely, that even our most abstract concepts are rooted in our sensorimotor experiences—to the notion of the aesthetic as sensuous perception (or, as Kant defined it, as the science that treats the conditions of sense perception). In other words, since what we know is derived from our senses, then our cognitive capacities cannot be separated from our aesthetic ones—even though we may no longer be consciously aware of our underlying body-based conceptual foundations.

Like Dewey, Johnson also sees the aesthetic as being deeply intertwined with other human capacities such as affect and cognition. However, Johnson stresses the way in which all the things that "go into meaning—form, expression, communication, qualities, emotion, feeling, value, purpose" (p. 212) are also rooted in bodily perceptions. Also like Dewey, Johnson sees the arts as the culmination of the aesthetic dimension of human experience, and thus proposes to study the arts as a way to locate the bodily sources of meaning. However, Johnson wants to use human artistic expression as an opportunity to probe human understanding: "[a]esthetics is not just art theory, but rather should be regarded broadly as the study of how humans make and experience meaning" (p. 209).

Johnson's conception of the aesthetic stretches very broadly (perhaps too broadly, in the sense that one could infer that all cognition is aesthetic). He seeks to replace the traditional focus of philosophy on language with a focus on the body as the bearer of human meaning. As a result, fully acknowledging the aesthetic involves going beyond linguistic meanings, and accepting embodied meanings as well—and not just in the arts, but also in other disciplines. Johnson's argument is that embodied meanings of art—such as the rhythms of poetry or the textures of paintings—are body-based meanings that underlie more abstract understandings as well. Trying to identify these meanings, and even describing them, presents new challenges to researchers attempting to locate the aesthetic underpinnings of students' mathematical understandings. As I discuss later, some mathematics educators are already doing this, though without using the construct of aesthetics explicitly.

3.3 The aesthetic as a dimension of inquiry

The art historian E. Gombrich (1979), in his study of decorative arts around the world, emphasises the human need to find some kind of order or pattern in the flux of experience. He calls this a drive for "a sense of order". Humans are thus biased in their perception for straight lines, circles, and similarly ordered configurations rather than with the random shapes encountered in the chaotic world. Gombrich emphasizes that the order hypothesis is the condition that makes learning possible, since without some initial system, a first guess, no "sense" could be made of the millions of ambiguous stimuli incoming from the environment.

More recent research by cognitive scientists has also posited a mechanism through which humans look for order and pattern. E.O.Wilson (1998) argues that humans have predictable, innate aesthetic preferences they use in making sense of their environments. He notes that basic functioning in the environment depends on discerning patterns, such as the spatial patterns involved in perceiving surfaces and objects, and the rhythmic patterns involved in detecting temporal change. The continued and improving ability to discern such patterns gives rise to what Wilson calls "epigenetic rules," that is, to inherited regularities of development in anatomy, physiology, cognition and behaviour. He argues that such rules account for many predispositions and preferences. For instance, studies in human facial recognition show that humans are particularly sensitive to looking for right/left symmetry (as opposed to looking for up/down symmetry or not attending to symmetry at all). Finding such symmetry provides the simplest (shortest) descriptions of faces, and even of bodies—and thus makes such stimuli easier to encode and recall.

Wilson provides a concrete example of a universally shared aesthetic preference. He describes a study tracing arousal response to a variety of visual images in which the most arousing are those that cognitive psychologists call "optimally complex". Although researchers have a method for quantifying complexity, a qualitative description will suffice here: "optimally complex" designs are those that contain enough complexity to engage the mind but that do not overwhelm it with incomprehensible irregularity or diversity. If too many variations or distortions are made, such that little or no redundancy and repetition can be detected, the design moves from too simple to too complex to provoke arousal. However, if the stimulus is just complex enough, the perceiver is most aroused since, as Gombrich explains, "delight lies somewhere between boredom and confusion" (p. 9).

Dewey and Peirce both offered more philosophical perspectives of the role of the aesthetic in inquiry. Both saw the aesthetic playing a crucial role at the initial stage of inquiry, and as providing the guiding impetus for understanding and solving problems. Both also saw the aesthetic as being imaginative, intuitive and non-propositional. For Peirce (1908/1960), aesthetic responses feature strongly in the free exploration of ideas that gives rise to abductions, which were the only kind of inference to produce new ideas in scientific inquiry. Anticipating Dewey's account of the architecture of inquiry, he elaborates that inquiry begins with "some surprising phenomenon, some experience which either disappoints an expectation, or breaks in upon some habit of expectation of the *inquisiturus*" (6.469).

Dewey's (1938) logic of inquiry offers a similar, but more compelling account of the fundamentally aesthetic nature of inquiry. He claims that there is an aesthetic quality that belongs to any inquiry, be it scientific or artistic: "The most elaborate philosophical or scientific inquiry and the most ambitious industrial or political enterprise has, when its different ingredients constitute an integral experience, esthetic quality" (p. 55). What is this aesthetic quality? Dewey maintains that it relates to the human's inevitable tendency to arrange events and objects with reference to the demands of complete and unified perception.[2] For Dewey, inquiry also starts with surprise, or the feeling of something being problematic. He maintains that a problem must be "felt" before it can be stated; the problematic quality is felt or "had" rather than thought. It cannot be expressed in words. An inquirer is aware of quality not by itself but as the background, thread, and the directive clue in which she acts. Dewey suggests that the types of exclamations and interjections such as "Oh!" "Yes," or "Alas" that open most every scientific investigation supply perhaps the simplest examples of qualitative thought.

For both Dewey and Peirce, the aesthetic of inquiry is linked to the non-propositional, qualitative and felt experience of a situation, which provides the basis for further distinction, conceptualization, and articulation. Dewey saw the aesthetic quality as pervading the whole process of inquiry, and providing the basis for the evaluative judgement made by the inquirer at its close.

Despite the fact that both philosophers believed that the process of inquiry could be studied empirically, and was not simply a succession of mental states that were somehow unobservable or transcendental (as Poincaré might have argued), it has been challenging for researchers to operationalise concepts such as qualitative unity, or even abduction. The most important consequence of their theories has been to underline the important role that the initial stage of inquiry plays, in either providing new ideas or formulating a persistent quality.

In mathematics, there have been few studies of the process of inquiry, with the book *Thinking Mathematical* by Mason, Burton and Stacey (1982) being a notable exception. These authors dwell on the initial part of inquiry, and on the qualitative responses it will give rise to, as exemplified by the importance they accord to "recognizing and harnessing to your advantage the feelings and psychological states that accompany [mathematical enquiry]". Interviews with mathematicians, as well as their autobiographies, have confirmed the way in which qualitative responses, and the exploitation of feelings contribute significantly to the posing and solving of problem (see Albers, Alexanderson & Reid 1990; Davis, 1997; Hofstadter, 1997; Sinclair, 2002; Weil, 1992).

Dewey's and Peirce's ideas have not generated much interest in the mathematics education community. One may hypothesise that aesthetic engagement in mathematics may be dependent on opportunities to engage in the full process of inquiry, which includes exploring situations without specific goals in mind and posing problems that arise out of this exploration (see Hawkins, 2000). Under this assumption, it is not that students are incapable of aesthetic engagement in mathematics, but, rather, that school mathematics offers few opportunities for the kind of mathematical inquiry described by Dewey and Peirce. While some may argue that engaging in mathematical inquiry would be the essence of 'acting like a

[2] Langer (1957) emphasizes this fact by describing how the merest sense-experience is a process of formulation; human beings have a tendency to organise the sensory field into groups and patterns of sense-data, to perceive forms rather than a flux of light-impressions. They promptly and unconsciously "abstract a form from each sensory experience, and use this form to conceive the experience as a whole, as a thing" p. 90. For Langer, this unconscious appreciation of forms is the primitive root of all abstraction, which in turn is the keynote of rationality; so it appears that the conditions for rationality lie deep in pure animal experience—in the human power of perceiving, in the elementary functions of eyes and ears and fingers.

mathematician,' and therefore strongly desirable, Dewey might instead argue that manufacturing situations in which learners can have aesthetic experiences, the most valuable and satisfying kind of experience possible, should be the driving goal of mathematics education.

In contrast to the focus on inquiry, Gombrich and Wilson both point to possibilities of aesthetic engagement in the more common activities of the mathematical classroom such as solving problems or making sense of ideas. Interestingly, despite the previous emphasis on the subjective, contextual nature of aesthetic judgements, their work may support the conjecture that human beings share many significant penchants and preferences. For example, they often seem to organise their perceptions around symmetry or balance, either because it reduces complexity, as White would argue, or because of its connection to our own bodily symmetries, as Johnson would argue. However, while symmetry acts as an organising principle, Gombrich draws attention to a range of preferences humans have expressed around symmetry: while western decorative art tends to value the presence of symmetric configurations, much of the decorative arts of the east prefer breaking symmetry.[3] Similarly, perceptions of confusion and boredom are highly personal and contextual.

3.4 The aesthetic as an evolutionary imperative

The anthropologist Ellen Dissanakye (1992) takes a unique approach to conceptualising the aesthetic in her book *Homo Aestheticus: Where Art Comes From and Why*. Instead of rooting the aesthetic in the human body and the sensory organs, Dissanayake links the aesthetic to more evolutionary concerns. She is concerned with understanding why people everywhere, in different cultures and historical time periods, spend so much time decorating and adorning themselves and their surroundings. The amount of time spent on these activities seems to contradict evolutionary assumptions about survival—no tattoed arm, elaborate dance ritual, or decorated door mat can answer the need for food and shelter. Dissanayake thus sees these aesthetic productions as ways of "making special". The human aesthetic capacity—which she sees as being on par with other capacities that such as the emotional, the cognitive and the practical—is nothing more than the need to identify things in the flow of experience as worthy of attention and embellishment.

Dissanayake's approach offers some rather different insights for mathematics education, focusing as it does on the need to highlight and embellish as a means of avoiding either monotony or chaos. There are several ways to see how this kind of need plays out in the mathematics classroom—not all of them conceptually relevant!—whether it's doodling in the notebook to break the monotony of a lecture or seeking repeatable rules to overcome the perceived chaos of algebraic manipulation.

Pinker (1997), another scholar interested in the aesthetic dimension of human behaviour, explains how human emotions become so deeply implicated in aesthetic responses. He focuses on the adaptive responses of human beings to selective pressures in an evolutionary context and, in particular, on responses to the set of "enabling acts" which increase their ability to survive within environmental and social constraints. Some subconscious part of the mind, he argues, registers those highly enabling acts—such as using symmetry to perceive and gather information on family members or hunted animals—through a sensation of pleasure. This pleasure in turn alerts us to, or brings to our consciousness, the advantages of such acts. Enabling acts occur through obtaining information about the improbable, information-rich, consequential objects and forces that dominate everyday lives. Whereas these may have once been acts of predicting rains, fertile hunting grounds, or generosity in other humans, modern humans face very different situations. Nevertheless, Pinker argues that the pleasure-alerting mechanisms function in the same way. When confronted by information-rich and potentially consequential stimuli—the ominous foreign subway map separating me from my hotel, for example—I derive pleasure from being able to discern its underlying pattern.

In contrast with Wilson and Gombrich, who are concerned with the causal dimension of human behaviour, Pinker and Dissanayake examine the consequential dimension of human behaviour, namely, the way in which "making special" or "registering enabling acts" forms the basis for aesthetic sensibility (similar to the way in which Johnson, in his focus on embodiment, attempts to define the basis for aesthetic sensibility). While their approach may help provide persuasive arguments about the centrality and importance of the aesthetic in human thinking and behaviour, it also offers different interpretations of the aesthetic that broaden its relevance to learning, and to mathematics education.

From a theoretical point of view, the work of Pinker suggests a strong connection between affect and aesthetics, and, in particular, the possibility of identifying "enabling acts" through the cue of pleasure responses. More empirically, instead of investigating what students find beautiful (or not), researchers might study the range of "enabling acts" that can occur in problem solving. What sets of actions or transformations can give rise to the kind of

[3] The physicist Freeman Dyson (1982) also distinguishes between two types of scientists, namely, the 'unifiers' and the 'diversifiers,' the former finding and cherishing symmetries, the latter enjoying the breaking of them.

pleasure response described by Pinker? To what extent are these acts shared across different learners? From a pedagogical level, it reinforces Brown's critique about "false aesthetic unity" of the mathematics classroom, in which things always work out, in whole numbers, or in orderly patterns. One of the many reasons for the negative affect that is so preponderant amongst students may be linked to the lack of opportunities they have to register such enabling acts. In this respect, Pinker's work links strongly with issues of motivation in mathematics education.

Dissanayake's work leads in a different direction, in that her conception of aesthetics does not have the same close links to cognition. However, it does emphasise the interplay between the aesthetic and the affective in emphasizing the satisfaction that comes from the successful manifestation of the basic "making special". What might constitute an act of making special in mathematics, or in the mathematical classroom? Might researchers be able to use the construct of "making special" in order to assess the engagement of the aesthetic? In the discipline itself, are there characteristic ways in which mathematicians "make special" and are these relevant to mathematics learning?

4 What the aesthetic can mean in mathematics education

Stepping back now, it is clear that the conceptions of the aesthetic used by the scholars cited above vary quite widely. Dewey aims to use the aesthetic as a way of challenging classical distinctions between the cognitive, the affective and the artistic, and to locate the aesthetic as a theme in human experience. Johnson concerns himself with the link between embodied cognition and the aesthetic, also attempting to challenge traditional conceptions of cognition as disembodied and emotion-free. Dissanayake and Pinker want to understand why people engage in activities. Gombrich and White are interested in empirically-derived tendencies in human perception.

In terms of mathematics education, all points of view insist on conceiving the learner as being in possession of aesthetic sensibilities and values needing to be exercised. This is radically divergent from current trends in mathematics education research, which most often ignore the aesthetic dimension of mathematics teaching and learning. In the following sections, I explore some of the reasons for the prevailing gulf between mathematics education research and current theories related to the aesthetic. In particular, I consider the following three factors: (1) the cognitivist orientation to research in mathematics education; (2) the lack of connection between the current theories of aesthetics and the fundamentally social nature of the mathematics classroom; (3) the power dynamics around mathematics education and the accompanying elitist views of the mathematical aesthetic.

4.1 Integrating the cognitive, affective and aesthetic

A major theme of the scholarship discussed above involves the extremely close connection between cognition, affect and aesthetics. Some see the aesthetic as the unifying principle of meaning and experience (see also Schiralli, 2006), but all agree that the human aesthetic plays a role in learning about the world and is intimately related to pleasure and satisfaction. Despite this, the predominantly cognitive approaches in mathematics education acknowledge the existence of affect and aesthetics, but ascribe them both a rather epiphenomenal role in cognitive processing. Moreover, these approaches do not generally take into consideration the cultural and historical aspects of human meaning-making that I have argued are central to understanding the role of aesthetics in mathematical thinking.

Goldin's (2000) work on affective representational systems stands out from other approaches, be they cognitive, affective or sociocultural, in that he explicitly links affective and cognitive representations in his model of problem-solving competence. In particular, he analyses relationships between affective states and heuristic configurations and posits certain pathways through affective states that different problem solvers might take, and that might lead to different types of heuristics. For Goldin, affect comprises a tetrahedral construct which includes (1) beliefs, (2) attitudes, (3) emotional states and (4) values, ethics, and morals. As such, both aesthetics and ethics are subsumed within the affective domain.

The conflation of affect and aesthetics, which defies long-standing distinctions in philosophy, would also be refuted by each of the contemporary interpretations developed above. Most fervent opposition would come from the embodiment viewpoint, which might instead subsume affect under the aesthetic, given that feelings and emotions rely on sensory perception. From the evolutionary viewpoint, the aesthetic, as a form of "making special" or as expressed through enabling acts, involves an attention to values—to what is worthwhile in experience and action. From the inquiry viewpoint, the aesthetic functions as a non-logical form of knowing, which aligns itself much more with cognition (broadly viewed) than with affect. Dewey points to the way in which affective responses might alert the inquirer to the presence of certain perceptions and inferences, but those perceptions and inferences cannot be reduced to feelings, or even beliefs.

Drawing on the last perspective, I would also challenge Goldin's subsumation of the aesthetic. As Dewey would argue, the problem-solver becomes alert to aesthetic responses *through* affective states. Silver and Metzger

(1989), in their study of research mathematicians, also support this view: "decisions or evaluations based on aesthetic considerations are often made because the problem solver "feels" he or she should do so because he or she is satisfied or dissatisfied with a method or result" (p. 70). Positive or negative feelings can arise from a perception, or an awareness, of something being worthwhile, important or interesting. In other words, the aesthetic and the affective domains each *function* differently in the problem-solving process: the aesthetic draws the attention of the perceiver to a phenomenon (a pattern, a relationship, a contradiction), while the affective can bring these perceptions to the conscious attention of the perceiver.

From a pragmatic point of view then, in terms of describing and explaining mathematical problem solving, the aesthetic and the affective should retain conceptual distinctiveness, despite their obvious interconnections. Further, theories of affect in mathematics education cannot explain the derivation of aesthetic values, their propagation within different cultures, and their role in guiding the growth of the discipline. Aesthetic responses and values do not exist as biological configurations, which is how Goldin describes affective states. They are socially and historically evolved, contextualized by shared practices within a community, and they exert themselves by determining what should be considered worthwhile, important and useful.

The above discussion suggests some intermingling between aesthetics and affect (feelings arising in relation to perceptions of pleasure, beauty, worthiness, and so on), but the aesthetic can also be tightly coupled with the cognitive. At one level, and perhaps a rather cerebral one, we might talk about perceptions of simplicity, structure, conciseness and lucidity. However, Johnson's view of aesthetics offers a more visceral link. Drawing on Lakoff and Núñez (2000) *Where Mathematics Comes From,*[4] researchers have studied the way in which more body-based experiences can help support the development of abstract mathematical ideas. For example, the work of Radford (2003), which focuses on the *direct* connection between bodily movement and abstract mathematical conceptualisations, has a strong affinity with this aesthetic approach. In particular, Radford shows how the rhythmic utterances of students are used to construct meaning for algebraic patterns they are studying—the rhythm, and not the actual words, act as semiotic markers of generalisation. Recall that for Johnson, rhythm is an aesthetic form of meaning-making, so that the link to cognition occurs through the body.

In fact, several mathematics educators have become interested in the role of bodily actions and gestures in the elaboration of concepts (see Arzarello and Robutti, 2001; Nemirovsky, 2003; Núñez, 2004). This research, while acknowledging the role of body-based meanings, has tended to privilege meanings that are highly cognitive in nature. This stands to reason, given the ultimate interest mathematics educators have in coordinating body-based meanings with abstract mathematical symbolism. However, it does compromise both Johnson's and Dewey's more comprehensive approaches to aesthetics since it usually overlooks both affective and axiological dimensions of meaning.

It may well be close to impossible to coordinate the range of meanings arising from episodes of student mathematical work. Nonetheless, taking Radford's example above, it might be fruitful to examine how the perception and construction of rhythm also relates to affective meanings of comfort and security. Alternatively, might the perception and production of rhythm relate to a heightened sense of interest in, or perceived worthiness of, algebraic patterns? While these considerations may not be immediately germane to the cognitive concerns of researchers, they seem extremely relevant to understanding the full range of meanings that learners attach to mathematical ideas.

In sum, while categories such as cognition, affect and aesthetics provide useful and fruitful analytical tools when considered separately, they clearly intermingle in important ways in mathematical thinking and learning. However, even if assumptions about the primarily cognitive nature of mathematics were to be successfully challenged, the aesthetic dimension of human experience will always be more challenging to study, given their fuzzy, implicit, and ephemeral nature. An important first step would involve forging connections between aesthetics and existing theories in mathematics education. As I argue in the next section, this will require further consideration of the aesthetic dimension of mathematics enculturation.

4.2 Social considerations

A second reason for the lack of attention to aesthetics relates to considerations that are much more social in nature. While the contemporary perspectives described above draw attention to the importance of the aesthetic in human experience and perception, they focus almost exclusively on individual capacities and tendencies than on the way in which aesthetic values and sensibilities are developed, shared, communicated, and disputed in human

[4] In this book, Lakoff and Núñez offer a very stimulating perspective on the genesis of mathematical ideas, based on their theories of embodied cognition. While many scholars (including cognitive science, mathematicians and educators) have expressed reservations about their specific claims (see, for example, Schiralli and Sinclair, 2002), variations of the ideas expressed in this book have motivated many studies in mathematics education that are relevant to an embodied perspective on aesthetics.

interaction. This more social perspective cannot be ignored in mathematics education, where issues of communication and enculturation are central.

The issue of enculturation is especially interesting in mathematics, where the sharing of aesthetic values has traditionally been rather secretive—or at least implicit—and elitist: the general practice is to begin aesthetic enculturation at the PhD level, when students are, often for the first time, having to choose a novel dissertation-level problem. Further, unlike other aesthetically-driven disciplines such as the visual arts, literature, or music, mathematics has no practice of public criticism and thus no mechanism through which aesthetic values might be articulated, defended, or socially mediated. While some philosophers have pointed to the dangers of this for the discipline itself (see Corry, 2001; Csiszar, 2003; Tymoczko 1993), the repercussions for mathematics education may be even worse because they lead to the belief that aesthetic values are either intrinsic to mathematics, or to the mathematicians who control them (see Sinclair and Pimm, forthcoming).

Bishop (1991) argues that mathematics education should go beyond developing students' conceptual understanding, and should include the teaching of the history and values of the discipline. He links the educational importance of making these values explicit in the classroom to improving the affective environment of the classroom. However, these values also play an important part in determining what mathematicians count as important or interesting in mathematics—these questions being aesthetic in nature (see Sinclair, 2006a). The impact of values extends beyond the emotional, and to the broader activities of mathematics such as inquiry and communication.

The notion of mathematical enculturation, which involves immersion in and reflection on the values of the mathematics culture, offers a more socially oriented opportunity for aesthetics in mathematics education. In particular, it suggests that aesthetic values should be explained and shared at the classroom level, and that the process of doing so may require longer periods of discussion and negotiation. Note that Bishop's perspective of mathematics enculturation is a critical one in that it is meant to expose students to the underlying values that not only drive the conceptual development of the discipline, but that also may interact with other social goals and discourses.

By taking the mathematics classroom as the unit of analysis—rather than the individual learner—researchers such as Yackel and Cobb (1996) have been able to study the way in which various normative values become established in a classroom. They have been particularly interested in mathematical norms that involve decisions about what counts as different when students discuss and offer solutions to problems (although they also acknowledge other mathematical values such as efficiency, elegance and sophistication). They show that it takes time, as well as strong guidance from the teacher, in order for students to understand what it means to be *mathematically* different. In the same way, the normative understandings of mathematical efficiency, elegance, sophistication, and other aesthetic values, would require a certain period of enculturation.

Also taking a classroom-based unit of analysis, Sinclair (2008) studies the ways in which aesthetic values are being communicated in a classroom where the teacher is not necessarily focussed on aesthetic enculturation. The research was based on the assumption that such values would be at least implicitly communicated, as hypothesised by the contemporary research described above. As predicted, while the teacher hardly ever used words such as "beauty" or "elegance" in reference to mathematical ideas, he did appeal to aesthetic values quite frequently, in terms of drawing students' attention to what counts as interesting in mathematics, or what kinds of things mathematicians like to do. For example, in presenting different ways to solve an algebraic equation involving fractions, the teacher talked about how they could "defeat the algebra beast" and turn the fractions into whole numbers. On one hand, the teacher is communicating the fact that techniques that can reduce complexity or "beastliness", are valued in mathematics. On the other, the teacher offers an image of some of the negative aesthetic responses that can be had in mathematics, namely, that fractions are ugly and beastly.

Sinclair also finds a range of responses from the students to the teacher's appeal to aesthetic values. Unless the teacher was able to elicit emotional responses from the students, his appeals to the aesthetic seemed to go unnoticed by the students, and were even interpreted as further supporting the anaesthetic vision of mathematics held by many people. This analysis thus provides some insight into the way in which aesthetic values are negotiated at the classroom level, and how easily unquestioned values from the teacher's perspective might override the aesthetic sensibilities and capacities of learners. Further research might focus more explicitly on students' interpretations of the aesthetic values evinced by teachers (as well as by textbooks and other materials.

4.3 Critical perspectives: power and aesthetics

If one adopts the viewpoint of mathematicians such as Krull and Poincaré, namely, that the discipline of mathematics is fundamentally and crucially an aesthetic enterprise, then one must concede that most learners do not currently have the opportunity to do mathematics. The current elitist (or frivolous) positioning of the aesthetic in mathematics education has important repercussions when it comes to access and power. In fact, Sinclair and Pimm, (forthcoming) propose that the recent emphasis on "mathematics for all" may in fact be compromised by this very positioning.

In discussing the role of power in mathematics education, Valero (2005) points out that power cannot lie in the discipline of mathematics itself, nor in its practitioners, but must, instead, be seen as "a relational capacity of social actors to position themselves in different situations, though the use of various discourses" (p. 10). In terms of aesthetic considerations then, one cannot blame mathematics itself for its inaccessibility: mathematical objects are not beautiful in and of themselves, and they cannot transfer their beauty to potential learners. Nor can one blame mathematicians, even the ivory-tower, eminent mathematicians who are often seen as controlling the exchange of power. Instead, Valero proposes that power transactions evolve out of ever-changing and often subtle practices and discourses.

What might these subtle practices and discourses, which are disempowering learners in their aesthetic engagement with mathematics, look like? As I will show below, they are wide-ranging and surprisingly disparate. For example, consider the current school practice of assigning the aesthetic development of students to arts education—a practice that has developed over many centuries, and differs from schooling practices of the ancient Greeks. The allotment of aesthetic development to the arts makes it acceptable and reasonable to ignore aesthetic development in mathematics (and, of course, art teachers are not supposed to concern themselves with the logical development of their students). This practice seems deeply entrenched (as does the discourse around the purposes of mathematics and arts education more generally), and influential not only in mathematics, but also across the whole schooling system.

A more localised and recent example of the kind of practice that relates to power issues around the mathematical aesthetic involves the decline of geometry in the mathematics curriculum (and the corresponding ascendance of "numeracy," and the attention to fractions and algebra) over the past half-century. Limited experiences with the mathematics of shape and space reduces the range of sensory-based interactions that learners have with mathematical ideas, representations and phenomena, and thus limits learners' aesthetic engagement. This particular practice grows out of a more general turn toward the numeric, the analytic and the algebraic, or the tendency to talk in numbers about almost anything, and the underlying desire for generality, rigor and objectivity (see Sfard, 2008, for a discussion of how these desired properties are actually misleading). The privileging of number not only pushes out the spatial, visual and continuous, thus compromising embodied meanings, it also leads to a certain discourse about what counts are more valuable knowledge, thus affecting judgements about what is interesting, worthy, or even true.

Indeed, there are a wide range of discourses and practices that contribute to current power dynamics underlying the elitist view of the mathematical aesthetic, but I would like to highlight one in particular that is especially germane given its direct relation to aesthetic considerations. It involves the positioning of mathematics (especially by mathematicians) as an artistic discipline rather than a scientific one. For example, the mathematician Sullivan (1925/1956) claims that mathematicians are impelled by the same incentives as artists, citing as evidence the fact that the "literature of mathematics is full of aesthetic terms" and that many mathematicians are "less interested in results than in the beauty of the methods" (p. 2020) by which those results are found. There is also the argument that, unlike with the sciences, mathematics does not have to compare itself against an outside reality—thus, the implication being mathematicians have choice and freedom when it comes to selecting their objects of interest. Sullivan described mathematics as the product of a free, creative imagination and argued that it is just as "subjective" as the other arts. Related to this creative aspect, the mathematician G.H. Hardy also viewed mathematics (the kind he liked anyway) as an art: "I am interested in mathematics only as a creative art" (1967/1999, p. 115).

While one might expect this comparison of mathematics to the arts to enhance its accessibility, I propose that is actually has the opposite effect. The characteristics that mathematics supposedly share with the arts—creativity and free choice, as well as the use of "aesthetic terms"—may sound alluring to non-mathematician, who can recognise them as familiar in other (less exclusive) experiences. Tell a mathematics-fearing artist that the discipline is really about ambiguity (see Byers, 2007), creativity and freedom, and their ears will likely perk up. However, these very characteristics only serve to remove the accessibility of mathematics from the non-mathematician further since, like the aesthetic sensibilities of Poincaré, they only belong to a privileged few.

It may be more fruitful to consider the differences between mathematics and the arts in understanding the power dynamics involved. Indeed, the philosopher Thomas Tymoczko (1993) may well have pointed out the most operative difference between aesthetic judgments in mathematics and those at work in the arts: the mathematics community does not have many "mathematics critics" to parallel the strong role played by art critics in appreciating, interpreting and arguing about the aesthetic merit of artistic products.[5] Lakatos also alludes to this in his *Proofs and Refutations*, when Gamma, exasperated by the never-ending complexities added to a simple equation in order to deal with "monsters", asks "Why not have mathematical critics just as you have literary critics, to develop mathematical taste by public criticism?" (Lakatos, 1976, p. 98). Gamma realises that truth cannot operate separately from taste when it comes to mathematical discovery: not every fact is worth proving.

Mathematics may well be a discipline of freedom and creativity, but in other disciplines that are driven by aesthetics, there are critics to interpret and negotiate the meaning and place of creative new products. In mathematics, however, virtually no one stands on that border between the productive and interpretive aspects of creative work for mathematics (see, for instance, Corfield, 2002, on Lakatos's legacy in this regard). This is not just a problem for non-mathematicians, who have little help in assessing the importance of new developments in mathematics; it has been problematic within mathematics itself.

Rota's claim that mathematicians use language full of aesthetic terms to hide the fuzziness of their experiences (and perhaps of their truths) deserves further consideration. This will likely require the introduction of new discourses, ones like Thurston (1994) offers, gently expose rather than hide fuzziness.

5 Some final remarks

The philosophical tides are changing, as scholars become increasingly interested in the axiological dimension of philosophy and, in addition, in articulating the more porous borders between knowledge, feelings and values. The question for mathematics education research is whether these new directions in philosophy help solve any of the perennial problems of mathematics education. The most obvious relevance of aesthetic considerations in mathematics education research relates to student motivation, which persists as one of the greatest problems faced in mathematics education. Many researchers have proposed ways in which to address this problem, ranging from a focus on providing students with a better rationale for why they should study mathematics to finding ways in which to promote students' confidence. By in large, these proposals minimise aesthetics, and in cases where they do not, non-mathematical ideas or activities are frequently used to provide aesthetic values. Further research on the role that aesthetics can play in student motivation deserves urgent attention. This may involve investigating the extent to which students' aesthetic engagement leads to increased interest or intrinsic motivation. The notion of aesthetic engagement may vary depending on which of the contemporary approaches is adopted: from an embodied cognition point of view, for example, researchers might determine whether embodied experiences with mathematical ideas motivate students. In contrast, from a Deweyian perspective, promoting students' aesthetic engagement would involve a very different orientation toward the goals and purposes of mathematics education.

Using the embodied perspective described above, aesthetic considerations in research may also help solve more specific problems such as understanding how students can learn fractions better. Such problems have been studied through mostly cognitive approaches, and they may never be resolved without a broader concept of human understanding to guide the research questions and methods. The fact that students tend to want to avoid fractions probably has some non-cognitive origins, if we accept the theories outlined above. How does the belief that "fractions are hard" relate to the strange way in which they are written, the lack of embodied experiences they give rise to, or the lack of opportunities students have to encounter them in the context of satisfying experiences?

In addition to addressing recognised problems in mathematics education, the attention to aesthetic considerations in research—especially in terms of power dynamics—may also have more transformative influence, helping to suggest new possibilities, draw attention to problems that have not been recognised, or whose solutions have been taken for granted. In particular, it may suggest new ways of positioning school mathematics with respect to research mathematics—not as a mere servant, or as a separate discipline, but as an explicator, mediator and critic.

[5] It might be argued that journal editors play the role of the art critic in mathematics, but their work is done within a small community of mathematicians, rather than being available or addressed to those outside that community. Textbook authors also play a role similar to that of the art critic, in that they seek to organize, explain and even interpret mathematical products for an outside audience; however, they rarely actually criticize or question these products.

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