

# A contemporary analysis of the six “Theories of Mathematics Education” theses of Hans-Georg Steiner

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**Abstract** In this contribution we discuss the six theses presented by Hans-Georg Steiner (1987), which were instrumental in the community becoming interested in theories and philosophies of mathematics education. We discuss overlooked aspects of this seminal paper particularly in light of recent developments in the field of mathematics education. Nearly 20 years later, we reflect on the development of Steiner’s program for theory development and examine if any progress has been made at all on the open questions that Steiner (1987) posed to the community.

## 1 Introduction

Philosophy has been both an outstanding and an integral part of mathematics for a long time and was regarded as a sub-discipline. It was also implicitly accepted that a person’s philosophical positions can influence his or her view on mathematics and its teaching. In 1973, the famous mathematician Thom

(1973) proclaimed at the Second International Congress on Mathematical Education that “whether one wishes it or not, all mathematical pedagogy even if scarcely coherent, rests on a philosophy of mathematics”. (p. 204) Simply put, philosophy of mathematics is the important framework for teaching (and learning) mathematics.

It was Hans-Georg Steiner who became aware of the importance of this correlation at a time when the mathematics education community was not paying explicit attention to this line of philosophical enquiry. His work was deeply rooted in the development of IDM and its aiming at theoretical foundations of didactics of mathematics (Steiner 1975). Others engaged in this line of enquiry were Confrey (1980), Dawson (1969), Lerman (1983), Nickson (1981) and Rogers (1978). The often used and cited example is Paul Ernest’s exemplary formulation of research questions regarding the impact on the teaching of mathematics in 1989. In addition to these other important papers, the Steiner-paper (1987) titled *Philosophical and epistemological aspects of mathematics and their interaction with theory and practice in mathematics education* can be regarded as one of the key papers for the development of theories on mathematics learning and—as well as what is called today—beliefs theory. Thus, nearly 20 years later, it is time to reflect on the development of the Steiner program 1987 and examine if any progress has been made at all on the open questions that Steiner (1987) posed to the community.

The development of mathematics didactics in Germany is in a sense inseparably connected to the name of Hans-Georg Steiner. The question is what among his many contributions should be most

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appreciated by the community. The authors of this article have chosen to focus on an overlooked and visionary contribution to the field, namely his 1987 paper which was based on earlier work. Steiner's approaches to theories of mathematics education were concerned both with the philosophy of mathematics, the processes surrounding its instruction, and the need to internationalize the basic problems of the field. And so we would like to appreciate this exemplary individual in the context of this particular FLM article, whose original drafts date back to 1984. As stated earlier, many of today's theories which address beliefs, psychological and epistemological issues can be viewed in light of the theses presented in his article.

## 2 Background of the 1987 article

The 1987 article is based on lectures, which Steiner held at the 5th ICME in Adelaide in 1984, PME 8 in Sydney, and at the University of Georgia in 1985. On the basis of these lectures, Steiner initiated the formation of TME (Theory of Mathematics Education Group), which was concerned with basic problems confronting the field, namely to systematize the foundations, theories and methodologies for mathematics education research, development and practice (see Steiner 1985, 1992; Steiner et al. 1984).

If one carefully examines Steiner's articles which led up to this particular 1987 publication, the work comes across as an axiomatic approach to delineating the foundational problems of the field. We believe this approach might probably be shaped predominantly by Steiner's career trajectory with formative experiences in the 1960s, at which time axiomatic drafts were en vogue (see e.g. Steiner 1966a, b, 1974). A starting point was also the book "*Mathematiker über die Mathematik*" edited by Otte (1974), where the paper of René Thom was published in German! Our reference to the axiomatic approach may be termed misleading by senior scholars who are also familiar with Steiner's massive works in German. On the one hand that may mean that Steiner was aiming at a quasi-axiomatic philosophy of mathematics underestimating the need of empirical studies of beliefs etc, which is true for a certain period. But this is not to be confused with his earlier attempts to make axiomatics accessible at school level. The 1966a publication rather points to another source of Steiner's interest in philosophy. During the new math reform Steiner tried to construct a new math philosophy including mathematics as activity (as mathematizing as axiomatizing). The philosophical shortcomings in the new math reform let him later consider the need for

a much broader philosophy of mathematics including human and social aspects.

Steiner essentially dedicated himself to the development of a meta-theory which took into consideration the philosophical and epistemological aspects of mathematics education. We pose the question as to why epistemological and philosophical considerations are important for theories of mathematics education? Epistemology is a branch of philosophy concerned with the nature of knowledge and justification of belief. Sierpinska and Lerman (1996) state:

Epistemology as a branch of philosophy concerned with scientific knowledge poses fundamental questions such as: 'What are the origins of scientific knowledge?' (Empirical? Rational?); 'What are the criteria of validity of scientific knowledge?' (Able to predict actual events? Logical consistency?); 'What is the character of the process of development of scientific knowledge?' (Accumulation and continuity? Periods of normal science, scientific revolutions and discontinuity? Shifts and refinement in scientific programs?). (p. 828)

The question: What is mathematics? for teaching and learning considerations brings into relevance the need to develop a philosophy of mathematics compatible with mathematics education. In order to answer this question for mathematics education, we think it is necessary to examine the writings of mathematicians who are sympathetic to the issues of teaching and learning mathematics, as opposed to simply doing original research. Hersh (1979) defined the "philosophy of mathematics" accordingly to the working philosophy of the professional mathematician, a philosophical attitude to his work that is assumed by the researcher, teacher, or user of mathematics and especially the central issue—the analysis of truth and meaning in mathematical discourse. Much later, Hersh (1991), wrote

Compared to "backstage" mathematics, "front" mathematics is formal, precise, ordered and abstract. It is separated clearly into definitions, theorems, and remarks. To every question there is an answer or at least, a conspicuous label: "open question". The goal is stated at the beginning of each chapter, and attained at the end. Compared to "front" mathematics, mathematics "in back is fragmentary, informal, intuitive, tentative. We try this or that, we say "maybe" or "it looks like". (p. 128)

So, it seems to us that Hersh is not concerned with dry ontological statements about the nature of mathematics and mathematical objects, but is more concerned

with the methodology of doing mathematics, which makes it a human activity.

### 3 The six theses

Given this background, we are now ready to examine the six theses presented by Steiner and our interpretation of the basic message in each of them.

Thesis 1: *Generally speaking, all more or less elaborated conceptions, epistemologies, methodologies, philosophies of mathematics (in the large or in part) contain—often in an implicit way—ideas, orientations or germs for theories on the teaching and learning of mathematics.*

Message 1: Epistemology/philosophy have implicit didactic assumptions or implications.

Thesis 2: *Concepts for the teaching and learning of mathematics—more specifically: goals and objectives (taxonomies), syllabi, textbooks, curricula, teaching methodologies, didactical principles, learning theories, mathematics education research designs (models, paradigms, theories, etc.), but likewise teachers’ conceptions of mathematics and mathematics teaching as well as students’ perceptions of mathematics carry with them or even rest upon (often in an implicit way) particular philosophical and epistemological views of mathematics.*

Message 2: Non-philosophy is also a philosophical position.

**Corollary 1** *There is no neutrality in ones philosophical and epistemological views about mathematics.*

We refer to Davis (1972); we also assume that Steiner considered at that time the current publications of Davis and Hersh, e.g. see Davis and Hersh (1986). The papers of Hersh (1979) and Davis and Hersh (1980) also belong to this tradition as well as representing a spirited reaction of working mathematicians to various pompous aberrations of the New Math. Lastly, the book by Hersh (1997), *What is Mathematics, really?* constitutes a plea for a deliberate epistemological positioning, with a particular view towards the importance of epistemological perspectives for teaching and learning. This message is best illustrated by Hersh’s (1997) critique of a formalist view towards mathematics. He writes:

The devastating effect of formalism on teaching has been described by others. **I haven’t seen the effect of Platonism on teaching described in print.** But at a teachers’ meeting I heard this: “Teacher thinks s/he perceive other wordly mathematics. Students is convinced teacher really does perceive other wordly mathematics. No way does student believe he’s about to perceive other wordly mathematics (p. 238).

If we turn the message of thesis 2 positively, then one can refer to Schoenfeld (1998), and his theory of teaching-in-context, in which one interprets teacher behavior rationally based on a teacher’s dominant goals and beliefs. Much earlier, Fenstermacher (1978) predicted that the single most important construct in educational research, are beliefs (see also Pajares 1992). Around the time Steiner published his work, Thompson (1982, 1992) was laying the foundations of a theory to explain teacher’s actions in a mathematics classroom based on their beliefs about mathematics. What we are suggesting is that is an instance of the development of a local philosophy based on the particular problems in the domain of beliefs. Thompson (1992) wrote:

I think we will get further evidence on the role of teachers’ views of mathematics when we go into more detail and investigate their understanding of different domains of mathematics, of specific components such as the meaning of mathematical concepts, proof, definition, theorem, conjecture, variable, symbols, rule, formula, axiom, problem, problem solving, application, model, computation, graphical representation, visualization, metaphor, etc., both with respect to the various sub-domains of mathematics as well as in a more general sense. (p. 142)

Today we usually speak of teachers’ beliefs, which are generally formulated as “views about mathematics” (see e.g. Grigutsch 1996; Pajares 1992). It is assumed that different beliefs about mathematics have different associated philosophies and/or epistemologies. Törner (2002) explores this question somewhat more in detail, unaware of the work of Steiner (1987) and that of Aguirre (2006). In contrast, in the literature from psychology the focus is placed on very domain specific epistemological beliefs (Buehl et al. 2001; Paulsen and Wells 1998) generally ignoring the discussion in mathematics education.

Thesis 3: *There is no distinguished, constant, universal philosophy of mathematics. One should evaluate philosophies of mathematics according to their fruitfulness for particular goals and purposes and develop criteria for evaluation.*

Message 3: A relativistic stance towards a philosophy of mathematics (in the sense of Perry resp. Ernest) is necessary dependent on the particular problem context/framework being addressed. Another implication that Steiner places in this thesis is that research mathematics might be a different (higher) instance than that of school mathematics when he accepts to take particular goals and purposes into consideration.

Our subjective interpretation is that this creates a co-existence of belief systems with those held by mathematicians. Hersh (2006) grants or certifies schizophrenia to the mathematicians. Thus are mathematicians in terms of their philosophical orientation sometimes chameleons?

For instance Hersh (1997) writes:

The working mathematician is a Platonist on weekdays, a formalist on weekends. On weekdays, when doing mathematics, he's a Platonist, convinced he's dealing with an objective reality whose properties he's trying to determine. On weekends, if challenged to give a philosophical account of the reality, it's easiest to pretend he doesn't believe it. He plays formalist, and pretends mathematics is a meaningless game. (p. 39)

Does this mean that the community should accept that no universal philosophy of mathematics education is therefore possible? We turn to the next thesis to further develop this line of inquiry.

Thesis 4: *For mathematics education one should prefer and elaborate philosophies of mathematics which especially respect the following aspects: different forms and conditionalities of mathematical knowledge, means and modes of representation and activities, relations between subjective and objective developments of knowledge (complementarity, obstacles, dynamics), relation of mathematical knowledge to, other knowledge, special fields and applications; the personal, social and political dimension of mathematics.*

Message 4: We think this thesis is a difficult one to directly interpret. Our interpretation is that focusing on how learning occurs may develop a more favorable philosophy of mathematics.

Again Steiner is not concerned with mathematics education directly. A philosophy of mathematics in his sense should provide a very comprehensive meta-theory of mathematics including all the mentioned aspects, including the genesis of mathematics etc. And if we have developed such a theory, we can better ground mathematics education in it. In a sense the 1987 paper was not (yet) concerned with TME but with previous preliminary notions of TME. TME later was aimed as a framework for guiding and structuring mathematics education research, which was really something very different and much broader and ambitious than the 1987 paper. In Biehler et al.'s (1994) introduction to chapter 7 of the *Didactics of Mathematics as a Scientific Discipline*, one sees an attempt to relate Steiner's work to other developments including the historical dimension, related to epistemology.

The two dominant philosophies that arose in the 1980s and 1990s were radical constructivism (see von Glasersfeld 1983) and social constructivism (Ernest 1989, 1991). With a very instrumental view of mathematics—understandably—the classical “Stoffdidaktik” tradition in Germany asserts the need to continually develop the pedagogy of mathematics. However, there were some inherent problems in each of these philosophies as is pointed out by Goldin (2003).

Radical constructivism helped overthrow dismissive behaviorism, rendering not only legitimate but highly desirable the qualitative study of students' individual reasoning processes and discussions of their internal cognitions (but with the unfortunate provision that no ‘objective validity’ could be claimed for the conclusions of research). It led to many in-depth, observational studies that have been of value to those who have advocated meaningful, guided discovery-oriented mathematical learning. Social constructivism pointed to the importance of social and cultural contexts and processes in mathematics as well as mathematics education, and postmodernism highlighted functions of language and of social institutions as exercising power and control. And ‘mind-based mathematics’ emphasized the ubiquity and dynamic nature of metaphor in human language, including the language of mathematics. Unfortunately, in emphasizing its own central idea, each of these has insisted on excluding and delegitimizing other phenomena and other constructs, even to the point of the words that describe them being forbidden—including central constructs of mathematics and science—or, alternatively, certain meanings being forbidden to these words. Yet the ideas summarized here as comprising the ‘integrity of knowledge’ from mathematics, science, and education are not only well-known, but have proven their utility in their respective fields. There are ample reasoned arguments and supporting evidence for them. (p. 196)

Goldin then gives an interesting analogy between researchers working in mathematics education and those working in mathematics. He writes:

Educational researchers who might be characterized as ‘behaviorists’ or ‘neo-behaviorists’, as well as those who might be termed ‘ultrarelativists’, have performed groundbreaking work. Mathematicians who might be characterized as ‘Platonists’ or ‘formalists’, as well as those holding quite different views, have achieved important

mathematical insights, and argued for attention to important educational priorities. (p. 197)

Although Goldin’s (2003) analysis of mathematics and mathematics education in their mutually dependent conditions is helpful, it should be noted that he does not go all the way in taking a completely relativistic stance. We argue that for Goldin, mathematics remains a last instance of a monolithic super structure. He writes:

My thesis here is that the chasm that has opened is in part attributable to the long fashionableness of certain epistemologies or theoretical ‘paradigms’ in mathematics education, that dismiss or deny the integrity of fundamental aspects of mathematical and scientific knowledge. (p. 174)

The question then is does Goldin really acknowledge that paradigm shifts which occur even in mathematics<sup>1</sup>? (see Stewart 1995)

The message in the next two theses of Steiner’s (1987) article are quite self evident:

Thesis 5: *Such philosophies of mathematics should become an ingredient of a form of reflective mathematics teaching and learning, and contribute to the development of an adequate meta-knowledge not only for teachers but also for students.*

Thesis 6: *Mathematics education needs comprehensive approaches and meta-theories which should comprise an adequate philosophy of mathematics. For a metatheory which is built on a systems approach based on human activity and social interaction, an adequate philosophy of mathematics should view mathematics itself as a system from the point of view of human object-related cooperative activities.*

#### 4 Where are we today? Where should we go?

The question now is why is Steiner’s (1987) article of relevance today? It seems to us that Steiner was pointing to the (in-) differences of mathematics educators and psychologists<sup>2</sup> to the epistemologies used by each side on what constitutes knowledge and learning. For instance, every paper dealing with the topic of epistemology in mathematics education cites the survey article of Sierpinska and Lerman (1996) addressing this issue. On the other side, every psychologically

oriented publication in general reference papers e.g. of Buehl et al. (2001); Clarebout et al. (2001); Hofer and Pintrich (1997); Schommer (1990); Schommer et al. (1992) and especially that of Muis (2004). What are the reasons that these two research areas have never met and hardly communicated? Rolka (2006) can be seen as a small contribution toward an attempt to bridge these two strands. To find a rationale for this ‘ignorance’ we note that mathematics educators themselves have been intensively dealing with beliefs as a hidden variable for more than 30 years (Leder et al. 2002). Although, their discussions are tightly intertwined with epistemological considerations, this is seldom recognized in most of their papers, and the term ‘epistemology’ is rarely mentioned explicitly. In this paper, we approach this issue in a pragmatic way. On the one hand, we focus on epistemology in general and show that its role has always been a prominent and integral topic in the history and philosophy of mathematics. On the other hand, we draw on the findings of various psychologists. Last but not least, it should be noted that our research focus does not exclusively dwell on academic quality, which might be viewed as a purely philosophical issue, however, epistemological orientations and precepts play an important role in the everyday life of teaching and learning (see e.g. Köller et al. 2000; Noss 2002; Noss et al. 1999; Sierpinska 1992).

Another example in the differences of epistemologies comes in the epistemology of what constitutes “mathematical creativity”. For instance, in mathematics education, the epistemology of “mathematical creativity” is typically drawn from analyzing the writings and reflections of eminent mathematicians. Recently Burton (2004) proposed an epistemological model of “mathematician’s coming to know” for consideration by the community, consisting of five interconnecting categories, namely the person and the social/cultural system, aesthetics, intuition/insight, multiple approaches, and connections. This model is grounded in the extensive literature base of mathematics, mathematics education, sociology of knowledge and feminist science, and purports to address four dominant challenges, namely “the challenges to objectivity, to homogeneity, to impersonality, and to incoherence.” (p. 17). Burton empirically tests this model by interviewing 70 mathematicians and qualitatively analyzing the data to generate the dominant themes. On the other hand in mainstream psychology, the approach to studying creativity uses quantitative methods and a systems approach to understanding the construct. Prominent among these are the “historiometric approach” (Simonton 1984, 1994); the “systems

<sup>1</sup> Goldin has been interpreted by some scholars to say that knowledge is to a certain extent independent from the “philosophy” of the researcher either in mathematics or in mathematics education.

<sup>2</sup> Although Steiner does not explicitly mention psychologists, the field of mathematics education was under the growing influence of theories of cognition from the domain of psychology.

approach” (Csikszentmihalyi 1988, 2000); and the “investment theory approach” (Sternberg and Lubart 1996, 2000) to studying creativity generated from historical and empirical data. These confluence theories both circumscribe and complement the categories of Burton’s model. Burton’s findings about the practices of mathematicians within the field of mathematics do cohere with the confluence theories of creativity but interestingly enough the references from the field of psychology are absent in the list of the book’s references and vice versa. It seems to us that there is a further schism in the epistemologies of mathematics educators and psychologists conceptions of creativity.

In the domain of proofs in mathematics education, the DNR<sup>3</sup> theory created by Harel (2006a) attempts to bridge the epistemologies of what constitutes proofs in the professional mathematics and mathematics education. Harel (2006a) writes:

Pedagogically, the most critical question is how to achieve such a vital goal as helping students construct desirable ways of understanding and ways of thinking. DNR has been developed to achieve this very goal. As such, it is rooted in a perspective that positions the mathematical integrity of the content taught and the intellectual need of the student at the center of the instructional effort. The mathematical integrity of a curricular content is determined by the ways of understanding and ways of thinking that have evolved in many centuries of mathematical practice and continue to be the ground for scientific advances. To address the need of the student as a learner, a subjective approach to knowledge is necessary. For example, the definitions of the process of “proving” and “proof scheme” are deliberately student-centered. It is so because the construction of new knowledge does not take place in a vacuum but is shaped by one’s current knowledge. (p. 23, pre-print)

Harel’s views echo the recommendations of William Thurston, the 1982 fields medal winner, who contributed a chapter entitled On Proof and Progress in Hersh (2006). Thurston outlines for the lay person (1) what mathematicians do, (2) how (different) people understand mathematics, (3) how this understanding is communicated, (4) what is a proof, (5) what motivates mathematicians, and finally (6) some personal experiences. Thurston stresses the human dimension of what it means to do and communicate mathematics. He also gives numerous insights into the psychology of math-

ematical creativity, particularly in the section on what motivates mathematicians. Mathematics educators can draw great satisfaction from Thurston’s writings, particularly on the need for a community and communication to successfully advance ideas and the very social and variant nature of proof, which depends on the sophistication of a particular audience.

It seems to us that although there are dissonances in the terminology used by psychologists, mathematics educators and mathematicians when speaking about the same construct, there are some similar elements which can lay the foundation of a common epistemology. But several hurdles exist in creating common epistemologies for the diverse audience of researchers working in mathematics education. Harel (2006b) in his commentary to Lester’s (2005) recommendations to the mathematics education research community (for developing a philosophical and theoretical foundation), warns us of the dangers of oversimplifying constructs that on the surface seem to be the same. Harel (2006b) writes that a major effort has been underway for the last two decades to promote argumentation, debate and discourse in the mathematics classroom. He points out that scholars from multiple domains of research have been involved in this initiative, i.e., mathematicians, sociologists, psychologists, classroom teachers, mathematics educators and such. Harel (2006b) writes:

However, there is a major gap between “argumentation” and “mathematical reasoning” that, if not understood, could lead us to advance mostly argumentation skills and little or no mathematical reasoning. Any research framework for a study involving mathematical discourse ...[w]ould have to explore the fundamental differences between argumentation and mathematical reasoning, and any such exploration will reveal the critical need for deep mathematical knowledge. In mathematical deduction one must distinguish between status and content of a proposition (see Duval 2002). Status (e.g., hypothesis, conclusion, etc.), in contrast to content, is dependent only on the organization of deduction and organization of knowledge. Hence, the validity of a proposition in mathematics—unlike in any other field—can be determined only by its place in logical value, not by epistemic value (degree of trust). Students mostly focus on content, and experience major difficulties detaching status from content. As a consequence, many propositions in mathematics seem trivial to students because they judge them in terms of epistemic values rather than logical values. Another.. [s]ource of difficulty for

<sup>3</sup> DNR = *duality, necessity, and repeated-reasoning*.

students, is that in the process of constructing a proof, the status of propositions changes: the conclusion of one deductive step may become a hypothesis of another. These are crucial characteristics that must hold in any form of mathematical discourse, informal as well as formal (!). In argumentation and persuasion outside mathematics, on the other hand, they are not the main concern: the strength of the arguments that are put forward for or against a claim matters much more... (pp. 60–61)

## 5 Concluding points

We take a relativistic stance akin to Perry (1970), namely a scientific-sociological view and decide that in the long term the field has to develop a locally appropriate philosophy and an associated epistemology depending on the area of investigation (such as beliefs or creativity or proofs etc). Following is a representation how he conceptualized his scheme<sup>4</sup> (see also Perry 1970). It should be noted too that Perry based the essential features of his scheme on interviews with Harvard students at a time when the population was overwhelmingly white, privileged, and male. Many researchers who have attempted to apply his model have found little evidence of his higher levels in the undergraduate populations of less elite institutions. Nevertheless, Perry’s analysis of the undergraduate “journey” has not really been challenged as the American university’s view of itself and ideal experience for students. We also list some meaningful metaphors for the various levels which can also be found in the reference.

1. *Acknowledges absolute knowledge handed down by authority.* Knowledge is an objective, definite, and organized body of facts that constitute the truth about a subject, to be distinguished from opinion, which is subject and cannot be proven as true. (‘in port’)
2. *Acknowledges differences of opinion that are the result of poorly qualified authority.* Knowledge consists of facts, principles, axioms, etc. that can be proved, although it may be difficult to carry out the proof. Overcoming this difficulty is the expert’s challenge, and some are more expert than others. (‘in safe harbor’)
3. *Acknowledges uncertainty as temporary.* Knowledge consists of facts, principles, axioms, etc. that

can be proved, although it may be difficult to carry out the proof. The coherence and completeness of the system may vary across disciplines, some being more advanced than others. (‘crossing the bar’)

4. *Acknowledges relativistic knowledge as the exception to the rule.* Knowledge is not secure but is any person’s organization and interpretation of available information. One interpretation is as good as another. But people with power can assert their interpretations over those of others. (‘entering the open’ and ‘losing sight of the shore’)
5. *Acknowledges absolute knowledge as the exception to the rule.* It can be shared but not “measured” or counted upon to remain the same. (‘entering rough waters’)
6. *Apprehends the need for personal commitment in a relativistic world.* Knowledge is not something that is external and definite but something that each individual constructs according to his/her experience, background, etc. (‘Weathering storms; losing bearings’)
7. *Initial commitment is made.* Knowledge is the world view one has constructed from learning and experience, along with the ethical implications of this view, synthesized into a consistent philosophy. (‘Getting back one’s bearings; navigating successfully’)
8. *Exploring commitment.* Knowledge is a creative resolution between uncertainty and the need to act, which makes it a dynamic means of transaction between the self the environment, requiring both stability and flexibility. (‘Progressing into new regions’)
9. *Acknowledges commitment as an ongoing, complex, and evolving process.* Knowledge is the evolution of awareness, best expressed as ascending levels of consciousness, in which the individual must break through to new perspectives and discard those no longer useful. (‘Discovering anticipated and unforeseen destinations or destinies’)

It seems to us that the Steiner’s theses are an attempt to bridge the world of the working mathematician with that of the student and teacher of mathematics. Steiner’s stance is quite relativistic (particularly #3 and #4). How would one resolve the writings of Goldin (2003) in light of Perry’s positions? Would Goldin reject such a categorization as inappropriate for mathematics? Again, Hersh (2006), continuing in the tradition of Lakatos (1976) presents mathematics as a cultural activity, which can be fallible at times. In particular he explains more about how mathematicians work, and acknowledges a kind of

<sup>4</sup> <http://www.indiana.edu/~1506/theoryframe/506Model.htm>

relativistic conception in the world of the working mathematician (where people comfortably go back and forth between Platonism and Formalism). We think that Platonism is more of a psychological construct for today's mathematician as opposed to a philosophical construct. The relevance of Steiner's theses clearly is the fact that mathematics education has not succeeded in arriving at any philosophy that fits well with teaching and learning processes somewhat akin to futile attempts of squaring the circle! The great debate between social and radical constructivists did not really solve any problems. What we suggest is the creation of local philosophies based on the particular problems such as the success of researchers working in the domain of mathematical beliefs and proof schemes. We must grant also a limited validity period to such local philosophies given the volatile nature of mathematics education. We are reminded of the quote by Euclid, which says that there is really no high road to mathematics. Similarly there can never be any final curriculum or schoolbook which will alleviate the basic problems confronting the field.

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