ADVANCED REAL TIME OPTICAL IMAGING



In Situ Observations of Agglomeration of Non-metallic Inclusions at Steel/Ar and Steel/Slag Interfaces by High-Temperature Confocal Laser Scanning Microscope: A Review

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The agglomeration behavior of non-metallic inclusions in the steelmaking process is important for controlling the cleanliness of the steel. In this work, the observation of agglomeration behaviors of inclusions at steel/Ar and steel/ slag interfaces using a high-temperature confocal laser scanning microscope (HT-CLSM) is summarized. This HT-CLSM technique has been applied to observe phase transformation during solidification and heat treatment and the engulfment and pushing behavior of inclusions in front of the solidified interface. In the current work, the inclusion agglomeration behavior at steel/ Ar and steel/slag interfaces is summarized and discussed. Subsequently, the development of the theoretical work investigating inclusion agglomeration at steel/Ar and steel/slag interfaces including the initial capillary force model and Kralchevsky-Paunov model is described. Finally, the Kralchevsky-Paunov model is applied to investigating nitride inclusion agglomeration at highmanganese steel/Ar interfaces. This work aims to give a critical review of the application of HT-CLSM in secondary refining as well as a better control of inclusion elimination for clean steel production.

INTRODUCTION

The cleanliness of steel is of vital importance in meeting the increasing demands for high-quality steel grades with superior mechanical properties. To improve steel cleanliness, the agglomeration behavior of inclusions in steel must be controlled accurately. Steel plants aim to produce steels with as much cleanliness as possible to avoid nozzle clogging, improve mechanical properties, and ensure economical production.¹⁻³ Against this background, the field of 'inclusion engineering' has been developed,⁴ which deals with control of the morphology, size distribution, amount, and composition of non-metallic inclusions formed in liquid steel during refining and casting processes.

The agglomeration behavior of inclusions has been investigated experimentally by using the high-temperature confocal laser scanning microscope (HT-CLSM). The pioneering work on the application of HT-CLSM dates back to the work of Emi's group,^{5,6} with the observation of crystal growth during the solidification of steel⁵ as well as engulfment and pushing behaviors of inclusions in steel the melt/solid interface.⁶ HT-CLSM has also been used to observe the phase transformations in solid steel during continuous cooling.^{7,8} Moreover, previous HT-CLSM work focusing on the observation of inclusion agglomeration behavior at steel/Ar and steel/slag interfaces has been reported in Refs. 9–26.

In the current work, an overview of in situ observation of non-metallic inclusion agglomeration behavior is presented. Furthermore, the theoretical attractive force for inclusion agglomeration was initially calculated by Emi et al.^{9,10} and subsequently by Nakajima and Mizoguchi,¹¹ Kimura et al.,¹² Wikström et al.,^{23,26} and Mu et al.^{13,14} using the Kralchevsky–Paunov model.^{27,28} The calculated results can provide a reasonable fit with the experimental data of attractive force.^{11–14,23} Subsequently, the agglomeration potency of different nitride inclusions at a high-manganese steel/Ar interface is quantitatively evaluated, based on the calculation results of capillary force. The present work aims to consolidate and rationalize the extensive observations in the literature with the objective of better control of inclusion elimination and improve the steel quality in secondary refining and casting, according to the concept of inclusion engineering.

EXPERIMENTAL WORK OF IN SITU OBSERVATION OF INCLUSION AGGLOMERATION AT STEEL/AR AND STEEL/SLAG INTERFACES BY HT-CLSM

Inclusion Agglomeration at the Steel/Ar Interface

Several researchers have investigated inclusion agglomeration behaviors at steel/ Ar^{9-22} and steel/ slag interfaces^{16,19,23–26} by HT-CLSM, as summa-rized in Table I. Emi et al.^{9,10} observed the agglomeration behavior of solid inclusions (Al₂O₃, A80S, CA80S, CAS95, CA80), semi-solid inclusion (CA60), and liquid inclusions (CA60S, CA50S, CA50) at the steel/Ar interface, where CaO is abbreviated as C, Al₂O₃ as A, SiO₂ as S, and MgO as M. The numerals after the abbreviated letters represent the average content in each inclusion, for example, CA60S represents $60 \text{ mass}\% \text{Al}_2\text{O}_3\text{-xCaO-ySiO}_2$. Thev reported that solid-solid inclusion pairs would form as (1) intermediate aggregates, (2) loose structured clusters, and (3) densified compact clusters more easily than other types of inclusion pairs. The attractive capillary force was in the range of 10^{-16} – 10^{-13} N. An illustration of the calculation of the attractive force of inclusion agglomeration based on the HT-CLSM video from previous studies^{9,10,12} is shown in Fig. 1.

$$a_i = (V_{i+1} - V_i) / \Delta t_i = (d_{i+1} / t_{i+1} - d_i / t_i) \Delta t_i$$
 (1)

$$\Delta t_i = t_{i+1} - t_i \tag{2}$$

$$m_2 = \rho \times V_2 \tag{3}$$

$$F_{A,i} = m_2 \times a_2 \tag{4}$$

$$F'_{A,i} = m_2 \times a_i \times m_1/(m_1 + m_2) \tag{5}$$

Equations 1–4 show the calculation methods for one inclusion (guest) moving toward a stagnant inclusion (host) where m_1 is the mass of the guest inclusion and a is the acceleration of the guest inclusion toward the host inclusion. Also, d_1 , d_2 , and d_3 are the distances between two inclusions at each time. The time interval, Δt , equals 1/30 of a second. Furthermore, if two inclusions approach each other, a revised parameter of $m_2/(m_1 + m_2)$ was introduced to replace m_1 . In that case, Eq. 5 was introduced. It should be noted that the inclusion is assumed to have a disk shape in Refs. 9, 10 and 12; it is also assumed to have a spherical shape in Refs. 13 and 23. The volume of inclusions can be calculated based on the different geometries.

It is reported that there is no collision between the globular liquid-liquid inclusions. The strength of the attraction force between two inclusions in pairs at the steel/Ar interface can be listed as follows: liquid/liquid pair < liquid/semi-liquid pair < semi-liquid/semi-liquid pair < liquid/solid pair < semi-liquid/solid pair < solid/solid pair.¹ The detailed composition, attractive force, morphology, and agglomeration characteristics of inclusions are summarized in Table II. In addition, Yin et al. claimed that strong long-range attraction extended as far as 50 μ m between Al₂O₃ inclusions and about 40 μ m between 80%Al₂O₃·20%SiO₂ inclusions.⁹ Subsequently, Nakajima et al.^{11,12} continued this

research work with reporting data for inclusion agglomeration in 16Cr Al-Si-killed and 16 Cr Sikilled stainless steel.¹¹ They concluded that an attractive force existed between pairs of inclusions of a similar kind, such as solid/solid, solid/semisolid. Specifically, the solid/solid inclusion pair had the strongest attraction, and the liquid/liquid inclusion pair had the weakest. However, a repulsive force exists between the complex-liquid inclusion pair. This conclusion is similar but not identical to that reported by Yin et al.^{9,10} Subsequently, these researchers claimed that attractive forces between 93%Al₂O₃·7%MgO inclusions and those between MgO inclusions were quite similar in the range of 5×10^{-18} – 5×10^{-16} N,¹² approximately 1/10 of the force between Al₂O₃ inclusion pairs reported by Yin et al.⁹ The maximum acting length of the force for 93%Al₂O₃·7%MgO inclusion pairs and MgO inclusion pairs was $21-22 \ \mu m$,¹² which is much shorter than the maximum acting length for Al₂O₃ and $80\%Al_2O_3 \cdot 20\%SiO_2$. This acting distance is affected by the inclusion size. It is worth noting that the aforementioned 93%Al₂O₃.7%MgO and MgO inclusions, which have a small value of acting distance, had a radius $< 5 \mu m$; however, this acting distance can be over 150 μm for Al_2O_3 inclusions with a radius > 40 $\mu m.^{13-15}$

In situ observations of inclusion agglomeration have been continuously reported in Refs. 16–19 and 23. Some researchers named 'the steel/Ar interface' as 'the molten steel surface,' and these two terms have the same meaning in the HT-CLSM work. Vantilt et al.¹⁷ observed the agglomeration of Al₂O₃·MnO (sol.), Al₂O₃·MnO·SiO₂ (liq.), and Al₂O₃ (sol.) inclusions at the (Mn,Si)-killed steel/Ar interface and found that solid inclusions move freely to form clusters and liquid inclusions are forced to agglomerate, affected by the fluid flow, which is a similar conclusion to that reported by Nakajima et al.^{11,12} Also, Coletti et al.¹⁶ reported that the clustering behavior of liquid CaO·Al₂O₃ inclusions (30%CaO·70%Al₂O₃ or 25%CaO·75%Al₂O₃) was not found at the surface of Ca-treated Al-killed low-

Table I. A	summ	ary of in situ observ	ations of inclusion agglom	eration behavior at th	e steel/Ar interface and steel/slag interfaces ⁹⁻²⁶
Interface					
type	Year	Authors	Agglomerated inclusion	Slag system	Steel type (mass%)
Steel/Ar interface	1997	Yin et al. ^{9,10}	Al ₂ O ₃ , Al ₂ O ₃ -SiO ₂ , CaO-Al ₂ O ₃ , CaO- Al ₂ O ₃ -SiO,	I	LCAK steel; Fe-3%Si; HSLA steel; Si-killed steel:
	9001	Nolroiimo and			HC-Ca-treated steel EQ 0.017/0.06702, C. 16.302/Ca
	2001	Mizoguchi ¹¹ Mizoguchi ¹¹ Kimura et al ¹²	Solid 93% Alo7% MeO	1 1	Te-U.047/0.025/0.6%Mn-0.3%Si-0.0022/0.0048%O (16CrS.S.) 0.25/0.6%Mn-0.3%Si-0.0022/0.0048%O (16CrS.S.) Fe-0.04%C-0.02/0.21% Mn (Me-added Al-killed steel Me-killed
	1007		and MgO		$D_{2} = 0.000 \text{ m} C_{1} + 200 \text{ m} C_{1} + $
	2002	Coletti et al.	CaU-A12U3	I	re-u.usa%c1.40%ип-U.134%Si-U.usa%Al-U.uuusu%O (Са- treated. Al-killed steels)
	2004	Vantilt et al. ¹⁷	Al ₂ O ₃ , Al ₂ O ₃ - MnO, Al ₂ O ₃ - MnO-SiO ₃ .	I	Fe-0.047%C-0.64%Mn-0.27%Si-0.006%Al-0.031%Cr-0.006%N (Si-Mn-killed low-C steel)
	2006 2008	Liang et al. ¹⁸	$CaO-MgO-Al_2 \widetilde{O_3}^{-}SiO_2$	I	Fe-17%Cr-8.36%Ni-0.06%C-0.63%Si-0.7%Mn (AISI 304 S.S.)
	2008	Wikström et al.	Al ₂ O ₃ -CaO Al ₂ O ₃ -CaO	1 1	$F_{e} \sim 30\%$ Fe-13 $\sim 20\%$ Cf- 0.1%AH-0.002 $\sim 0.1.7\%$ Ce 0.18 Si-0.48%Mn-0.001%Al-0.0012%Ca-0.01%Cu-0.003%O (HC-Ca-treated steel). the same composition
	2011	Kang et al. ²⁰	Al ₂ O ₃ , MgO·Al ₂ O ₃ , CaO·2Al ₂ O ₃ ,	I	as in Ref. 8 Fe-0.39%C-1.0%Si-0.4%Mn-5.3%Cr-1.3%Mo-0.9%V
	2016	Du et al. ²¹	liquid inclusion Al ₂ O ₃ , MgO·Al ₂ O ₃	I	Fe-0.4%C-1.0%Si-0.3%Mn-5%Cr-1.2%Mo-0.9%V-0.14%Ni-
	$\begin{array}{c} 2016 \\ 2017 \end{array}$	Michellic et al. ²² Mu et al. ^{13–15}	CaO-xAl ₂ O ₃ (CAx) Al ₂ O ₃ containing	11	U.01/0.024%A1-00124%A3-001017%Mg Fe-0.15%C-0.02%Si-1.07%Mn-0.04%A1-0.002%Ca-0.004%S Fe-0.16%C-0.84%Mn-0.15%Si-0.008%Ti-0.01%Al-0.03%S
Steel/slag	2000	Misra et al. ²⁴	trace 11 oxide Al ₂ O ₃ ·CaO·SiO ₂	50% CaO-50% Al ₂ O ₃	Fe-1.3%Si-1.1%Mn-1.2%Al
Intertace	2001	Misra et al. ²⁵	NiT	39.5%SiO ₂ - 33.4%CaO- 19.5%Al ₂ O ₃ -	85.4%Fe -13.52%Cr -0.17%Ti-0.23%Ni-0.16% Al- 0.39%Si - 0.11%Ca
	2003	Coletti et al. ¹⁶	Al ₂ O ₃ ,Al ₂ O ₃ -SiO ₂ - containing	7.3% MgO C55.3A46.5S0.1, C40A20S40,	Fe-0.093%C-1.46%Mn-0.134%Si-0.039%Al-0.00030%O (Ca- treated, Al-killed steels)
	2004	Vantilt et al. ¹⁷	mclusion Al ₂ O ₃ -CaO- MnO-SiO ₂	C33A20540Mg7 C50A50, C40A20S40, C55.3A46.5S0.1, C52.5A46.5S0.1,	Fe-0.047%C-0.64%Mn-0.27%Si-0.006%Al-0.031%Cr-0.006%N [(Si-Mn)-killed LC steel]
	2008	Wikström et al. ^{23,26}	Liquid CaO- Al ₂ O ₃	CaO-420340M1g1 20.7%Al ₂ O3-36.5% CaO-42.8%SiO ₂	Fe-0.75%C-0.18%Si-0.48%Mn-0.001%Al-0.0012%Ca-0.01%Cu- 0.003%O (HC-Ca-treated steel)

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Fig. 1. Schematic illustration to calculate the attractive force from the HT-CLSM video, adapted from Refs. 9, 10, and 12.

carbon steel.¹⁶ Similar phenomena regarding the motion of complex Al₂O₃·SiO₂·CaO·MgO inclusions at the molten steel surface and the δ -ferrite/liquid interface of AISI 304 stainless steel were reported by Liang et al.¹⁸ Appelberg et al.¹⁹ reported the agglomeration behavior of alumina and cerium oxide (Al₂O₃·Ce₂O₃) inclusions in liquid Fe-20%Cr ferric stainless steel. Different types of alumina and cerium oxide inclusions have been observed according to the different ratios of [Ce]/[Al] in the steel. However, all the types of $Al_2O_3 \cdot Ce_2O_3$ inclusions could form clusters with radii up to approximately $20 \ \mu m$.¹⁹ Wikström et al.²³ reported the agglomeration behavior of two types of Ca-Al oxides, liquid 50%Al₂O₃·50%CaO (CA50) and semi-liquid 62%Al₂O₃·38%CaO (C38A62) inclusions, at the surface of high-carbon Al-killed steel with Ca treatment. They claimed that the collide type between the semi-liquid C38A62 inclusion pair was 'free' and that between the liquid-liquid CA50 inclusions was both 'free' and 'forced.' This conclusion means that the liquid-liquid inclusion pair could not collide as freely as the liquid/semi-liquid inclusion pair, which is similar to the conclusion reported in previous studies.⁹⁻¹² However, the liquid-liquid inclusion pair can be forced to agglomerate at the steel/Ar interface, driven by the steel flow when the temperature increases. Besides, MgO·Al₂O₃ is a technologically important inclusion type, which may lead to nozzle clogging. Regarding this type of inclusion, Kang et al.²⁰ reported that Al_2O_3 inclusions attracted each other and agglomerated. However, MgO·Al₂O₃ spinel inclusions and solid calcium aluminate inclusions did not show any sign of attraction or agglomeration. Du et al.²¹ reported Al₂O₃ and MgO·Al₂O₃ spinel behaviors in Fe-0.4C-1Si-0.3Mn-5Cr-1.2Mo-0.9V Al-killed steels with and without Mg addition. They observed agglomerated MgO·Al₂O₃ spinel clusters of 20–60 μ m; however, their number and size were much smaller than the Al₂O₃ clusters in the same steel grade without Mg addition. To date, the exact reason for the different agglomeration behaviors of MgO·Al₂O₃ spinel inclusions reported by different researchers^{20,21} is unclear; it may be related to the different physical properties of various steel grades or the number density of the MgO·Al₂O₃ spinel inclusions presented in the steels.

Inclusion Agglomeration Behavior at the Steel/Slag Interface

The use of HT-CLSM to investigate the inclusion agglomeration behavior at the steel/slag interface dates back to the work of Sridhar's group. ^{16,17,24,25} A schematic illustration of the steel/Ar and steel/slag interfaces before in situ observation experiments is shown in Fig. 2. To observe the inclusion agglomeration behavior at the steel/slag interface, an optically transparent slag disc should be used. ^{16,17,23,26} Another required condition is the melting point of the slag should be lower than that of the steel. According to these two requirements, a 50% CaO·50% Al₂O₃ slag^{16,23} and several CAS slags with or without MgO^{19,24–26} were selected in previous studies.

Detailed information on the agglomeration behavior of inclusions at the steel/slag interface is shown in Table I. Misra et al.²⁴ reported the agglomeration behavior of different types of Al_2O_3 ·CaO·SiO₂ inclusions at the interface created by a (Mn,Si)-killed steel and a 50%CaO·50%Al₂O₃ slag. Irregular inclusions (solid) with sizes $< 10 \ \mu m$ would occasionally attract and absorb irregular inclusions with smaller size to form aggregates. This qualitative observation is quite similar to the case of inclusion behavior at the steel/Ar interface.⁹⁻¹¹ Thereafter, the agglomeration of different morphologies of TiN inclusions at a molten stainless steel/CASM slag interface was reported by the same authors.²⁵ Loose clusters with hexagonal-shaped TiN precipitates could be observed at 1489°C. However, triangular-shaped TiN precipitates were found to form clusters at $1589^{\circ}C.^{25}$

Quantitative analysis of the attractive force of inclusion agglomeration at the steel/slag interface has been continuously reported.^{16,17,23,26} Coletti et al.¹⁶ and Vantilt et al.¹⁷ reported that the same types of 40%CaO·40%SiO₂·20%Al₂O₃, 53.3%CaO·46.5% Al₂O₃·0.1%SiO₂, and 33%CaO·20%Al₂O₃·40%SiO₂·6 7%MgO slags were used to create the steel/slag interface. Moreover, (Mn,Si)-killed low-carbon steel.¹⁷ abbreviated as LC-SMn steel, and Catreated Al-killed low-carbon steel,¹⁶ abbreviated as LC-CA steel, were chosen. For the case of LC-SMn steel, complex liquid Al₂O₃·CaO·MnO·SiO₂ inclusions were found, and the agglomeration of this type of inclusion was reported to be inhibited by a counteracting force at the steel/slag interface. This differs from the behavior of similar types of inclusions at the steel/Ar interface, where solid inclusions (Al₂O₃ and Al₂O₃·MnO) were observed to move freely to form a cluster, and liquid inclusions $(Al_2O_3 \cdot MnO \cdot SiO_2)$ were forced to agglomerate under the influence of fluid flow.¹⁷ For the case of LC-CA steel, the inclusion is identified as the solid Al_2O_3 and Al₂O₃-SiO₂-containing phase.¹⁶ Wikström et al.^{23,26} reported the clustering behavior of liquid Al_2O_3 CaO inclusions at the interface between

		Compos	ition (m	lass%)			;	Attractive			•
Inclusion	CaO	Al_2O_3	SiO_2	MgO/Ce ₂ O ₃	State in liq. steel	Shape	Kadius (µm)	force (N)	Initial product of collision	Steel type	Refer -ences
${ m Al}_2{ m O}_3$ A80S	1 1	> 2000980) < 20	11	Solid Solid	Irregular with tips Irregular with	$\sim 2 \ 1 \sim 3$	$1.8\mathrm{E}{-15}$ $7\mathrm{E}{-16}$	Loose cluster Dense cluster	LCAK Fe-3%Si	66
CA60S C30A60M1 CA80S	${}^{>30}_{>32} > 30$ < 10	${\sim 60 \atop \sim 61 \atop \sim 61}$ 75 ~ 90	$\begin{array}{c} < 10 \\ 0 \\ 10 \sim 20 \end{array}$	-7 (MgO) -	Liquid SolLiq. Solid	smooth surface Globular with tips Irregular with	$egin{array}{c} 5 \sim 10 \ 1.5 \sim 15.5 \ 1 \sim 3 \ 1 \sim 3 \end{array}$	$\stackrel{-}{1\mathrm{[E-14$}\sim1\mathrm{E-17$}}$	No collision Collision Dense cluster	HSLA 16Cr Al–Si killed Si-killed steel	10 11 10
CAS95	02 ℃	۲ 2	> 95	I	Solid	smootn suriace Near globular with	$5\sim 10$	$8.2\mathrm{E}-15$	Coarse cluster		
CA50S C8A92 CA80	$egin{array}{c} 20 \sim 30\ 7 \sim 9\ < 20\ < 20 \end{array}$	$egin{array}{c} 40 \sim 60 \ 91 \sim 92 \ > 80 \end{array}$	$20\sim 30 < 1 < 1$ -	< 1 (MgO)	Liquid Solid Solid	nard surface Globular Irregular Irregular with	$egin{array}{c} 5 \sim 10 \ 2.5 \sim 25 \ 3 \sim 5 \ 3 \sim 5 \end{array}$	$\stackrel{-}{18-14} \sim 1E-18 \ 6.5E-14$	No collision Collision Coarse cluster	16 Cr Si-killed HC-Ca	11 10
CA60	$30 \sim 35$	$60 \sim 65$	Ι	I	Sol liq.	coarse tips Irregular with	$5\sim 10$	$1\mathrm{E}-16$	Larger globular		
CA50	$40\sim 60$ $44\sim 55$	$40 \sim 60 45 \sim 56$	11	1 1	Liquid Liquid	coarse tips Globular Globular	$5\sim 10$ $10\sim 40$	1 1	No collision Forced collision		19
C40A55M5 A93M7	$38 \sim 45$	$\frac{1}{48} \sim \frac{55}{55}$	۱ <mark>۲</mark>	$5 \sim 7 \ (\mathrm{MgO})$ $7 \ (\mathrm{MgO})$	Liquid Solid	Globular -	< 40		Cluster Coagulation,	AISI 304 S.S. LCAK	11 21
${ m MgO}_{{ m Al}_2{ m O}_3\cdot{ m Ce}_2{ m O}}$	I I "	- 79	1 1	$100 < 21 (Ce_2O_3)$	Solid Solliq.	- Semi-globular and	$1 \sim 9$	No exact value	eaker unan A1 ₂ O3 Cluster	LCMK Fe-20%Cr S.S.	$\frac{12}{18}$
	I	$20 \sim 70$	I	$55 \sim 80 \; (Ce_2O_3)$	Solid	Irregular, some	$1\sim 11$		Cluster	Fe-20%Cr S.S.	18
	I	< 15	I	$> 85 (Ce_2O_3)$	Solliq.	wuu ups Semi-globular and irregular	$1 \sim 10$		Cluster	Fe-20%Cr S.S.	18

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Ca-treated Al-killed high-carbon steel, abbreviated as HC-CA steel, and the 40%CaO·20%Al₂O₃·40%-SiO₂ slag.

Even if the steel compositions are different, several conclusions are still in agreement. First, the steel would form a pronounced meniscus, penetrating into the slag. A schematic illustration is shown in Fig. 3. The inclusions could be driven together at the top of the hemispherical meniscus because of the buoyancy force. Second, there is an attractive force working at relatively long distances



Fig. 2. Schematic illustrations of the (a) steel/Ar and (b) steel/slag interfaces, which were prepared before in situ observation, adapted from Ref. 23.



Fig. 3. Illustration of the steel/slag interface after the HT-CLSM experiment, adapted from Ref. 17.

between inclusions or clusters to force them to converge. Thereafter, a repulsive force seems to exist holding back inclusions when they approach within about 150 μ m. However, the repulsive force is quite weak, and the inclusions will finally agglomerate at the steel/slag interface.^{16,17} Coletti et al.¹⁶ and Vantilt et al.¹⁷ reported the repulsive force could be observed at the interface between low-carbon steel and 33%CaO·20%Al₂O₃·40%SiO₂·6 7%MgO slag; see Fig. 4a. More specifically, Fig. 4b indicates that the repulsive force is larger when the size of the approaching inclusion is larger.¹⁷ According to these authors' understanding, whether the force is attractive or repulsive depends on the wetting behavior between the inclusion and the liquid steel and between the inclusion and slag at the interface.

In the original Refs. 16 and 17, the repulsive force was defined as positive and the attractive force as negative. In this article, the attractive force is defined as positive and the repulsive force as negative; see Fig. 4b. The value of the repulsive force is in the range of 1E-17 to 1E-16 N, which is quite weak. In this case, the inclusions can finally agglomerate, driven by the slag flow forces. Experimental evidence is shown in Fig. 5. It is found that the distance between inclusions decreases with prolonged time. Moreover, Wikström et al.^{23,26} reported a similar phenomenon at the interface between HC-CA steel and 40%CaO-20%Al₂O₃-40%SiO₂ slag. Figure 6 shows the distance between pairs of liquid 50%CaO·50%Al₂O₃ inclusions as a function of time. For convenience, each individual inclusion is labeled I, II, and III, the resulting pairs being I and II, I and III, and II and III. The free attraction behavior can be seen between 0 s and 4.5 s for inclusions I and II. Thereafter, the inclusions reach a neutral position until 16 s. After this, the liquid inclusions are forced together by the slag flow forces, as observed by Wikström et al.²³ For the



Fig. 4. Comparison of the attractive force and repulsive force between the inclusion and cluster in the low-carbon steel/33%CaO·20%A- I_2O_3 ·40%SiO₂·6 ~ 7%MgO slag interfaces.



Fig. 5. Comparison of distance between inclusions at the interfaces between LC-CA and LC-SMn steels and 33%CaO·20% Al_2O_3 ·40% SiO_2 ·6 ~ 7%MgO slag.



Fig. 6. Comparison of distance between inclusions at the interfaces between HC-CA steel and 40%CaO·20%Al₂O₃·40% SiO₂ slag.

other pairs of inclusions, I-III and II-III, the forced agglomeration is directly observed. By adjusting the starting time of the inclusion agglomeration in Fig. 5 to be the same as for inclusion pairs I-III and II-III, the agglomeration in Fig. 5 can also be considered the forced type because of the overlapping data. It should be noted that the term 'forced agglomeration' is used in Refs. 23 and 26, but not in the Refs. 16 and 17. Finally, the inclusion compositions were reported to change because of the reaction between the inclusion and the slag; however, no comment was made regarding the effect of this change.

Summary of In Situ Observation Experiments of Inclusion Agglomeration by HT-CLSM

In situ observations of inclusion agglomeration at the steel/Ar interface can be summarized as follows: Pairs of inclusions of like phases exhibit attraction, and the solid/solid pair shows the strongest attraction followed by the semi-liquid/semi-liquid pair and liquid/liquid pair. For the case of inclusion pairs with different phases, both an attractive force and a repulsive force exist. The difference depends on the physical properties, especially the contact angle between the inclusion and the liquid steel. For the inclusion agglomeration at the steel/slag interface, the force is more complicated. There is a longdistance-range attractive force and a repulsive force when the spacing is $< 100-150 \ \mu m$. Inclusions at the steel/slag interface can be forced to agglomerate by the slag flow.²³ The range of distance for different inclusion agglomerations at the steel/slag interface is different, but no explanation for this has been offered. This issue is worthy of detailed investigation in future work.

Besides the case of the steel/Ar and steel/slag interfaces, the inclusion agglomeration behavior in slag was reported by Wikström et al.²³ as well, and they claimed that the agglomeration of liquid inclusions was enhanced remarkably compared with inclusion agglomeration behavior at the steel/slag interface. However, dissolution of inclusions in the slag was not observed by these researchers. This may be due to the inclusion time because the inclusion in the slag is too short for observation. Lee et al.,²⁹ Miao et al.,³⁰ Monaghan et al.,³¹⁻³³ and Feichtinger et al.³⁴ observed various inclusions dissolved in the slag after it transferred across the steel/slag interface by using CLSM. The more systematic in situ observation work in combination with the inclusion agglomeration and dissolution in liquid slag can be considered in future work.

According to previous studies, the agglomeration behavior of Al_2O_3 , Al_2O_3 ·SiO₂, different types of Al_2O_3 ·CaO, MgO, Al_2O_3 ·MgO, Al_2O_3 ·Ce₂O₃, and complex inclusions containing Al, Si, Ca in low-/ high-carbon steels, and specific grades of stainless steels (AISI304, 16Cr) have been reported. However, the agglomeration behavior of other types of inclusions, such as Ti oxides and Ti–Al spinel, has not been reported extensively. Moreover, inclusion agglomeration behaviors in other kinds of high-alloy steels, for instance, high-manganese steel, have not been found in the open literature. In addition, physical property data, especially the contact angle for inclusions at the steel/Ar and steel/slag interfaces, are required to calculate the interaction force.

THEORETICAL STUDY OF INCLUSION AGGLOMERATION AT THE STEEL/AR AND STEEL/SLAG INTERFACES

To compare with the experimental evidence of inclusion agglomeration at the steel/Ar and steel/ slag interfaces observed by HT-CLSM, a theoretical study on summarizing the calculation methods of the attractive capillary force is introduced in this section. Subsequently, the coagulation coefficients for inclusion agglomeration as well as the Kralchevsky–Paunov model applied in high-manganese steel melts are calculated. The hybrid calculations, combining attractive capillary force and the coagulation coefficient, aim to give a quantitative comparison of the agglomeration/dispersion potencies of different inclusions in the steels.

Initial Theoretical Attractive Force Model

Yin et al.^{9,10} were the first to report that the difference of capillary pressure between the inside and outside of the inclusion pair can push the two inclusions toward each other when they are sufficiently closed; this is called capillary attraction. At that time, a quantitative model to calculate the capillary force between two inclusions at the steel/ Ar interface had not been developed, and Eqs. 6 and 7 were used as a preliminary estimation for the capillary force calculation.

$$F = 0.5g \times (\rho_L - \rho_G) \times w \times \Delta h^2 \tag{6}$$

$$\Delta h = 2\gamma \times \cos\theta/g \times (\rho_L - \rho_G) \times \delta \tag{7}$$

where Δh is the difference in the liquid surface height between the inside and outside of the inclusions, w is the width of the surface roughness of the particles, θ is the contact angle between the inclusion and liquid metal, γ is the surface tension, δ is the separation distance, and ρ_L and ρ_G are the densities of the liquid metal and the gas; however, ρ_G is always negligible.

This model gives the first method to evaluate the capillary force for inclusion agglomeration; however, it does not work properly. For instance, it is reported that Δh is calculated to be 0.6 m if using 50 μ m of δ , 7000 kg/m³ of ρ_L , 1.54 N/m of γ , 133° of θ , and 1.25 of surface roughness of Al₂O₃. To get a reasonable capillary force (3.5×10^{-14} N), 0.32 μ m of Δh is estimated.⁹ In this case, the model does not provide a reasonable evaluation of the attraction capillary force more accurately, the Kralchevsky–Paunov model was introduced.^{27,28}

Kralchevsky-Paunov model

According to the calculation of the attractive capillary force, Kralchevsky et al.²⁷ derived a general mathematical model for the energy and force balances between two spherical particles floating on the surface of a liquid phase, and this model is used at room temperature. Subsequently, Paunov et al.²⁸ provided simplified equations to calculate the capillary interaction between the two floating particles at the interface between liquid metal and Ar. Nakajima et al.^{11,12} were the first to apply this model to process metallurgy, calculating the capillary force for inclusion agglomeration at the liquid steel/Ar interface. However, the inclusions in the calculation are only defined as solid, liquid, and complex particles. The quantitative analysis for



Fig. 7. Schematic illustration of the capillary meniscus around two spherical inclusions at the steel/Ar interface.

specific kinds of inclusion was not made. Subsequently, Mu et al. 13,14 applied this model to compare with experimental data from in situ observation of large Al₂O₃ inclusions containing minor Ti-oxide inclusions, and reasonable agreement was obtained. Based on the agreement, parametric studies on the effects on the capillary force, inclusion size, surface tension of metal, inclusion density, and contact angle between the inclusion and metal were carried out, and finally the order of the capillary force for a range of oxides (Ce2O3, Al2O3, Ti2O3, MgO, CaO, TiO_2 , SiO_2 , different spinel oxides) at the interface between Ar and pure iron and medium carbon steel was evaluated quantitatively. In addition, this model has been applied in the in situ dynamic study of the bending deformation of an Al₂O₃ chain aggregate. It was concluded that the capillary force between two particles in an Al₂O₃ chain aggregate is the main driving force for the aggregate bending at the steel/Ar interface.¹⁵ The main equations for the model derivation are summarized here, and the details can also be found elsewhere.¹¹⁻¹⁴ Figure 7 shows the schematic illustration of the interaction force existing on a pair of spherical particles with radii R_1 and R_2 floating at the steel/Ar interface.

Equation 8 shows the capillary interaction energy, *W*, between spherical inclusions.

$$\Delta W = -\pi\gamma \sum_{k=1}^{2} (Q_k h_k - Q_{k\infty} h_{k\infty}) \left(1 + O\left(q^2 R_k^2\right)\right) \quad (8)$$

$$q = \sqrt{\frac{(\rho_{\rm I} - \rho_{\rm II})g}{\gamma}} \approx \sqrt{\frac{\rho_{\rm I}g}{\gamma}}, \quad \text{where } \rho_{I} \gg \rho_{II} \qquad (9)$$

where $\rho_{\rm I}$ and $\rho_{\rm II}$ are the densities of liquid iron/ steel(I) and Ar(II). q is the capillary length, defined by Eq. 9. g is acceleration due to gravity. γ is the surface tension of the liquid metal. The subscript krepresents inclusion 1 or 2 in a pair. O(x) is the zero function of the approximation. Subsequently, the capillary charges and height differences of the meniscus (Q_k and $Q_{k\infty}$, and h_k and $h_{k\infty}$) can be calculated using the following equations.

$$Q_k = rac{1}{2} q^2 b_k^2 igg(R_k - rac{1}{3} b_k igg) - rac{4}{3} D_k R_k^3 - r_k^3 h_k ~~(10)$$

In Situ Observations of Agglomeration of Non-metallic Inclusions at Steel/Ar and Steel/Slag Interfaces by High-Temperature Confocal Laser Scanning Microscope: A Review

$$\begin{split} h_k = & (\tau_k + 2\ln(1 - \exp(-2\tau_k))) - (Q_1 + Q_2)\ln(\gamma_e q a) \\ & + (Q_1 - Q_2) \left(A - (-1)^k \sum_{n=1}^{\infty} \frac{2}{n} \frac{\exp(-n\tau_k)\sinh n\tau_j}{\sinh n(\tau_1 + \tau_2)} \right) \end{split}$$
(11)

where *j* and *k* = 1 and 2 and $j \neq k$; $(qR_k)^2 \ll 1$. According to Fig. 7, R_k is the radius of inclusion *k*, b_k is the immersion depth, r_k is the capillary meniscus radius, ϕ is the angle according to the slope of the meniscus, α_k is the contact angle between the inclusion and the liquid metal at the steel/Ar interface, and *L* is the distance between the inclusions at the interface. γ_e is the Euler–Masceroni constant and is reported to equal 1.78.^{11,12,23} D_k is the density ratio. The parameters *A*, τ_k , and *a* are used for the simplification of Eq. 11. The calculation of each parameter described above can be seen in Eqs. 12–17.

$$A = \sum_{n=1}^{\infty} \frac{1}{n} \frac{\sinh n(\tau_1 - \tau_2)}{\sinh n(\tau_1 + \tau_2)}$$
(12)

$$\tau_k = \ln\left(\frac{a}{r_k} + \left(\frac{a^2}{r^2} + 1\right)^{0.5}\right) \tag{13}$$

$$a^{2} = \left(L^{2} - (r_{1} + r_{2})^{2}\right) \left(L^{2} - (r_{1} - r_{2})^{2}\right) / (2L)^{2} \quad (14)$$

$$D_k = R_k (1 + \cos(\alpha_k + \varphi_k)) \tag{15}$$

$$\varphi_k = rc \sin(Q_k/r_k)$$
 (16)

$$r_k = 0.5 \Big(R_k \sin lpha_k + \left(R_k^2 \sin^2 lpha_k + 4 Q_k R_k \cos lpha_k
ight)^{1/2} \Big)$$
(17)

Equation 11 can be simplified as Eq. 18 for the case of agglomeration of two inclusions with the same composition and size.

$$h'_{k} = Q_{k}(\tau_{k} + 2\ln(1 - \exp(-2\tau_{k}))) - (Q_{1} + Q_{2})\ln(\gamma_{e}qa)$$
(18)

When L approaches ∞ , $Q_{k\infty}$ and $h_{k\infty}$ become Eqs. 19 and 20.

$$Q_{k\infty} = \frac{1}{6}q^2 R_k^3 \left(2 - 4D_k + 3\cos\alpha_k - \cos^3\alpha_k\right) \quad (19)$$

$$h_{k\infty} = r_{k\infty} \sin \alpha_k \varphi_{k\infty} \frac{4}{\gamma_e q (1 + \cos \varphi_{k\infty})}$$
(20)

For the different values of L, the capillary force can be calculated as follows.

$$F = \frac{d(\Delta W)}{dL} \tag{21}$$

Here, the capillary interaction energy between two inclusion particles (ΔW) is expressed by the wetting contribution (ΔW_w) , meniscus surface tension

contribution (ΔW_m) , and gravity contribution (ΔW_g) ; see Eqs. 22–24. The detailed derivation has been reported by Kralchevsky and Paunov.^{27,28}

$$\frac{d\Delta W_w}{dL} = -\pi\gamma \sum_{k=1}^2 (qr_k)^2 R_k \cos \alpha_k \frac{dh_k}{dL} \times \left[1 + O\left(q^2 R_k^2\right)\right]$$
(22)

$$rac{d\Delta W_m}{dL} = \pi\gamma\sum_{k=1}^2 \Big[Q_k + (qr_k)^2 R_k \cos lpha_k\Big] rac{dh_k}{dL} imes \Big[1 + Oig(q^2 R_k^2)ig]$$
(23)

$$\frac{d\Delta W_g}{dL} = -\pi\gamma \sum_{k=1}^2 2Q_k \frac{dh_k}{dL} \left[1 + O(q^2 R_k^2)\right]$$
(24)

$$F = -\pi\gamma \sum_{k=1}^{2} Q_k \frac{dh_k}{dL} \left[1 + O(q^2 R_k^2) \right]$$
(25)

Here, a simplification has been made by Paunov et al.²⁸ The following Eq. 26 has been used.

$$h_k = h_{k\infty} + Q_j K_0(qL) \quad (j,k = 1, 2; j \neq k; r_k \ll L)$$
(26)

where the function $K_0(x)$ represents the modified Bessel function of zero order.³⁵

By substituting Eq. 26 into Eq. 25, the expression of Eq. 27 can be obtained.

 $F = 2\pi Q_1 Q_2 q K_1(qL) \left[1 + O(q^2 R_k^2)\right]$ $(r_k \ll L)$ (27) where $K_1(x)$ is the modified Bessel function of the first order, and its analogous formula can be seen in Eq. 28.

$$K_1(x) = \frac{1}{x} + O(x \ln x) \quad (x \to 0)$$
 (28)

By substituting Eq. 28 into Eq. 26, the simplification of the attractive capillary force calculation has been made. Paunov et al.²⁸ reported the final simplified equation can be expressed as Eq. 29; more details can be found in Ref. 28. This equation is made when the distance between two particles, L, is between r_k and q^{-1} .

$$F = 2\pi\gamma \frac{Q_1 Q_2}{L} \quad \left(r_k \ll L \ll q^{-1}\right) \tag{29}$$

$$O(x\ln x) \approx 0 \quad (x \to 0)$$
 (30)

Here, the assumption defined by Eq. 30 has been used in Ref. 28. Even if this is not directly reported, this assumption can be judged from expression of Eq. 29.

Mu et al.¹³ reported a revised approximation according to the L'Hôpital's rule,³⁷ and the expression of the capillary force can be seen in Eq. 31, which is the most recently developed equation to calculate the capillary force for inclusion agglomeration at the steel/Ar interface.

$$F = \frac{2\pi Q_1 Q_2 \left(1 - q^2 L^2\right)}{L} \quad (r_k \ll L) \qquad (31)$$

Application of the Capillary Force Model for Inclusion Agglomeration at the Interface Between Ar and High-Manganese Steel

The above-mentioned Kralchevsky–Paunov capillary force model has been applied to calculate inclusion agglomeration in pure iron and medium



Fig. 8. Comparison of the attractive capillary force of TiN and AIN at the interface between Ar and high manganese steel.



Fig. 9. Comparison of coagulation coefficient of different TiN and AIN in high manganese steel matrix.

carbon steel.^{13,14} The agglomeration tendency of different inclusions at the pure iron/Ar and medium carbon steel/Ar interfaces seems to be similar because of the similarity in the physical properties of pure iron and medium carbon steel, including the surface tension, density of liquid metal, and contact angle and interfacial energy between inclusion and liquid metal. Here, the capillary force model is applied to calculate the inclusion agglomeration behavior in high-alloy steel, and thus the agglomeration of nitride inclusions at the interface between Ar and high-manganese steel is discussed in this section, since nitride inclusions can be formed in the liquid state of high-manganese steel, whereas generally nitrides only precipitate in solid-state lowalloy steel. Specifically, AlN has been observed to be one of the major inclusion types in solidified highmanganese TWIP steel (Fe-16.83%Mn-0.58%C-2.1%Al-0.0041%N).³⁸ Moreover, equilibrium calculations show that AlN can form in the liquid state in high-manganese steel.³⁹ Park et al. reported the morphology of agglomerated AlN inclusions.⁴⁰ Also, Kikuchi et al.⁴¹ claimed the existence of TiN in lowcarbon high-manganese steel. In this case, the attractive capillary forces and coagulation coefficient representing the inclusion agglomeration potency in the liquid metal matrix of AlN and TiN are calculated, and the obtained results are shown in Figs. 8 and 9. The physical parameters used for this calculation were collected from the published literature $^{42-47}$ and are summarized in Table III. The calculation method of coagulation coefficient can be seen in Ref. 14. TiN was found to agglomerate more readily than AlN, and the calculation result of attractive capillary forces is in agreement with the result of the coagulation coefficient. The agglomeration behavior of oxide inclusions in highmanganese steel cannot be quantitatively discussed, because the relevant physical properties are not available in the open literature, which can be considered in future work.

SUMMARY

High-temperature confocal laser scanning microscope (HT-CLSM) is an effective technique to observe inclusion agglomeration in situ at steel/Ar and steel/slag interfaces. Previous studies focusing on this topic have been summarized in this article. The HT-CLSM methodology can be applied in a comprehensive study of the behaviors of various

Table III. Physical parameters used for the calculation of the capillary attractive force and the coagulation coefficient of inclusions in high-manganese steel $^{42-47}$

Inclusion	$\rho_1 ~(\mathrm{kg/m^3})$	$\rho_2 ~(\mathrm{kg/m^3})$	$\alpha_1(^\circ)$	α ₂ (°)	$\rho_{iron} \ (kg/m^3)$	$\gamma_{\rm M}~({\rm J/m^2})$	$\gamma_{\rm I} ({\rm J}/{\rm m}^2)$	$\gamma_{\rm IM}~({\rm J/m^2})$
TiN AlN	$\begin{array}{c} 5400\\ 3260\end{array}$	$\begin{array}{c} 5400\\ 3260\end{array}$	$\begin{array}{c} 123 \\ 132 \end{array}$	$\begin{array}{c} 123 \\ 132 \end{array}$	6600 6600	$\begin{array}{c} 1.686\\ 1.304\end{array}$	$\begin{array}{c} 1.29 \\ 0.88 \end{array}$	$\begin{array}{c} 2.221 \\ 1.768 \end{array}$

types of inclusions at different steel/Ar and steel/ slag interfaces in the future work. Besides experimental work, a modified agglomeration model previously proposed by the current authors is also used to calculate the agglomeration behavior of nitride inclusions at the high-manganese steel/Ar interface, and the result shows that TiN agglomerates more easily than AlN in high-manganese steel. Calculations for oxide inclusions in high-manganese steel are not available because of the lack of physical properties. The measurement of the contact angle and interfacial energy between the inclusion and liquid steel could assist in developing a comprehensive understanding of inclusion agglomeration behavior in different steel grades.

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