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REVIEW ARTICLE

Review of Quantum Image Processing

Zhaobin Wang[1] · Minzhe Xu[1] · Yaonan Zhang[2]

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Abstract

As an interdisciplinary between quantum computing and image processing, quantum image processing provides more possibilities for image processing due to the powerful parallel computing capabilities of quantum computers. In recent years, quantum image processing attracts more and more researcher's attention. In order to allow researchers to better understand quantum image processing technology, we have reviewed relevant literature in recent years in the paper. First, the background and mathematical concepts of quantum computing are introduced. Then, the research progress of quantum image processing is sorted out and summarized in the fileds of quantum image representation, geometric transformation, image encryption, edge detection, image segmentation, filtering and compression. Finally, we have discussed the advantages and disadvantages of quantum image processing, and pointed out the potential future research.

1 Introduction

In the past few decades, the computing power of electronic computers has increased exponentially in accordance with Moore's Law [1]. However, due to the limitations of many objective factors, the computing power of single-core CPUs has not increased significantly in recent years. Therefore, it is necessary to explore other ways to increase computing power. In 1982, Feynman proposed a new computing model, named quantum computer, which is based on quantum mechanics and can use the superposition and entanglement characteristics of quantum mechanics to store, process and transmit information, and its computing power is much higher than classical computer [2]. In 1994, Shor proposed a quantum algorithm for prime factor decomposition [3], and in 1996, after Grover proposed a quantum search algorithm [4], the computing power of quantum computers was proved, and quantum computing began to be paid more and more attention, and appeared in various fields of computer science.

In order to solve the problem of image processing on quantum computers, classical image processing algorithms are accelerated by quantum computing, and quantum image processing algorithms are gradually emerging.

Digital image processing is an important part of information science and the core program of many applications. Image processing algorithms have the characteristics of many parallel operations. In classical algorithms, a large number of operations are required. In recent years, with the rapid development of pattern recognition, image understanding, and image sensors, the number of images has become more and more, and the size of images has increased, the algorithms become more and more complex. It makes classical image processing algorithms often require a lot of time, and the hardware requirements of image processing algorithms are gradually increasing. Therefore, it is necessary to find high-performance methods to store and process these images. Unlike classical image processing, quantum computers store images in qubits. Utilizing the superposition state and entangled state of quantum, quantum computers have good parallel processing capabilities. The complexity of storing n-bit long sequences on a classical computer requires $O(n \times 2^n)$, but only $O(n)$ on a quantum computer. When performing operations on sequences, such as bitwise inversion, classical computers need to calculate sequentially, with a complexity of $O(2^n)$, or a large number of repeated arithmetic circuits, and quantum computers can operate

✉ Zhaobin Wang
zhaobin_wang@hotmail.com

Yaonan Zhang
yaonan@lzb.ac.cn

1 School of Infomation Science and Engineering, Lanzhou University, Lanzhou 730000, China

2 National Cryosphere Desert Data Center, Northwest Institute of Eco-Environment and Resources, Chinese Academy of Sciences, Lanzhou 730000, China

on each bit at the same time. Only $O(1)$ is needed, which is helping to speed up image processing algorithms with many parallel calculations. The space required to store the image and the circuit scale required to process the image are greatly reduced, so the efficiency of the algorithm is greatly improved.

Quantum image processing roughly requires three parts. The digital image is represented on a quantum computer, and then the image is processed on the quantum computer using a quantum algorithm, and finally the processed quantum image is converted into a classical image. To process images on a quantum computer, the images need to be stored in a quantum system. In order to solve such problems, many quantum image representation methods have been proposed. First of all, in 2003, the Qubit Lattice representation method was proposed [5], which is similar to the classical method that treats the image as a matrix, and each qubit stores only one pixel. It is the first attempt at quantum image representation, which is closer to the classical way, but it does not use quantum superposition and is more complex. In 2005, Real Ket's representation method was proposed [6], which used quantum superposition state for the first time.

The entangled image model proposed in 2010 uses the entangled state [7]. In the same year, a flexible representation of quantum images (FRQI) was proposed [8], which represented grayscale information as an angle, stored in the probability amplitude of a qubit, used n qubits to represent the pixel number of a square image, and represented the grayscale qubits are entangled with them so that the coordinates correspond to the grayscale. The multi-channel representation for quantum images (MCRQI), which extends it to RGB, was proposed the following year [9]. This method makes full use of quantum superposition and entanglement, and can perform the same operation on all pixels at the same time, thereby solving the real-time calculation problem in image processing applications. Since then, many image processing algorithms have also been proposed based on it. However, FRQI only uses one qubit to store the grayscale information of each pixel in the image, so some complex color operations are not easy to perform on the basis of FRQI. The novel enhanced quantum representation (NEQR) [10], which was proposed in 2013, uses eight qubits to represent grayscale information. It stores the grayscale information in an eight-bit binary format in the qubit sequence. Although the use of qubits is increased, it solves the disadvantage that FRQI grayscale information cannot be accurately measured within a limited number of times, and facilitates color operations. It has also become a more widely used image representation model in quantum image processing. Since then, more improvements have also been proposed. On the basis of these image representation methods, many quantum image processing algorithms have been proposed. Quantum versions of some classical image algorithms are proposed. Compared with their classical versions, they have an exponential reduction in complexity. Many algorithms have been proposed for image translation, image scaling, morphological transformation, edge detection, image encryption, watermarking, filtering, compression, and more complex image matching and quantum machine learning.

Quantum computing can bring huge space savings and speed improvements to digital image processing. Quantum image processing is an emerging field that still needs to be developed and supplemented. With the gradual advancement of quantum computers, the need for quantum image processing is becoming more and more urgent. With the efforts of researchers, it is believed that more representation methods and processing algorithms will be proposed.

The remaining part of the paper proceeds as follows. A brief introduction to basic concepts is provided in Sect. 2. And the survey will review the methods related to quantum image representation in Sect. 3; geometric transformation and morphological transformation in Sect. 4; quantum image protection in Sect. 5; edge detection and image segmentation in Sect. 6; similarity analyses and image matching in Sect. 7; quantum image filtering and compression in Sect. 8; quantum machine learning in Sect. 9. And other algorithms will be reviewed in Sect. 10. An analysis of possible future work is presented in Sect. 11. Finally, we conclude in Sect. 12.

2 Basic Concepts

2.1 Vector

In quantum computing, we pay attention to the state of quantum. A quantum state can be represented by a vector in Hilbert space. The standard quantum mechanical notation for a vector in Hilbert space is as follows:

$$|\psi\rangle = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \tag{1}$$

where the symbol $|\cdot\rangle$ is called Dirac symbol, which is a standard symbol of state in quantum mechanics, used to indicate that an object is a vector. ψ is the label of the vector. $|\psi\rangle$ is called *ket*, and the corresponding *bra* is $\langle\psi|$, which is the conjugate transpose of the *ket*.

2.2 Tensor Product

The tensor product is a way to put vector spaces together to form a larger vector space. It satisfies the following basic properties:

(1) For any scalar z, any element $|v\rangle$ in V space, any element $|w\rangle$ in W space

$$z(|v\rangle \otimes |v\rangle) = (z|v\rangle) \otimes |w\rangle = |v\rangle \otimes (z|w\rangle) \tag{2}$$

(2) For any $|v_1\rangle$ and $|v_2\rangle$ in V space, any $|w\rangle$ in W space

$$(|v_1\rangle + |v_2\rangle) \otimes |w\rangle = |v_1\rangle \otimes |w\rangle + |v_2\rangle \otimes |w\rangle \tag{3}$$

(3) For any $|v\rangle$ in V space, any $|w_1\rangle$ and $|w_2\rangle$ in W space

$$|v\rangle \otimes (|w_1\rangle + |w_2\rangle) = |v\rangle \otimes |w_1\rangle + |v\rangle \otimes |w_2\rangle) \tag{4}$$

Assuming that A is a $m \times n$ matrix and B is a $p \times q$ matrix, then $A \otimes B$ is shown in Eq.5

$$A \otimes B \equiv \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & A_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{bmatrix} \tag{5}$$

In addition, $A^{\otimes k}$ means that matrix A has done k tensor product operations on itself.

2.3 Qubit

Bit is the basic concept of classical computing. Quantum computing is based on a similar concept, namely qubit. In most cases, qubits are treated as abstract mathematical objects. In this way, the theory of quantum computing can be constructed freely, without relying on specific systems for implementation.

A classical bit has two states, 0 and 1. The two possible states of a qubit are $|0\rangle$ and $|1\rangle$, which correspond to states 0 and 1 of the classical bit. The difference between a bit and a qubit is that a qubit can be in a state other than $|0\rangle$ or $|1\rangle$. It can exist in the continuous state between $|0\rangle$ and $|1\rangle$. Called superposition state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{6}$$

where, α and β are called probability amplitudes. The state of a qubit is a vector in a two-dimensional complex vector space. The states $|0\rangle$ and $|1\rangle$ are called calculation base states, and constitute a set of orthonormal basis of this vector space.

When measuring the superposition state $|\psi\rangle$, only one component state can be gotten and other states disappear. The probability of $|0\rangle$ being measured is $|\alpha|^2$, the probability of $|1\rangle$ being measured is $|\beta|^2$. And:

$$|\alpha|^2 + |\beta|^2 = 1 \tag{7}$$

In quantum computing, $|0\rangle$ and $|1\rangle$ are usually expressed as:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{8}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{9}$$

Suppose there are two qubits. A two-qubit system has four calculation ground states, which are $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. A pair of qubits can exist in the superposition of these four states, so the state vector describing the two qubits is

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle\alpha_{10}|10\rangle\alpha_{11}|11\rangle \tag{10}$$

where α_x is the probability amplitude corresponding to state $|x\rangle$ and satisfies:

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1 \tag{11}$$

Similar to the case for a single qubit, for a two qubit system:

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

$$|10\rangle = |1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 1 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{14}$$

$$|11\rangle = |1\rangle \otimes |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 1 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{15}$$

There is a special case called entanglement. For example, two two-quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ are shown below.

$$|\psi_1\rangle = \alpha_1|00\rangle + \beta_1|11\rangle \tag{16}$$

$$|\alpha_1|^2 + |\beta_1|^2 = 1 \tag{17}$$

$$|\psi_2\rangle = \alpha_2|10\rangle + \beta_2|11\rangle = |1\rangle \otimes (\alpha_2|0\rangle + \beta_2|1\rangle) \tag{18}$$

$$|\alpha_2|^2 + |\beta_2|^2 = 1 \tag{19}$$

In $|\psi_1\rangle$, if the first quantum is in state $|0\rangle$, the second quantum also is in state $|0\rangle$; if the first quantum is in state $|1\rangle$, the second quantum also is in state $|1\rangle$. They entangle togther. But in $|\psi_2\rangle$, the first quantum is in state $|1\rangle$, the second quantum also is in state $(\alpha_2|0\rangle + \beta_2|1\rangle)$.

2.4 Gate and Circuit

Similar to logic gates in classical digital circuits, logic gates are also used in quantum circuits to manipulate information, and logical transformations are achieved by unitary transformations of quantum states. The quantum gate can be represented in matrix form.

A quantum logic gate that only requires one qubit to participate is called a single quantum gate. As shown in the Fig. 1, the horizontal line represents a wire, which represents a qubit, and U represents a single quantum gate. Input a qubit $|\psi\rangle$ and output $U|\psi\rangle$.

Expressed in matrix form:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, U = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{20}$$

$$U|\psi\rangle = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{bmatrix} \tag{21}$$

Table 1 shows the commonly used single quantum gates.

Hadamard gate:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{22}$$

If applying it to $|0\rangle$ or $|1\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \tag{23}$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \tag{24}$$

Therefore, the H gate has the ability to convert $|0\rangle$ into a superposition state with equal probability of $|0\rangle$ and $|1\rangle$.

Pauli-X:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{25}$$

It can flip the state of a single qubit, so it is also called a NOT gate:

$$NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{26}$$

Corresponding to a single quantum gate, there are also gates that require multiple qubits to participate. As shown in the Table 2, there are commonly used multiple quantum gates. When multiple qubits participate, there is a relationship between control and being controlled, such as CNOT gate.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{27}$$

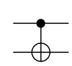
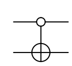
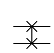
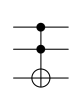
Fig. 1 Single quantum gate

$$|\psi\rangle \;-\boxed{U}-\; U\,|\psi\rangle$$

Table 1 Commonly used single quantum gates

Gate type	Circuit	Matrix
Hadamard	$-\boxed{H}-$	$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Pauli-X	$-\boxed{X}-$	$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
NOT	$-\oplus-$	$NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y	$-\boxed{Y}-$	$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Phase	$-\boxed{S}-$	$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
Identity	$-\boxed{I}-$	$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Table 2 Commonly used multiple quantum gates

Gate type	Circuit	Matrix
CNOT		$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
0CNOT		$0CNOT = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Swap		$swap = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli		$Toffoli = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

The first qubit is a control qubit, and the second qubit is controlled by the first qubit and is called a target qubit. When the first qubit is $|1\rangle$, the second qubit is reversed. As shown in Eqs.(28-31):

$$CNOT|00\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle \tag{28}$$

$$CNOT|01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle \tag{29}$$

$$CNOT|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle \tag{30}$$

$$CNOT|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle \tag{31}$$

Such properties can be used to implement XOR operations.

On the contrary, there are also 0CNOT gates:

$$0CNOT = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

If the control qubits of the CNOT gate are increased to two, this kind of gate is called Toffoli gate. If the number of control qubits continues to increase to n, it is called an n-CNOT gate.

In addition, there is a swap gate that can exchange the states of two qubits:

$$swap = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

2.5 Quantum Fourier Transform

The quantum Fourier transform is defined as a linear transformation on n qubits [11], and its mapping is as follows

$$|F\rangle \longrightarrow \frac{1}{\sqrt{2^n}} \sum_{f=0}^{2^n-1} e^{2\pi jfF/2^n} |f\rangle, \quad j = \sqrt{-1} \tag{34}$$

The inverse transformation mapping relationship is

$$|f\rangle \longrightarrow \frac{1}{\sqrt{2^n}} \sum_{F=0}^{2^n-1} e^{-2\pi jfF/2^n} |F\rangle \tag{35}$$

Express $F = F_1 2^{n-1} + F_2 2^{n-2} + \cdots + F_n 2^0$ as $F = F_1 F_2 \cdots F_n$, and express $F_\varrho/2 + F_{\varrho+1}/2^2 + \cdots + F_m/2^{m-\varrho+1}$ as $0.F_\varrho F_{\varrho+1} \cdots F_m$, then the quantum Fourier transform of $|F\rangle = |F_1 F_2 \cdots F_n\rangle$ is expressed as:

$$\begin{aligned} |F_1 F_2 \cdots F_n\rangle \rightarrow & \frac{1}{2^{n/2}} \left(|0\rangle + e^{2\pi j 0.F_n}|1\rangle\right) \\ & \cdot \left(|0\rangle + e^{2\pi j 0.F_{n-1}F_n}|1\rangle\right) \cdots \\ & \cdot \left(|0\rangle + e^{2\pi j 0.F_1F_2\cdots F_n}|1\rangle\right) \end{aligned} \tag{36}$$

3 Quantum Image Representations

To process images on quantum computers, the images need to be stored in a quantum system. For this reason, various quantum image representations have been proposed. First in 2003, Qubit Lattice was proposed [5], similar to the classical approach, the image is stored as a matrix, each qubit stores only one pixel. It is the first attempt at quantum image representations. In 2005, Real Ket was proposed [6], which used quantum superposition for the first time. Entangled image proposed in 2010 uses quantum entanglement [7]. In the same year, FRQI was proposed by Le et al. [8], it stores the color information as an angle, and stores the coordinate information in an n-qubit sequence. This model represents the image as follows:

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} \left(\cos\theta_i|0\rangle + \sin\theta_i|1\rangle\right) \otimes |i\rangle \tag{37}$$

$$\theta_i \in [0, \frac{\pi}{2}], i = 0, 1, \cdots, 2^{2n} - 1 \tag{38}$$

where $|i\rangle (i = 0, 1, \cdots, 2^{2n} - 1)$ are 2^{2n} computational basis quantum states, and $\theta = (\theta_0, \theta_1, \cdots, \theta_{2^{2n}-1})$ is the vector of angles encoding colors. Gray scale information is encoded with $\cos\theta_i|0\rangle + \sin\theta_i|1\rangle$, and coordinate information is expressed with $|i\rangle$. This allows the grayscale information and coordinate information to correspond accurately. The superposition of each pixel information reduces the use of storage space.

The following year, Sun et al. [9] extended it to the RGB space and MCRQI was proposed. Three qubits are used to store RGB color information and opacity information. Represent the image as:

$$|I(\theta)\rangle = \frac{1}{2^{n+1}} \sum_{i=0}^{2^{2n}-1} |c^i_{RGB\alpha}\rangle \otimes |i\rangle \tag{39}$$

$$\begin{aligned} |c^i_{RGB\alpha}\rangle = &\cos\theta_{Ri}|000\rangle + \cos\theta_{Gi}|001\rangle \\ &+ \cos\theta_{Bi}|010\rangle + \cos\theta_{\alpha i}|011\rangle \\ &+ \sin\theta_{Ri}|100\rangle + \sin\theta_{Gi}|101\rangle \\ &+ \sin\theta_{Bi}|110\rangle + \sin\theta_{\alpha i}|111\rangle \end{aligned} \tag{40}$$

where $\theta_{Ri}, \theta_{Gi}, \theta_{Bi}, \theta_{\alpha i}$ are the vectors encoding the colors of the R,G,B and α channels respectively. In a $2^n \times 2^n$ image, the color information is encoded with the probability amplitude of three qubits, and the coordinate information is represented with $2n$ qubits.

FRQI uses a normalized superposition to store all the pixels in an image, the same operations can be performed simultaneously on all pixels, and therefore FRQI can resolve the real-time computation problem of image-processing applications. Since then, many image processing algorithms have been proposed based on it. However, because FRQI uses only a single qubit to store the gray-scale information for each pixel in an image, some complex color operations cannot be done easily on the basis of FRQI. The NEQR proposed in 2013 uses two entangled qubit sequences to store the grayscale and position information, and stores the whole image in the superposition of the two qubit sequences. It stores the grayscale information in binary form in q qubits [10]. Because it binary encodes grayscale, this model is more convenient to use. The range 2^q binary grayscale information is stored in a q-qubit sequence, and the coordinate information of the $2^n \times 2^n$ image is stored in a $2n$-qubit sequence. The expression for a quantum image can be written as:

$$|I\rangle = \frac{1}{2^n} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} \bigotimes_{i=0}^{q-1} |c^i_{yx}\rangle |yx\rangle \tag{41}$$

In the preparation procedure for NEQR, the first step is to prepare $q + 2n$ qubits in $|0\rangle$, and apply I-gates and H-gates to the initial state. Then, set the gray-scale value for all pixels by $2n$-CNOT gates. For example, when preparing a grayscale image with a size of 2×2 as shown in Fig. 2 into NEQR model. First apply the H-gate to two of the ten $|0\rangle$ qubits to store the coordinate information, and then use the 2-CNOT gate to set the grayscale information in turn. The circuit is shown in Fig. 3.

Although more qubits are used, NEQR solves the problem that the FRQI grayscale information cannot be accurately measured within a limited number of times, and it facilitates color operations. It has also become a more commonly used model in quantum image processing. Unfortunately, these models have a disadvantage that they can only store square images. To solve this problem, The improved novel enhanced quantum representation (INEQR) changed the horizontal and vertical coordinates to unequal lengths [12]. The generalized quantum image representation (GQIR) uses logarithmic coordinates to the point of being able to represent rectangular images of arbitrary size [13], but this inevitably leads to redundancy. In addition, The novel quantum representation for log-polar images (QUALPI) realizes the representation of polar coordinates [14]. In 2014, Li et al. proposed a new representation method for multidimensional color images, called n-qubit normal arbitrary superposition state (NASS). They store grayscale in angles and use states to represent multiple dimensions. For the first time on a quantum computer to achieve a multi-dimensional color image compression [15]. In 2016, Li et al. [16] improved the FRQI, a flexible representation for quantum color image (FRQCI) is proposed. This model uses probabilistic amplitude (Eqs. 42–45) or phase (Eqs. 46–47) to store the RGB information of the pixels.

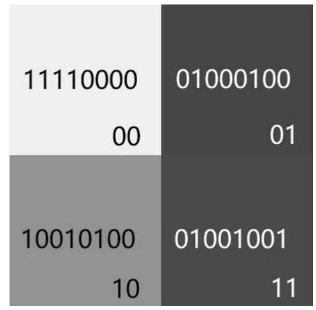


Fig. 2 The original image

$$|I(\theta,\phi)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} |i\rangle |c_i\rangle \tag{42}$$

$$|c_i\rangle = \cos\frac{\theta_i}{2}|0\rangle + e^{j\phi_i}\sin\frac{\theta_i}{2}|1\rangle \tag{43}$$

$$\theta_i = \frac{\left(c^R_i \times 2^{16} + c^G_i \times 2^8 + c^B_i\right)\pi}{2^{24}-1} \tag{44}$$

$$\phi_i = 0 \tag{45}$$

$$\theta_i = 0 \tag{46}$$

$$\phi_i = \frac{\left(c^R_i \times 2^{16} + c^G_i \times 2^8 + c^B_i\right)2\pi}{2^{24}-1} \tag{47}$$

In the same year, Sang et al. [17] improved MCRQI and NEQR and proposed a novel quantum representation of color digital images (NCQI). This model changes the q

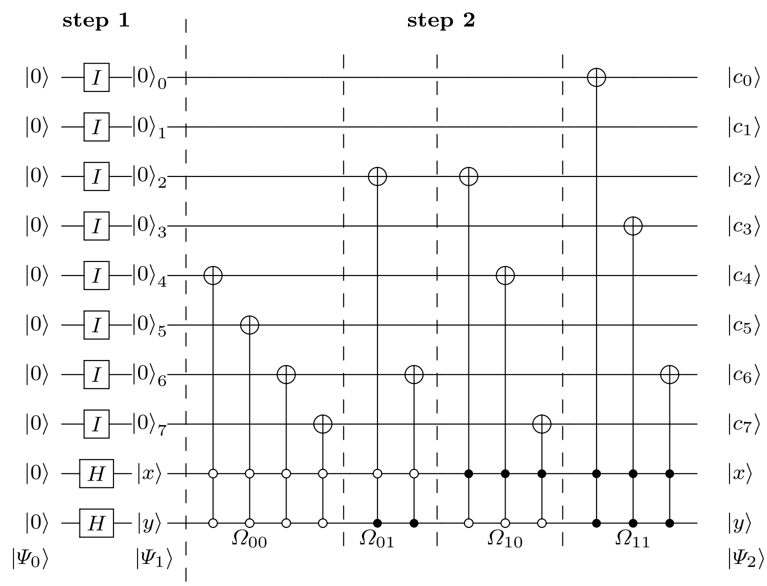


Fig. 3 Quantum circuit for NEQR preparation [10]

qubits representing gray scale to $3q$ on the basis of NEQR, which is used to represent the three channels of RGB. The time complexity of preparing the NCQI quantum image experiences an approximately quadratic decrease compared to MCRQI, and many color operations can be executed conveniently based on NCQI, especially some complex color transformation. But this model has the disadvantage of using too many qubits. In order to solve this drawback, in 2018, Liu et al proposed an optimized quantum representation for color digital images (OCQR) [18]. Compared with NCQI, OCQR uses nearly one-third times the qubits to store the pixel value. The time complexity of preparing these two models are almost the same. Not only does OCQR use fewer qubits to save computing resources, but it also allows some color transformations to be performed more efficiently. In 2017 Abdolmaleky et al. [19] expanded NEQR to RGB, a new quantum multi-channel Red–Green–Blue (RGB) representation (QMCR) is proposed. This model uses more qubits than MCRQI, but reduces the complexity of preparation procedure and can accurately retrieve images. Jiang et al. [20] proposed a three-dimensional quantum representation for digital images, named quantum point cloud. This model stores 3D images as:

$$|P\rangle = \frac{1}{2^{\frac{m_i}{2}}} \sum_{i=0}^{N-1} |i\rangle \otimes |x_i y_i z_i\rangle \tag{48}$$

$$m_i = \begin{cases} \lceil \log_2 N \rceil, & N > 1 \\ 1, & N = 1 \end{cases} \tag{49}$$

where, i is the counting number, x_i, y_i, z_i are the three dimensional coordinates.

In 2018, Li et al. [21] proposed a bitplane representation of quantum images (BRQI), and based on BRQI proposed complement of colors, reverse of bitplanes, and translation of bitplanes based on BRQI. This model improves NEQR, divides the gray value into 8 binary bits, decompose the grayscale image into 8 binary images. Three qubits are used to represent the number of the bitplanes, n qubits are used to represent the coordinate information, and one qubit is used to store the binary grayscale information of each bitplane.

When representing grayscale images:

$$|\Psi_B^8\rangle = \frac{1}{\sqrt{2^{n+3}}} \sum_{l=0}^{2^3-1} \sum_{x=0}^{2^{n-k}-1} \sum_{y=0}^{2^k-1} |g(x,y)\rangle |x\rangle |y\rangle |l\rangle \tag{50}$$

where $|x\rangle$ and $|y\rangle$ represent coordinates, $|l\rangle$ represents the number of the bitplanes, $|g(x,y)\rangle$ represents the gray level at this coordinate of the bitplane.

When representing RGB images:

$$|\Psi_B^R\rangle = \frac{1}{\sqrt{2^{n+3}}} \sum_{l=0}^{2^3-1} \sum_{x=0}^{2^{n-k}-1} \sum_{y=0}^{2^k-1} |g_R(x,y)\rangle |x\rangle |y\rangle |l\rangle \tag{51}$$

$$|\Psi_B^G\rangle = \frac{1}{\sqrt{2^{n+3}}} \sum_{l=0}^{2^3-1} \sum_{x=0}^{2^{n-k}-1} \sum_{y=0}^{2^k-1} |g_G(x,y)\rangle |x\rangle |y\rangle |l\rangle \tag{52}$$

$$|\Psi_B^B\rangle = \frac{1}{\sqrt{2^{n+3}}} \sum_{l=0}^{2^3-1} \sum_{x=0}^{2^{n-k}-1} \sum_{y=0}^{2^k-1} |g_B(x,y)\rangle |x\rangle |y\rangle |l\rangle \tag{53}$$

$$|\Psi_B^{24}\rangle = \frac{1}{\sqrt{3}}(|\Psi_B^R\rangle|01\rangle + |\Psi_B^G\rangle|10\rangle + |\Psi_B^B\rangle|11\rangle) \tag{54}$$

The preparation procedure for BRQI is similar to the NEQR. BRQI requires $n+4$ qubits with a state of $|0\rangle$. First apply Hadamard gates to $|x\rangle$, $|y\rangle$ and $|l\rangle$, then use $(n+3)$–CNOT gates to operate on each pixel.

When preparing RGB images, as shown in Fig. 4. Two qubits are added to distinguish the three planes of R, G, and B, $|01\rangle$ represents R, $|10\rangle$ represents G, and $|11\rangle$ represents B. The $R_x[\arctan(\sqrt{2})]$ gate and the 0CNOT gate and the controlled H gate set the probabilities of the three states of $|01\rangle$ and $|10\rangle$ and $|11\rangle$ to be equal. $U_R U_G U_B$ is composed of $(n+3)$–CNOT gates, which are used to operate the color information of the RGB plane. Compared with NEQR and NCQI, this model can reduce the qubits of stored images. When performing some special image processing algorithms, such as inverting and operating on the entire bitplane, it can greatly reduce the complexity. Compared with NEQR and INEQR, the storage capacity of BRQI improves 16 times. For color images, compared with NCQI, it omproves 2^{18} times.

Coincidentally, Wang et al. [22] also utilized a bitplane in the quantum representation model of color digital images in 2019, called QRCI, where the color information is encoded by the basis states of qubit sequences.

$$|I\rangle = \frac{1}{\sqrt{2^{2n+3}}} \sum_{l=0}^{2^3-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} |R_{lyx} G_{lyx} B_{lyx}\rangle \otimes |lyx\rangle \tag{55}$$

QRCI model utilizes $2n+6$ qubits to store a color digital image with size $2^n \times 2^n$. Compared with the existing NCQI representation model, the storage capacity of QRCI improves 2^{18} times. Compared with the way of storing RGB separately in BRQI, fewer qubits are used and the preparation procedure is more concise. In 2018, in order to solve the storage of multi-spectral images on quantum computers, Sahin et al. [23] proposed quantum representation of multi-wavelength images (QRMW). This model uses a quantum sequence to specifically store the channel number.

$$|I\rangle = \frac{1}{\sqrt{2^{b+n+m}}} \sum_{\lambda=0}^{2^b-1} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^m-1} |f(\lambda, y, x)\rangle \otimes |\lambda\rangle \otimes |yx\rangle \tag{56}$$

The number of qubits used in MCQI model is less than in the QRMW model for fewer channels. However, as the number of channels increases, the number of qubits used in the QRMW model is less than in the MCQI models, more suitable for multi-spectral images with a large number of channels. Li et al. [24] proposed an improved FRQI model called the improved FRQI model of representing the color image (FRQCI), and provided several simple image processing operators for color and pixel position.

In 2019, after analyzing FRQI and NEQR, Khan proposed an improved flexible representation of quantum images (IFRQI) [25], which expressed each two bits as an angle in the probability amplitude, and successfully used a qubit to encode 2-bit grayscale information. It assists in accurate retrieval of original image information. In the same year, Wang et al. [26] proposed a quantum indexed image representation method (QIIR). A quantum indexed image consists of a quantum data matrix and a quantum palette matrix. Each data structure is based on the basic states of qubit sequence to represent information, including pixel positions and pixel values in the data matrix, and indexes and color values in the palette matrix. Indexed images are usually used to store images with simple color requirements, and images with complex colors require true colors. Xu et al. [27] proposed order-encoded quantum image model (OQIM). Li et al. [28] proposed four models of quantum signal representation (QSR) for integer, real, and complex signals. These models provided the foundation of more complex applications in quantum signal processing. In 2020, Grigoryan et al. [29] proposed an Fourier transform representation (FTQR), which stored images in a new way. Wang et al. [30] proposed a double quantum color images representation model (DQRCI) which can store two color digital images simultaneously into a quantum superposition state.

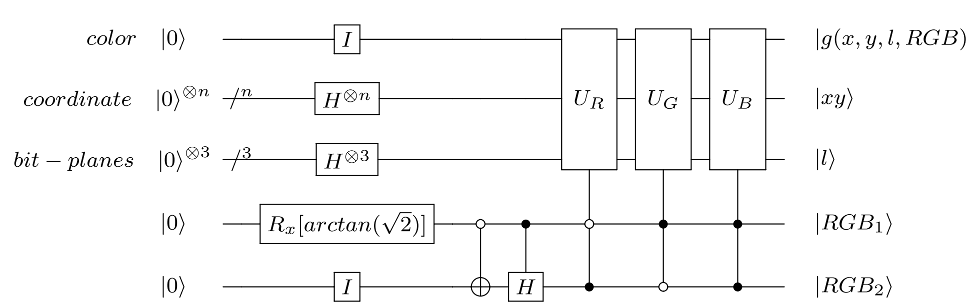


Fig. 4 The implementation circuit of BRQI for RGB color images [21]

4 Geometric Transformation and Morphological Transformation

Geometric transformation and morphological transformation are relatively mature operations in classical image processing, and almost all image processing software has these features. For example, image translation, image scaling. Many image processing also use two basic morphological transformations, erosion and dilation.

In 2011, Le et al. [31] proposed three design strategies for constructing new geometric transformations on quantum images from other transformations. The strategies focus on the affected areas in the images, the separability, and smoothness of the transformations by exploiting a representation of images on quantum computers extensively. Image translation, which maps the position of each image element into a new position, is a basic image transformation. Although it has been deeply researched and widely used in classical image processing, its quantum version is a vacancy. In 2015, Wang et al. studies the quantum image translation for the first time. Two types of quantum image translation entire translation and cyclic translation based on NEQR was proposed [32]. In 2016, Fan et al. [33] proposed a quantum algorithm for geometric transformation based on NASS (two-point switchings, symmetric flip, local flip, orthogonal rotation and translation). They provide global operators and local operators respectively, and prove that, contrary to classical images, global operators are faster than local operators in quantum image processing. In 2017, Zhou et al. designed two kinds of quantum image translation based on FRQI, including global translation and local translation. Global translation is implemented by adder modulo N, and Gray code is employed in local tranlstion [34], global translation is presented in Fig. 5, local translation is presented in Fig. 6.

Image scaling is one of the most frequently used and basic operations in image processing, can adjust the size of digital images. In image scaling, it is necessary to use interpolation methods to create new pixels when the image is scaled up, or to delete redundant pixels when the image is scaled down. The commonly used interpolation methods include nearest-neighbor interpolation, bilinear interpolation and bicubic interpolation [35]. Scaling up the image can make the details clearer, and scaling down the image can save image storage space.

In 2017, Zhou et al. [36] proposed quantum multidimensional color image scaling based on nearest-neighbor interpolation and the bilinear interpolation method for NEQR [37], which extended FRQI to multi-dimensional color model. Expressed as

$$|I(\theta)\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} \left(\cos\theta_i|0\rangle + \sin\theta_i|1\rangle\right) \otimes |i\rangle \tag{57}$$

$$|i\rangle = |d_1\rangle|d_2\rangle \cdots |d_k\rangle \tag{58}$$

$$\theta = \frac{\pi}{2} \times \frac{c}{2^{24}-1}, c = R \times 2^{16} + G \times 2^8 + B \tag{59}$$

Zhou et al. [35] proposed a quantum algorithm to scale up and scale down quantum images based on bilinear

Fig. 5 Global translation [34]

Fig. 6 Local translation [34]

(a) Original image (b) Bilinear interpolation

Fig. 7 Scaling up [38]: **a** Original image (90 × 80); **b** The image after Scaling up (192 × 224)

(a) Original image (b) Bilinear interpolation

Fig. 8 Scaling down [38]: **a** Original image (256 × 256); **b** The image after Scaling down (60 × 100)

interpolation with integer scaling ratio based on GQIR. Quantum images with arbitrary size $h \times w$ can be scaled to $h' \times w'$. Compared with nearest-neighbor interpolation, bilinear interpolation and bicubic interpolation have better effect on target image. In 2019, Zhou et al. improved their algorithm. A quantum algorithm based on bilinear interpolation with arbitrary scaling ratio is proposed to resize quantum images [38]. Firstly, a quantum image with arbitrary size $h \times w$ is described by GQIR. Then, utilizing bilinear interpolation, new pixels are created when scaling up, or redundant pixels are deleted when scaling down as shown in Figs. 7 and 8.

$$y = \left\lfloor y' \times \frac{h}{h'} \right\rfloor, x = \left\lfloor x' \times \frac{w}{w'} \right\rfloor \tag{60}$$

$$d_y = \frac{h}{h'} y' - y, d_x = \frac{w}{w'} x' - x \tag{61}$$

This algorithm breaks the previous situation where quantum image scaling ratio was just an integer. Change the Eq.62 in the previous algorithm to Eq.63.

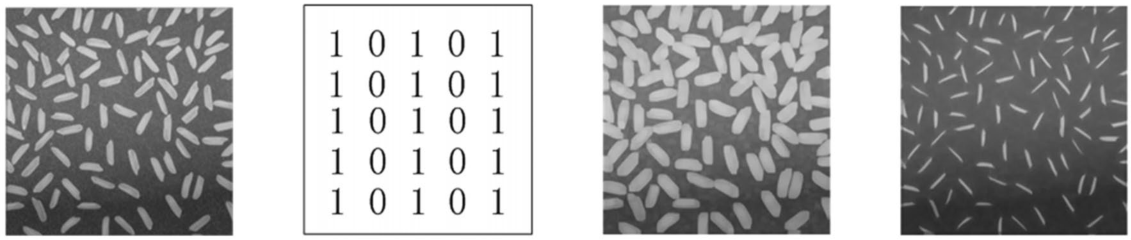
$$|c_{y',x'}\rangle = \frac{\left\{\begin{array}{c} [r_y - (y' - y \times r_y)] \times [r_x - (x' - x \times r_x)] \times |c_{y,x}\rangle \\ +(y' - y \times r_y) \times [r_x - (x' - x \times r_x)] \times |c_{y+1,x}\rangle \\ +[r_y - (y' - y \times r_y)] \times (x' - x \times r_x) \times |c_{y,x+1}\rangle \\ +(y' - y \times r_y) \times (x' - x \times r_x) \times |c_{y+1,x+1}\rangle \end{array}\right\}}{(r_y \times r_x)} \tag{62}$$

$$|c_{y',c'}\rangle = \frac{\left\{\begin{array}{c} [h' - (hy' - h'y)] \times [w' - (wx' - w'x)] \times |c_{y,x}\rangle \\ +(hy' - h'y) \times [w' - (wx' - w'x)] \times |c_{y+1,x}\rangle \\ +[h' - (hy' - h'y)] \times (wx' - w'x) \times |c_{y,x+1}\rangle \\ +(hy' - h'y) \times (wx' - w'x) \times |c_{y+1,x+1}\rangle \end{array}\right\}}{(h' \times w')} \tag{63}$$

In 2018, Li et al. designed a new method based on quantum Fourier transform for bilinear interpolation of images. It uses QFT to implement addition and multiplication operations [39]. In the same year, Zhou also proposed the nearest neighbor value (NNV) interpolation algorithm for INEQR [40].

The current image scaling research mainly focuses on nearest neighbor interpolation and bilinear interpolation, and there is a lack of related research on bicubic interpolation. The bilinear interpolation method of FRQI and its related research are also lacking.

Morphological image processing is a relatively mature image processing method in classical image processing, dilation and erosion operations are fundamental to morphological operations. Commonly used in gradient calculations, edge detection and other algorithms. But in quantum image processing, there are relatively few studies on morphology. In 2016, Yuan et al. [41] proposed two kinds of improved quantum dilation and erosion operations to reduce the time complexity of quantum morphology operations. Utilizing quantum position shifting transformation, and make the neighborhood information store in a quantum image set. The time complexity is greatly reduced compared with the previous quantum dilation and erosion operations as show in Fig. 9. In 2019, Li et al. [42] designed the quantum circuits of the two basic operations of dilation and erosion for binary images and grayscale images. On this basis, three morphological algorithms for quantum circuits are proposed for binary images. The noise removal, boundary extraction and skeleton extraction are designed in detail. In the same year, Fan et al. proposed the flat grayscale dilation and erosion operations are proposed for NEQR. Furthermore, through combining these two morphology operations, they further realize the morphological gradient operation [43].



(a) Original image (b) Flat structuring element (c) Dilation operation (d) Erosion operation

Fig. 9 Morphological transformation [41]: **a** The original image; **b** The flat structuring element for dilation and erosion; **c** Image after dilation operation; **d** Image after erosion operation

$$\begin{aligned} MF &= (F \oplus B)(s,t) - (F \ominus B)(s,t) \\ &= [\max F(s+x,t+y) - \min F(s-x,t-y)], (x,y) \in D_B \end{aligned} \tag{64}$$

5 Quantum Image Protection

In classical image processing, image protection has been very mature. It is to protect image information from theft or unauthorized use. Similar to classical image processing, quantum images may also be attacked in many ways, so image protection is also essential for quantum images. Image protection is divided in two categories. One is image encryption, and the other is quantum data hiding including quantum watermarking and quantum steganography. Image encryption is to transform the images into meaningless form, and the watermark is to hide image information by embedding it into the carrier images. In addition, there is an image encryption technology similar to watermark, called steganography, whose purpose is to communicate secretly. In the quantum image steganography process, the original secret quantum image is first encrypted, and then the encrypted image is embedded into the quantum cover image. In this section, quantum image encryption, quantum watermarking and quantum steganography are introduced respectively.

5.1 Quantum Data Hiding

Quantum watermarking is a technique which embeds the invisible quantum signal such as the owners identification into quantum multimedia data (such as audio, video and image) for copyright protection [44].

In recent years, many watermarking strategies have been proposed. In 2015, Yan et al. [45] utilizing a stockpile of efficient transformations consisting of channel of interest, channel swapping, and quantum Fourier transforms, proposed a duple watermarking strategy onmulti-channel quantum images. The objective in most of watermarking strategy is accomplished by embedding some secret messages, and this strategy can embed the watermarked image into both spatial and frequency domains. When preprocessing the watermark, two keys are generated utilizing quantum measurements and owner assignation respectively. They are used for scrambling the watermark in the embedding procedure and descrambling in the extraction procedure.

In 2016, Miyake et al. [46] proposed a new quantum gray-scale image watermarking scheme by using simple and small-scale quantum circuit. First, expand the watermark of $n \times n$ size 8-bit grayscale to $2n \times 2n$ size 2-bit grayscale image. Then, the expanded image is scrambled by the SWAP gate controlled by the keys. The scrambled image is embedded into the carrier image by the XOR operation. In 2016, Heidari et al. [44] proposed a new quantum watermarking protocol including quantum image scrambling based on Least Significant Bit (LSB).

In 2017, in order to solve the problem of embedding the watermark into the quantum color image, Li et al. [47] proposed an improved scheme of using small-scale quantum circuits and color scrambling. First, utilize a controlled rotation gates to scramble the color of pixels in the watermark image, and then expand the scrambled watermark with $2^n \times 2^n$ image size and 24-qubit grayscale to an image with $2^{n+1} \times 2^{n+2}$ image size and 3-qubit gray scale. Finally, the expanded watermark image is embedded into the carrier image by the CNOT gate. In 2017, Naseri et al. proposed a new watermark strategy for quantum images. In this scheme and with the aim of data hiding, in addition to using the least significant bit (LSB), the most significant bit (MSB) is also employed [48]. Compared with the previous protocol,this scheme indicates not only the better resistance against attacks and noises, but also less resources and complexity, because it exhibits a better PSNR, and it is more secure.

In the same year, Qu et al. [49] proposed a new quantum watermark algorithm based on quantum log-polar image (QUALPI). This algorithm utilized LSB modification technique for embedding. The least significant bit (LSB) replacement method provides an easy way to hide secret data in the cover image. In order to realize quantum watermark embedding, the LSB of the quantum carrier image is replaced by the quantum watermark image. Compared with other quantum watermarking algorithms, this algorithm effectively utilizes two important properties of log-polar sampling, i.e., rotation and scale invariances. By combining QUALPI representation with the LSB modification technique, the new algorithm make quantum watermark image extracted have a good robustness when it was subjected to geometric attacks.

Unfortunately, this algorithm does not consider more complex attacks such as filtering and compression. In 2018, Luo et al. [50] proposed an enhanced quantum watermarking scheme on the basis of INEQR. In contrast to previous quantum watermarking scheme, this scheme utilizes the edge pixels of a carrier image, which cannot be noticed visually as the embedding region. This watermarking scheme is keyless and blind, with higher visual quality and better robustness. As human vision is insensitive to the edge regions of an image, the watermark is embedded into the edge region of a carrier image by LSB modification technique. This quantum watermarking scheme has high visual effects. However, the payload for this scheme is very low, which only has 1/16 bits per pixel, and the watermark image is not scrambled, which is not conducive to enhancing the robustness of the watermarked images.

To improve the payload and the robustness of the quantum watermarked image, in the same year Hu et al. proposed a LSBs-based quantum color images watermarking algorithm in edge region. First, utilize the nearest neighbor interpolation method to resize the original quantum watermark image to an appropriate size and then implement Gray code transformation to scramble the resized watermark image. In order to scatter the watermark image qubits embedded into the edge regions of the quantum color image LSB and second LSB, two quantum embedded key images are generated.

And in the same year, in order to break through the limitations of quantum encryption algorithms based on square images, Zhou et al. [51] presented a watermarking scheme based on INEQR, and a quantum color image watermarking scheme based on the improved novel quantum representation of color digital images model (INCQI) [52]. This algorithm employs a fast bitplane scrambling method, which can transform the secret image into a disordered form. Both the carrier image and the watermark image are non-square color images, the embedding procedure adopts dual embedding algorithm.

In 2019, Luo et al. [53] proposed a adaptive LSB quantum watermarking method using tri-way pixel value differencing. First, the quantum cover image is partitioned into 2×2 blocks with four non-overlapping pixels. In order to classify the block as a smooth area or an edge area, the tri-way pixel value differences are calculated and compared with a predefined threshold. Then, according to the level k of each block, the expanded and encrypted quantum watermark image is embedded into the quantum cover image by the k-bit LSB replacement method. In addition, the algorithm utilizes the pixel value difference (PVD), therefore, the method embeds more secret data inedge areas to acquire more payload, and embeds less data in the smooth area to preserve visual quality. And in order to enhance the robustness and security of the method, parity bits are utilized in the embedding process. As show in Fig. 10, they do not show any visible marks to be suspected.

Steganography is the study and application of techniques to hide messages within carriers, so that people cannot perceive their existence. one may pass the message unnoticed to everyone except for the receiver. Thus, the information is protected and the existence of the communication itself is concealed [54]. Steganography is similar to watermarking but has a different purpose. Quantum image watermarking aims to use the signal embedded in the cover image to protect the copyright of the image. In the case of quantum steganography, its objective is to securely communicate the secret data by embedding the secret data in the quantum cover image without triggering any suspicions from the unauthorized third parties [55]. Moreover, steganography is a mode of communication, so large amounts of data needs to be embedded. And it is important to make sure the image

(a) Watermark (b) Original image (c) Extracted watermark (d) Extracted watermark

Fig. 10 Watermarking [53]: **a** Watermark image; **b** Original image; **c** The image with embedded watermark; **d** The extracted watermark image does not exhibit any traces of hidden data. On the contrary, the watermark has no limitation on the amount of embedded data. And the watermark only embeds the copyright information of the image.

In 2016 Laurel et al. [54] proposed a steganography on quantum pixel images using shannon entropy. They embed secret information in the least significant bit (LSB) from the code of the most significant bit information (MSBI). The Shannon entropy is used to determine the optimal LSB from the MSBI, which improves the hiding security of the system. Images altered in the LSB from the MSBI, display a better performance than simply altering the LSB. Jiang et al. [56] proposed two blind LSB steganography algorithms in the form of quantum circuits based on the novel enhanced quantum representation (NEQR). One algorithm is plain LSB which uses the message bits to substitute for the pixels' LSB directly. The other is block LSB which embeds a message bit into a number of pixels that belong to one image block. The algorithm has good invisibility, and the balance between the capacity and the robustness can be adjusted according to the needs of applications.

In 2017, Heidari et al. [57] suggested three quantum steganography procedures which utilize all RGB channels in the cover image. The first algorithm employs only one channel to cover secret data. The second algorithm is based on LSB XORing technique, and the last algorithm utilizes two channels to cover the color image to hide secret quantum data.

In 2018, El-latif et al. [58] proposed a highly secure quantum image steganography scheme based on the logistic chaotic map. The quantum secret image is encrypted using a CNOT gate. Then the encrypted secret image is embedded into the quantum cover image utilizing the two most and least significant qubits. In addition, they presented a quantum image watermarking approach. The quantum watermark image is scrambled by utilizing Arnold's cat map, is then embedded into the quantum carrier image using the two least and most significant qubits. Sahin et al. [59] proposed a novel quantum steganography algorithm based on LSBq for multiwavelength quantum images. Compared with other methods, this method does not need to use the quantum comparator operations. Thus, this allows us to reduce time complexity. And they proposed that both position and channel qubits are considered as a position and the pixels in this new position are used for embedding by a certain modulo value, not in order.

Li et al. [60] proposed a novel quantum steganography scheme using color images as cover images. First, the secret information is divided into 3-bit segments, and then each 3-bit segment is embedded into the LSB of one color pixel in the cover image according to its own value and using Gray code mapping rules. In the same year, in order to improve the embedding capacity of quantum image steganography, they proposed a novel LSB-based steganography using reflected Gray code for colored quantum images, and the embedding capacity of this scheme is up to 4 bits per pixel [61].

In 2018, Zhou et al. [62] proposed a novel quantum image steganography scheme based on the NEQR and LSB scheme. This model uses bit-plane scrambling method to scramble the original secret image, expands the scrambled image and then scrambling with the Arnold scrambling. In 2019, Qu et al. proposed a novel quantum image steganography algorithm based on an efficient embedding technique of exploiting modification direction. This embedding technique is referred to the exploiting modification direction (EMD) embedding [63]. The merit of the EMD embedding is that it provides good image quality. However, the EMD embedding has room for further improvement of its embedding capacity.

Luo et al. [55] proposed an quantum steganography scheme using inverted pattern approach. Before embedding the secret image, each pixel of the quantum secret image will be inverted or not. The decisions are recorded by a quantum key image. Then, the quantum secret image is embedded in the k least significant bits of the quantum cover image. The computational complexity of this scheme is lower than its classical counterpart and other quantum shorthand schemes. And this scheme has outperformed other methods in terms of the visual quality and embedding capacity. In the same year, they proposed two-level information hiding for quantum images using optimal LSB. The first level is to hide the encrypted quantum secret image into a quantum watermark image, and the second level is to embed the quantum watermark image into a quantum cover image. Using the optimal LSB-based algorithm, the double embedding can make the position of embedding have a certain randomness, thus increasing security [64]. Under the same embedding capability, it can provide better visual effects than existing quantum steganography solutions, as show in Fig. 11. In 2020, Su et al. [65] proposed a novel information hiding method based on the NEQR quantum image and Bell states.

5.2 Quantum Image Encryption

Image encryption can transform the image into a meaningless form according to the key by a variety of encryption models, so that third parties cannot obtain the image content.

The bitplane consists of 8 binary images formed by separating the grayscale information, and the image can be scrambled by operating on the bitplane. In 2018, Heidari et al. [66] proposed a quantum representation of a digital scrambling algorithm for quantum NCQI color images. This method is divided into two sub-algorithms. First, algorithm manipulates qubits of the RGB channels by XOR and XNOR operations for bitplane scrambling. Then, the pixel-plane is scrambled to disorder pixels of the image. Zhou et al. used the quantum channel swapping operation and hyper-chaotic

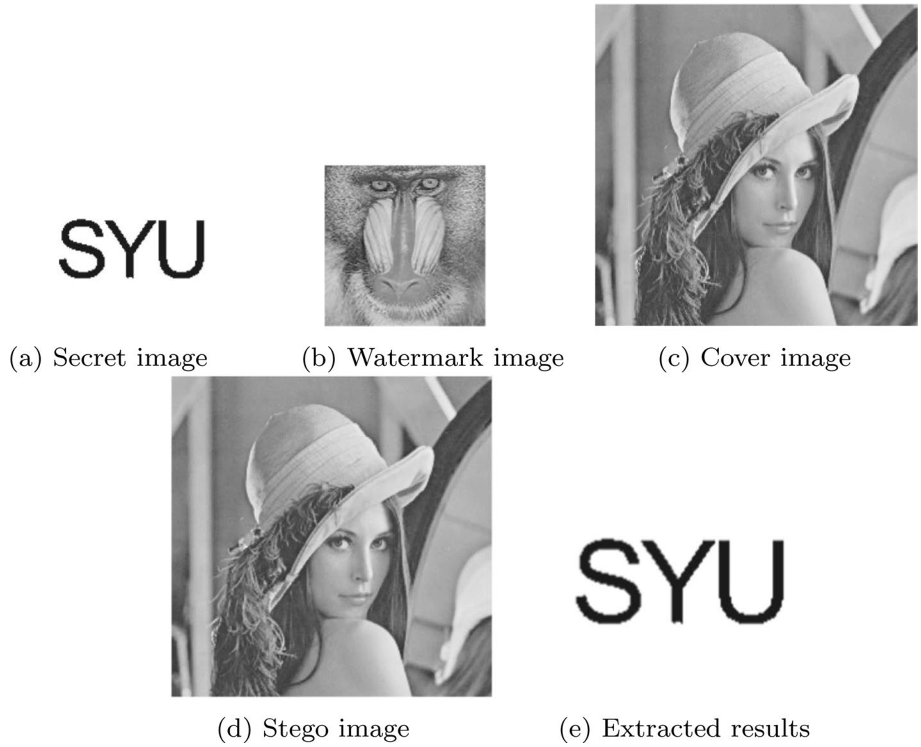


Fig. 11 Steganography [64]: **a** The image that needs to be hidden; **b** Watermark image; **c** Cover image; **d** The image embedded with hidden information; **e** The extracted results

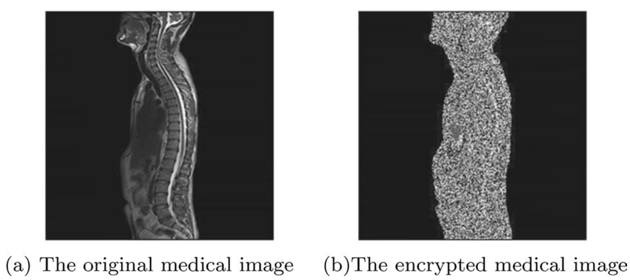
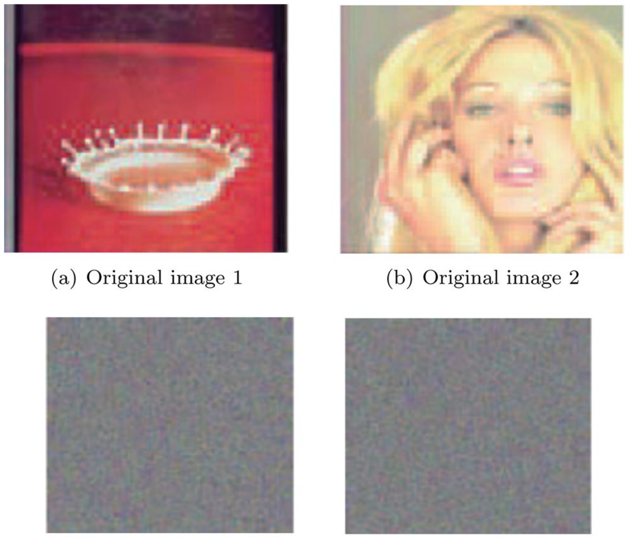


Fig. 12 Encrypting the region of Interest of the medical image by operating on the bitplane [64]

system at the same time to enhance the scrambling effect. A bit-level quantum color image encryption scheme by exploiting quantum cross-exchange operation and a 5D hyper-chaotic system is proposed [67]. First, the algorithm exchanges the bitplanes and RGB channels, and then uses 5D hyper-chaotic system and XOR operation to encrypt. El-Latif et al. [68] proposed a novel approach for the efficient quantum image encryption of healthcare media. The quantum image is scrambled by bitplane and Gray code. Then, the scrambled quantum image is encrypted by a XOR operation based on a key generated by the logistic-sine map. In 2019, Heidari et al. [69] proposed a novel quantum selective encryption method for medical images based on BRQI images. Region of interest of the medical image can be effectively encrypted by operating on the bitplane, as show in Fig. 12. Liu et al. [70] proposed a quantum image encryption schemeby using the inter–intra bit-level permutation strategy. This method utilizes both intra and inter bit permutation to scramble pixels, and chaotic diffusion with logistic map is also performed. Li et al. [71] proposes a block-based image scrambling scheme for the generalized model of novel enhanced quantum representation (GNEQR). This scheme employs geometric transformations and an operation of bitplane scrambling to perform position and pixel scrambling, respectively. And it suit for rectangular images, so this scrambling scheme is more general.

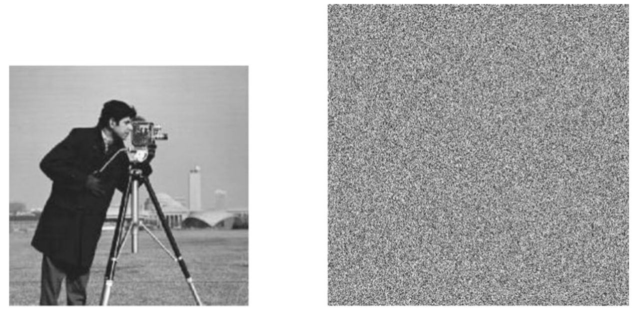
The quantum rotation gate can rotate the qubit on the Bloch sphere at any angle, it can be used in image encryption. In 2017, Li et al. [72] proposed a simple encryption scheme for quantum color image. Qubits are respectively transformed from a basic state into a balanced superposition state according to the key by employed the controlled rotation gates. In 2018, Liu et al. [73] proposed a novel color image encryption/decryption method based on random rotation of qubit and quantum Fourier transform (QFT). Qubit rotation operates once in spatial and frequency domains respectively, with the help of quantum Fourier transform. The encryption effect is shown in Fig. 13. In the same year, a novel double quantum image encryption approach based on quantum Arnold transform (QAT) and qubit random rotation is proposed [74]. The algorithm is extended to double quantum on the basis of the previous. And before the qubit rotation operates in spatial and frequency domains, QAT operations would be performed in the spatial domain. In 2018, Khan et al. [75] proposed an innovative encryption



(a) Original image 1 (b) Original image 2

(c) Double encrypted 1 (d) Double encrypted 2

Fig. 13 Encryption algorithm employing rotation gate [73]: **a**, **b** Original images; **c**, **d** Encrypted images



(a) The original image (b)The encrypted image

Fig. 14 Encryption algorithm utilizing QFT [79]

scheme for digital data based on quantum spinning and rotation operators.

Fourier transform is a commonly used operation in signal processing, and QFT can also be applied in quantum image encryption. The algorithm proposed by Liu et al. [74] in 2018 uses QFT . In 2013, Yang et al. [76] proposed a novel gray-level image encryption/decryption scheme which is based on quantum Fourier transform and double random-phase encoding technique. This algorithm applies two phase codings in spatial and frequency domains to perform double quantum image encryption, and the two random-phase encodings are used as the keys. They generalized the double random-phase encoding technique to quantum scenarios for the first time.

In 2014, Yang et al. [77] expanded the algorithm to RGB on this basis, and proposed a quantum cryptographic algorithm for color images using quantum Fourier transform and double random-phase encoding. El-Latif et al. proposed a new color image encryption scheme based on quantum chaotic system is proposed. Firstly, scramble only the Y (Luminance) component of low frequency subband. Then mix the features of horizontally and vertically adjacent pixels with the help of adopted quantum chaotic map. Finally, generate an intermediate chaotic key stream image with the help of quantum chaotic system [78]. In 2016, Tan et al. [79] proposed a quantum color image encryption algorithm based on a hyper-chaotic system and quantum Fourier transform. A sequence is generated using the Chen's hyper-chaotic system, which is scrambled and diffused with the original image, and use quantum Fourier transform to complete encryption, the encryption effect is shown in Fig. 14. In 2018, Gong et al. [80] proposed a new single channel quantum color image encryption algorithm based on HSI model and QFT, where the color components are converted to HSI and the logistic map is employed to diffuse the relationship of pixels in color components and use quantum Fourier transform to complete encryption.

Arnold transform can effectively scramble the image, expand the key space, and is widely used in image encryption. In 2015, Zhou et al. [81] proposed a novel quantum image encryption algorithm based on generalized Arnold transform and double random-phase encoding. The image is scrambled by the generalized Arnold transform, and the gray level information is encoded by the double random-phase operations.

In 2017, Hu et al. proposed a novel quantum multi-image encryption algorithm based on iteration Arnold transform with parameters and image correlation decomposition [82]. Perform 2-D discrete wavelet transform on each image respectively, randomly splice the corresponding low-frequency image to one image. scramble the new image by the iteration Arnold transform with parameters, and then use the quantum image correlation decomposition to encode. Zhou ey al. suggested a new quantum image encryption scheme by using the iterative generalized Arnold transforms and the quantum image cycle shift operations [83]. The image pixels are scrambled by the iterative generalized Arnold transform, and the quantum image cycle shift operations is used to altere the values of the pixels, where the times of shift operations are controlled by a new 4D hyperchaotic system.

In 2018, Zhou et al. [84] devised a novel quantum multi-image encryption scheme by combining quantum 3D Arnold transform and quantum XOR operations with scaled Zhongtang chaotic system. This encryption scheme could encrypt multiple images simultaneously. In 2019, Luo et al. [85] propose a novel quantum secret image-sharing scheme. The images are scrambled through the Arnold transform, and then the quantum shared images are constructed by the swap operations and the CNOT gates. Liu et al. [86] proposed a quantum block image encryption scheme based on quantum

Arnold transform. First, a quantum block image representation (QBIR) model for block image is proposed. Then, QAT is applied to scramble the position of image blocks. Finally, the pseudo-random sequence is generated by SCM, and the encrypted image is obtained through quantum XOR operations.

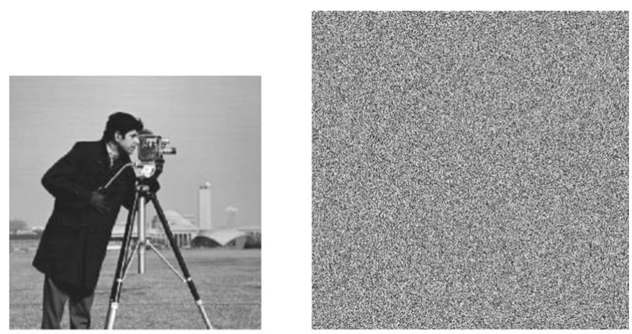
In addition, there are many other encryption algorithms, which use various chaotic systems to help encryption. The behavior of chaotic systems is very complex and highly sensitive to initial conditions, making it difficult to predict. In 2019, Abd-El-Atty et al. proposed a encryption and decryption algorithms for NEQR images based on discrete quantum walks on a circle. Discrete quantum walk can be regarded as a nonlinear mapping between quantum state and position probability distribution, so it can be used to generate encryption keys [88]. The algorithm uses the key image generated by QWs to XOR the original image. In 2020, EL-Latif et al. [87] presented a method to build pseudorandom number generators based on quantum walks and used this result to encrypt quantum color images. This algorithm utilize controlled alternate quantum walk to create PRNG. The image is encrypted by the CNOT gate based on the key sequence generated by the pseudo random number generator (PRNG). The encryption effect is shown in Fig. 15.

In 2017, Kadir et al. [89] proposes a color image encryption scheme based on coupled hyper chaotic Lorenz systems. This algorithm randomly inject impulse signals into coupled Lorenz system during iterations to enhance the complexity of trajectory. In 2018, Ran et al. [90] proposed a quantum color image encryption scheme based on coupled hyperchaotic Lorenz system with three impulse injections. In order to prevent the behavior of the short-period chaotic system after too many iterations from causing degeneration of dynamics, three impulse signals values are injected into coupled hyperchaotic Lorenz system during iterations.

In 2012, Akhshani et al. [92] proposed an implementation of image encryption scheme based on the quantum logistic map, it is the first attempt to apply quantum map in the construction of chaotic cryptographic systems. In 2014, Cao et al. [93] designed a new image encryption scheme based on quantum chaos. An disturbing mechanism is introduced to realize a one-time running-key stream, and reduce the dynamical degradation of digital chaos. In 2016, Liang et al. [94] devised a novel quantum image encryption algorithm combining the generalized affine transform with logistic map. The key is generated by the logistic map, and the gray level information is encrypted by XOR operation, while the position information is encoded by generalized affine transform.

Gong et al. [91] proposed a novel encryption algorithm for quantum images based on quantum image XOR operations. The hyper-chaotic sequences generated with the Chen's hyper-chaotic system is utilized to control the CNOT gate to encode gray-level information, the encryption effect is shown in Fig. 16. Yang et al. proposed a novel quantum gray-scale image encryption algorithm based on one-dimensional quantum cellular automata. The quantum image encryption algorithm is realized by subtly constructing the evolution rules of one-dimensional quantum cellular automata [95]. In 2018, Naseri et al. [96] proposed a new bi-step quantum image cryptography algorithm. The scheme is consisted of four different coding algorithms. According to the pixels and key, select the algorithm.

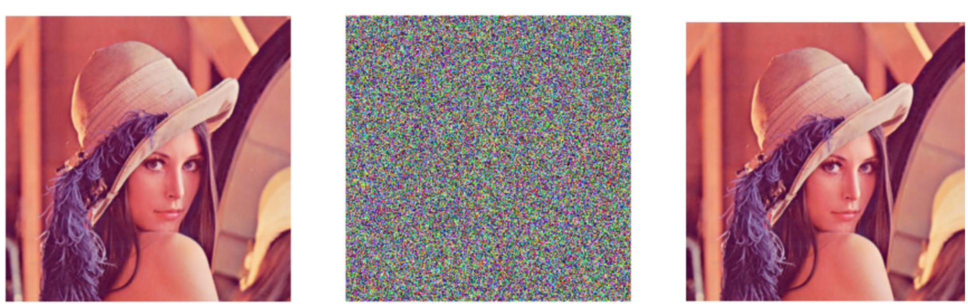
In 2019, Li et al. [97] proposed an algorithm of quantum image encryption based on NASS by using quantum geometric transform, phase-shift transform, and quantum Haar wavelet packet transform. Firstly, pixels are exchanged along



(a) The original image (b) The encrypted image

Fig. 16 The original image is encrypted by Chen's hyper-chaotic system [91]

Fig. 15 The effect of QWs encryption scheme [87]: **a** The original image lena; **b** The encrypted image **c** The decrypted image



(a) Priginal image (b) Encrypted image (c) Decrypted image diagonal lines. Next, keys randomly distributed are encoded by using phase-shift transform. Then, image information is stored in low frequency using quantum Haar wavelet packet transform (QHWPT). Liu et al. [98] proposed a new algorithm of image encryption based on random selection of crossover operation and mutation operation. In order to obtain the high complexity and unpredictability further, quantum chaotic map is coupled with nearest-neighboring coupledmap lattices (NCML). After performing XOR operation, adjacent pixels are carried out bit-level crossover operation. Then two different bits of each pixel are employed to perform mutation.

Xu et al. [99] proposed a novel encryption algorithm based on quantum chaotic map and four-wing complex system. Firstly, quantum logistic map and Arnold transform is utilized to disrupt position information. Then, a complex hyperchaotic system is used to diffuse the value of each pixel. Wang et al. [100] proposed a improved QISS scheme, which is implemented on both quantum gray image and quantum color image. Wang et al. [101] presented a novel quantum image encryption algorithm based on quantum key image. The secret image does the XOR operations with a quantum key image bit by bit, which is generated by a cryptographic algorithm. Liu et al. [102] proposed a novel n out of n quantum visual secret sharing (QVSS) scheme. Jiang et al. [103] proposed a quantum image encryption based on Henon mapping. In 2020, Luo et al. [104] proposed an image encryption scheme that is based on hyper-chaos and quantum coding. First, perform the bit-level adjacent-exchange operation. Then, the image is diffused by the quantified sequences where the self-adapting parameters are applied to quantify the sequences that are generated by Chen's hyperchaotic system. Finally obtain the final cipher-image by a novel scrambling method that is based on quantum location coding.

6 Edge Detection and Image Segmentation

6.1 Edge Detection

Edge detection is an important part of image processing. It refers to sharp discontinuity localization process in an image, the classical edge detection algorithms are mostly based on the numerical derivative near the pixel of the image [105]. Edge detection can extract most of the important features of the image, which is very useful on image segmentation and object identification.

In 2016, Abdel-Khalek et al. [105] proposed a novel edge detection algorithm based on quantum entropy. This algorithm utilizes quantum entropy to take correlations among quantum bases into the calculation of entropy and uses the quasi-threshold that leads to the maximum quantum entropy as the optimal threshold to obtain the maximum amount of information. In 2017, Yao et al. proposed a highly efficient quantum algorithm for detecting the boundary between different regions of a image. It requires one single-qubit gate in the processing stage, independent of the size of the image [106]. In order to try to resolve the realtime problem of image edge extraction in practice image processing, in 2018 Fan et al. [107] proposed a quantum image edge extraction for NEQR based on classical Sobel operator, and an enhanced quantum edge detection algorithm based on NEQR, which combines the classical Laplacian operator and zerocross method [108].

And in 2019, Zhou et al. [109] proposed a quantum image edge extraction algorithm based on improved sobel operator for the generalized quantum image representation (GQIR). This scheme can achieve more accurate edge extraction, especially for diagonal edges. In the same year, in order to solve the coarse edge detection and false edge detection caused by artificial selection of threshold in the traditional Prewitt edge detection algorithm, Zhou et al. [110] proposed a quantum image edge extraction for NEQR based on improved Prewitt operator, which combines the non-maximum suppression method and adaptive threshold value method. As shown in Fig. 17. However, these algorithms are still sensitive to noise.

In 2020, Li et al. [111] proposed a quantum scheme of classical Marr–Hildreth edge detection. This scheme is insensitive to the noise of mixed images.

6.2 Image Segmentation

Image segmentation is the process of separating the foreground of one or more objects from the background in a digital image [112]. It is a key step in image processing and plays an important role in computer vision.

In 2015, Youssry et al. [112] proposed a novel and generic framework based on quantum mechanics for image processing and applied it to image segmentation. Caraiman et al. [113] discussed the development of a quantum

(a) The original image (b) The result image

Fig. 17 Edge detection algorithm [110]: **a** The original image; **b** The result image of this quantum image edge extraction version for the image segmentation operation. In 2016, Karmakar et al. presented the application of quantum dot gate nonvolatile memory (QDNVM) in image segmentation. Zhao et al. [114] proposed a novel side scan sonar image segmentation algorithm integrating neutrosophic set (NS) with quantumbehaved particle swarm optimization (QPSO). In order to implement image segmentation precisely and efficiently, Wang et al. [115] present an image thresholding method based on the criteria of global quantum entropy maximization (GQEM), and the quantum lossy-encoding based entropy maximization (QLEEM) approach is used to deal with the time consumption problem of thresholding. In 2017, in order to get a reasonable threshold for threshold segmentation, Huo et al. [116] proposed an improved algorithm of Bloch spherical coordinates for quantum artificial bee colony, and applied it to image threshold segmentation. In 2020, Huo et al. [117] proposed an improved Bloch quantum artificial bee colony by combining the Bloch spherical coordinates of qubit with artificial bee colony (ABC) algorithm, and applied it in multilevel image threshold segmentation.

7 Similarity Analyses and Image Matching

Image matching is the process of searching for a small image in a big image [118]. It is widely used in computer vision, face detection, and so on. As the fundamental research of quantum image matching, similarity analyses between quantum images are so essential. In 2016, Iliyasu et al. [119] proposed an enhanced quantum-based image fidelity metric, the enhanced quantum-based image fidelity metric (QIFM) as a tool to assess the "congruity" between two or more quantum images. Jiang et al. [118] provided a quantum image matching solution which does not compute similarity. This scheme can get answer by being processed and measured only once, which helps to drop the scheme's complexity. But the paper only matched one pixel, instead of an area. If more than one pixels in the big image are the same as the one at the upper left corner of the small image, the algorithm will randomly measure one of them. In 2017, Dang et al. [120] presented an improved version which takes full advantage of the whole matched area to locate a small image in a big image. In 2018, Zhou et al. [121] proposed a similarity analyses scheme based on a novel quantum image representation and quantum amplitude amplification algorithm. Quantum amplitude amplification algorithm can avoid measuring many times. And in order to store two quantum images with internal relation more conveniently, two images are stored into a novel quantum image representation that shares coordinate information.

$$|I\rangle = \frac{1}{2^n}\sum_{y=0}^{2^n-1}\sum_{x=0}^{2^n-1}|yx\rangle|f_A(y,x)\rangle|f_B(y,x)\rangle \tag{65}$$

These schemes only consider exact matching without considering fuzzy matching of quantum images. Luo et al. [122] proposed a fuzzy quantum image matching scheme based on gray-scale difference. This scheme evaluate the gray-scale difference between two quantum images by thresholding. If all of the obtained gray-scale differences are not greater than the threshold value, it indicates a successful fuzzy matching of quantum images. In 2019, Liu et al. [123] applied quantum counting to five algorithms for assessing the similarity of quantum images of binary image gray image and color image respectively, brings an advantage to the number of quantum measurement.

8 Quantum Image Filtering and Compression

8.1 Quantum Image Filtering

In classical image processing, image filtering is a common preprocessing operation, which is generally achieved by correlating the image with a filter mask. In computer vision, it is often used to eliminate noise in images. Although it is difficult to perform convolution operations in quantum image processing and bring difficulties to filtering operations, many feasible solutions have been proposed.

In 2017, Yuan et al. [124] proposed a framework of quantum image filtering in the spatial domain. In order to avoid the quantum multiplication, they employed quantum addition operation. But this method is only suitable for integer filter coefficients, and before each filtering behavior the value of the filter coefficients should be knew exactly. The filtering effect of salt and pepper noise is shown in Fig. 18.

In 2018, in order to overcome these shortcomings, Yuan et al. [125] proposed an improved version which employs

(a) Salt and pepper noise (b) Images after filtering

Fig. 18 Quantum image filtering [124]: **a** Images with salt and pepper noise; **b** Images after filtering quantum multiplication. In 2017, Li et al. [126] investigated design method of quantum weighted averaging filter and its application in image de-noising. In 2018, Li et al. [127] improved this method, investigated the use of quantum Fourier transform (QFT) in the field of image processing, developed a quantum version for the color image filtering operation. In the same year, they proposed a quantum image median filtering method in the spatial domain [128]. In this method, the original image is first translated by one unit in eight directions, then the median of nine pixels with the same position is calculated. In 2019, Jiang et al. [129] improved the method, utilizing the extremum detection approach to distinguish between noises and normal signal points proposed a improved quantum image median filtering in the spatial domain. The filtering effect of salt and pepper noise is shown in Fig. 19. This scheme stores only the color information of the neighborhood pixels entangled with position information of the original image.

8.2 Quantum Image Compression

Image compression algorithm is a widely used algorithm in classical image processing. In recent years, several quantum image compression methods have been proposed.

In 2018, Jiang et al. [130] proposed a novel quantum image compression method based on JPEG. This method inputs the quantized JPEG coefficients into qubits and then convert them into pixel values. Because the data amount of JPEG coefficients are less than the data amount of pixel values, the JPEG scheme can reduce the number of quantum gates used in the GQIR model. Li et al. [131] devised a quantum gray image encryption-compression scheme based on quantum cosine transform and 5-dimensional hyperchaotic system. Image is compressed by the quantum cosine transform and Zigzag scan coding, the compressed image is encrypted by the 5-dimensional hyperchaotic system. In 2019, Pang et al. [132] presented a quantum discrete cosine transform algorithm (QDCT), and used it to develop and realize a quantum image compression technique.

9 Quantum Machine Learning

Machine learning is a field with broad applications. It can perform some complex tasks that only the human brain can do, such as image recognition, language translation, and decision making, and it can process tasks faster. With the advent of quantum technology, designing quantum neural networks for quantum machine learning is a crucial task. Quantum machine learning is a very promising emerging research field. To form a deep neural network, the selection of learning algorithms and its parameters is very important. There have been many attempts in this field in recent years.

In 2013, Pudenz et al. [133] developed an approach to machine learning and anomaly detection via quantum adiabatic evolution. This method uses classical preprocessing. In 2016, Dunjko et al. [134] propose an approach for the systematic treatment of machine learning, from the perspective of quantum information, which covers supervised, unsupervised, and reinforcement learning. Konar et al. [135] proposed an efficient technique for binary object extraction in real time from noisy background using quantum bi-directional self-organizing neural network (QBDSONN) architecture. Lau et al. [136] generalized quantum machine learning to infinite-imensional systems, and proposed the critical subroutines of quantum machine learning algorithms for an all-photonic continuous-variable quantum computer.

In 2017, Montanaro et al. [137] suggested a quantum algorithm which achieves a super-polynomial separation from classical computation for the basic problem of pattern matching on average case inputs. Benedetti et al. [138] proposed the quantum-assisted Helmholtz machine, which is a hybrid quantum–classical framework with the potential of tackling high-dimensional real-world machine learning datasets on continuous variables. This method uses deep learning to extract low-dimensional binary representations of data, and then uses quantum deep learning to train unsupervised generative model.

In 2018, Patel et al. [139] proposed a quantum-inspired stacked autoencoder-based deep neural network learning algorithm, which uses stacked auto-encoder to form a deep

(a) Original image (b) Salt and pepper noise (c) Images after filtering

Fig. 19 Quantum image median filtering [129]: **a** The original image; **b** Images with salt and pepper noise; **c** Images after filtering neural network. In order to avoid improper selection of learning algorithm parameters, a representation and evaluation function characterized by the representation of population dynamic is used. Liu et al. [140] presented a practical gradient-based learning scheme to train quantum circuit Born machine as a generative model of discrete data.

In order to optimize the efficiency of image classification, Dang et al. proposed an image classification scheme based on quantum K Nearest Neighbor algorithm [141]. After extracting the feature vectors on a classical computer, the feature vectors are input into quantum computer for parallel computing of similarity. Piat et al. [142] proposed a framework to accomplish the task of processing large scale data on small quantum devices. First, a classical autoencoder is trained to compress image to a size that can be loaded on a quantum device. Then, use the compressed data to train the RBM on the D-Wave device, and then use the weights from the restricted Boltzmann machine (RBM) to initialize the neural network for image classification.

Potok et al. [143] proposed a new deep learning architecture based on the unique capabilities of the quantum, high performance, and neuromorphic approaches. The high performance computer is used to create a well performing CNN on image type data. The final layer or two is processed by quantum computers using the limited Boltzmann machine (LBM) network. Employing the spiking neural network (SNN) to model the temporalaspects of the data, then merge the ensemble models and produce output. Wiebe et al. [144] suggested a number of ways that quantum information can be used to help make quantum classifiers more secure or private. In 2019, Huggins et al. [145] proposed quantum computing approaches to both discriminative and generative learning, with circuits based on tree and matrix productstate tensor networks.

Because of the intractability of deep quantum circuits, it is difficult to simulate classical deep learning models. Thus, in 2020, Chen et al. [146] designed a quantum algorithm for quantum machine learning for noisy intermediate scale quantum (NISQ) devices. This work explores variational quantum circuits for deep reinforcement learning. This work is the first to prove that the variational quantum circuits to approximate the deep q-value function for decision making and policy selection reinforcement learning with experience replay and target network. Yang et al. [147] presented a quantum deep learning scheme based on multi-qubit entanglement states, including computation and training of neural network in full quantum process. The distance between the unknown unit vector and the known unit vector is calculated by the measurement based on the Greenberger-Horne-Zellinger entanglement. It also provided the quantum scheme corresponding to the multilayer feedforward neural network.

Li et al. [148] investigated a quantum deep convolutional neural network (QDCNN) model based on the quantum parameterized circuit for image recognition. Illustrates the architecture of a quantum convolutional layers sequence followed by a quantum classified layer. Inspired by the variational quantum algorithms, a quantum classical hybrid training scheme is proposed for the parameter updating in the QDCNN. First, the input image is prepared as the quantum state with basis encoding. Then a sequence of parameter-dependent unitary transformations is employed to realize the corresponding quantum evolution, which can be divided into quantum convolution layer and quantum classified layer. Finally, a quantum measurement is performed on the specified qubit to obtain the category label.

Kerstin et al. [149] proposed a truly quantum analogue of classical neurons, which form quantum feedforward neural networks capable of universal quantum computation, and describe the efficient training of these networks using the fidelity as a cost function. This method allows for fast optimisation with reduced memory requirements. They proposed a natural quantum perceptron and define it to be a general unitary operator acting on the corresponding input and output qubits, and proposed a training algorithm for this quantum neural network. Mixed state ρ^{out} for the output qubits is:

$$\rho^{out} \equiv tr_{in,hid}\left(\mathcal{U}\left(\rho^{in} \otimes |0\cdots 0\rangle_{hid,out}\langle 0\cdots 0|\right)\mathcal{U}^{\dagger}\right) \tag{66}$$

where $\mathcal{U} \equiv U^{out}U^{L}U^{L-1}\cdots U^{1}$ is the quantum neural network quantum circuit, $\mathcal{U}^{l}$ are the layer unitaries, comprised of a product of quantum perceptrons acting on the qubits in layers $l-1$ and l.

10 Other Algorithms

In addition to the previously introduced algorithms, many other excellent algorithms have been proposed.

In order to extend image stabilization to the quantum computing domain. In 2016, Yan et al. [150] explored a novel quantum video framework and proposed a method based on it to perform the image stabilization by utilizing the quantum comparator and quantum image translation operations. This method can estimate the camera motion during the exposure process and compensate for the video jitter based on it. By Schmidt decomposition and symmetric states permutation, Yue et al. [151] proposed a method can got the quantum image with high retrieval performance.

For binary images, or bitplanes of non-binary images, Chapeau-Blondeau et al. [152] investigated a quantum image coding method with a reference frame independent scheme. Jiang et al. [153] presented a quantum image location algorithm, which modifies the probability of pixels to make the target pixel to be measured with higher probability. In order to dealing with the fusion task of infrared and visible images, Kong et al. [154] proposed a novel fusion method based on improved quantum theory model. Compared with quantum theory model (QTM), a new qubit state 0.5 is added in improved quantum theory model (IQTM). It's helpful to represent the gray information more accurately. Yang et al. discussed the application of quantum Hash function to image encryption. It is found that quantum Hash function can act as a hash function for the privacy amplification process of quantum key distribution systems. And they proposed a novel image encryption algorithm [155].

In 2017, Naseri et al. [156] proposed a novel quantum version of Hilditch algorithm for quantum image thinning. Skeletonization or thinning process is an important step in pre-processing phase. This algorithm can be considered as the first quantum image thinning algorithm. Liu et al. proposed three noise removal algorithms based on NEQR. These algorithms are the first attempts to detect and deal with noise using quantum image representation model in spatial domain [157].

In 2019, Du et al. [158] realized the synthesis of two quantum images theoretically by constructing a specific phase rotation transform. Xia et al. [159] designed a novel multi-bit quantum comparators, and realized quantum image binarization based on it. This multi-bit quantum comparators can compare more bits with only one auxiliary bits in a less quantum cost. Liu et al. [160] proposed a novel multi-modality image fusion algorithm based on QWT and quantum version of SML. Quantum images are transformed with QWT to capture salient features of source images. Fusing the wavelet coefficients by sum-modified-laplacian (SML) rule. The final fused image is obtained by using inverse quantum wavelet transform. Heidari et al. proposed a new general model for quantum image histogram, which is based on NEQR and NCQI [161].

11 Discussion

In recent years, quantum image processing has developed rapidly. Many quantum image processing algorithms have been proposed. These algorithms make use of the superiority of quantum computing to parallel computing, exponentially accelerate and simplify the classical image processing algorithms. However, compared with classical image processing, quantum image processing is a brand new research field, and existing algorithms still urgently need to be improved.

For quantum image representations, the proposed representation model stores images whose side length can only be an integer power of 2. Otherwise, images of arbitrary size can be stored at the cost of added redundancy. Therefore, quantum images need to have more efficient representation methods. And the image representation models containing error correction codes can also be proposed to improve robustness.

For morphological transformation, erosion and expansion operations have been proposed, but quantum implementations of more complex morphological transformations need more research. And more applications can be enriched based on these basic operations. In quantum scale operations, algorithms using bilinear interpolation are still relatively rare, and algorithms using bicubic interpolation have not yet been proposed.

Although image encryption models have been greatly developed, most algorithms are for grayscale images. The encryption of color images, double-quantum images, and multiple images is still relatively rare. In terms of watermarking. The robustness of watermarking under noise needs to be improved. And frequency domain watermarking algorithms are still relatively few.

In the field of edge detection and image segmentation, relatively few algorithms have been proposed. Although quantum algorithms provide lower complexity, there is not enough edge continuity in the proposed methods and the algorithms are not robust enough to noise. There are also few algorithms for image filtering. Since convolution is not easily used for quantum computation, there is a need to find quantum image spatial filtering methods that do not depend on convolution. In addition, in image matching research, some exact matching methods have been proposed, but fuzzy matching algorithms are rare. And algorithms for image classification can be designed based on quantum machine learning.

In addition, the research on quantum image processing is still in its infancy. Although many quantum image processing algorithms have been proposed, it is still not rich enough. Relatively complex algorithms such as edge detection, image segmentation, filtering, frequency domain processing, image compression are relatively few. There are also few algorithms for image synthesis, binarization, and image grayscale histograms. Therefore, more algorithms are needed to increase the richness of quantum image processing. Moreover, the algorithms that have been proposed use auxiliary qubits, and it is always the question that how to reduce the use of auxiliary qubits to reduce the complexity. Also, quantum image processing has potential for machine learning, pattern recognition, etc. In addition, the development of algorithms requires the support of quantum computers. As algorithms increase in size and complexity, algorithm simulations will be limited by classical computers and it will be more efficient to simulate quantum systems on quantum computers.

12 Conclusion

Quantum processing algorithms proposed in recent years are sorted and summarized in the paper. On the basis of mathematical concepts of quantum computing, we have emphatically reviewed the research progress in the fields of quantum image representation, geometric transformation, morphological transformation, quantum image protection, edge detection and image segmentation, image matching, filtering, compression, quantum machine learning. Based on the current research progress, we discussc and point out several exisitng problems and challenges in the field of quantum image processing, which will also be one of the future works.

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Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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