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ORIGINAL PAPER

Particle Swarm Optimization Variants for Solving Geotechnical Problems: Review and Comparative Analysis

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Abstract
Optimization techniques have drawn much attention for solving geotechnical engineering problems in recent years. Particle swarm optimization (PSO) is one of the most widely used population-based optimizers with a wide range of applications. In this paper, we first provide a detailed review of applications of PSO on different geotechnical problems. Then, we present a comprehensive computational study using several variants of PSO to solve three specific geotechnical engineering benchmark problems: the retaining wall, shallow footing, and slope stability. Through the computational study, we aim to better understand the algorithm behavior, in particular on how to balance exploratory and exploitative mechanisms in these PSO variants. Experimental results show that, although there is no universal strategy to enhance the performance of PSO for all the problems tackled, accuracies for most of the PSO variants are significantly higher compared to the original PSO in a majority of cases.

1 Introduction

Engineering problems are challenging because, very often, they have a large number of design variables, are non-convex within their solution domains, contain multiple local minima in their search space, and involve multiple constraints. Global optimization algorithms allow us to handle these challenges when solving different types of engineering problems [1–5]. The complex nature of civil engineering problems has led to the use of optimization algorithms in a wide range of application areas such as earthquake engineering [6], structural engineering [7, 8], geotechnical engineering [9–13], transportation [14, 15], construction management [16, 17], and water resource engineering [18, 19].

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Inconsistent behavior of soil and rock, as well as conditional reaction to different stimulations, have made geotechnical engineering problems difficult to solve with conventional optimization techniques. Artificial intelligence (AI) has thus become an alternative for solving such problems efficiently. The common approaches in geotechnical problems can be broadly categorized into analyzing (e.g., [20] and [21]), designing (e.g., [22]), predicting (e.g., [23]), and classification (e.g., [24]). One of the issues in analyzing problems is examining the stability of a given heterogeneous soil slope. A large number of studies have focused on finding the most critical failure surface of soil slopes [25]. Methods proposed to tackle this problem include the harmony search (HS) algorithm [26], support vector machine (SVM) [27], artificial neural network (ANN) [28], fuzzy logic [29–31], relevance vector machine [32], genetic algorithm (GA) [33], artificial bee colony (ABC) [34], gravitational search [35], ant colony optimization (ACO) [36], cuckoo search as well as other evolutionary-based optimization algorithms [37, 38].

Back analysis of different geotechnical problems has also attracted much attention from the AI community. For example, Ledesma et al. [39] estimated parameters in geotechnical back analysis based on a maximum likelihood approach. Wei [40] proposed to use particle swarm optimization (PSO) for back analysis in geotechnical engineering. Cheng et al. [41] utilized a hybrid approach for handling pile driving back analysis. Yu et al. [42] used an ANN for displacement back analysis of earth-rockfill dams. Hashash et al. [43] used optimization-based inverse analysis for excavation response. Rechea et al. [44] performed an inverse analysis for parameter identification in simulation of excavation support systems using optimization algorithms. Moreira et al. [45] used an evolution strategy for back analysis of geomechanical parameters in underground work.

In terms of designing geotechnical structures (e.g., retaining structures, shallow foundations, pile foundations), many studies have focused on finding optimal design of concrete retaining walls. For example, Camp and Akin [46] tackled the problem using a big bang-big crunch approach. Other methods used include the ACO [47], an enhanced charged system search algorithm [48], biogeography-based optimization algorithms [49], evolutionary optimization algorithms [50], and the teaching learning-based optimization algorithm [51]. Ponterosso and Fox [52] applied a GA for optimization of reinforced soil embankment. Basudhar et al. [53] studied the design of geosynthetic reinforced earth retaining walls. Basha and Babu [54] tackled the external stability of geosynthetic reinforced soil using a reliability-based approach. Manahiloh et al. [55] solved this problem by the HS algorithm. Ghiassian and Aladini [56] dealt with reinforced earth walls with metal strips using a GA. Kashani et al. [57] attempted to find optimum design of reinforced earth wall using evolutionary optimization algorithms.

When it comes to prediction related geotechnical engineering problems, several studies have used ANNs to model the capacities of axial and lateral loads of pile foundations [58–64]. Settlement and the load-settlement response of pile were predicted using ANNs by some other researchers [65–67]. Khajehzadeh et al. [68] found the optimum design of shallow foundation by means of gravitational search. Camp and Assadollahi [69] utilized a hybrid big bang-big crunch algorithm for shallow footing optimization. Gandomi and Kashani [70] explored the efficiency of a number of swarm intelligence-based algorithms for optimal cost design of shallow foundations. Other related applications include predicting liquefaction [71–76], mining [77, 78], rock mechanics [79, 80], site characterization [81], tunneling [82, 83], deep excavation [84], and classification problems [85–87].

In this paper, we focus on the PSO algorithm because of its wide applications in engineering optimization problems. PSO has drawn considerable attention in the field of civil engineering in general and geotechnical engineering in particular. We first provide a comprehensive review of the different applications of PSO on geotechnical engineering problems. Then, we employ PSO and its varaints to solve three benchmark geotechnical engineering problems, namely the slope stability, retaining wall, and shallow footing problems. Natural and artificial soil slopes as a prevalent structure in various construction projects, retaining walls as a kind of instrument for increasing the stability of unstable soil slopes, and shallow footing as one of the most impactful parts of a structure for conveying effective forces to the earth, are all of high importance in civil engineering.

Slope stability analysis examines the stableness of a soil slope by defining a factor of safety (FOS). This problem follows a nonlinear and nonconvex function with strong local minima within the solution domain [37], which makes finding the optimal solution using classical optimization algorithms nearly impossible. For the retaining wall and shallow footing problems, two key criteria have to be met during the design procedure: geotechnical stability and structural strength. Furthermore, the final costs of projects, as well as the volume of consumed materials, have to be minimized. Since the objective function engages a large number of design variables, satisfying the above-mentioned criteria is very difficult. Metaheuristic algorithms have proven to be helpful for handling these problems [88–90]. Our focus in this study is to assess the performance of the PSO variants, especially their information-sharing mechanisms in controlling exploration and exploitation. Through simulation experiments, we study the convergence histories and carry out statistical analysis over the results obtained.

The rest of this paper is organized as follows. In Sect. 2, we present an overview of PSO. In Sect. 3, we describe the geotechnical engineering problems in detail, including their objective functions. PSO algorithms are then discussed in Sect. 4. In Sect. 5, we review the application of PSO on a wide range of geotechnical engineering problems. In Sect. 6, simulation experiments and results are discussed. Finally, Sect. 7 concludes the work and suggests future research directions.

2 Particle Swarm Optimization Overview

PSO is one of the most popular metaheuristic algorithms, known for its ability to solve a variety of challenging problems [91–94]. Due to its stochastic nature, PSO produces varying solutions in each trial and is computationally more expensive than exact mathematical methods in general. A key issue here is to balance between *diversification (exploration)* and *intensification (exploitation)*. A suitable trade-off between the two is essential for maximizing the algorithm's performance [95]. In the following, some related work on PSO is presented and analyzed.

Shi and Eberhart [96] proposed an inertia weight w to balance intensification and diversification for the original PSO. Smaller values of w push PSO toward local search while larger values lead to global search. The main reason behind this is that large values of w decrease the independence of particles from the initial solutions and make them free to search new areas. Cognitive and social learning factors have also been proven to play a significant role in balancing global and local search. Increasing the cognitive coefficient provides more concentration on global search, and a large social coefficient strengthens local search.

Cui et al. [97] presented an improved PSO (IPSO) with three non-linear time-varying strategies for cognitive c_1 and social c_2 component adjustment. The underlying approach is based on providing more global search in the intial iterations and proceeding toward local search in the final iterations. Therefore, two of those three schemes proposed a decreasing pattern for cognitive coefficient and one proposed an increasing pattern. Comparison of the proposed decreasing routines with the original PSO and linear time-varying strategy exhibited slower convergence in the intial runs versus better convergence in the final iterations.

Ziyu and Dingxue [98] utilized an exponentially time-varying acceleration function for updating c_1 and c_2, resulting in a modified PSO (MPSO). This modification was successful in striking a balance between exploration and exploitation for solving some benchmark functions. However, results proved that this method may not be as consistent, since it failed to solve the Sphere benchmark function. Bao and Mao [99] applied an asymmetric time-varying trend for acceleration coefficient adjustment to the MPSO. In their study several patterns were proposed where the variation of c_1 and c_2 did not follow the same pace. Their proposal proved to be efficient in enhancing PSO.

A comprehensive learning particle swarm optimizer (CLPSO) was proposed by Liang et al. [100, 101]. Instead of following the global best solution (g_{best}), all particles' best solutions (p_{best}) have the possibility to serve as an exemplar for updating the velocity. The main concept behind this is to benefit from sharing the achievements of all the particles. Comparison of the results obtained by CLPSO with several variations of PSO showed superiority of this algorithm in handling the tackled benchmark functions.

Ngo et al. [102] attempted to modify the particle motion based on selecting an exemplar other than the global best solution. In other words, all particles, from the best to the worst, have the chance of being a leader in the next iteration. They called their algorithm the extraordinary particle swarm optimization (EPSO). EPSO was tested over several constrained and unconstrained (i.e., unimodal and multimodal) functions, and its performance was assessed against other algorithms inclusing other PSO variants. Even though EPSO did not provide the best solutions, its results were close to the global optimum and considerably comparable to those generated by the best algorithms.

A more recent variant of PSO based on the comprehensive learning strategy can be found in the heterogeneous comprehensive learning particle swarm optimization (HCLPSO) algorithm [101]. In HCLPSO, the population is divided into two subpopulations, conducting exploration and exploitation independently without interfering with each other. Another improvement in HCLPSO is the use of adaptive control parameters in order to boost exploration and exploitation.

Other PSO variants include an evolutionary extension developed by Tillett et al. [103], known as Darwinian PSO (DPSO), which employs many swarms that work independently at any time; the fractional-order DPSO (FDPSO) proposed by Couceiro et al. [104], which employs fractional calculus for evaluating the velocity; and an improved random drift PSO (IRDPSO) by Elsayed et al. [105], which imitates the free electron model. The IRDPSO was based on two modifications applied to the original RDPSO: (1) adding a crossover operator; and (2) using local best instead of the mean best position.

3 Geotechnical Engineering Problems

3.1 Slope Stability Analysis

To examine the stability of a soil slope, valid trial slippery surfaces must be constructed by considering predefined rules. The critical failure surface has to be concave upward without fluctuation. In this paper, the proposed method by Cheng [106] is utilized to produce a valid slip surface, and the Morgenstern-Price method [107] is utilized to evaluate the FOS for every potential failure surface.

3.2 Retaining Wall Design

Concrete cantilever retaining wall is an important geotechnical structure because of a wide range of applications, massive and costly construction operations, and serious consequences of collapse. Successful functioning of retaining walls necessitates meeting the following geotechnical measures: (1) overturning safety factor (FOS_O); (2) sliding safety factor (FOS_S); (3) bearing capacity safety factor (FOS_B). In this study, the Mononobe–Okabe method is used to evaluate the active and passive forces under a seismic loading case [108]. For more details on the design procedure, see [12].

3.3 Shallow Footing Design

Another key geotechnical structure is the shallow foundation. Any structure or megastructure would be unable to successfully function unless the loads directed to the earth are effective. A foundation can be remodeled by the footing length (L), width (B), thickness (H), and depth from the ground to the bottom of the footing (D).

The final design must be checked for two fundamental criteria: geotechnical stability and structural strength. Geotechnical stability measures are bearing capacity and settlement. Additionally, a lot of structural requirements are defined based on ACI 318-05 [109] to guarantee enough strength for providing serviceability. These requirements can be listed as: one-way shear capacity, two-way shear capacity, flexural capacity, the column's bearing capacity, dowels, and footing, and the reinforcement development length. More details on such restrictions and constraints can be found in ACI 318-05 [109] and Camp and Assadollahi [69, 88].

3.4 Objective Function Formulation

As a necessary part of working with an optimization algorithm, an objective function should be defined. In slope stability analysis, the value of FOS is considered as the fitness function in a minimization procedure.

However, for the retaining wall minimization problem, the final cost and weight of the structure are subject to Eqs. (1) and (2), respectively.

$$f_{cost} = C_s W_{st} + C_c V_c \tag{1}$$

$$f_{weight} = W_{st} + 100 V_c \gamma_c \tag{2}$$

where C_s and C_c indicate unit costs of steel and concrete, respectively, W_{st} is the steel's weight, V_c represents the concrete volume, and γ_c is the concrete unit weight scaled by a factor of 100 in a similar manner to Saribas and Erbatur [110].

For the shallow footing optimization problem, minimum cost design is considered based on Eq. (1).

4 Optimization Algorithms

4.1 Particle Swarm Optimization

As a swarm intelligence algorithm, PSO imitates the collective behavior of a school of fish or birds [111]. In PSO, a group of particles searches the solution space via sharing their best-found information. Each particle schedules its next movement by taking its own best finding and the overall best achievement into account. PSO defines each particle's position as follows:

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{3}$$

where X_i^{t+1} shows the the position of the ith particle in $t+1$th iteration, X_i^t indicates the current position (in tth iteration), and V_i^{t+1} is the velocity calculated for the $t+1$th iteration.

Shi and Eberhart [96] proposed Eq. (4) for evaluating the velocity.

$$V_i^{t+1} = \omega V_i^t + C_1 r_1 (P_i - X_i^t) + C_2 r_2 (P_g - X_i^t) \tag{4}$$

In this equation, ω is the inertia weight within [0, 1.2] to balance exploration and exploitation, P_i and P_g are the personal and global bests of particles, respectively; C_1 and C_2 are constants typically set to a number in the interval of [0, 2], and r_1 and r_2 are random numbers in the interval of [0, 1].

4.2 Comprehensive Learning Particle Swarm Optimization

The CLPSO was proposed to improve exploratory and exploitative behavior in PSO [112]. By changing how the velocity is updated, the best experiences of all particles will be shared. Based on this method, for each dimension, a random number will be produced and compared to a learning probability function, P_{ci}. In the case of having a number lower than P_{ci}, the dth variable of P_i will be updated using other particles' best-found solutions. Otherwise, their own best experience will be selected for updating P_i. The velocity term and probability function are defined by Eqs. (5) and (6), respectively.

$$V_{i,t+1}^d = wV_{i,t}^d + c \times rand_i^d \times (pbest_{f_i(d)}^d - X_{i,t}^d) \tag{5}$$

where $f_i(d)=[f_i(1), f_i(2), \dots, f_i(D)]$ indicates if the ith particle follows its own or others' $pbest_i^d$ for each dimension d.

$$P_{C_i} = a + b \times \frac{(\exp((10(i-1))/(ps-1)) - 1)}{(\exp(10) - 1)} \tag{6}$$

In Eq. (6), 'ps' represents the population size, $a=0.05$, and $b=0.45$.

4.3 Heterogeneous Comprehensive Learning Particle Swarm Optimization

In continuing PSO improvement, a heterogeneous version of CLPSO was introduced with a different scheme for updating the velocity term [113]. In this method, the population is categorized into two separate subpopulations with different tasks: exploration and exploitation. The exploitation subpopulation uses Eq. (7) for updating the velocity while the exploration subpopulation will move to the next step accordingly:

$$V^d_{i,t+1} = wV^d_{i,t} + c_1 \times r1^d_i \times (pbest^d_{f_i(d)} - X^d_{i,t}) + c_2 \times r2^d_i \times (gbest^d - X^d_{i,t}) \tag{7}$$

where the inertia weight w decreases linearly with respect to run time from 0.99 to 0.2. A time-dependent acceleration factor, c, is defined, which varies between 1.5 and 3 to boost the exploration ability of sub-population-group 2. c_1 and c_2 in Eq. (7) are set within [0.5, 2.5] to improve the exploitation capacity.

Another important fact about HCLPSO is that the exploration subpopulation does not have access to the exploitation information. This way, HCLPSO prevents a rapid information flow. HCLPSO thus benefits from a satisfactory balance between exploration and exploitation.

4.4 Extraordinary Particle Swarm Optimization

In this algorithm, similar to many other efforts, the main focus is on balancing exploration and exploitation. By updating the particles' movement directions based on their best-found solutions (p_{best}) and the global best (g_{best}), it is highly possible that the algorithm will face premature convergence and be trapped in a local optimum solution. On the other hand, using information from other individuals for producing new solutions enables the algorithm to explore the solution space with higher diversity. Taking this into account, Ngo et al. [102] proposed a new variant of PSO, the EPSO.

With EPSO, all the particles take part in updating the velocity term stochastically by the following equation:

$$\vec{V}_{i,t+1} = C(\vec{X}_{Ti,t} - \vec{X}_{i,t}) \tag{8}$$

where $X_{Ti,t}$ is the determined target of the ith particle at the tth iteration, and C is a combined coefficient including cognitive and social factors.

The utilized stochastic approach for selecting the target individual, $X_{Ti,t}$, follows the following basic rules:

1. the upper limitation of T_{up} is defined by a user-defined coefficient α multiplied by population size N_{pop} according to:

$$T_{up} = round(\alpha \times N_{pop}) \tag{9}$$

2. a target is randomly produced according to:

$$T = round(rand \times N_{pop}) \tag{10}$$

3. if the produced T in step 2 is between 0 and T_{up}, the particle will move toward the target using Eq. (10). Otherwise, it will move randomly.

4.5 Fractional-Order Darwinian PSO

Tillett et al. [103] proposed a new Darwinian-based variation of PSO called DPSO. In this algorithm, sub-swarms are formed and used in conjuction with the selection, as an evolutionary operator.

The most important feature of DPSO is enlisting the fundamental Darwinian theory of survival of the fittest to evade the local optima. DPSO tries to search the solution domain using different swarm sets, which work simultaneously in parallel for a given test problem. In 2012, Couceiro et al. [104], by implementing fractional calculus using *Grünwald–Letnikov* definition, proposed Eq. (11) for updating the velocity:

$$\begin{aligned} v^n_{t+1} = {} & \alpha v^n_t + \frac{1}{2}\alpha v^n_{t-1} + \frac{1}{6}\alpha(1-\alpha)v^n_{t-2} \\ & + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)v^n_{t-3} + \rho_1 r_1(\breve{g}^n_t - x^n_t) \\ & + \rho_2 r_2(\breve{x}^n_t - x^n_t) + \rho_3 r_3(\breve{n}^n_t - x^n_t) \end{aligned} \tag{11}$$

where α is a fractional coefficient, ρ_1, ρ_2, and ρ_3 assign weights to the inertial influence, $\breve{g}^n_t$ is the global best, $\breve{x}^n_t$, n^n_t is the neighborhood best.

4.6 Improved Random Drift PSO

The RDPSO developed by Sun et al. [114] is an improved version of PSO in which a modified equation for velocity is proposed. In this algorithm, the velocity term uses the mean best position at each iteration to update the position. In fact, RDPSO mimics the free electron model in which the electron thermal motion and electric field cause random movements. Elsayed et al. [105] proposed an improved version of this algorithm called IRDPSO by taking two modifications into account: (1) adding a crossover operator to the original RDPSO; and (2) replacing the mean best position in the population with one of the locally optimal solutions obtained. The proposed velocity equation in IRDPSO is presented as follows:

$$v^n_{t+1} = \alpha\left(Y^{t-1}_{i,j} - X^{t-1}_{i,j}\right)\delta^t_{i,j} + \beta\left(Z^t_i - X^{t-1}_i\right) \tag{12}$$

where $Y^{t-1}_{i,j}$ is the jth element of the personal best position for particle i at iteration $t-1$, α is the thermal coefficient, β is the drift coefficient, $\delta^t_{i,j}$ is a random number with a similar distribution to the jth element of particle i at iteration t, and

Z_i^t is the local focus position of i-th particle in t-th iteration.

4.7 Improved Particle Swarm Optimization Based on Dynamic Parameter Setting

The main parameters of PSO are the weighting factor (w), cognitive coefficient (c_1) and social coefficient (c_2). Similar to many other optimization algorithms, fine-tuning of these parameters will affect its performance especially in dealing with complex problems. The important fact in adjusting c_1 and c_2 is that a greater value of c_1 than c_2 guides the algorithm toward global search while a greater value of c_2 leads the algorithm toward local search. Besides, c_1 should be changed sharply to provide exploration and c_2 should be changed gently to strengthen PSO's ability to evade from the local minima. Dynamic parameter settings have been proposed as an efficient alternative to reach an appropriate adjustment of acceleration coefficients (c_1 and c_2) [97–99]. In this paper, three improvements of PSO based on time-dependent parameter settings are utilized accordingly.

4.7.1 An Improved PSO equipped with Time-Varying Accelerator Coefficients

In 2008, an improved version of PSO was proposed by Cui et al. [97], based on a time-varying strategy for adjustment of c_1 and c_2, known as IPSO in short. They proposed three different non-linear methodologies for updating the cognitive coefficient: the upward, concave, and exponential functions. It is mentioned in the original study that the sum of c_1 and c_2 will be 3.0, therefore, by defining c_1 the relevant value of c_2 can be evaluated easily. Here, the following settings are considered for c_1 and c_2:

$$c_1(t) = 2.5 + 2\left(\frac{t}{T}\right)^2 - 2\left(\frac{2t}{T}\right), \quad c_2(t) = 0.5 - 2\left(\frac{t}{T}\right)^2 + 2\left(\frac{2t}{T}\right) \tag{13}$$

In Eq. (13), t shows the current iteration and T represents the predefined maximum iteration.

4.7.2 A Modified PSO with Adaptive Acceleration Coefficients

Considering the impact of acceleration coefficient in exploring the solution space effectively, Ziyu and Dingxue [98] proposed another modified version of PSO named TACPSO. TACPSO uses Eqs. (14) and (15) to encourage the algorithm toward global search in the initial iterations and more concentration on local search in the later iterations.

$$c_1 = c_{\min} + (c_{\max} - c_{\min}) \cdot \exp[-(4k/G)^2] \tag{14}$$

$$c_2 = c_{\max} - (c_{\max} - c_{\min}) \cdot \exp[-(4k/G)^2] \tag{15}$$

where G represents the maximum iteration time and k shows the present iterative time, and c_{max} and c_{min} are 2.5 and 0.5, respectively.

4.7.3 PSO with Asymmetric Time Varying Acceleration Coefficients

Another effort to enhance the effectiveness of PSO via a time-varying acceleration coefficient approach can be found in a study by Bao and Mao [99]. In this version of PSO, called MPSO, the acceleration is updated according to Eqs. (16) and (17):

$$c_1 = c_{1\max} - k \times (c_{1\max} - c_{1\min})/k_{\max} \tag{16}$$

$$c_2 = c_{2\min} + k \times (c_{2\max} - c_{2\min})/k_{\max} \tag{17}$$

where k is the current iteration and k_{max} is the maximum iteration. In this study, the parameter settings utilized in [30] are considered for Eqs. (16) and (17).

4.8 Autonomous Particle Groups for Particle Swarm Optimization

Autonomous particle groups for particle swarm optimization (AGPSO) is a new approach based on defining autonomous groups that explore the solution space independently [115]. AGPSO mimics the diverse obligations of individuals in a termite colony based on their different abilities and potentials. Mirjallili et al. [115] attempted to find a balance between diversification and intensification in their modified version of PSO by defining four groups of particles that explore the solution space autonomously. Similar to the original PSO, global best (g_{best}) and personal best (p_{best}) of the particles are evaluated at each iteration. Then, each group of particles will have their specific strategy for updating c_1 and c_2, [115]. After calculating c_1 and c_2, the velocities and positions of particles will be updated following the conventional PSO rules using Eqs. (3) and (4).

In this study, three different variations of AGPSO developed in the original paper [115], named AGPSO1, AGPSO2, and AGPSO3, are used. These versions are differentiated by different strategies for updating c_1 and c_2.

5 PSO Applications in Geotechnical Engineering

In this section, we provide a comprehensive review of the different applications of PSO algorithms for geotechnical engineering problems. In the following sub-sections, a detailed review is presented based on the following categories: slope stability, retaining walls, shallow foundations, pile foundations, reinforced soil, tunneling, and others.

5.1 Slope Stability

One of the most crucial challenges in geotechnical engineering is examining the stability of a slope. These structures are prevalent in many construction projects such as highways, mines, tunneling, embankments, etc. Hence, any fault in stability assessment of the slopes can end up in a catastrophic disaster. However, finding the most critical failure surface and its relevant FOS is a complicated task. Hui et al. [116] in 2004, used the PSO algorithm for finding the critical slip surface in soil slopes for the first time. In 2005, an improved version of PSO was proposed for slope stability analysis by Li et al. [117], in which a mutation operator was mounted to the original PSO.

Cheng et al. [118], in 2007, proposed a methodology for finding the non-circular slip surface in two-dimensional soil slopes. Such a consideration is more compatible with non-homogeneous soil slopes and will result in more realistic solutions. In their study, a limit-equilibrium based method by vertical slices was suggested for evaluating the FOS. They handled this problem by enlisting the original PSO and a modified version of PSO. Based on this modification, the number of flies for the particles became limited to a maximum band and better particles have more chances to fly more than once. The proposed methods were assessed over several benchmark case studies. Li et al. [119], in 2007, also proposed a discontinuous flying PSO to improve the performance of this algorithm and reduce the operation time for time expensive problems. The proposed algorithm was examined for analyzing a wide range of soil slopes. In another study by Cheng et al. [120] in 2007, six different heuristic algorithms were employed for handing slope stability problem, with the PSO demonstrating more stable performance among them. In this study, the authors proposed well-adjusted values for necessary parameters of PSO (i.e., inertia weight and stochastic weighting factors) by conducting a sensitivity analysis. Wang et al. [121] proposed a methodology using a finite element approach for evaluating the stability of a given soil slope. In this work a PSO algorithm was attributed to find the solution. In 2009, Tian et al. [122] attempted to handle the problem of finding minimum FOS based on the stress field resulting from a finite element simulation by means of the PSO algorithm. Li et al. [123], in 2009, proposed a combined method based on HS and PSO for finding the most critical failure surface of a soil slope. Khajehzadeh et al. [124, 125] incorporated uncertainties of the soil material into the FOS evaluations. In 2010, Li et al. [126] proposed an improvement on the PSO algorithm based on the same strategy utilized by Cheng et al. [118] and Li et al. [118]. Li and Chu [127], in 2011, used a hybrid approach based on PSO and HS to estimate the stability of 2D slope. In 2012, Kalatehjari et al. [128] utilized the PSO algorithm for exploring the stability of homogeneous soil slopes. Cheng et al. [41] developed a hybrid approach based on merging PSO and HS for slope stability optimization. Khajehzadeh et al. [129] concentrated on locating the critical slip surface in 2D soil slopes by enlisting a modified PSO algorithm. Based on this modification, the inertia weight decreased in the course of time. Furthermore, the content of a randomly chosen particle was shared for updating the velocity. In a study by Johari and Sahebkar [130] in 2013, the PSO algorithm was used to handle the problem of slope stability. Wang et al. [131] considered a limit equilibrium method based on the locations of the trailing edge and shear outlet; they considered the circular slip surface and utilized the PSO algorithm to solve the problem. In 2013, Khajehzadeh et al. [132] applied a modified version of PSO by considering a chaotic sequence for updating the inertia weight to analyze the 2D soil slope. Kalatehjari et al. [133] considered the effects of two different methods (i.e., conventional method (CM) and triple-point method (TPM)) for generating trial circular slip surface on the final results obtained by PSO in slope stability analysis. Kalatehjari et al. [134, 135] analyzed 3D soil slopes by an FEM-based model in PLAXIS coupled with the PSO algorithm. In 2015, Gandomi et al. [136] considered the problem of 2D soil slopes using some swarm intelligence-based algorithms among which PSO performed satisfactorily for handling the tackled problems. Li et al. [137] proposed a hybrid approach based on quantum-behaving PSO integrated with a least square SVM for slope stability analysis. Chen et al. [138] synchronized the standard landslide analysis program (STABL) with PSO for slope stability assessment. They tested the proposed methodology for handling homogeneous soil slopes considering different values for the inertia weight. Xue [139] developed a hybrid technique based on the least square SVM (LSSVM) and PSO algorithm for defining the stability of slopes. Their proposed methodology was validated through two numerical case studies. Kang et al. [140] conducted a reliability analysis of soil slopes based on the υ-SVM. Kang et al. [21] in another study used the LSSVM for reliability analysis of the soil slopes. The hyper-parameters of both υ-SVM and LSSVM were adjusted using PSO. Gordan et al. [141] attempted to solve the homogeneous slope stability problem using a combined PSO and ANN approach. In that study, 699 cases were simulated using GeoStudio software and their associated FOS was computed. Those case studies remodeled a wide range of possible conditions while varying the influential parameters such as slope height, gradient, cohesion, friction angle and peak ground acceleration. Moreover, Gordan et al. [141] studied the impact of stochastic weighting factors on the function of their proposed technique via sensitivity analysis. Reale et al. [142, 143] conducted comprehensive studies for reliability analysis of the soil slopes. In their study, the main aim was finding the location of the most critical failure surface with the minimum reliability index as well as finding all possible discrete failure mechanisms. To that end, the authors used two multimodal versions of PSO named locally informed PSO (LIPS) and standardized LIPS (SLIPS). Wang et al. [144] studied the stability of rock slopes during the excavation of the surface. They conducted a reliability analysis for this problem using the Monte Carlo method. In this study, a binary PSO was utilized for finding the minimum shear strength in a 3D shear zone. Johari and Mousavi [145] took the uncertainties of the effective parameters on soil slope stability (i.e., angle of shearing resistance (φ), cohesion intercept (c), and unit weight (γ) of soil) into account while height and inclination were frozen. Four limit equilibrium-based methods were utilized for FOS evaluation. In that study, PSO was assigned to automate the procedure of finding minimum reliability index. In one of the most recent studies, Pandit et al. [146] provided a review of the application of various deterministic and stochastic algorithms including PSO for slope stability analysis. Additionally, an overview of both deterministic and reliability analysis of the soil slope stability was provided. Sharma et al. [147] matched GeoStudio software with PSO for finding the most critical failure surface in 2D slopes. They considered the bishop method with circular slip surface in their study. Himanshu and Burman [148] studied locating the critical slip surface of slopes using PSO in 2019. In their study, Bishop's method with a different number of vertical slices was utilized for evaluating the FOS based on seepage and seismic loading considerations. Koopialipoor et al. [149] compared the performance of ANN for handling the stability analysis of homogeneous soil slope affected by static and dynamic loads. They used four optimization algorithms, including the GA, PSO, imperialistic competitive algorithm, and artificial bee colony, for adjusting weight and bias of ANN. Luo et al. [150] developed a hybrid approach combining the PSO and cubist algorithm called PSO-CA to predict the stability of soil slopes. A comparative study was provided by applying other tools such as the SVM, k-nearest neighbor (kNN), and classification and regression trees (CART). Shinoda and Miyata [151] used the PSO algorithm for stability analysis of unreinforced and reinforced soil slopes, where a comprehensive sensitivity analysis was conducted on the effect of inertia weight, stochastic weighting factors, and the number of particles. Singh et al. [152] solved the problem of slope stability using three optimization algorithms, namely the GA PSO, and biogeography-based optimization (BBO) algorithm. Yuan and Moayedi [153] used multilayer perceptron (MLP) for classifying the soil slopes and failure recognition, where they combined several optimization algorithms(GA, PSO, ACO, BBO, evolutionary strategy, and probability-based incremental learning) with MLP. Table 1 summarizes the application of different techniques for evaluating the slope stability problem.

5.2 Retaining Wall

Retaining walls are important whenever dealing with a naturally unstable tranche is a concern in a construction project. However, due to their huge bulk, they cause extravagant expenses for any given project. Therefore, much effort has been devoted to the optimal design of retaining structures. Here we discuss those studies that used the PSO algorithm for handling the retaining wall optimization problem. Zhao and Ru [154] presented a hybrid method based on SVM and PSO for optimum design of retaining structures of deep pits. In 2009, Ahmadi-Nedushan and Varaee [155] applied the PSO algorithm for minimum-cost and minimum-weight design reinforced concrete cantilever retaining walls (RCC wall). Khajehzadeh et al. [156, 157] considered the optimal design of retaining walls using particle swarm optimization with passive congregation (PSOPC) and MPSO. In those studies, eight different design variables for describing a wall without a base shear key were considered. In 2012, Pei and Xia [158] applied some heuristic algorithms (GA, PSO, and simulated annealing (SA)), including a random direct search algorithm called the complex method (CM), to the problem of minimum-cost retaining wall optimization. In 2014, Kaveh and Soleimani [159] utilized improved harmony search (HIS), colliding bodies optimization (CBO) and democratic PSO (DPSO) for optimum design of retaining walls. In their study, the minimum cost design of the wall based on ACI 318-05 [109] considerations, was proposed. In 2015, Gandomi et al. [160] utilized four

Table 1 Review of the application of PSO to slope stability problems

References	Year	Objectives	Algorithms	Methodology	Findings
Cheng et al. [118]	2007	Finding the most critical failure surface	SA, GA, PSO, SHM, MHM, ACO, Tabu	Limit equilibrium method was used for evaluating FOS (Spencer method) Noncircular slip surface was considered The effect of different number of vertical slices was studied The impact of necessary parameters of each algorithm on their performance was studied through a sensitivity analysis	For normal and simple problems with less than 20 control variables, HS and GA were the most efficient algorithms while Tabu and ACO did not work uniformly For normal and simple problems where the number of design variables exceeds 20, MHM and PSO were more efficient PSO was the best algorithm for large-scale problems For steep slope with tension crack and soil nail, SA and PSO were recommended
Cheng et al. [120]	2007	Finding the most critical failure surface	PSO, MPSO	Limit equilibrium method for computing FOS (Morgenstern–Price method) Noncircular slip surface was considered A modified PSO developed based on defining a termination criterion other than reaching maximum number of evaluations (defining a tolerance of the search)	The obtained results by the proposed algorithm were verified using a pattern search methodology proposed by Cheng [106] PSO and MPSO performed satisfactorily for all the relatively difficult examples within acceptable solution times
Tian et al. [122]	2009	Finding the most critical failure surface	PSO	Using stress filed calculated by a FEM method	The proposed method proved to be feasible based on its results over numerical simulations
Li et al. [123]	2009	Finding the most critical failure surface	Combine PSO and HS	Limit equilibrium method for computing FOS (Morgenstern–Price method) Noncircular slip surface was considered	The proposed hybrid approach performed better than both PSO and HS
Li et al. [126]	2010	Finding the most critical failure surface	DPSO	Noncircular slip surface was considered Different number of slices were considered	PSO reached the global optimum by increasing the population size DFPSO outperformed PSO in terms of accuracy and cost effectiveness

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Khajehzadeh et al. [124]	2010	Calculating the minimum reliability index and corresponding critical probabilistic slip surface	PSO, MPSO	Limit equilibrium method was used for evaluating FOS (Spencer method) Noncircular slip surface was considered Probabilistic analysis conducted by considering the following random variables: the effective friction angle, effective cohesion, unit weight and pore water pressure ratio Advanced First-Order Second-Moment (AFOSM) was utilized for reliability analysis	Comparison of the results of the proposed methods and previous studies demonstrated the superiority of MPSO over other utilized strategies MPSO showed much faster convergence rather than PSO Based on the outputs there is no guarantee that the critical failure surface with minimum FOS occurred within the area with maximum probability of failure The critical failure surface obtained by deterministic and probabilistic analysis are close in homogeneous slopes whereas those are completely different in heterogeneous slopes
Li and Chu [127]	2011	Finding the most critical failure surface	Hybridized PSO with HS	Noncircular slip surface was considered HSPSO proposed based on dynamic parameter setting for the algorithm Different parameter setting for the necessary parameters of PSO and HSPSO was conducted	Dynamic parameter setting proved to be effective for finding the critical slip surface The proposed HSPSO was much less sensitive to the choice of parameters compared with original PSO
Khajehzadeh et al. [125]	2011	Calculating the minimum reliability index and corresponding critical probabilistic slip surface	PSO, Hybrid Chaotic PSO and HS (CPSOHS)	Limit equilibrium method was used for evaluating FOS (Morgenstern–Price method) Noncircular slip surface was considered Probabilistic analysis conducted by considering the following random variables: the effective friction angle, effective cohesion, unit weight and pore water pressure ratio Advanced First-Order Second-Moment (AFOSM) was employed to perform reliability analysis	The superiority of the proposed algorithm CPSOHS was demonstrated through some numerical examples Results indicated the direct relationship between angle of friction and reliability index. Cohesion and the angle of friction of soil have a direct relationship with the reliability index Reliability index decreased by increasing soil strength The distribution of random variables affected the reliability of the slopes to some extent

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Kalatehjari et al. [128]	2012	Finding the most critical failure surface	PSO	Limit equilibrium method was used for evaluating FOS (Bishop) Circular slip surface was considered A sensitivity analysis for swarm size and number of iterations was conducted	Based on a sensitivity analysis, the optimum values of iteration number and swarm size were defined to be 70 and 50, respectively Based on the results of two numerical simulations, PSO handled the problem of slope stability satisfactorily
Cheng et al. [41]	2012	Finding the most critical failure surface	PSO, Hybrid MPSO and HS	Limit equilibrium method was used for evaluating FOS (Spencer method) Noncircular slip surface was considered Hybridizing a modified PSO based on limiting the number of flights to a maximum band and giving more chance of flying to the more fitted particles	The efficiency of the proposed hybrid algorithm was tested over several complicated numerical examples The obtained results for more complicated problems proved the efficieny of this algorithm while for simpler problems it was not much effective
Khajehzadeh et al. [132]	2013	Finding the most critical failure surface	PSO, MPSO	Limit equilibrium method was used for evaluating FOS (Spencer method) Noncircular slip surface was considered A chaotic based approach was proposed for updating the inertia weight (logistic map)	The efficiency of the proposed algorithm was assessed through three hypothetical soil slope examples and a real slope in Malaysia The observation verified that MPSO converged to the lowest value of FOS in lower course of iterations
Johari and Sahebkar [130]	2013	Finding the most critical failure surface	GA, PSO	Limit equilibrium method was used for evaluating FOS (Morgenstern–Price method) Noncircular slip surface was considered Using PLAXIS software for analyzing one of the numerical examples	GA and PSO found better solution than PLAXIS Obtained results by GA and PSO were comparable to other algorithms GA and PSO proved to be stable in solving the tackled case studies
Wang et al. [131]	2013	Finding the most critical failure surface	PSO	Limit equilibrium method based on self-weight stress field was used for FOS evaluation	Stress deformation in rigid body was removed and a reasonable convergence rate was recorded The critical failure surface for left bank in Xiluodu Hydropower Station in this study was well-matched with the actual engineering

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Kalatehjari et al. [133]	2014	Finding the most critical failure surface	PSO	Limit equilibrium method was used for evaluating FOS (Simplified Bishop and Spencer method) Circular slip surface was considered Two different strategies were proposed for generating trail failure surface: Conventional method (CM) and triple-point method (TPM)	The simulation results proved that TPM was more successful than CM Based on the results CM failed to converge to valid solutions in some cases while TPM was more stable Based on the results the proposed methodology outperformed previously recorded studies
Kalatehjari et al. [134]	2014	Finding the most critical failure surface in 3D soil slopes	PSO	Limit equilibrium method was used for evaluating FOS based on mobilized shear force Ellipsoidal slip surface was considered Plaxis-3D model of one of the numerical examples was utilized for comparison	The best settings for swarm size, coefficients, and inertia weight of velocity equation were examined Two numerical models were simulated. The first one was remodeled and compared with Plaxis-3D results. The other one was handled only by PSO and compared to other studies Both numerical simulations proved that PSO successfully located the critical surface in 3D slopes
Kalatehjari et al. [135]	2014	Finding the most critical failure surface in 3D soil slopes	PSO	Limit equilibrium method was used for evaluating FOS based on mobilized shear force Ellipsoidal slip surface was considered Direction of sliding was assessed Plaxis-3D model of one of the numerical examples was utilized for comparison The verification of the model was conducted through a physical modeling of a small-scale 3D soil slope under vertical load in the lab	The proposed approach was verified by two numerical examples The first example was solved by the proposed method based on PSO as well as a FEM model in Plaxis-3D. The results were compatible satisfactorily The second example was reused from previous studies and the result of PSO-based method was compatible with previous studies The results of a computer model of constructed model were compatible with the observations in the lab

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Reale et al. [142]	2015	Deterministic and probabilistic multi-modal analysis of slope stability	PSO, LIPS	Limit equilibrium method was used for evaluating FOS and reliability index (simplified Bishop method) Circular slip surface was considered Multi-modal PSO was utilized to determine multiple critical slip surface for each failure mode Three soil slopes were to be solved using the proposed method (i.e., two of them were borrowed from the previous studies, and one based on an Australian open cut coal mine)	The proposed method converged to all significant extrema It was mentioned that there are many critical failure surfaces with similar failure probabilities other than viable slip surface The results obtained for solving the third example using the proposed method were very similar to the one obtained by Slope/W An advantage of using LIPS was finding several risky failure surfaces The differences between deterministic and probabilistic slip surfaces were declared based on the simulation results
Li et al. [137]	2015	Analyzing the stability of slopes	LSSVM, PSO-LSSVM, QPSO-LSSVM	A FOS was attributed to a given slope based on unit weight, cohesion, angle of internal friction, slope angle, height, and pore water pressure The proposed artificial intelligence-based tools were trained using the provided datasets	The proposed methodology facilitated the slope stability problems without a need for highly-nonlinear analysis of slopes QPSO outperformed NDWPSO and LDWPSO PSO-LSSVM and QPSO-LSSVM accomplished more efficient training and testing Considering the training time and convergence property, QPSO-LSSVM performed better than PSO-LSSVM
Chen et al. [138]	2015	Finding the most critical failure surface	PSO combined with the standard landslide analysis program (STABL)	Limit equilibrium method was used for evaluating FOS Circular/Noncircular slip surface was considered A homogeneous slope was considered	The proposed methodology was compared to the ones utilized in previous studies, and it outperformed them
Gandomi et al. [136]	2015	Finding the most critical failure surface	PSO, FA, CS, LKH	Limit equilibrium method was used for evaluating FOS (Morgenstern–Price method) Noncircular slip surface was considered	Based on the results, although PSO was stable and performed satisfactorily, CS and LKH outperformed PSO

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Kang et al. [21]	2016	Reliability analysis of slopes	Combined LSSVM and PSO	Monte Carlo simulation was utilized for probability analysis Computer-aided generated samples was utilized for developing LSSVM model to approximate the limit state function based on the response surface PSO was used for adjusting hyper parameters of LSSVM	Failure probability was computed using the PSO-LSSVM response surface in combination with Monte Carlo simulation The proposed methodology worked efficiently in terms of computational effort and accuracy
Gordan et al. [141]	2016	Analyzing the stability of slopes	ANN, ANN-PSO	A series of 699 homogenous slopes were analyzed considering seismic loading GeoStudio software was utilized for analyzing the slopes The effective parameters for defining the problems were slope height, gradient, cohesion, friction angle and PGA	The obtained results show that ANN-PSO performed more effectively than the ANN itself Different combinations of the dataset from the whole available data were tested. Results showed that all the models were applicable to the problem, though ANN-PSO was the best choice for higher accuracy
Reale et al. [143]	2016	Deterministic and probabilistic multi-modal analysis of slope stability	PSO, SLIPS	Limit equilibrium method was used for evaluating FOS and reliability index (simplified Bishop method) Non-circular slip surface was considered Multi-modal PSO was utilized to determine multiple critical slip surface for each failure mode Three soil slopes were subject of numerical simulation: two were selected from previous studies and one was an Australian open cut coal mine	It was mentioned that there are many critical failure surfaces with similar failure probabilities other than viable slip surface A formula was developed for evaluating the correlation matrix between different failure modes in polar co-ordinate system Three numerical cases were analyzed with different failure modes. The critical failure mode may vary under different conditions. In the first two examples, one dominant failure mode for each existed. In the third example, a real slope was considered. SLIPS found some viable slip surface including one where the soil slope failed.

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Xue [139]	2016	Analyzing the stability of slopes	Combined LSSVM and modified PSO	Input variables for describing a soil slope and its relevant FOS were considered as: cohesion, angle of internal friction, pore-water pressure coefficient, unit weight of soil, slope angle, and height of slope (H) A modified equation was proposed for the inertia weight of PSO	Two case studies were considered A sensitivity analysis was conducted to reach a well-adjusted combination for PSO parameters as well as finding optimum value for swarm size and maximum iterations The predicted results showed that the PSO-LSSVM model worked effectively and relatively reliably for the tackled problem
Kang [140]	2016	Reliability analysis of slopes	ν-SVM combined with PSO and ACO	ν-support vector machine (ν-SVM) was used for a surrogate model PSO was used to optimize the ν-SVM This surrogate model was used to improve Monte Carlo simulations	This study gave an overview of ν-SVM surrogate model for system reliability analysis of soil slopes In case of well-adjusting hyper parameters, ν-SVM could find a proper relation between the FOS and uncertain variables of soil slope The performance of ν-SVM was highly dependent on the hyper parameters Both PSO and ACO were effective in optimizing the hyper parameters Based on the results, $N=15D$ was a suitable number of sampling for achieving an effective ν-SVM surrogate model using Latin hypercube sampling
Wang et al. [144]	2018	Reliability analysis of 3D rock slopes	BPSO	Shear strength of rock mass was evaluated using BPSO The probability of failure behind excavation surfaces of various sizes was computed using Monte Carlo simulation	BPSO, thanks to its high speed and simplicity, improved the processing speed The orientation, width, and length of the shear zone were considered to be the effective factors for the shear strength in rock masses. Enlarging the wedge size decreased the stability coefficient initially, but increased after that because of rock bridges The failure probability was enhanced for deeper and longer excavation

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Johari and Mousavi [145]	2018	Probabilistic analysis of slopes	PSO	Several limit equilibrium methods were selected for evaluating FOS (simplified Bishop, simplified Janbu, Spencer's methods, and Morgenstern–Price) Probabilistic analysis was handled using Jointly distributed random variables (JDRV) method Uncertainties in soil parameters (angle of shearing resistance, cohesion intercept, and unit weight of soil) was considered	In this study Janbu's method had higher probability of failure compared to the other methods with and without considering the correlation between angle of shearing resistance nad cohesion intercept The cases with correlation coefficients resulted in larger reliability index than those without considering cross correlation While considering the time for finding the same probability by JDRV and Monte Carlo simulation, results showed that JDRV was more efficient with lower elapsed time. There was a direct corelation between the probability of failure and unit weight while the relation was inverse between the internal friction angle and cohesion Probability of failure's curves experienced the steepest inclination by changing the internal friction angle among other parameters.
Sharma et al. [147]	2018	Analyzing the stability of slopes	PSO	Limit equilibrium method was used for evaluating FOS (Bishop method) GeoStudio software was utilized for analyzing Circular slip surface was considered	In this study the effect of different number of slices was explored. Three kinds of analysis were conductd; GeoStudio without optimization, GeoStudio with optimization, using PSO Using MATLAB PSO optimization proved to be the most efficient approach

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Koopialipoor et al. [149]	2019	Analyzing the stability of slopes under static and dynamic loading	Hybrid approaches based on the ANN and each of ICA, PSO, GA, ABC	A series of 699 homogenous slopes were analyzed considering static and seismic loading Hyper parameters of ANN were adjusted by GA, ABC, PSO, and ICA The FOS was attributed to five important parameters which determine a slope including slope gradient, slope height, soil cohesion, peak ground acceleration, and friction angle of soil	A new measure was introduced for ranking the tackled algorithms called the color intensity rating PSO-ANN was the most efficient technique in this study
Himanshu and Burman [148]	2019	Analyzing the stability of slopes considering seepage and seismic loading	PSO	Limit equilibrium method was used for evaluating FOS and reliability index (Bishop method) Circular slip surface was considered The effect of swarm size, number of iterations, and number of slices was studied	Two benchmark problems were borrowed from previous studies and resolved The results of this study matched those from previous studies This study proposed that swarm size and iteration number be greater than 75 and 80, respectively PSO dealt efficiently with the slope stability analysis under different types of loading conditions
Luo et al. [150]	2019	Analyzing the stability of slopes	PSO-CA	450 simulations from GeoStudio were utilized as a database Cubist algorithm (CA) was utilized to predict the stability of the slope and PSO was tackled to adjust its hyper parameters The results from PSO-CA compared to other algorithms such as SVM, CART, and kNN	The proposed algorithms handled the slope stability problem successfully PSO optimized the hyper parameters of CA efficiently Among all the algorithms tested, PSO-CA was the best algorithm

Table 1 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Shinoda and Miyata [151]	2019	Analyzing the stability of unreinforced and reinforced slopes	PSO	Limit equilibrium method was used for evaluating FOS (Spencer method) Noncircular slip surface was considered A sensitivity analysis was conducted on inertia weight and stochastic weighting factors Th different number of nodes constituting the slip surface was explored	The inertia weighting coefficient was 0.6 and c1 and c2 were set to 1.9. This configuration led to finding the optimum parameter setting of PSO for unreinforced soil In case of handling the reinforced soil the best performance of PSO observed with inertia weighting coefficient of 0.7 and the same value for c1 and c2. The initial maximum number of nodes were four, and and seven for linear or circular critical slip surface, and nonlinear critical slip surface, respectively
Singh et al. [152]	2019	Analyzing the stability of slopes	GA, PSO, BBO	Limit equilibrium method was used for evaluating FOS (Fellenius, Bishop, Janbu, and Janbu corrected method) Circular slip surface was considered	The results showed that GA, PSO, and BBO handled the problem successfully BBO confirmed to be the best algorithm
Yuan and Moayedi [153]	2019	Analyzing the stability of slopes	MLP combined with BBO, ACO, GA, ES, PSO and PBIL	FEM-based Optum G2 software was utilized for stability analysis of the slopes to provide a dataset The stability of a slope was examined A single-layered cohesive soil slope was used as a numerical example.	BBO, ACO, GA, ES, PSO, and PBIL were utilized for training MLP The hybrid approach based on BBO and MLP was the best algorithm

Table 2 Review of the application of PSO to retaining wall optimization

References	Year	Objectives	Algorithms	Methodology	Findings
Ahmadi-Nedushan and Varaee [155]	2009	Minimum-cost and minimum-weight design of retaining walls	PSO	A 4.5 m-tall retaining wall without a base shear key was borrowed from a study by [169] for simulation Seven design variables were considered among which four designed the geometry of the wall and three others determined the reinforcements (for stem, heel, and toe)	PSO efficiently solved the problem PSO provided a decrease of 12% in concrete volume and 6% decrease in reinforcement. PSO provided 12% and 2% reduction in cost and weight, respectively
Khajehzadeh et al. [156]	2010	Minimum-cost and minimum-weight design of retaining walls	PSO, PSOPC	Two retaining walls (3 m-tall and 5.5 m-tall) were considered in this study Eight design variables were proposed to describe a given trail wall, five of them for geometry and three for reinforcements (stem, heel, and toe) Final cost minimization was considered as the objective function	The obtained results demonstrated that PSOPC performed better than PSO by achieving better optimal solutions
Khajehzadeh et al. [157]	2011	Minimum-cost design of retaining walls	PSO, MPSO	A 3 m-tall retaining wall without a base shear key was considered for numerical simulation Eight design variables were proposed to describe a given trail wall, five of them for geometry and three for reinforcements (stem, heel, and toe) A modified PSO was proposed based on sharing the content of a randomly selected particle for updating the velocity term. Moreover, a time-dependent equation for inertia weight was proposed Minimum cost of the wall was considered as the objective function A sensitivity analysis was conducted	A comparison between the results achieved with MPSO, PSOPC, and PSO was presented MPSO was the best algorithm based on the final results A nonparametric statistical test was utilized for analyzing the algorithms' performances more accurately The sensitivity analysis confirmed that the main parameter in the optimization of RCC retaining walls was friction angle of the retained soil, especially when the height of the wall increased

Table 2 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Pei and Xia [158]	2012	Minimum-cost design of retaining walls	GA, PSO, SA and a random direction search (CM)	Nine design variables were defined for describing the wall, five of them for geometry and four for reinforcement (stem, toe, and heel in addition to non-overall-length bar for the vertical wall stem) A 3 m-tall retaining wall without a base shear key was studied Penalty approach was mentioned for handling the constraints	The results showed that CM was not efficient for handling the proposed problem GA and PSO successfully approached the proposed problem with penalty approach SA was not successful in handling the constraint using penalty technique
Kaveh and Soleimani [159]	2014	Minimum-cost design of retaining walls	PSO, HIS, CBO, DPSO	A wall with a base shear key was considered for simulations Seven design variables were attributed to the geometry and four design variables for reinforcements Pseudo-static loading cases were applied to the wall based on six combinations of vertical and horizontal acceleration coefficients Two types of backfill material were considered for static analysis and one type for pseudo-static	The bearing capacity of soil considering and impacted by the toe and the shear strength of critical section of toe were determined to be the impactfull factors in seismic design Increasing the horizontal earthquake factor resulted in more expensive solutions while vertical factor had an inverse effect DPSO and CBO were the best algorithms in this study
Gandomi et al. [160]	2015	Minimum-cost and minimum-weight design of retaining walls	PSO, FA, APSO, CS	A 3 m-tall retaining wall without a base shear key and a 4.5 m-tall with and without a base shear key were considered for analyzing Eight design variables were attributed to the geometry and three of them were eliminated for a wall without a shear key For a wall without and with a base shear key, two sets of discrete variables were used with three and four variables. A sensitivity analysis was conducted over variation of surcharge load, backfill inclination, and base soil friction angle Each algorithm was operated in a series on 101 runs	FA was the poorest algorithm while PSO and CS obtained the best solutions CS performed better than PSO thanks to lower standard deviation values Sensitivity analysis confirmed that the parameter variations affected low-weight-based desings less than low-cost Adding a base shear key ended up decreasing the final cost and weight design for more intensive loading cases The most sensitive parameter in shorter walls was surcharge load while the least sensitive one was base soil friction angle In taller walls, backfill was the most sensitive parameter and soil friction angle was the lest sensitive parameter.

swarm intelligence-based algorithms for optimum design of retaining walls (PSO, FA, APSO, and CS), where the minimum-cost and minimum-weight analysis of the walls were considered based on the ACI 318-05 [109] rules. A short review of the application of PSO to retaining wall optimization is provided in Table 2.

5.3 Reinforced Soil

Due to the lack of tensile strength of soil materials, there are many cases where they cannot provide a satisfying demanding service. One solution in those conditions is utilizing reinforcements. In those situations, the cost of utilized reinforcements or finding an appropriate design would not be an easy task. Artificial intelligence-based techniques and in specific PSO have been the subject of many solutions in the literature. For instance, Li et al. [161] attempted to determine proper reinforcement parameters to provide stability of Zhongjiawu slope with lowest possible construction cost. In their study, a hybrid approach based on PSO and SVM handled an objective function based on construction cost. The stabilizing method in the mentioned investigation was the anti-sliding piles utilized for a high cut slope and FOS of the slope applied to the design procedure as a constraint. Shinoda and Miyata [162] conducted a probabilistic analysis of soil slopes affected by seismic loading, where the noncircular failure surface was considered based on the limit equilibrium method. PSO was selected for handling the objective function and evaluating the FOS while quasi-Monte Carlo simulation was tackled to conduct a probabilistic analysis. One of the most recent studies on the basis of reinforced soils was conducted by Yalcin et al. [163] in 2019. In their study, four optimization algorithms (i.e., GA, PSO, ABC, and DE) were enlisted to handle the optimum cost design of mechanically stabilized earth walls (MSEW) with geosynthetic. Three objectives were followed in this study to reach optimum cost design: (1) reinforcement type, (2) length, and (3) layout of MSEWs. The design procedure in the study was developed based on the Federal Highway Administration (FHWA) requirements. Sereshki and Derakhshani [164] considered the optimum design of MSEWs reinforced with metal strips using PSO, GWO, and salp swarm algorithm (SSA). FHWA regulation was again the foundation of the design procedure and the design variables were the length, width, thickness, vertical spaces, and horizontal spaces of reinforcements. Numerical simulations were conducted by resolving a case study presented by FHWA for four different heights of the wall with different combinations of soil specifications. As depicted in Table 3, Shinoda and Miyata [151] considered the stability analysis of reinforced slopes by applying PSO to a limit equilibrium-based procedure. Table 3 provides a review of the application of PSO to reinforced soil-related problems.

5.4 Shallow Foundation

Shallow foundation is one of the key elements that guarantees serviceability of any given structure by directing the effective loads to the earth. Many researchers have focused on different aspects of the problem of shallow foundation, though focus on their optimal cost design is a relatively new research area. In the following, we present a review of the different application of PSO to shallow foundation and how PSO was helpful in addressing the issue.

In 2010, Zhao and Yin [165] introduced a hybrid approach based on combining SVM and CPSO for predicting the shallow foundations' bearing capacity. In their study, CPSO was utilized to adjust the hyper-parameters of SVM. The algorithm was trained using 50 datasets by considering ultimate bearing capacity based on some factors such as the width of footing, depth of footing, etc. Khajehzadeh et al. [124] examined PSO, PSOPC and MPSO for optimum design shallow foundations, and also proposed an improved version of PSO based on passive congregation where a randomly selected particle for updating the velocity term in addition to the best-found solutions would be considered. Modified PSO (MPSO) that utilizes a time-varying equation for inertia weight was proposed. The objective function was defined as the final cost of the footing.

In 2013, Jing et al. [166] utilized a hybrid approach based on an ANN and improved PSO to predict dam foundation uplift pressure. Marto et al. [167] proposed a hybrid predictive method based on the ANN for estimating the bearing capacity of shallow foundations, where PSO was utilized for the optimum hyper-parameter setting of ANN. 40 recorded samples in granular soils from full-scale axial compression load test on shallow foundations were chosen from the literature. Ultimate axial bearing capacity was defined as a function of footing length and width, embedded depth of the footing, average vertical effective stress of the soil, friction angle of the soil, and groundwater level. The optimum parameter setting of PSO and architecture of ANN was achieved by a sensitivity analysis. Nazir et al. [168] used a hybrid approach based on the ANN and PSO to predict the settlement of a shallow foundation on cohesionless soil. The decision variables were length, width, and depth of embedment in addition to friction angle, stiffness, and effective stress below footing. Eighty footing load tests on cohesionless soils collected from the literature to constitute the database.

Table 3 Review of the application of PSO to reinforced soil

References	Year	Objectives	Algorithms	Methodology	Findings
Li et al. [161]	2012	Optimization of the construction cost for stabilizing Zhongjiawu slope	SVM-PSO	In this study Zhongjiawu high cut slope in China was considered Two rows of piles were considered as reinforcements Length of pile A, length of pile B, width of pile section, height of pile section, pile spacing, and factor of safety were considered as input variables	The optimal deign obtained was compared to the previously built system Considering the same value of FOS, the proposed methodology decreased the construction cost of the anti-sliding pile reinforcement from 3.0008 million RMB to 2.55 million RMB (about 15%)
Shinoda and Miyata [162]	2017	Reliability-based analysis of reinforced soils	PSO	Seismic stability of reinforced slopes was studied Non-circular slip surface was tackled for analysis Minimum FOS was applied as a constraint to the design procedure Limit equilibrium method was used for stability analysis (Spencer method) based on PSO Quasi-Monte Carlo method was utilized for reliability analysis	The obtained results confirmed the efficiency of the utilized method for calculating the limit state exceedance probability
Yalcin et al. [163]	2019	Metaheuristic-based approach for optimum cost design of stabilized soils	GA, PSO, ABC, DE	This study optimized reinforcement type, length and layout of MSEs Federal Highway Administration guidelines were utilized to control design procedure To address variability of the layout, three different types of wall with their specific reinforcement combinations were considered	Some benchmark problems were analyzed with the proposed design procedure Incorporating a robust algorithm to design procedure resulted in better solutions in terms of quality and reliability All the utilized algorithms were capable of proposing valid and feasible solutions The quality of the results were dependent on the problem's complexity. (the more design variables, higher the complexity level) Although PSO proved to be effective occasionally, DE performed as the best algorithm over all the cases in this study Difference in type of the wall affected the final cost to some extent for tall walls

Table 3 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Sereshki and Derakhshani [164]	2019	Metaheuristic-based approach for optimum cost design of stabilized soils	PSO, GWO, SSA	The objective function was defined in terms of construction cost Design variables were length, width, thickness, vertical spaces, and horizontal spaces of reinforcements A sensitivity analysis was conducted over the variation of cost factor, height of the wall, inclination of the backfill, and soil parameters (friction angle and density)	Utilizing optimization algorithms resulted in 13–26% reduction in final costs where PSO and GWO caused more reduction than SSA Increasing the height of the wall decreased rate of reduction in construction costs For the shorter walls, PSO was more efficient while for taller walls, GWO was better Sensitivity analysis showed that the price of the construction material affected the final cost more than earthwork Internal friction angle variations led to significant changes in the final cost values Unit weight of soil variations was serious in short wall while it was effective parameter in taller ones Backfill slope proved to be important for taller walls

Rezaei et al. [169] applied a GA- and PSO-based ANN for predicting the bearing capacity of thin-walled shallow foundations. 145 recorded test results in addition to some experimental tests by the authors provided the required datasets. Results declared a good performance of PSO-base ANN in handling this problem. In 2017, Debnath and Ghosh [170] utilized the PSO algorithm to solve limit equilibrium equations that govern the seismic bearing capacity of shallow foundations. The results were reported in a tabular form to be applicable for examining the seismic bearing capacity of any given shallow foundation. In 2018, Gandomi and Kashani [70] enlisted several swarm-intelligence-based algorithms for optimum cost design of shallow foundations. In the study, PSO, APSO, FA, LKH, whale optimization algorithm (WOA), antlion optimizer (ALO), grey wolf optimizer (GWO), moth-flame optimization algorithm (MFO), and teaching–learning-based optimization algorithm (TLBO) were analyzed. The design procedure was automated based on ACI 318-05 [109] requirements. Moayedi et al. [171] proposed several hybrid approaches for predicting the ultimate bearing capacity of shallow foundation on two-layered soil. The utilized algorithms were an ANN, combined GA, PSO, and differential evolution (DE) with ANN, adaptive neuro-fuzzy inference system (ANFIS), general regression neural network (GRNN), and feedforward neural network (FFNN). Input variables in the study were footing width, top and bottom soil layer properties, thickness of top layer, and width of foundation. Among all the utilized techniques ANN-PSO was the most efficient method. A summary of the application of PSO to shallow foundation is provided in Table 4.

5.5 Pile Foundations

Pile foundations have numerous applications in a wide range of civil engineering projects where the shallow foundation is incapable of directing the effective force to the earth appropriately. Guo and Liu [172] utilized the PSO algorithm for analyzing the reliability of bearing capacity of multi-pile foundations. The reliability index was evaluated based on the first-order second-moment approach that minimizes the distance from the origin to the limit-state surface. Three numerical cases were studied, and the results were compared to Monte Carlo simulations. Ismail and Jeng [173] studied the prediction of single piles' settlement using PSO based higher order neural network (HONN-PSO). Cheng et al. [41] proposed a hybrid approach based on HS and PSO to address the evaluation of pile capacity and pile driving's control based on a back-analysis approach. Ismail et al. [174] applied a hybrid approach based on PSO and back propagation (BP) to predict the load-deformation behavior of piles. The predicted results based on this hybrid approach satisfactorily matched the actual data. Armaghani et al. [175] proposed a hybrid approach based on PSO and

Table 4 Review of the applications of PSO to shallow foundations

References	Year	Objectives	Algorithms	Methodology	Findings
Zhao and Yin [165]	2010	Predicting the bearing capacity of shallow foundation	PSO-SVM, CPSO-SVM	Chaotic PSO was utilized to adjust hyper parameters of SVM In CPSO a chaotic map was utilized for updating inertia weights and random parameters involved in the velocity term A series of 50 datasets was used in this study of which 40 datasets were utilized for training and 10 for testing	CPSO was faster and superior in terms of searching compared to PSO CPSO-SVM enhanced the performance of SVM and achieved more accurate prediction than PSO-SVM
Khajehzadeh et al. [124]	2011	Optimum design of shallow footing	PSO, PSOPC, MPSO	Six design variables were utilized for modeling the problem including the length of footing, width of footing, thickness of footing, depth of embedment, long direction reinforcement, and short direction reinforcement Two numerical examples were considered for simulations, one with vertical loads, and the other with eccentric loads A sensitivity analysis was conducted to consider the effect of soil properties on final cost	The results proved that MPSO found lower cost values in fewer iterations than PSO and PSOPC Nonparametric test analysis confirmed the superiority of MPSO over PSO, and PSOPC From the sensitivity analysis, Young's modulus and effective friction angle of the base soil were identified as the main parameters for optimum design of spread footing For small friction angle, the factor of safety was the control parameter while for large friction angle, the allowable settlement was crucial
Marto et al. [167]	2014	Predicting the bearing capacity of shallow foundation	ANN-PSO	A back-propagation neural network was utilized PSO was considered for improving the efficiency of ANN 40 datasets were selected from the literature 75% of the data was used for training and 25% for testing	The results generated by the proposed PSO-ANN method were very close to the measured bearing capacity.

Table 4 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Nazir et al. [168]	2014	Predicting spread foundations' settlement in granular soils	ANN-PSO	80 samples of footing founded on cohesionless soils selected from the literature were used for training the model The effective parameters were geometrical properties and soil properties	ANN-PSO results had a satisfying agreement with the measured settlements that proved the accuracy and efficiency of the utilized method
Rezaei et al. [169]	2016	Predicting the bearing capacity of thin-walled shallow foundations	ANN-GA, ANN-PSO	145 case studies of related footing load tests were selected to constitute the dataset The input variables were friction angle, unit weight of sand, footing width, and thin-wall length to footing width ratio Several experimental loading tests were added to the dataset to provide diversity	A good agreement between the predicted results and observations demonstrated the efficiency of ANN-PSO ANN-PSO was the best algorithm among the utilized methodologies (PSO, ANN-GA, and ANN-PSO) Based on the laboratory tests, increasing wall length had a positive effect on the bearing capacity. By increasing the length from 0.5 times to 1.12 times the width, the bearing capacity improved 0.5 times.
Debnath and Ghosh [170]	2017	Seismic bearing capacity of a shallow strip function founded in two-layered soil	PSO	The weaker layer was considered to be at the top A sensitivity analysis was performed for the variation of seismic bearing capacity with respect to soil parameters	By keeping the bottom layer's value constant, and by increasing the values of soil properties of top layer (such as unit weight of the soil, cohesion, and soil friction angle) seismic bearing capacity was increased and vice versa Increasing the horizontal and vertical seismic acceleration coefficients resulted in decreasing the seismic bearing capacity

Table 4 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Gandomi and Kashani [70]	2018	Optimum design of shallow foundations	PSO, APSO, FA, LKH, WOA, ALO, GWO, MFO, TLBO	Four design variables described the geometry of the footing and five design variables defined the reinforcements The effect of the location of the column of final cost design was studied by adding two design variables that changed the position of the column Three different cases (i.e., uniaxial loading, a combination of uniaxial loads and moments, the effect of the column's position) were studied through numerical simulations using various algorithms A sensitivity analysis for soil parameters was conducted to assess their effect on final cost design	Adding flexural moments increased the final cost Changing the position of the column at the top of the footing reduced the final cost by up to 37.8% PSO performed very well on the fist case study, however, it was not as successful for the second and third cases
Moayedi et al. [171]	2019	Predicting the bearing capacity of shallow foundation	ANN, ANN-GA, ANN-PSO, ANN-DE, ANFIS, GRNN, FFNN	Eight different sandy soils with considerably different properties were considered for simulations Different range of internal friction angle, dilation angle, poison's ratio, and modulus of elasticity were considered 3515 cases of shallow foundations were simulated using FEM analysis and the above mentioned factors to provide the required datasets	Fourteen models were developed by applying the proposed methodologies Comparing the results using the statistical tests proposed that five out of six techniques performed satisfactorily with high level of prediction accuracy The most efficient algorithm in this study was ANN-PSO
Guo and Liu [172]	2010	Reliability analysis of multi-piles foundations	PSO	PSO was utilized to handle the first-order second-moment approach The efficiency of the proposed method was explored by providing a comparative study resolved by Monte Carlo simulation over three case studies	PSO-based method proved to be more efficient by providing high performance and accuracy while reducing the operation time significantly

Table 4 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Ismail and Jeng [173]	2012	Predicting the settlement of single piles	HONN-PSO	A model was proposed for predicting load-settlement relationship for a single pile under axial load Static pile loading tests were tackled to constitute a dataset The input parameters were SPT blow counts, soil type along the pile embedment, type of pile installation, geometric parameters and elastic modulus of the pile Two types of piles were examined: driven and bored	Comparison of the results of training and testing data resulted in high values of correlation coefficient that confirmed the acceptance and efficiency of the model A comparison between the results from common practice approach (t-z model) was conducted and the method proposed in the study demonstrated a substantial improvement in settlement prediction
Cheng et al. [41]	2012	Pile driving back analysis	HSPSO	A back-analysis procedure was proposed for evaluating pile capacity and control pile driving The objective was minimizing the variance between the evaluated and the measured force values	The proposed method was found to be effective for handling this problem
Ismail et al. [174]	2013	Predicting the load-deformation behavior of axially loaded piles	BP-PSO	Full-scale pile loading tests were used to provide a database for developing a model in this study PSO was employed to adjust hyper parameters of BP A series of 115 static load tests on piles with 1285 data points was collected from FHWA The design variables were pile stiffness, the shear resistance of the soil around the pile shaft, and the bearing resistance of the soil at the pile base Two types of piles (driven and bored) with different materials (concrete, H-steel, pipe) were considered	The proposed hybrid technique with BP achieved more accurate prediction of the load–deformation curve for axially loaded piles than PSO and the existing PSO–BP hybrid methods Results of the proposed approach also had better agreement with the observed tests than the other utilized techniques and common practice approach (t-z model)

Table 4 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Armaghani et al. [175]	2017	Predicting the bearing capacity of rock-socketed piles	ANN, ANN-PSO	The design variables in this study were soil length to socket length ratio, total length to diameter ratio, uniaxial compressive strength, and standard penetration test The utilized dataset was constituted by conducting 132 pile driving analyzer (PDA) tests on rock-socketed piles used in Klang Valley Mass Rapid Transit (KVMRT) project in Malaysia 5 different models were developed using ANN and ANN-PSO	The best models obtained by each of ANN and ANN-PSO were selected using simple ranking method The prediction ability of ANN-PSO proved to be better than ANN on both training and testing dataset based on r-squared measure A sensitivity analysis over the input variables showed that uniaxial compressive strength was the most effective input parameter
Moayedi et al. [176]	2019	Calculating the friction capacity ration in driven piles	ANFIS, ANFIS-GA, ANFIS-PSO	The design variables were pile diameter, pile length, relative density, and cone penetration test (CPT) ANFIS was trained using a learning fuzzy-based algorithm 20 datasets were used for developing the mode	The results using the proposed methods in this study were compared to the observed ones based on in situ cone penetration test A satisfying conformity between the predicted and measured data was observed ANFIS-GA proved to be the best model based on the statistical analysis

ANN to predict the bearing capacity of rock-socketed piles. The bearing capacity was attributed to soil length to socket length ratio, total length to diameter ratio, uniaxial compressive strength, and standard penetration test. The simulations results demonstrated that PSO-ANN had high reliability in estimating ultimate bearing capacity. Moayedi et al. [176] tried to predict friction capacity ratio in driven shafts using Adaptive Neuro-Fuzzy Inferences System (ANFIS). Their main focus was on optimizing the performance of ANFIS by incorporating GA and PSO in the training procedure. Table 5 tabulates the previous efforts on the application of PSO to pile foundations.

5.6 Tunnels

Tunnel engineering is another field that has benefited from artificial intelligence to some extent. Xing et al. [177] automated the adaptive control of tunnel excavation using PSO. This procedure utilized to back-analyze rock mechanics parameters and to select the optimal construction schemes. Liu et al. [178] applied PSO based LSSVM to the problem of Optimal earth pressure balance control. This is done for shield tunneling. Jiang et al. [179] proposed an AI-based alternative for feedback analysis of tunnel construction, where a two-step procedure based on a combination of PSO and SVM was proposed for handling the problem. Annan and Zhiwu [180] applied an improved PSO (IPSO) to the problem of optimizing the supporting parameters (anchor and spay layer parameters) of metro tunnel. Bahmanikashkooli et al. [181] proposed utilizing the SPO algorithm for determining the critical depth of horseshoe cross-section channels. Conducting numerical simulations showed the efficiency of PSO in handling this problem. Hasanipanah et al. [182] developed a hybrid approach based on ANN and PSO for examining the surface settlement that might take place during tunneling. Hou et al. [83] utilized an exponential tuning mechanism for the inertia weight immune PSO (EAIW-IPSO) for selection of shield tunneling parameter values. A comparative study was provided by applying linear decreasing inertia weight particle swarm optimization (LDIW-PSO), random inertia weight particle swarm optimization (RIW-PSO), and exponentially decreasing inertia weight particle swarm optimization (EDIW-PSO) to this problem. Moosazadeh et al. [183] developed a combined algorithm based on PSO and ANN to assess the building damage caused by tunneling. A summary of different applications of PSO to tunnel related problems can be found in Table 6.

5.7 Miscellaneous Applications

Chen and Feng [184] attempted to automate back analysis of displacement to estimate rheological parameters of a soft and weak rock mass. To this end, a modified version of PSO with contracted ranges in search space and velocity (CSV-PSO) was considered for handling the problem. This modification was based on considering a parallel strategy where a parallel CSV-PSO with master–slave mode was developed called PCSV-PSO. Zhang [185] utilized a multi-objective approach to find a tradeoff between equipment and configurations of earthmoving operations. In 2009, Chen et al. [186] optimized the construction time of a secant pile wall. A comparative study based on the performance of self-organizing map-based optimization (SOMO) and PSO algorithms were considered for handling the problem. Zhao and Yin [187] estimated the geomechanical parameters based on back analysis approach and monitored displacements. They utilized a combined strategy based on SVM and PSO to handle the back-analysis approach. Jiang and Wan [188] proposed a methodology based on combining colony density PSO (CDPSO) and FLAC for optimizing the length and interval of anchor and the thickness of shotcreting, which consequently resulted in finding optimal cost and time. Yunkai et al. [189] proposed a prediction model for soil erosion using coupled SVM and PSO based on monitoring the data of sand production. Sadoghi Yazdi et al. [190] developed a model based on neuro-fuzzy model and PSO for calibration of soil parameters used in a linear elastic-hardening plastic constitutive model. This model was used in conjunction with the Drucker-Prager yield criterion. A short review of the above-mentioned studies is provided in Table 7.

Roshani and Farsadizadeh [191] utilized PSO for optimizing the dimension of clay core in non-homogeneous earth fill dams. Wan [192] utilised clustering analysis to generate landslide susceptibility maps for Shei Pa National Park in Miao Li, Taiwan. In the study, two different classifiers were used for handling the problem: (1) a combination of entropy-based classification (EBC) and K-mean with PSO (KPSO); and (2) self-organizing map (SOM). Zhang et al. [193] utilized hybrid moving boundary PSO (hm-PSO) for parameters identification in Barcelona Basic Model (BBM). This methodology presented an automatic back analysis approach that minimized the difference between examined and observed values on the cavity pressure-cavity strain curve. Piliounis and Lagaros [194] studied reliability analysis of geostructures using some metaheuristic optimization techniques. Choobbasti et al. [195] developed a hybrid method based on PSO and MLP to locate a trench layer around a pipeline to reach the minimum liquefaction potential. Mirzaei et al. [196] incorporated weighted LSSVM (WLS-SVM) and PSO algorithms to handle the optimal design of homogeneous dams with oblique and horizontal drains. Kutanaei and Choobbasti [197] enlisted a PSO algorithm-based method to examine

Table 5 Review of the application of PSO to pile foundations

References	Year	Objectives	Algorithms	Methodology	Findings
Xing et al. [177]	2010	Adaptive control of tunnel excavation based on numerical simulation and particle swarm optimization	PSO	Two optimization procedures were conducted: (1) identifying the parameters of the model, (2) optimizing the control variables PSO was employed to find optimal values for mechanics parameters. A solution strategy was proposed based on combining PSO and 3D fast Lagrange numerical method Gezhenpu tunnel in Dalian was tackled for verification and examination of the proposed model	The obtained optimal solution kept surrounding rock stable while reduced material and labor cost
Liu et al. [178]	2011	Predict earth pressure balance control during excavation	LSSVM-PSO	A predictive model was built for earth pressure balance during excavation During the optimization procedure, the difference between estimated earth pressure and target pressure was minimized The design variables were parameter variables (advance speed, screw conveyor speed, jack thrust, cutter rotational speed, earth pressure in chamber at current time) and control variables (the optimal advance speed and screw conveyor speed at the next time) A metro project in Guangzhou was targeted for data collection	The simulation verified that the proposed method dealt with the problem of controlling earth pressure balance efficiently with high precision
Jiang et al. [179]	2011	Feedback analysis of tunnel construction	SVM-PSO	This problem was based on: (1) feedback analysis of the mechanical parameters of the surrounding rock, (2) optimization of the supporting scheme based on recognized rock parameters PSO was utilized in two stages: (1) for training the SVM, (2) for handling the back analysis In the first step, SVM was trained using PSO to predict the displacement of the surrounding rock at the key point In the second step, PSO was applied to do this iteratively until finding the minimum difference between the predicted and monitored values The input parameters in this study were rock parameters (Young’s modulus, shotcrete thickness, shotcrete Young’s modulus, cable diameter, and cable length)	SVM model accurately modeled the nonlinear relations between displacement and rock parameters SVM parameter settings affected the prediction’s error which proved the necessity of PSO for parameter setting The results demonstrated that the predicted displacements showed acceptable agreement with really measured ones. Additionally, the obtained shotcrete parameters satisfactorily controlled the deformation of surrounding rock

Table 5 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Annan and Zhiwu [180]	2011	Optimizing supporting parameters of metro tunnel	IPSO	The main aim of this study was optimizing anchor and spay layer parameters of the tunnel as supporting parameters of the tunnel An improved PSO was developed by proposing a time-dependent updating formulation for inertia weight The proposed method was applied to a metro tunnel of Dalian City of China	The proposed method proved to be efficient in handling the tackled problem and provided satisfying results
Bahmanikashkooli et al. [181]	2014	Finding critical depth of horseshoe cross section tunnel	PSO	The critical depth is corresponding to the minimum specific energy for a given discharge in an open channel An objective function was defined by considering cross sectional area of flow, top width, and flow discharge	Evaluating the proposed method based on using PSO for finding the critical depth demonstrated that this solution procedure was accurate and simple
Hasanipanah et al. [182]	2016	Predicting surface settlement due to tunneling	ANN, ANN-PSO	A combined algorithm based on ANN and PSO was proposed to evaluate maximum settlement of surface because of tunneling The design variables in this study were horizontal to vertical stress ratio, cohesion and Young's modulus The model was established using 143 data sets obtained from the line No. 2 of Karaj subway in Iran	Numerical simulations proved that ANN-PSO was superior than ANN Sensitivity analysis proved that the most impactful parameter was horizontal to vertical stress ratio

Table 6 Review of the application of PSO to tunnel problems

References	Year	Objectives	Algorithms	Methodology	Findings
Hou et al. [86]	2019	Selection of shield tunneling parameters	PSO, LDIW-PSO, RIW-PSO, EDIW-PSO, EAIW-IPSO	A time-dependent exponential formula was proposed for inertia weight Experimental study was delivered using 12 benchmark functions (6 unimodal and 6 multi-modal) as well as a real case study GA-BP neural network was employed as a predictor for the ground settlement and selected engineering parameters. EAIWIPSO was used to optimize tunneling parameters under specific geometric and formation conditions based on predictive model	The results demonstrated that the proposed modified algorithm improves the efficiency of the selection of tunneling parameters
Moosazadeh et al. [183]	2019	Building damage estimation due to tunneling	ANN-PSO	Two-stage methodology estimated the damages: (1) ground movement in the greenfield condition was estimated empirically; (2) a method based on structural mechanic principles was used to assess the damage A database was collected from a total of 44 data sets from Line No. 2 of the Karaj Urban Railway Project in Iran Ten inputs (building height, width, length, and stiffness ratio, eccentricity, inflection point, maximum settlement, horizontal strain, axial stiffness ratio, and bending stiffness ratio) and two outputs (number of cracks and crack width) were tackled in this study Sensitivity analysis was done to reach optimum setting of PSO parameters Finally, a model with one hidden layer and 13 nodes were selected.	The results obtained based on the proposed model were in good agreement and consistent with filed measurements, with high accuracy

Table 6 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Chen and Feng [184]	2007	Back analysis of rheological parameters of rockmass	PCSV-PSO	Five benchmark functions (Sphere, Rosenbrock, Rastrigrin, Griewank, and Schaffer's f6) were analyzed to evaluate the performance of PCSV-PSO A sensitivity analysis on the effect of parameters of PCSV-PSO (random seed, stagnancy number, and α_0) was conducted Back analysis method for rheological parameters of rockmass based on FLAC3D was done PCSV-PSO was selected for handling this procedure No. 72 testing tunnel of left bank slope, Longtan Hydropower station, China, was selected for numerical simulations	The observations from sensitivity analysis demonstrated that random seed, stagnancy number and constant α_0 determining flying velocity of particles had a significant impact on the performance of PCSV-PSO algorithm Well-adjusted random seed accelerated convergence rate while no pattern for setting this parameter was found Too small or too large stagnancy number ruined the convergence and stability of the algorithm The smaller, the value of α_0, the poorer the optimizing ability of the algorithm Results from numerical simulations of the tackled geotechnical case study showed high efficiency of PCSV-PSO in handling the problem
Zhang [185]	2008	Multi-objective optimization for earthmoving operations	PSO	Six activities were attributed to earthmoving operation: load of soil, haul of truck, dump of soil, return of truck, spread of dumped soil, and compact of spread soil Four kinds of resources were involved in the earthmoving operation: loader, truck, dozer and compactor Multiple performances such as project duration and total cost were computed and transformed to a total attribute	The numerical analysis demonstrated that the method could determine optimal equipment-configuration for an earthmoving operation

Table 6 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Chen et al. [186]	2009	Minimizing the construction time of a secant pile wall	PSO, SOMO	A database was compiled based on 207 primary and secondary bored piles for a secant pile wall The detailed construction time was measured in minutes and broken down into 16 work activities for each unit of the wall The obtained results were further analyzed to estimate optimal construction sequence and time	Both algorithms were efficient in handling the problem A time-saving of about 125 and more hours was recorded SOMO with 3.42% more time-saving proved to be better than PSO
Zhao and Yin [187]	2009	Predicting geomechanical parameters	SVM-PSO	An intelligent back-analysis approach was considered for handling the problem The verification of the model was done through a numerical modeling based on the following parameter settings: Poisson's ratio $\mu = 0.25$, equal distribution of initial geostress with $\sigma_x = \sigma_z = 0.98$ MPa, $\tau_{xz} = 0$ and Young's modulus $E = 98$ MPa A real case study was conducted by considering the permanent shiplock as one of the major components of Three Gorges Project in China Six input parameters were utilized for simulations: deformation moduli for moderately weathered zone, damaged zone, unloading deformation zone and slightly or non-weathered zone as well as geostress coefficients 50 datasets for training and 10 datasets for testing obtained based on monitoring the displacements of six different points	The proposed algorithm demonstrated a satisfying relation between rock mass parameters and displacements The efficiency and precision of back analysis was enhanced via the proposed algorithm

Table 7 Review of the other applications of PSO in geotechnical engineering problems

References	Year	Objectives	Algorithms	Methodology	Findings
Jiang and Wan [188]	2009	Optimization of the time, cost and deformation of a tunnel	CDPSO	Three benchmark functions were tested using the proposed optimization algorithm (CDPSO) to examine its efficiency FLAC software was utilized for this simulation Stability criteria for the tunnel were supposed to be sedimentation of arch crown in excavation, and the convergence deformation of side wall in excavation Design variables were the length and interval of anchor and the thickness of shotcreting	Based on the results CDPSO was found to be reliable and efficient in handling the proposed problem
Yunkai et al. [189]	2010	Predicting soil erosion modulus in small watershed	BP ANN, BP ANN-PSO, SVM-PSO	Huangfuchuan small basin was the subject for data collection and analysis The design parameters effective in soil erosion were terrain factors (channel density, ravine area, average Slope, land vegetation coverage, and topographical ratio), soil factors (sand rock proportion, sandy soil proportion, loess proportion, and chestnut soil proportion), vegetation factors (total vegetation coverage, slope vegetation coverage, and ravine vegetation proportion), and other factors (watershed area) Arcview GIS was used to gather the data 449 samples described by the above-mentioned design variables were collected randomly	The main reason for degeneration and even thorough destruction of the land resource was soil erosion The terrain, soil, runoff, land utilization, etc. were effective parameters in soil erosion of small basin With the average error of 3.85% the proposed SVM-PSO model proved to be efficient Because the model did not thoroughly consider the measure of water and soil conservation as well as the complexity and the uncertainty of corrosion which produced the sand, the application of PSO–SVM prediction model requires further research

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Sadoghi Yazdi et al. [190]	2011	Calibration of soil model parameters	Coupled neuro-fuzzy model and PSO	The neuro-fuzzy system was used to examine a nonlinear regression between the deviatoric stress and axial strain obtained from a consolidated drained triaxial test on samples of poorly graded sand Model parameters were set based on a triaxial test result in conjunction with a typical elastoplastic constitutive model	The proposed methodology proved its ability to determine model parameters with relatively high accuracy A second separate simulation with different set of data obtained under different confining pressure confirmed a close match with the same order of accuracy
Roshani and Farsadizadeh [191]	2012	Optimizing the clay core's dimensions of earth fill dams	ANN-PSO	The objective function was defined as the minimization of total cost of water loss and earthworks based on two phases In the first phase, water seepage volume through the dam core was calculated by combining FEM and ANN Evaluating the cost of earthworks based on the core volume A database constituted of 600 FEM-based modeling was considered for developing ANN model Output data from ANN went through the PSO to achieve optimized dimensions Linear and logarithmic regression were tackled as benchmarks Allavian earth fill dam considered as a real-world case study to validate the proposed method A sensitivity analysis studied the effect of each parameter on the final design	The proposed dimensions for the tackled project were bigger than the project constructed in the real-world, which led toward reduction in total waste water This method was capable of decreasing the final cost of project by about 4 percent Comparison of the utilized method with linear and logarithmic regression demonstrated the higher efficiency of the ANN-PSO based method

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Wan [192]	2013	Generation of landslide susceptibility maps using classification approach	EBC-KPSO and SOM	Digital elevation modeling and remote sensing data were used for developing the model Input variables for the developed model were distance to river, distance to road, elevation, slope, normalized difference vegetation index, band ratio square root of band ratio, and vegetation index K-fold cross-validation was used for evaluating the original database 20 datasets driven from cross-validation were utilized for training course NanKeng River area, which had a lower level of uncertainty because of its soil type was selected for this study	The proposed methods successfully handled the problems with high levels of accuracies EBC-KPSO provided 86% accuracy, which was better than SOM with 77% accuracy
Zhang et al. [193]	2013	Characterization of parameters of unsaturated soils based on a back-analysis	hm-PSO	BBM parameters values were determined using a back-analysis of the cavity pressure-cavity strain relationship FEM based model was utilized to calculate the parameters Nelder-Mead local search algorithm combined with hm-PSO was utilized to minimize the difference between FEM results and in-field measured parameters	The proposed model was validated for artificially generated data The number of parameters to be identified significantly impacted the accuracy. If the number of BBM parameters were six, the results were satisfactory A practical approach for parameter determination was found to depend on a mixing laboratory tests, filed pressuremeter tests and the hm-PSO algorithm

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Piliounis and Lagaros [194]	2014	Reliability analysis of geostructures	ANN-MCS in conjunction with GA, PSO, DE, HS, ES, ABC, covariance matrix adaptation (CMA), and elitist covariance matrix adaptation (ECMA)	The reliability analysis was done by means of MCS and first order reliability method (FORM) ANN was utilized for limit state approximation to enhance MCS performance Seven optimization algorithms (GA, PSO, DE, ES, HS, ABC, CMA, and ECMA) were combined with FORM to solve the reliability problem Liquefaction, concrete dam, embankment, and pile foundation were considered for analysis Hasofer-Lind reliability index was used for examining the performance of the proposed methods	The results with the proposed hybrid approach were compared to those with MCS in terms of efficiency and robustness The results with the two methods closely matched each other. The proposed method proved to be more time-efficient than MCS by two or more orders of magnitude The results demonstrated that DE and ABC were superior than other algorithms
Choobbasti et al. [195]	2014	Obtaining the minimum liquefaction potential via optimally locating a trench layer around a pipeline	MLP-PSO	Local radial basis function differential quadrature method (LRBF-DQ) was utilized to deal with governing equations of seismic accumulative excess pore pressure The gathered data from the previous step was used to train the ANN algorithm PSO was utilized to find the position of trench layer, which minimized the liquefaction potential	The obtained results showed that there was a linear relation between the location of the pipeline and the optimum position of the trench layer The optimum location of the trench in terms of depth was recommended to be under the pipeline where the liquefaction probability was minimal

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Mirzaei et al. [196]	2015	Minimization of the seepage through the body and the weight of homogeneous earth dam	Hybrid WLS-SVM and PSO	The design variables in this study were the upstream and downstream slopes of earth dam, the length of oblique and horizontal drains and angle among the drains The FOS against failure of upstream and downstream slopes were applied to the design procedure as constraints Seep/W and Slope/W were utilized to compute hydraulic responses of the dam and constitute the database of 200 samples	The study confirmed the successful application of the developed AI-based technique as an alternative to the FEM-based model Among the tackled objective functions the seepage through the dam body was shown to be more significant than the weight of the dam
Kutanaei and Choobbasti [197]	2015	Prediction of mechanical properties of improved sand with fibers and cement	PSO	Experimental studies were conducted by means of laboratory modeled samples Cement and polyvinyl alcohol (PVA) fiber were added with different combination percentages to enhance the performance of sand A PSO-based approach was developed to predict unconfined compression strength, modulus of elasticity, and axial strain at peak strength	The observations demonstrated a good agreement between the experimental tests and PSO-generated ones Effects of cement on sand resulted in more brittle behavior while enhancing the modulus of elasticity and unconfined compression strength Although adding fiber to cemented sand decreased modulus of elasticity, it strengthened the unconfined compression Deformability index was directly related to fiber content
Nama et al. [198]	2015	Evaluating pseudo-static active earth pressure coefficient	PSO, HS, and TLBO	Limit equilibrium method was utilized for evaluating the earth pressure coefficient Inclination of the bottom side varied with respect to the vertical coefficient, backfill varied with respect to horizontal coefficient, earthquake vertical and horizontal coefficients	Results showed that optimization algorithm handled the problem successfully and effectively Among the evaluated algorithms, TLBO provided the best performance

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Hasanipanah et al. [199]	2017	Finding an exact equation for predicting flyrock	PSO	Five effective parameters (burden, spacing, stemming, powder factor, and rock density) on flyrock were considered as input variables Three different measures were proposed to evaluate the efficiency of the method (i.e., root-mean square error, Nash and Sutcliffe, and coefficient of multiple determination The results obtained by the PSO-based method were compared to those by multiple linear regression (MLR) 76 blasts from three quarry sites in Malaysia were considered for the dataset	The results revealed that the proposed PSO-based method performed better than the MLR method From the sensitivity analysis it was found that the most effective parameter was rock density
Samareh et al. [200]	2017	Predicting the ground vibration due to mining blasts	ANN, NLRA, and NLRA optimized model by GA-PSO	A mathematical model was developed for predicting PPV using the properties of the wave emission environment A regression analysis on 95 calculated seismic mapping proposed four out of eleven impactful parameters on vibrational wave velocity Some describing models for PPV generated by NLRA and ANN A hybrid approach based on GA and PSO applied to the NLRA model to generate an optimized model 18 seismic mapping were tackled for validating the model Sarcheshmeh Copper Mine was the subject of numerical studies for validation of the produced model	Three model were developed in this study: ANN, power, and optimized power The obtained results indicated that the optimized power model was capable of predicting PPV with more accuracy than the power model Although there was no considerable difference between optimized power and ANN, the optimized power was recommended thanks to much easier usage

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Yin et al. [9]	2017	Identifying soil parameters	GA, PSO, SA, DE, and ABC	Objective function was defined as the difference between experimental and numerical results The objective error was outlined in terms of deformation and stress as two extremely significant factors in mechanical behavior of soils Weighted combinations of different errors constituted the objective function Several synthetic cases and two real synthetic pressuremeter tests (PMTs) were tackled for validation of the model and examining the optimization the performance of the algorithms	The results showed that DE was the best algorithm for handling the tackled problem thanks to the smallest objective error DE algorithm in spite of satisfying results suffered from slow convergence rate A hybrid approach based on DE and Nelder-Mead simplex (NMS) was developed to accelerate the convergence pace
Fatty and Li [201]	2018	Back analysis of geotechnical parameters of a rock slope	PSO	The objective function was defined as the difference between the targeted unit weight and the predicted unit weight The finite element upper bound limit analysis method was used in conjunction with the Hoek–Brown failure criterion A simple rock slope with the inclination of 45° was considered for numerical simulation Back analysis was performed over rock unit weight and degree of disturbance	The obtained results proved the effectiveness of the proposed method

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Hasheminejad et al. [204]	2018	Predicting collapsibility of unsaturated soils	ANFIS-PSO	Collapsibility potential was defined as the function of six input parameters (i.e., dry unit weight, moisture content, applied pressure, clay percentage, silt percentage, uniformity coefficient, and soil plastic parameters) Gaussian membership functions were tackled for handling fuzzy phase 266 datasets were used for training and 66 were used for testing The effect of each parameter on predicting collapsibility was studied via a sensitivity analysis	The results demonstrated a good agreement between real observation and estimated results Comparison of the results with previous studies showed superiority of the utilized method in this study over previous efforts Sensitivity analysis revealed that dry unit weight liquid limit and clay percentage were inversely related to collapsibility, whereas increasing slit percentage enhanced the collapsibility potential.
Bui et al. [23]	2018	Predicting soil compression coefficient	MLP-PSO	A database of 156 samples collected from geotechnical investigation of a high-rise building project in Hanoi (Vietnam) was tackled for developing the proposed model There were 12 design variables: depth of sample, sand percentage, loam percentage, clay percentage, moisture content percentage, wet density, dry density, void ration, liquid limit, plastic limit, plastic index, and liquidity index The proposed method was compared to conventional backpropagation MLP(BP-MLP), the radial basis function neural network (RBF-Neural Nets), the Support Vector Regression (SVReg), the random forest (RF), and the Gaussian Process used for regression analysis (GP)	The experimental results proved the significant superiority of MLP-PSO over other utilized techniques The main advantage of PSO-MLP was the conversion of the weight and bias matrices of the model to coordination of each particle in a swarm and vice versa, which provided a large number of trials for weight and bias matrices The main disadvantage of the proposed method was the limited search space of PSO due to determination of its parameters

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Nguyen et al. [202]	2019	Landslide susceptibility model prediction	ANFIS-PSO, ANN-PSO, and RFBFDT	12 describing factors were considered for model generation as follows: slope, aspect, elevation, curvature, slope length, valley depth, distance to rivers, distance to roads, distance to faults, Topographic Wetness Index (TWI), and Terrain Ruggedness Index (TRI), for landslide analysis and modeling 167 datasets from past landslides in Van Chan district, Yen Bai province, Vietnam constituted the database	The records indicated that RFBFDT was the best model compared to other utilized algorithms
Zhang et al. [203]	2019	Predicting peak particle velocity resulted from blast in open-pit mines	XGBoost-PSO	Based on this method XGBoost was selected as a machine learning tool The hyper parameters of XGBoost were optimized using PSO 175 recorded data were used for model development	The results confirmed the superiority of XGBoost-PSO for handling the tackled problem

Table 7 (continued)

References	Year	Objectives	Algorithms	Methodology	Findings
Xi et al. [205]	2019	Seismic-induced landslides hazard assessment	MLP and MLP-PSO	12 landslide conditioning factors were considered as the design variables in this study: elevation, lithology, slope degree, slope aspect, stream power index, peak ground acceleration, topographic wetness index, distance to river, distance to road, distance to fault, normalized difference vegetation index and plan curvature Susceptibility maps for the landslides were generated in a geographic information system (GIS) An area in Ludian, Yunnan province, China was considered as the subject of this study A database of 458 landslides collected from recorded information, aerial photos interpretation, and field monitoring using GPS was selected for developing the model Landslide risk assessment was described by a frequency ration (FR) The hybrid methodology based on ANN and PSO was developed in which PSO was utilized for optimizing the biases and weights of ANN A sensitivity analysis was conducted on different swarm size The obtained maps classified into five susceptibility groups namely, 'Very low', 'Low', 'Moderate', 'High', and 'Very high'	This study confirmed the positive impact of PSO for increasing MLP performance Results revealed that MLP with 6 hidden neurons and with 500 population size for PSO was the best configuration MLP assigned 23.04% of the landslides as the dangerous region, while this value for MLP-PSO was 19.88%

combined effects of fibers and cement on the mechanical properties of sand. Nama et al. [198] addressed the problem of evaluating active earth pressure in retaining walls. They tried to automate this procedure using PSO, HS, and TLBO optimization algorithms. Hasanipanah et al. [199] presented a PSO-based approach to find a precise equation for predicting flyrock due to blasting. Samareh et al. [200] developed a predictive method based on a hybridized PSO and GA. In the study, peak particle velocity (PPV) resulting from mining blasts were examined with nonlinear regression and ANN. Yin et al. [9] investigated the performance of different optimization algorithms for identifying soil parameters. Fatty and Li [201] used PSO to handle back-analysis procedure and investigate uncertain geotechnical parameters for a rock slope. Hasheminejad et al. [204] employed a hybrid approach based on adaptive neural fuzzy inference system (ANFIS) and PSO for examining collapsibility of unsaturated soils. Bui et al. [23] combined PSO and MLP for predicting soil compression coefficient. Nguyen et al. [202] utilized ANFIS enhanced by PSO, hybrid ANN-PSO, and best first decision trees-based rotation forest (RFBFDT) for landslide spatial prediction. Zhang et al. [203] employed a mixed technique based extreme gradient boosting machine (XGBoost) and PSO for predicting ground vibration in open-pit mines caused by blasting. Xi et al. [205] used a combined algorithm based on PSO and ANN for hazard assessment of earthquake-made landslide in Ludian area, China.

6 Numerical Simulation

In this section, we report on the efficacy of FDPSO, IRDPSO, MPSO, IPSO, AGPSO, CLSPO, HCLPSO and EPSO examined through computational experiments based on problems of three different geotechnical categories: concrete cantilever retaining walls, shallow foundations, and slope stability. To this end, three computer programs have been developed to simulate the basis of our problems as fitness functions based on Das [206] and ACI 318-05 [109] in MATLAB (2012Ra). For the retaining walls, two objectives are considered: to minimize the final cost and final weight of the projects. Shallow footing optimization deals with minimizing the final cost. For slope stability, the value of factor of safety is minimized to find the most critical failure surface.

Final results using the above-mentioned algorithms are compared to those obtained by PSO. The final results are reported in the form of Best, Mean and standard deviation (SD) from a series of 101 runs for retaining wall and shallow footing and 20 runs for slope stability problems for each algorithm. Moreover, a non-parametric Friedman statistical test is utilized for ranking the algorithms' performances based on their scores. The lower the scores, the better the performance of the algorithms. In the retaining wall and shallow footing problems, the population size of 50 and the number of iterations of 1000 were considered for all the algorithms. Besides, the slope stability problem was run with the population size of 50 and the number of iterations of 3000. The obtained results are collected in Tables 8–16 as well as Figs. 1–7. It is worth noting that the best-found observations are shown in bold.

6.1 Retaining Wall Minimum-Cost and Minimum-Weight Simulations

In this section, a 3 m wall presented as the first example in [11] is analyzed. This wall is affected by the nine combinations of seismic loading conditions. This example is resolved by eight variations of PSO. Based on the statistical results, the Mean and SD of each algorithm are tabulated in Tables 8 and 9, respectively. Rankings of the algorithms based on Friedman test results are presented in Tables 10 and 11.

Results of low-cost design reveal improvements to the PSO algorithm's performance with both HCLPSO and EPSO. RDPSO recorded the poorest results while HCLPSO has the lowest values of Mean solutions. Also, although HCLPSO found lower values of Mean and SD over all the cases, there were also slight differences between the results obtained by HCLPSO and EPSO. In contrast, for the minimum-weight design, there was no uniform pattern in the performance of the evaluated algorithms. It can be seen clearly that although in some cases EPSO and HCLPSO overcome the original PSO, in most of the cases there was no considerable advancement provided by HCLPSO and EPSO. In other words, the three algorithms performed identically in most of the cases. Considering all the algorithms together, we can say that PSO variations other than HCLPSO and EPSO performed inefficiently in this case. Moreover, RDPSO again recorded the poorest results in this case. Reviewing the results presented in Tables 8 and 9, it can be seen that for low-cost design, HCLPSO has achieved the lowest mean values again, while for low-weight design there is no definite algorithm that would be the best over all the cases.

In Table 10, the Friedman ranking results for low-cost design show that HCLPSO had the lowest scores. EPSO ranked second in all the cases except for cases 3 and 6, where the original PSO performed better. On the contrary, the worst algorithm over all but cases 4 and 7 was RDPSO. For cases 4 and 7, the poorest results were provided by AGPSO3 and AGPSO2, respectively. The Friedman test results in Table 11 again show that there was no consistent pattern between the evaluated algorithms. For weight design of the wall, HCLPSO was the best in all cases except cases 2, 3, and 8 where PSO proved to be the best. Ignoring cases 2 and 9 in which CLPSO and MPSO were the weakest algorithms, respectively, RDPSO was the weakest algorithm in the remaining cases.

Reviewing the convergence rate plots in Figs. 1 and 2, we see a more moderate level of convergence for HCLPSO and

Table 8 Mean and SD values in terms of cost for retaining wall numerical simulations

Case	PSO	HCLPSO	EPSO	AGPSO1	AGPSO2	AGPSO3	IPSO	MPSO	TAC PSO	CLPSO	FDPSO	RDPSO
1	68.95 ± 1.15	**62.51 ± 0.15**	62.86 ± 0.10	67.66 ± 5.51	67.65 ± 5.66	69.78 ± 5.94	68.34 ± 5.46	71.42 ± 6.38	65.28 ± 3.40	79.36 ± 11.11	68.32 ± 1.86	99.91 ± 20.42
2	84.47 ± 0.21	**83.72 ± 0.17**	84.11 ± 0.14	89.72 ± 5.48	89.57 ± 5.19	91.36 ± 5.41	90.88 ± 5.13	92.05 ± 5.29	87.11 ± 3.79	95.16 ± 8.27	89.99 ± 1.78	113.45 ± 13.89
3	117.46 ± 0.63	**116.73 ± 0.37**	117.40 ± 0.26	125.73 ± 6.30	127.12 ± 6.11	127.25 ± 6.25	127.32 ± 5.95	127.93 ± 6.55	125.59 ± 6.22	132.16 ± 11.94	123.21 ± 1.94	148.31 ± 10.75
4	65.60 ± 0.39	**59.42 ± 0.17**	59.47 ± 0.08	64.32 ± 6.07	64.47 ± 5.23	67.72 ± 7.81	65.76 ± 5.62	68.78 ± 8.35	62.24 ± 3.89	77.22 ± 13.14	66.21 ± 2.43	99.87 ± 19.56
5	80.12 ± 0.36	**79.27 ± 0.16**	79.57 ± 0.12	85.15 ± 5.21	85.86 ± 4.87	87.39 ± 5.58	86.70 ± 5.73	87.81 ± 5.52	83.17 ± 4.52	89.57 ± 9.90	85.33 ± 1.74	112.16 ± 15.18
6	119.96 ± 1.33	**119.01 ± 0.35**	119.85 ± 0.26	129.25 ± 6.24	130.52 ± 6.14	130.00 ± 6.27	129.43 ± 6.65	133.30 ± 9.23	126.22 ± 6.44	133.98 ± 10.42	125.74 ± 2.01	151.57 ± 8.96
7	62.99 ± 1.38	**56.95 ± 0.11**	57.02 ± 0.07	62.16 ± 6.21	61.56 ± 6.01	64.91 ± 7.16	63.13 ± 6.46	66.85 ± 8.06	59.86 ± 3.86	74.36 ± 10.31	65.44 ± 2.75	94.96 ± 19.15
8	76.00 ± 0.12	**75.20 ± 0.19**	75.37 ± 0.11	80.56 ± 5.14	80.83 ± 5.27	82.76 ± 6.11	80.13 ± 4.82	84.40 ± 6.55	79.18 ± 4.62	87.24 ± 10.02	81.27 ± 1.84	110.26 ± 16.22
9	132.36 ± 0.23	**131.23 ± 0.32**	132.19 ± 0.22	144.14 ± 7.38	144.49 ± 7.44	145.00 ± 6.84	144.20 ± 7.24	148.84 ± 8.25	142.32 ± 7.56	145.29 ± 9.79	138.74 ± 2.67	160.50 ± 9.20

Table 9 Mean and SD values in terms of weight for retaining wall numerical simulations

Case	PSO	HCLPSO	EPSO	AGPSO1	AGPSO2	AGPSO3	IPSO	MPSO	TAC PSO	CLPSO	FDPSO	RDPSO
1	2703 ± 0.23	**2481.16 ± 0.23**	2481.96 ± 0.22	2643.15 ± 206.64	2569.47 ± 160.31	2692.89 ± 216.17	2613.98 ± 181.65	2749.32 ± 242.96	2520.29 ± 85.02	3051.24 ± 509.57	3292.63 ± 662.41	3305.95 ± 726.19
2	**3050.22 ± 1.16**	3055.74 ± 0.83	3056.25 ± 0.38	3242.61 ± 240.66	3229.60 ± 238.63	3307.67 ± 244.68	3200.84 ± 207.42	3324.32 ± 271.66	3119.18 ± 145.89	3406.25 ± 303.83	3075.88 ± 4.20	3609.04 ± 471.69
3	**4205.08 ± 0.36**	4207.10 ± 0.19	4207.58 ± 0.22	4617.20 ± 313.43	4672.88 ± 299.74	4697.51 ± 300.24	4654.30 ± 307.58	4777.62 ± 380.96	4469.19 ± 272.22	4508.68 ± 229.51	4236.11 ± 6.33	4824.79 ± 326.32
4	2695.66 ± 0.43	**2471.40 ± 0.25**	2472.28 ± 0.32	2632.50 ± 232.04	2562.36 ± 151.89	2641.03 ± 216.10	2592.21 ± 172.29	2739.26 ± 275.11	2515.32 ± 92.49	2944.42 ± 357.09	2500.35 ± 7.58	3135.38 ± 605.29
5	**2947.83 ± 0.99**	2951.18 ± 0.56	2952.04 ± 0.39	3096.86 ± 205.06	3091.11 ± 202.57	3166.94 ± 239.56	3120.39 ± 217.06	3202.28 ± 259.06	3033.35 ± 182.66	3264.17 ± 326.50	2971.00 ± 4.49	3702.61 ± 652.62
6	4312.02 ± 26.72	**4307.06 ± 0.44**	4307.38 ± 0.25	4721.03 ± 313.84	4732.33 ± 327.45	4764.45 ± 317.76	4727.17 ± 294.73	4800.20 ± 345.57	4652.49 ± 294.48	4594.44 ± 209.06	4338.39 ± 5.81	4893.78 ± 252.39
7	2689.06 ± 13.26	**2465.42 ± 0.29**	2466.07 ± 0.24	2589.33 ± 177.05	2547.92 ± 145.70	2659.51 ± 256.95	2566.83 ± 174.66	2702.60 ± 249.88	2520.73 ± 135.31	3034.66 ± 389.88	2497.70 ± 8.78	3168.14 ± 591.10
8	**2856.87 ± 0.16**	2857.85 ± 0.17	2858.14 ± 0.24	3002.40 ± 219.36	2985.14 ± 211.66	3113.12 ± 271.38	2992.93 ± 224.13	3087.39 ± 277.00	2914.57 ± 129.80	3180.87 ± 332.33	2878.33 ± 4.11	3534.90 ± 541.23
9	4786.56 ± 1.88	**4779.37 ± 0.68**	4780.05 ± 0.3.5	5347.87 ± 298.05	5281.54 ± 324.20	5370.19 ± 302.60	5266.91 ± 321.86	5409.04 ± 348.64	5123.00 ± 292.26	5099.84 ± 226.92	4812.90 ± 18.66	5322.55 ± 190.62

Table 10 Friedman test scoring for minimum-cost design of retaining wall

Optimization algorithm	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
PSO	7.56	4.03	3.31	7.02	4.37	3.06	6.59	5.11	3.22
HCLPSO	**2.05**	**1.23**	**1.51**	**2.63**	**1.39**	**1.40**	**2.42**	**1.77**	**1.15**
EPSO	3.83	2.51	3.38	3.19	2.54	3.35	3.04	2.48	2.65
AGPSO1	5.48	6.76	6.51	4.76	6.53	7.12	4.82	6.40	7.31
AGPSO2	5.36	6.72	7.32	5.12	7.21	7.57	11.94	6.57	7.51
AGPSO3	6.90	7.79	7.47	11.82	7.78	7.55	6.19	7.57	7.86
IPSO	6.14	7.59	7.64	5.81	7.32	7.28	5.42	6.19	7.53
MPSO	7.86	8.08	7.81	6.91	8.12	8.35	6.77	7.99	8.95
TACPSO	4.31	5.37	6.58	3.80	5.60	5.79	3.54	5.85	6.77
CLPSO	9.93	8.94	8.59	9.03	8.21	8.51	9.05	8.97	7.70
FDPSO	6.83	7.18	6.18	6.97	7.12	6.33	7.41	7.31	5.98
RDPSO	11.75	11.81	11.69	10.93	11.81	11.69	10.81	11.79	11.38

Table 11 Friedman test scoring for minimum-weight design of retaining wall

Optimization algorithm	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
PSO	9.10	**1.66**	**1.00**	9.36	3.00	3.83	9.31	**1.03**	3.49
HCLPSO	**3.03**	5.59	2.99	**3.26**	**1.00**	**1.36**	**3.49**	3.64	**1.30**
EPSO	4.97	6.03	4.16	5.23	2	1.9	5.29	6.17	1.96
AGPSO1	5.73	6.82	7.41	5.69	6.86	7.88	5.30	6.35	8.79
AGPSO2	4.39	6.67	8.31	4.64	7.05	8.08	4.55	6.03	8.01
AGPSO3	6.75	8.00	8.47	6.00	7.89	8.35	6.23	7.81	9.18
IPSO	5.60	6.57	8.09	5.36	7.46	8.21	4.91	6.26	8.05
MPSO	7.86	8.09	8.91	7.45	8.39	8.51	7.22	7.11	9.19
TACPSO	3.31	5.50	6.16	3.58	6.47	7.04	3.65	5.09	6.76
CLPSO	10.14	9.82	7.55	10.15	9.47	7.57	10.60	9.68	7.45
FDPSO	6.06	8.02	5.34	6.28	7.53	5.45	6.50	7.78	4.77
RDPSO	11.06	5.22	9.60	11.01	10.88	9.82	10.95	11.04	9.05

EPSO than other variations of PSO. It can be observed that CLPSO could not converge to a valid solution until the latter iterations. Moreover, RDPSO and FDPSO reached solutions far away from the other algorithms' results. Mean convergence results confirm the better performance of HCLPSO and EPSO.

Different loading combinations from the static loading case (case 1) to dynamical ones (cases 2–9) showed that, by applying the horizontal loading factor, the final design saw an increase in either the cost value or weight value. However, by applying and increasing the vertical components the final design was reduced from 0 in cases 1 and 2 to 0.15 in cases 4 and 5, respectively, and from 0.15 in cases 4 and 5 to 0.3 in cases 7 and 8, respectively. Lower costs and weights in the final design, by increasing the vertical coefficient of earthquake loading, may be observed in the presence of horizontal loading component $k_h = 0.15$. On the contrary, this reduction was not observed if the horizontal loading factor of 0.3 was applied to the wall.

6.2 Shallow Footing Minimum-Cost Design

For the shallow footing case study, a foundation situated on a cohesionless soil under two different loading cases studied by Gandomi and Kashani [70] was considered. In the first case, this footing was subjected to transmission of uniaxial forces resulting from the combination of dead and live loads equalling 650 kN and 350 kN, respectively. The final results are summarized in Table 12 based on the best, worst, mean, SD, and median values. In this case, as shown in Table 12, all the variations except CLPSO, FDPSO, RDPSO and EPSO had the lowest solution of $43,442.27, though it can be seen that EPSO registered a better record based on the lowest values of Mean and SD of $47,152.33 and $2076.14, respectively. From the results, it can also be concluded that most of the improved algorithms had better performance than PSO because of the lower values of mean, SD, and Median. Analyzing the Friedman test results presented in Table 13, it can be seen clearly that HCLPSO performed

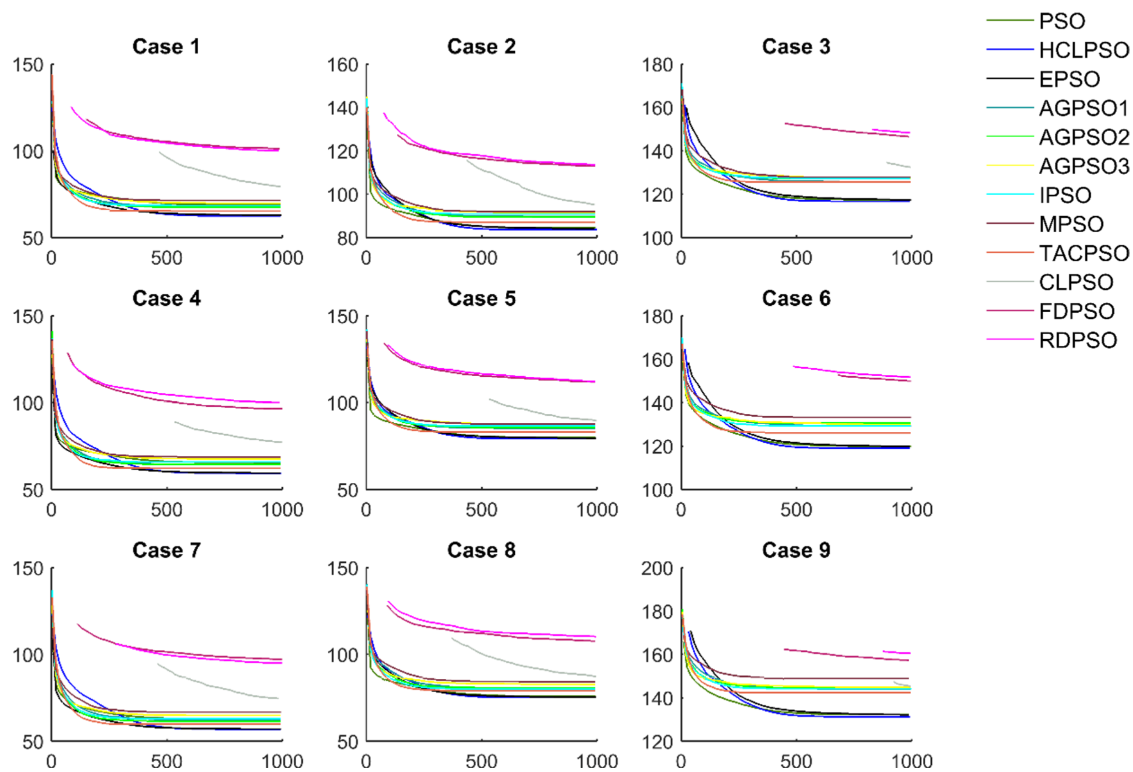

Fig. 1 Convergence rate plots based on best solutions of retaining wall numerical simulation for minimum cost design. (Color figure online)

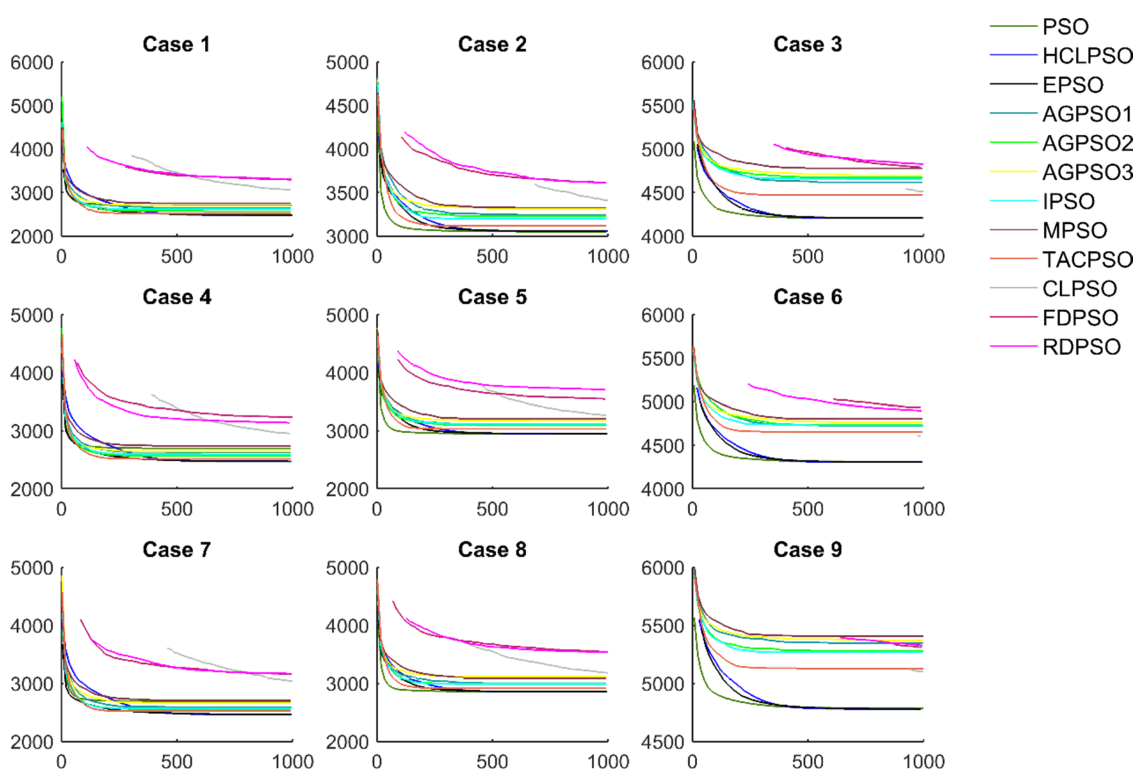

Fig. 2 Convergence rate plots based on best solutions of retaining wall numerical simulation for minimum weight design. (Color figure online)

Table 12 Design cost values for the shallow footing case I numerical simulations

Algorithm	Best	Worst	Mean	SD	Median
PSO	**43,442.27**	174,424.2	68,735.94	25,816.84	65,635.62
HCLPSO	**43,442.27**	54,799.31	48,439.09	2428.04	48,867.28
EPSO	**43,485.57**	**51,678.49**	**47,152.33**	**2076.14**	48,113.24
AGPSO1	**43,442.27**	92,818.81	54,182.15	13,798.72	**47,950.74**
AGPSO2	**43,442.27**	84,769.21	51,344.49	9166.555	48,867.29
AGPSO3	**43,442.27**	88,794.01	51,676.76	8425.636	48,867.29
IPSO	**43,442.27**	84,769.21	51,369.71	8323.933	48,867.29
MPSO	**43,442.27**	93,466.29	61,656.77	15,342.01	57,563.84
TACPSO	**43,442.27**	84,769.21	49,774.39	6899.736	48,867.29
CLPSO	**51,426.27**	66,373.16	59,541.56	6512.142	59,785.65
FDPSO	48,867.29	179,567.4	95,561.33	16,963.16	100,868.4
RDPSO	51,787.85	69,795.23	63,080.85	5966.359	65,239.66

Table 13 Friedman test scoring for minimum-cost design of shallow footing

Algorithm	PSO	HCLPSO	EPSO	AGPSO 1	AGPSO 2	AGPSO 3	IPSO	MPSO	TACPSO	CLPSO	FDPSO	RDPSO
Case I	6.21	**3.57**	5.57	5.14	5.36	4.07	4.57	7.64	4.57	10.57	9.29	11.43
Case II	–	2.33	**2.17**	5.22	4.84	5.46	5.03	6.78	4.34	–	8.84	–

Table 14 Design cost values for the shallow footing case II numerical simulations

Algorithm	Best	Worst	Mean	SD	Median
PSO	–	–	–	–	–
HCLPSO	71,256.61	78,707.9	72,755.26	1980.28	72,931.56
EPSO	71,351.24	**77,938.96**	**72,601.88**	**1461.44**	**72,102.83**
AGPSO1	71,256.61	170,332.5	81,634.31	13,495.7	78,451.58
AGPSO2	**71,225.33**	105,079.3	78,649.55	5603.179	78,340.99
AGPSO3	71,256.61	102,149.9	80,528.23	7073.776	78,451.58
IPSO	**71,225.33**	100,872.6	79,279.91	6349.778	78,347.51
MPSO	71,256.61	136,440.7	89,649.62	15,006.3	84,320.45
TACPSO	**71,225.33**	103,131.6	77,093.4	4341.954	77,835.71
CLPSO	–	–	–	–	–
FDPSO	100,073.7	114,711.2	110,119	3514.96	109,395.8
RDPSO	–	–	–	–	–

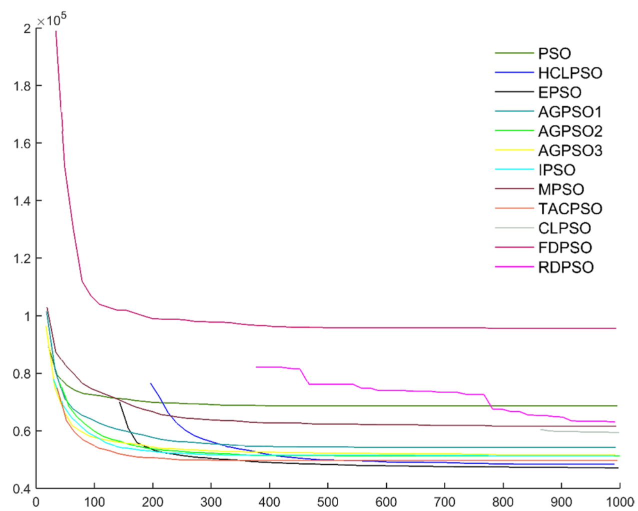

Fig. 3 Convergence rate plot based on best solutions for foundation design case I. (Color figure online)

better than EPSO, and AGPSO3 was the second best algorithm. Moreover, most of the algorithms performed better than the original PSO.

For the second case, in addition to the vertical forces in the previous case, a moment was applied at the center of footing, which is a combination of the dead and live loads of 400 kN m and 150 kN m, respectively. In this case, PSO, CLPSO, and RDPSO failed to converge to a valid solution. AGPSO2, ISPO, and TACPSO obtained the lowest design of $71,225.33, while EPSO had the lowest values of Mean, SD, and Median of $72,601.88, $1461.44, and $72,102.83, respectively. The Friedman test assessment and ranking of the algorithms based on their performances confirmed that EPSO was the best algorithm thanks to its lower score (Table 14).

Convergence rate curves of the shallow footing simulation based on the mean results are shown in Figs. 3 and 4, respectively. As expected, Fig. 3 shows a fast convergence

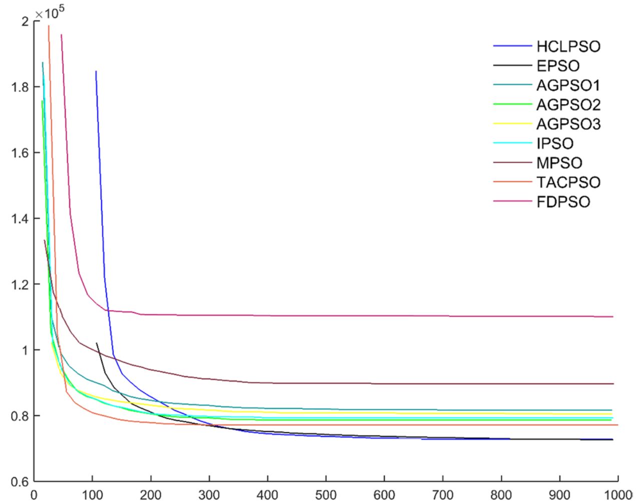

Fig. 4 Convergence rate plot based on best solutions for foundation design case II. (Color figure online)

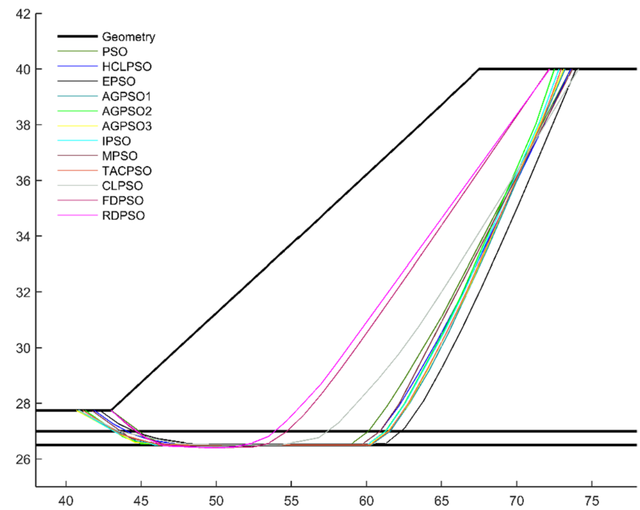

Fig. 6 Slope geometry and critical slip surface for case II. (Color figure online)

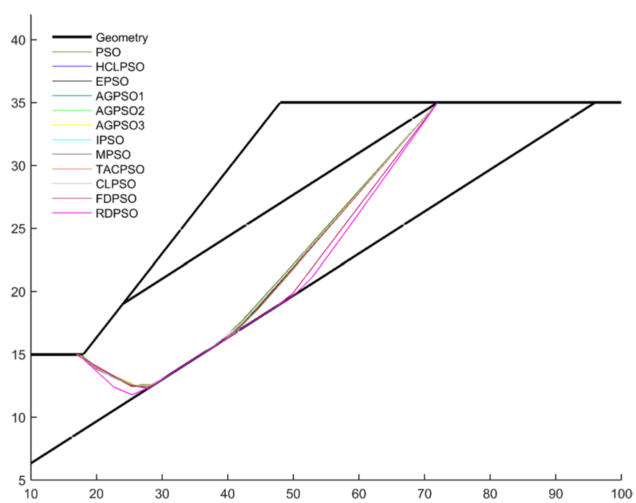

Fig. 5 Slope geometry and critical slip surface for case I. (Color figure online)

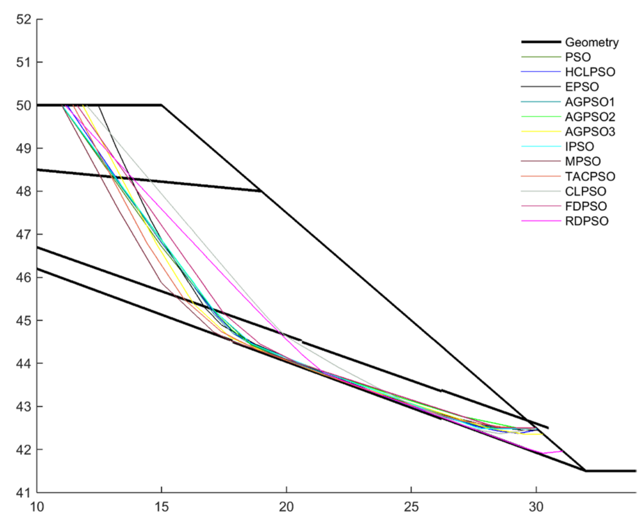

Fig. 7 Slope geometry and critical slip surface for case III. (Color figure online)

of PSO while other variations of PSO follow a gentler trend for convergence. The convergence pattern is a clear proof of stronger exploration in modified versions than the original PSO. In those figures, FDPSO and MPSO converged to the results far more than the optimal solutions. Original PSO, RDPSO, and CLSPO recorded weak performance in the first case study based on poor convergence and in the second case since they were unable to converge to the final solutions. According to the figures, EPSO and HCLPOS approached the final solution after the 100th iteration. The slower convergence rate of HCLPSO and EPSO in these cases proves the stronger exploration ability of HCLPSO and EPSO than other PSO variations.

6.3 Slope Stability Analysis Simulations

For the third numerical simulation, soil slope stability, which is one of the most complicated engineering problems was considered [26]. Three different soil slopes were resolved, as shown in Figs. 5, 6 and 7 borrowed from Arai and Tagyo [207], ACAD, a study by Donald et al. [208], and Zolfaghari et al. [209] as Cases I, II, and III, respectively. A brief comparison of the final results is shown in Table 15. Furthermore, the most critical slip surfaces of each slope are shown in Figs. 5, 6 and 7. In all of the case studies, we attempted to collect complicated slope samples in which a weak soil layer is situated between two stronger ones.

Table 15 Values of factor of safety for slope stability problems

Algorithm	Example 1			Example 2			Example 3		
	Mean	Best	SD	Mean	Best	SD	Mean	Best	SD
PSO	0.3937	0.3926	0.0009	1.4372	1.2462	0.0636	1.3959	1.1148	0.1564
HCLPSO	0.3921	0.3919	0.0002	1.2347	1.2285	0.0032	1.2558	1.0994	0.0773
EPSO	0.3919	0.3916	0.0002	1.2422	1.2273	0.0136	1.0949	1.0621	0.0213
AGPSO1	0.3919	0.3916	0.0003	1.2420	1.2133	0.0306	1.0862	1.0555	0.0427
AGPSO2	0.3920	0.3916	0.0003	1.2327	**1.2104**	0.0172	1.0710	1.0538	0.0165
AGPSO3	0.3924	0.3917	0.0012	1.2497	1.2134	0.0253	1.0799	1.0526	0.0229
IPSO	0.3920	**0.3915**	0.0005	1.2437	1.2115	0.0270	1.0756	1.0531	0.0354
MPSO	0.3924	0.3917	0.0009	1.2681	1.2243	0.0261	1.1194	1.0554	0.1026
TACPSO	0.3920	0.3916	0.0003	**1.2280**	1.2111	0.0174	**1.0674**	**1.0501**	**0.0109**
CLPSO	0.4016	0.3943	0.0058	1.4422	1.3289	0.1015	1.6261	1.3874	0.1956
FDPSO	**0.3917**	**0.3915**	0.0002	1.2521	1.2260	0.0200	1.2198	1.0640	0.1089
RDPSO	0.4107	0.3993	0.0192	1.7948	1.4525	0.4098	1.9218	1.4759	0.3670

The first example is the most straightforward case without sensible difference between the algorithms' results. In this case, the lowest FOS were obtained by FDPSO and IPSO, while FDPSO performed better with lower values of Mean and SD. Based on the Friedman test results too, the lowest score were provided by FDPSO. All the PSO variations except RDPSO and CLPSO were better than the original PSO in this case based on their lower Mean values and Friedman scores. For the second case, a more complex problem with a band of weak soil layer sandwiched between two strong layers was tackled. It can be seen that the final solution of FOS was improved considerably. The lowest values of FOS acquired by AGPSO3 and TACPSO are nearly identical and better than other algorithms. However, the Mean value of TACPSO was less than AGPSO3. The worst performances were recorded by RDPSO, CLPSO, and original PSO, respectively.

The Friedman test results, however, indicate that TACPSO was the best algorithm with the lowest score in examples 2. The worst algorithm, in this case, was again RDPSO. In the third case, TACPSO outperformed other PSO variants based on the lowest values of Best, Mean and SD. The Friedman test results further confirmed the superiority of TACPSO over other algorithms, while the worst performance was provided by the original RDPSO (Table 16).

7 Conclusion

There were two major objectives of this paper. First, we aimed to provide a comprehensive review of the application of PSO algorithms to solve a wide range of geotechnical engineering problems. Second, we wanted to investigate the use of some variants of PSO, including FDPSO, IRDPSO, MPSO, IPSO, AGPSO, CLPSO, HCLPSO and EPSO, to solve three geotechnical engineering problems.

Review of the literature on the application of PSO to geotechnical problems indicated that this algorithm approches difficult problems in different ways. There are several cases where PSO algorithms have shown satisfying performance to address difficulties of these problems. On the other hand, some other challenging cases proved to be beyond PSO algorithms' ability to find optimum solutions. In general two different strategies have been employed for using PSO to handle the problems: first, directly approaching the problems by consducting an optimization procedure; second, coupling with prective tools (e.g., ANN, SVM, etc.) for optimizing the hyper parameters. In addition, several reseachers have attempted to enhance PSO by proposing hybrid optimization algorithms or by introducing some modifications to PSO itself.

For the second objective, three different geotechnical engineering benchmark problems, which include the slope stability, retaining wall, and shallow footing problems, were

Table 16 Friedman test scoring for slope stability analysis

Algorithm	PSO	HCLPSO	EPSO	AGPSO 1	AGPSO 2	AGPSO 3	IPSO	MPSO	TACPSO	CLPSO	FDPSO	RDPSO
Example 1	9.80	6.10	3.45	4.75	5.45	6.35	5.10	7.05	4.30	11.30	**2.65**	11.70
Example 2	10.40	4.65	5.35	4.30	3.50	5.90	4.85	7.50	**2.90**	10.55	6.30	11.80
Example 3	9.70	8.45	5.80	4.30	3.40	4.25	3.45	5.55	**2.95**	11.20	7.25	11.70

tackled in this study. For the slope stability problem, the objective was to minimize the FOS against slipping, while for the retaining wall problem, two objectives—the total cost and weight minimization—were considered. For shallow footing, the total cost value was considered. All the three problems followed the stability criteria defined by ACI 318-05 [109], AASHTOO [108], and Das [206]. The algorithms' performances were examined via comprehensive simulation experiments on cantilever concrete retaining walls affected by pseudo-static loading cases, shallow footing, and soil slope stability. Experiments on each algorithm were repeated 20 times for every slope stability problem instance and 101 times for the retaining wall and shallow footing problems. We reported the best-found solutions, means and standard deviations of the results, as well as the amount of diversity. Non-parametric Friedman tests were carried out to ascertain the statistical significance of the results obtained.

Our results have clearly demonstrated that most of the PSO variations are capable of solving the problems at hand well. Comparisons of the mean and standard deviation results for the retaining wall problem showed substantial improvements achieved by HCLPSO and EPSO over other PSO variants in terms of minimizing the cost and weight. HCLPSO was the best algorithm for cost minimization of the wall. Moreover, this algorithm was also successful in achieving better results than other methods in terms of weight minimization in most of the cases. Friedman test results confirmed that the improvements were significant. Despite the results obtained by the algorithms, in terms of minimizing the weight, appearing nearly identical to the original PSO in some cases, Friedman test results indicated that HCLPSO's results were actually significantly better than the results provided by other algorithms. Shallow footing analysis has also shown significantly better performances of HCLPSO and EPSO over other variations of PSO based on the mean and standard deviation values as well as Friedman test results. For the slope stability problem though TACPSO performed better than other algorithms. TACPSO has obtained the lowest mean and standard deviation results in examples 2 and 3 while FDPSO emerged as the best performer in one of the case studies.

To sum up, balancing between exploration and exploitation is a necessary step to achieving the best possible performance of an optimization algorithm. This study reiterated this fact and showed that PSO variants can be successfully used to solve geotechnical engineering problems. It can be observed that most of the enlisted strategies for improving PSO's performance were successful in improving the results provided by the original PSO. In our future work, we will further investigate the potential of improving these PSO variants and use them to solve other civil engineering problems.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

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