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Game Theory Approaches for the Solution of Power System Problems: A Comprehensive Review

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Abstract

Deregulation and competition appearance in electric power systems and fundamental changes in control and operation structures of such systems require a strong tool for handling such issues. Game theory approach, which is defined as an analytical concept for dealing with the decision-making process in a variety of sciences, is vastly employed in power system problems. This paper provides a comprehensive review of the application of game theory approach to the solution of electric power system problems. The basic foundation of game theory approach and the basic concepts of such concept will be introduced to make the readers familiar with principals of game theory. Moreover, the introduction and a brief definition of main classifications of game theory including cooperative game, dynamic game, evolutionary game theory and strategic game will be studied. In addition, the implementation of different types of game theory approach to accomplish decision making process in power system problems will be reviewed. The main contributions of recent researches in the area of employment of game theory to power system problems are studied and discussed in details.

1 Introduction

The modern electric power systems are encountered with a series of challenging issues considering the appearance of deregulation and competition, which increased the hardness of decision-making process of the issues [[1\]](#page-19-0). On the other hand, a basic revision is demonstrated in such system from a vertical control and operation structure to a horizontal structure. Accordingly, the hardness level of issues related to power systems reliability, operation, control, and management increased [\[2](#page-19-0)]. Classical models face with trouble in dealing with the interdependent decision-making process of power system problems taking into account that such models treat the players as inanimate subjects [\[3](#page-19-0)]. Game theory is introduced as an effective tool for the

solution of modern power systems challenging issues. Game theory is an analytic tool, which economists use to make strategic interactions. In other words, game theory approach is an effective means to analyze strategic behavior of rational decision makers. In such approach, all players are supposed to be rational, which studies the strategic relation between rational players. In fact, a player's action depends on the other players' actions. Game theory is used in computer science, economics, biology and logic as well as political science, and psychology [\[4](#page-19-0)]. Today, game theory defines a wide range of interactions in humans and computers and is now a science of logical decision making. The natural field of application of the game theory approach is an economic theory; the economic system is seen as a huge game between producers and consumers, which transact through an intermediary market. Actually, players are assumed to act rationally. In noncooperative games and other games, the most famous of the theory is presented by John Nash that is the Nash equilibrium. In a set of strategies, the best response to the other strategies will be Nash equilibrium.

John von Neumann was the initiator of game theory at the scientific field in 1928 [\[5](#page-19-0)]. He has used Brouwer's fixed-point theorem on continuous mappings into compact convex sets. The published paper by him was referenced in ''Theory of Games and Economic Behavior'' book proposed by Oskar Morgenstern [\[6](#page-19-0)]. A method has proposed by Neumann and Morgenstern for solutions of twoperson zero-sum games. Game theory was applied to biology in the 1970s, explicitly. Game theory is an important tool in different fields. So far eleven economics Nobel Prize in the different application of game theory is received. To the application of game theory to biology and the evolutionarily stable strategy, John Maynard Smith was awarded the Crafoord Prize. Reinhard Selten presented a new solution concept of sub-game perfect equilibrium in 1965, which further is used in the Nash equilibrium. Also, the concepts of Bayesian games and incomplete information are presented by John Harsanyi in 1967s. The concepts of common knowledge were introduced and analyzed in [\[7](#page-19-0)]. Thomas Schelling and Robert Aumann as game theorists followed Selten, Nash, and Harsanyi as Nobel Laureates in 2005. Schelling worked on dynamic models. Aumann contributed more to correlated equilibrium and developing the common knowledge assumption and its consequences. In 2007, Leonid Hurwicz, Eric Maskin, and Roger Myerson were awarded the Nobel Prize in Eco-nomics for extensive of the mechanism design theory [\[6](#page-19-0)]. Hurwicz introduced and formalized the incentive compatibility concept of each player. In 2012, Alvin E. Roth and Lioyd S. Shapley were awarded the Nobel Prize in Economics for the extension of the market design concept in game theory.

To model the competition behavior of agents, game theory is used in economics and business [[8\]](#page-19-0). The applied models in this science included a wide range of economic approaches, such as auctions, bargaining, acquisitions pricing, oligopolies, duopolies [\[9](#page-19-0)], mechanism design [\[10](#page-19-0)], and voting mechanism [\[8](#page-19-0), [11\]](#page-19-0), etc. Different applications of the game theory method in the solution of problems in various sciences of electrical engineering include allocating resources Device-to-Device communications [\[12](#page-19-0)], power control concept for wireless data systems [\[13](#page-19-0)], multi-cell orthogonal frequency division multiple access (OFDMA) in order to distributed resource allocation [\[14](#page-19-0)], design of multiple input, multiple output (MIMO) systems [\[15](#page-19-0)] and cognitive radio networks considering power allocation strategy $[16]$ $[16]$. In addition, the fundamental sciences of human life utilize game theory method for analyzing cyber risks of bank transaction systems [\[17](#page-19-0)], analysis of relationship between students' grades and their teaching evaluation [\[18](#page-19-0)], allocating cost of the least-cost portfolio adaptation measures [\[19](#page-19-0)], behaviors of environmental supervisors and noise producers [[20\]](#page-20-0), automating cyber defence responses [\[21](#page-20-0)] and river water quality management [[22\]](#page-20-0).

In biology, to explain the evolution of the approximate 1:1 sex ratios game theory has been used. Fisher suggested which the 1:1 sex ratios are a result of evolutionary forces acting on individuals who could be seen as trying to maximize their number of grandchildren. Game theory plays an important role in logic and computer science. Computer scientists have used games to model interactive computations. Game theory has also challenged philosophers to think regarding interactive epistemology. Philosophers who have worked in this area include Bicchieri [[23\]](#page-20-0), Skyrms [\[24](#page-20-0)], and Stalnaker in the last decade of the twentieth century.

The aim of this paper is to provide a comprehensive review of applications of game theory approach to the solution of power system problems. The main classification of such approaches will be studied and analyzed, which include strategic games, extensive games, cooperative or coalitional games, evolutionary equilibrium, and mechanism design and market design. The main contributions of the published papers in the area of game theory approach for the solution of power system problems will be provided. Also, various proposed model in recent publications will be studied.

The remainder of this paper is organized as follows: Sect. 2 provides a brief definition of the proposed game theory approaches in recent studies. A comprehensive review on the implemented game theory methods on power system problems will be prepared in Sect. [3](#page-3-0). Section [4](#page-5-0) analyzes the performance of the game theory approached from different viewpoints, and finally, the paper will be concluded in Sect. [5.](#page-14-0)

2 Definition of the Game Theory Basics and Different Models

A game in game theory is defined as a situation in which intelligent decisions of individual players are interdependent. In this situation, utility payoff of a single player depends upon her own strategy and also the strategy of other agents. In game theory, all of the players are considered to be rational. Our optimal action will be depending upon what our opponent will do but as well as upon what we do.

Game components will be as follows:

Players Rational agents who participate in a game and try to maximize their payoff.

Strategy (action) An action which a player can choose from a set of possible actions in every possible situation.

Strategy profile A set of strategies including one strategy for each player.

Order of play Shows who should play when? At each history, the player may move simultaneously or sequentially.

Information set Information where players know about previous actions when they play.

Outcome For each set of actions, the players select optimal strategy as an outcome of the game.

Payoff The utility which a player receives in the action profile chosen is the payoff of a player.

Common knowledge concept Any player (decision maker) knows that other players are rational, and knows that every other player assumes that every player is rational.

Definition 1 One of the important concepts of game theory is a Nash Equilibrium. The optimal outcome of a game will be the Nash Equilibrium of a game so that no player will have an incentive to deviate from his chosen action with assuming other players' choice. In other words, the Nash equilibrium is a strategy profile such as a^* , if the other players fix their strategy and player i change a^* , no player have an incentive to change his strategy. If the game strategy profiles are shown with $a^* = \{a_1^*, a_2^*, \ldots, a_I^*\}$ so that a_i^* be the action set of player i'th in the game, then the strategy profile of opponent players will be $a_{-i}^* = \{a_1^*, a_2^*, \ldots, a_{i-1}^*, a_{i+1}^*, \ldots, a_l^*\}$. The strategy profile $a^* \in A$ is a Nash equilibrium in the strategic game if for each player *i* equation $u_i(a^*) \ge u_i(a_i, a_{-i}^*)$ be true where $u_i(\cdot)$ is a payoff function representing player *i*'s preferences.

Best response (BR) function concept The best response function of each player is the best reaction that he can do when his opponents do a specific action. So for each strategy of opponents will have the following relationship:

$$
BR_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \ge u_i(a'_i, a_{-i}); \forall a'_i \in A_i\}
$$
\n(1)

The Nash equilibrium will be one of the game BR. Therefore we will have

$$
a^* \text{ is a NE} \Leftrightarrow a_i^* \in BR_i(a_{-i}^*)
$$
 (2)

In this paper we have classified games to six general classes as follows.

2.1 Strategic Game with Perfect Information

In some of the literature, strategic games are also known as static game. A strategic game includes:

- A set of decision makers (players), N , in which players play simultaneously.
- A set of actions for each decision makers, $(A_i)_{i \in N}$.
- Ordinal preferences (a payoff function), $(u_i)_{i \in \mathbb{N}}$, over the set of strategy (action) profiles for each decision makers.

Definition 2 In games which preferences are ordinal, which means that actions order is important for us.

In some of literature game theory is classified into two main branches: cooperative game and non-cooperative game theory. Indeed, strategic game is a non-cooperative game [[25\]](#page-20-0).

In a non-cooperative strategic game, each player i want to choose an action $a_i \in A_i$ so as to maximize its payoff function $u_i(a_i, a_{-i})$ which it depends not only on the player i 's action choice but also on the action profile taken by the other players (a_{-i}) .

2.2 Strategic Game with Imperfect Information

In a perfect information game, the nth players will not only have the knowledge of its own payoff function but also knows its opponents' payoff functions. However, this is not usually the case in a competitive game, where each player would know its own payoffs function but may lack such information of other players. Therefore, each player would model its opponents into different types, ω [\[26](#page-20-0)]. Strategic games with imperfect information, which is also called Bayesian Game due to the calculation of the expected utility, addition to the above items includes the following components:

- For each player, a set of signals which receives, T_i .
- A signal function that corresponds a signal to each mode.

Pure strategy for a decision maker will be a function as $s_i: T_i \rightarrow A_i$..

Definition 3 The concept of Bayesian utility function:

A Bayesian game is a game in which the players have incomplete information on the other players, but, they have a known probability distribution on the opponent strategy profile. If a player is indicated as a set of pairs (i, ti) which i indicate player and ti be signal that he receives, then the expected utility of player (i, ti) shown as follows:

$$
\mathbf{u}_{i}(\alpha_{i}, \alpha_{-i}, \mathbf{t}_{i}) = \sum_{\omega \in \Omega} \text{prob}(\omega | \mathbf{t}_{i}) \cdot \mathbf{u}_{i}(\alpha_{i}, \hat{\alpha}_{-i}(\omega), \omega) \tag{3}
$$

2.3 Dynamic Game with Perfect Information

In some of the literature, dynamic games are also known as extensive game. The dynamic game is a strategic situation which players play during the time. In other words, the game is done as sequential. In any moment which the player are doing decision, he know action that opponent has already been selected. A dynamic game component consist of

- A set of players, N.
- A set of final history H.
- • A set of histories which are not final. They are called decision nodes and a player function defined for them $(p(h))$.
- For each player is defined a utility function based on each of the final histories which happen.
- A set of actions for each player, $A(h) = \{a_i(h, a)\}\.$

Then a dynamic game can be presented as follows:

$$
\Gamma = (N, H, p(h), u_i(h); A(h))
$$
\n(4)

Stackelberg game is an example of the dynamic games which first the leader decision maker moves, and then the follower decision makers move sequentially [[27\]](#page-20-0).

2.4 Dynamic Game with Imperfect Information

In this type of game, the player carries out games that exactly she/he does not know in which node or history has been located. This game is consists of the following components:

- A set of players, N.
- A set of final history H which will not be the subset of no mere history.
- Player function, p(h), corresponding to a player or nature which specifies the turn of game for each player.

Note: The nature is a player that plays based on the probability and is not defined any utility in the game for it.

- A probability distribution function is defined for every history which has assigned to the nature over the actions available at that history.
- For each player an information partition.

Signaling game is an example of a dynamic game with imperfect information [[28\]](#page-20-0).

2.5 Cooperative Game

Cooperative games allow to players investigate how the players can have an incentive for independent decision making. Players act together as any entity improve their payoff in the game. Each group of players is named a coalition (S), and the coalition of all players is created a grand coalition (N). This game consists of the following components:

- A set of players.
- For each coalition, a set of actions.
- For each player, preferences over the set of all actions of which she/he is a member.

Coalitional game theory converses with concepts like the core, Shapley value, bargaining set, von Neumann-Morgenstern solution, and nucleolus. Cooperative game theory is that it does not need an exactly defined structure for the actual game. It is enough what each coalition can achieve; no matter how it can. Some of the solution concepts for cooperative games are the core and the Shapley value and also the Nash-bargaining solution [[29](#page-20-0)].

2.6 Evolutionary Game Theory

The evolutionary game develops the non-cooperative game formulation by introducing the population concept, which relegates to a set of players [[30\]](#page-20-0). Mainly, the evolutionary game concentrates on the overall behavior of population that is different from a non-cooperative game. The evolutionary equilibrium in this game is similar to a NE in a strategic game. Therefore the evolutionary equilibrium will be a solution of this game and the population will not gain maximum payoff if is deviated from this point. Replicator is another important concept which demonstrates the selection dynamics of population modeled as a set of ordinary differential equations. Therefore, by proper design of a replicator, the population can achieve its evolutionary equilibrium, gradually [[30\]](#page-20-0). The evolutionary game has broad usage in engineering science due to its efficiency, especially for the multiple buyers-sellers scenarios [[31\]](#page-20-0). An evolutionary game component is consisting of player, population, strategy, and utility.

There are some other games which we have not put in these categories, but we will refer to them in the next section.

3 Classifications of Game Theory Approaches

Different types of game theory concepts are classified in Fig. [1](#page-4-0). In the following, the main classifications of game theory are introduced.

3.1 Cooperative/Non-cooperative

Game-theoretic classification can be done in cooperative and non-cooperative games, which it depends on the decision makers' behavior (players). In a non-cooperative game, players are in mutuality to each other, and they decide independently. Two most popular solution concepts used in these games are Nash and Stackelberg equilibria, which both of them are based on the best response functions analysis. Nash equilibrium is applied when the decision makers choose their strategies, simultaneously. While the Stackelberg equilibria will be appropriate when one player decides after the other. In cooperative games, decision makers will cooperate when they agree together. Basically, its outcome will be better than the Nash or Stackelberg equilibria [[32\]](#page-20-0).

3.2 Symmetric/Asymmetric

of games

In the symmetric game, all of the players know each other preferences. While in the asymmetric game, the players have asymmetric information to the preferences of each other. Figures 2 and [3](#page-5-0) demonstrate a simplified model of symmetric and asymmetric games, respectively.

It is assumed that player A knows the preferences of player B, whereas player B does not know his opponent's preference; in other words, player A has complete information, unlike his opponent. The Bayesian game can be used to model this situation [\[33](#page-20-0)].

Fig. 2 Symmetric game

3.3 Zero-sum/Non-zero-Sum

Zero-sum games are strictly competitive games, while in a non-zero game it is possible for the players to both wins. In a zero-sum game, the sum of all payoffs by a decision maker or group of players is equal to the sum of all losses for every possible outcome of that game. Non-zero-sum games refer to games that one player's gain does not necessarily equal to another player's loss. Another words the gains and the losses in the non-zero-sum game do not always added up to zero. The Prisoners' Dilemma is one of classic example in non-zero-sum games.

Unlike zero-sum games, non-zero-sum games will be not completely competitive so that they can contain all sorts of degrees of cooperation. Therefore the strategies of each player can change depend on cooperation degree of players in the game [\[34](#page-20-0)].

3.4 Simultaneous/Sequential

Depend on the nature of the game; decisions by players are sometimes taken simultaneously and or sequentially. Simultaneous decision making, contrary to how the problem is solved itself is relatively simple. In front of simultaneous decision making, Sequential games can be very complex so that those need to certain techniques to solve this problem. Sequential decision making is usually represented by tree-like diagrams [[25\]](#page-20-0). Sequential games will be games of either perfect or imperfect information.

Fig. 3 Asymmetric game

3.5 Perfect Information and Imperfect Information

In a game with perfect information, players choose their strategies, sequentially, and they are aware of what other players have already chosen, for example, chess. While in a game with imperfect information, the players are ignorance of one opponent's moves and merely anticipate what the other opponent will do [\[3](#page-19-0)].

4 Game Theory Approaches Applied to Power System Problems

In this section, the application of different game theory approaches to power system problems is studied. The application of game theory is classified to different methods including cooperative game, dynamic game, evolutionary game theory and strategic game for simplifying the review. The contributions of recent studies in the area of application of game theory approaches are studied and discussed in details.

4.1 Cooperative Game

The investigated issues in power system problems implementing cooperative game theory have been demonstrated in Fig. [4](#page-6-0).

4.1.1 Loss Allocation in Power Systems

The authors have applied a cooperative game theory based framework in [[35\]](#page-20-0) for comparing various methods used for loss allocation in transmission power systems. Equivalent bilateral exchanges (EBE) are considered in this reference for attaining an understanding of the problem difficulties.

An energy management model for power systems is proposed in [\[36](#page-20-0)] for reducing loss and maintain the stability of the system, which is based on two parts. The proposed model in this reference includes a game theory based part for loss reduction allocation (LRA) and a load feedback control. An optimal LRA solution is provided by maximizing total profit of the generators employing game theory concept.

In [[37\]](#page-20-0), a novel locational marginal price (LMP) is introduced for distribution networks, which is based on LRA. A cooperative game theory approach is implemented for allocating the loss share of DGs in LRA. The state estimation of the system is accomplished by integrating the proposed model and an iterative solution, and ANN is used for prediction of day-ahead demand and power market priced.

Authors have utilized Shapley value of the cooperative game theory in [[38\]](#page-20-0) for obtaining the solution of loss allocation in pool market. Shapley value is integrated to current injection, and equivalent constant impedance load models are introduced as feasible loss allocation approach.

4.1.2 Optimal Performance of Electricity Supply System by Market Agents

A cooperative game theory is used in [[2\]](#page-19-0) to solve competing interests in attaining optimal performance of electricity supply system by an optimal resource allocation of all network (local) actors. Such theory is incorporated with agent-based concepts to obtain the mentioned interests between producer/consumer agents and network agents.

A non-cooperative game with incomplete information is employed in [\[39](#page-20-0)] for obtaining the best strategy offered by generation units in power market. An electricity auction is studied considering the behavior of participants and the objective of ISO. The introduced concept in this study is based on max–min and probability distribution function.

A cooperative game theory is employed in [[40\]](#page-20-0) for allocating profit to independent power producers, where a novel method is proposed for formation of coalition in retail markets. The results analysis verifies that more profit will be attained by the cooperation of producers concerning their competition.

The authors have employed a cooperative game theory in [\[41](#page-20-0)] for the solution of fixed-cost allocation in bilateral transaction power market and a pool market. Nucleolus and the Shapley value are used in this study, which is selected based on the decision of the system operators.

4.1.3 Energy Management Framework of Smart Grids

An energy management framework of smart grids is proposed in [[42\]](#page-20-0) by competition game implementing grouping and scheduling approaches. In the proposed scheme in this reference, the groups are formed by exploiting different power appliances, and the cooperative concept is used for dynamic scheduling of the appliances in each group. The introduced model minimizes total cost of the system by satisfying the need for different appliances.

4.1.4 The System Reliability Performance

A cooperative game theory-based concept is introduced in [\[43](#page-20-0)] for determining the component criticality for overall reliability of the system. Shapely value is employed in this reference to define the contribution of each component on the system reliability performance. Also, this paper determines the relationship between system maintenance, reliability, aging of the components and total cost of the system.

Shapely value game theory is utilized in [\[44](#page-20-0)] to determine the contribution of each component in system reliability, which is a practical technique to distinguish maintenance focuses of the system. Reliability centered maintenance (RCM) is accomplished by the applied theory, which has successfully obtained critical components of the system.

A similar game theory for obtaining the contribution of each system component on the overall reliability of the system is employed in [\[45](#page-20-0)] to sort the generation units considering their criticality. In this reference, a connection of criticality measures to financial risk is taken into account according to their contribution in system failure.

Secondary voltage control (SVC) is studied in [\[46](#page-20-0)] by employing cooperative game theory, which considered the

possibility of coordination of control zones in SVC zones. In this reference, the control performance of control zones is improved by obtaining an efficient and stable cooperation of the zones.

4.1.5 Building Distributed Heating Supply System

Different frameworks of benefit allocation for building distributed heating system have been studied in [[47\]](#page-20-0) including Shapely, the Nucleolus, DP equivalent method and Nash-Harsanyi schemes. The proposed model, which is based on cooperative game theory, considers the consumers as stockholders aiming to maximize their benefit to attain fair economic solutions for the participants.

4.1.6 Supplying and Scheduling of Micro-grids Demand

In [\[48](#page-20-0)], a cooperative game theory is used to model a group of micro-grids suppling a zone, which are connected to the main grid. The proposed method allows the cooperation of micro-grids and formation of coalitions to buy or power to each other for meeting their demand. In a general overview, total loss of the system is minimized by introducing an optimal solution of power transfer within every coalition as well as the main grid.

A game theoretic strategy is proposed in [\[49](#page-20-0)] for distributed MGs to minimize the power loss related to power transfer between different MGs or transfer to the main grid. In this reference, the models decides to from or break the coalitions while obtaining the maximum objective functions of the MGs considering revisions in environmental aspects or changes of electrical energy demand. Also, the optimal number of MGs needed for an assumed area is obtained by the proposed model of this paper.

4.1.7 Distributed Energy Resources Participating in Energy Market

The profit allocation of distributed energy resources (DERs) participating in the energy market is taken into account in [[50\]](#page-20-0) aiming to obtain the maximum profit of the market. A bi-level scheme is proposed in this reference for providing a solution, where non-cooperative and cooperative game theory is implemented to maximize the profit of sources and provide profit allocation, respectively.

The Nucleolus and the Shapley concepts are used in [[51\]](#page-20-0) for specifying the obtained profits by virtual power plants (VPPs) to DERs. A new stochastic programming method is used to decide the participation of sources in the day-ahead market. The conditional value-at-risk (CVaR) is calculated by desired risk-aversion level of DERs, and the role of such sources in covering the risk and total profit is studied in this reference.

In [[52\]](#page-20-0), the management and pricing of reactive power ancillary service are studied by employing three different concepts (a) based on embedded cost techniques, (b) based on cost valuation of synchronous condensers and (c) cooperative game theory. Active power generation (energy), load following and reserve services (capacity) and reactive power service are considered as the products, which form the total cost. The proposed game aims to the simultaneous allocation of the cost to each of above-mentioned sectors.

4.1.8 Transmission Expansion Planning Cost Allocation

In [[53\]](#page-20-0), the authors have implemented Core and Nucleolus methods of cooperative game theory to allocate set of transmission expansion among the participants of the market. Transmission congestion as a constraint of the issue is relieved by transmission line expansion.

A novel cooperative game theory, which is based on the integration of corporation and coordination among agents, is applied in [\[54](#page-20-0)] to allocate cost in existing or expanding networks. Also, the physical and economic utilization of the agents are considered in the proposed method, and efficiency and fairness principles are implemented.

A cooperative game theory is used in [[55\]](#page-20-0) for handling conflicts associated with cost allocation problem in decentralized power networks. In this reference, a nucleolus is applied for minimizing the maximum regret of the participants, which will be acceptable and stable among them.

4.1.9 Renewable and Sustainable Energy Scheduling

A cooperative game theory and optimization control framework is introduced in [\[56](#page-20-0)] in order to obtain the maximum profit of an energy storage system (EES) combined with wind power generation plants. EES is effective to obtain a smooth power output of the wind generation plant by using two low-pass filters, where time constants of the two filter to attain the main goal. Game theory is applied in this reference considering wind power EESs and wind power capacitors as the game players, which coordinates and optimizes the above mentioned time constants.

4.2 Static Game

The studied issues in power system problems employing static game theory concept have been illustrated in Fig. [5.](#page-8-0)

4.2.1 Demand Side Management and Demand Response

A cloud computing framework for a group of smart energy hubs is represented in [\[57](#page-20-0)]. A non-cooperative static game theory is used to model the demand side management game theory

among the smart energy hubs. The hub attempts to minimize its own energy cost with the DSM setting. Simulation results show that at the Nash equilibrium, the load factor of the total electricity demand reduces significantly and as a result, the smart energy hubs will pay less for their energy bill.

In [[58\]](#page-20-0), a demand side management program proposed that is based on energy consumption scheduling and instantaneous load billing. The selfish consumer's behavior is formulated as an aggregative game to minimize their individual energy cost. The customers are connected, and they would like to share their estimated information with others. In this paper one timescale distributed iterative proximal-point algorithm, a distributed synchronous agreement-based algorithm, and a distributed asynchronous gossip-based algorithm to achieve the NE is developed.

In [\[59](#page-20-0)], authors have studied the demand response management (DRM) problem with considering competition among multiple utility companies and multiple residential users. In this game theory framework, the utility companies and the end users can exchange their information. Therefore knows the required information. DRM problem is as a two-level game. At the higher level, there is a competition among companies modeled by a non-cooperative game. The company's strategy is its generation amount, demands of users, and power price. While at the lower level, the residential user's competition formulated by the evolutionary game. The results show that the proposed methodology can significantly reduce the peak load and the power demand variation.

A non-cooperative game theoretical framework is applied in [\[60](#page-20-0)] to model demand side management problem along with energy storage devices. Each user tries to minimize its energy payment to an energy provider. A distributed algorithm is designed to each user to control its energy consumption scheduling, independently. The simulation results indicate that the proposed algorithm will minimize the peak-to-average ratio of power as well as the total energy cost.

4.2.2 Power Transactions of Generating Entities in Electricity Market

In [[61\]](#page-21-0), authors have proposed a non-cooperative static game with complete information to model the power transactions in a pool market. In the power transaction activities, each generating entity as a player does his best action to maximize the profits. The payoff of the each generating entity will be its profits and the bidding prices of each player compose its strategy. As a result, the proposed

method can provide useful information such as their optimal bidding strategies for each player.

The two models considering market assumptions are presented in [\[62](#page-21-0)] to address a bilateral market in which generators competes imperfectly to purchase transmission services from an ISO. In the first model, there is no arbitraging between different locations in the network so that generators can raise prices where competition is weak or demand is inelastic. In the second model, there are arbitragers who eliminate price differences between locations. It is equal to a POOLCO-based system considering locational marginal pricing.

Game theory and auction theory are applied in [\[63](#page-21-0)] to analyze the strategic behavior of a two-player static game with a big player and a small player. From the viewpoint of the effect of strategic bidding and market power are investigated the issue of uniform pricing and pay-as-bid pricing in electricity markets. The presented auction model follows the basic the sealed-bid multiple-unit auction framework. The total expected revenue for the NEs under uniform pricing, and pay-as-bid pricing was not equivalent.

A strategic game model represented in [[64\]](#page-21-0) to analyze an oligopolistic market economy consisted several dominant firms in an electric power network. Each firm submits bids to an ISO to maximize profits subject to reactions of opponent firms. The proposed model is based on DC optimal power flow (OPF), and their constraints are pricing strategies. In fact, oligopolistic price equilibria for linearized DC networks are calculated using the supply function conjectural variation.

An optimal bidding strategy problem represented in [[65\]](#page-21-0) to calculate the expected benefit of power suppliers by bidding a price higher than the marginal production cost. In other words, a methodology is extracted to the competitive power suppliers build optimal bidding strategies in dayahead auction-based electricity markets.

In [\[66](#page-21-0)], game theory has been applied to model the uncertainty of other participants' behaviors in a market for trading a new bilateral reserve market among the wind and conventional power producers. In the proposed model stochastic programming is used to generate optimal bidding strategies for wind power producers. The total profits of the wind and conventional power producers are maximized in both the energy market and a bilateral reserve market. The new trading mechanism allows the wind producers to buy energy from the bilateral reserve market to minimize the risk of losing money due to their production uncertainties.

A Cournot equilibrium based game framework is investigated in [[67\]](#page-21-0) to analyze three market players in a transmission-constrained network which it builds one step closer to realism. The presented model has considered nonconstant marginal cost instead of the constant marginal cost

that potentially lead to results that are not realistic. The market players are considered non-symmetric to model an actual market and both load and generation are modeled at each bus in the network.

A bidding strategy method based on the concept of conjectural variation (CV) is introduced in [\[68](#page-21-0)] to help generation firms to improve their strategic bidding and maximize their profits in real electricity markets with imperfect information. The conjecture of a firm is defined as its expectation of how its opponents will react to the change of its actions.

A game-theoretic framework is applied in [[69\]](#page-21-0) to model the power supplier behavior in the imperfect market to increase its own profit through strategic bidding. Each supplier submits linear bid functions. The power supplier behavior is modeled as unsymmetrical. The degree of imperfection of knowledge of each supplier relative to the other players leads to an unsymmetrical strategy. Simulation results shows which in competitive market condition, clearing price is lower and the market power will be reduced.

Competition among generating companies (GENCOs) considering transmission constraints is described in [\[70](#page-21-0)]. Each GENCO models its opponents by incomplete information. Problem formulation is modeled as a bi-level problem. Each GENCO maximizes its payoffs in the upper sub-problem and in the lower sub-problem, ISO minimizes consumers' payments. Due to the lack of enough transmission capacity, an electricity market cannot be a perfectly competitive market. The simulation results indicate that the transmission limits has a major impact on GEN-COs' bidding strategies.

Imperfect competition of strategic biding of participants in a deregulated electricity market is modeled in [[71\]](#page-21-0). The bimatrix approach is used to find the Nash equilibrium. The bimatrix approach determines pure and mixed equilibrium by the complementary pivot algorithm. The bimatrix approach is fast to finds Nash equilibrium as well as find mixed equilibrium.

4.2.3 Smart Home Energy Management

In [[72\]](#page-21-0), authors have proposed a non-cooperative static game to model the competition of between distributed residential electricity suppliers or as energy cells in the retail electricity market. The participation of individual residential suppliers can only be achieved by future power distribution infrastructure, i.e., energy internet. Each energy cells are comprised of local generators, energy storage, and controllable loads. Each energy cell can exchange the data as real-time with other cells by a twoway communication network infrastructure. Each

residential customer maximizes its benefits subject to some constraints.

A non-cooperative game theory is employed in [\[73](#page-21-0)] to model the end users behavior in response to any change in the electricity market price. In the proposed model, there is no communication or binding agreement between players. By using game theory and day ahead pricing strategy, each end user shift its unnecessary demand from peak hours with the high electricity price to off-peak hours with the lower price. Authors have considered a scenario with ten residential users with multiple elastic and non-elastic appliances. The simulation results show that the residential users can reduce their bills at least 25%.

In [\[74](#page-21-0)], a game theoretic framework is proposed to formulate energy consumption scheduling of residential customers in a smart grid. To minimize the cost of energy as well as to reduce the peak-to-average ratio of the total energy demand, an optimal incentive-based energy consumption scheduling algorithm is proposed. Each of players tries to maximize its own benefits in a game-theoretic framework. Nash equilibrium will receive the optimal performance regarding minimizing the energy costs.

4.2.4 Energy Management of Hybrid Energy System

In [[75\]](#page-21-0), a multi-agent modeling and control strategy is proposed to the energy management of the engine-generator/battery/ultra-capacitor in hybrid energy system (HES). The current control strategy is based on a non-cooperative game. Nash equilibrium, balances the different preferences of the engine-generator unit, battery and UC packs. The proposed energy management strategy of the multi-source is implemented experimentally on HES. The result shows improve on the flexibility, scalability, fault-tolerance, and reliability of the HESs.

A hybrid power system considering wind turbines, photovoltaic panels, and batteries storage units are planned in [\[76](#page-21-0)] using both non-cooperative and cooperative gametheoretic approach. The mentioned units are introduced as players and their life cycle income as payoffs. The Nash equilibriums of the game are determined by an iterative solution algorithm. The uncertainties of load demand, wind speed and sunlight are considered. Simulation result illustrated that in comparison with all the Nash equilibriums, the cooperation can bring out more payoffs compared to competition game.

4.2.5 Supplying and Scheduling of Micro-grids Demand

A non-cooperative static game-theoretic framework is proposed in [[77\]](#page-21-0) to study the interactions between microgrids that generate renewable energies. The strategic behavior of microgrids is characterized by the Nash equilibrium concept. The proposed framework incorporates both economic and technical aspects such as voltage angle regulations and the power flow constraints. Authors have developed a resilient and fully distributed phasor measurement unit enabled update algorithm for microgrids and designed its implementation control framework.

4.2.6 Strategic Bidding of VPPs

Two operation models are introduced in [\[78](#page-21-0)] included a dispatch model of virtual power plants (VPPs) and a game theoretic model to multi-VPP dispatch. With considering the interaction between VPP and consumers, time-of-use and interruptible loads based demand response are applied. In the market competition process, the strategic bidding of VPPs will be determined based on fuel cost and its affordable power output.

4.2.7 Distribution System (DS) Management

A decentralized cooperative algorithm based on gametheory is developed in [\[79](#page-21-0)] to manage distribution system (DS) operation in the presence of different participants. The results of the proposed algorithm was compared with the ''best response'' (BR) game-theoretic approach, which was used in similar DSs management to achieve a Nash equilibrium. Each participant will willingly participate in a game, should satisfy the DS operator needs. The proposed algorithm did not require players to know any information about the others players.

4.2.8 Renewable and Sustainable Energy Scheduling

The energy trading decisions and interactions of distributed storage units are studied in [[80\]](#page-21-0) based on a non-cooperative game theory. Storage units can be such as PHEVs, or an array of batteries to trade their stored energy. For this purpose, a double-auction market model is designed to allow to multiple buyers and multiple sellers to participate in power markets. Simulation results indicate that the proposed model enables the storage units to act strategically to improve their average payoff.

A strategic approach based on non-cooperative games theory is developed in [\[81](#page-21-0)] to coordinate the autonomous plug-in electric vehicles (PEVs) charging program in the smart grid. Because time-based and fixed price-based strategies difficulty fills the night-time valley, it is proposed real-time electricity price information. The optimal solution minimizes power generation costs by the PEV demand schedule to fill the overnight demand valley. Each PEV can choose and implement its charging control strategy to minimize its individual charging cost.

A game theoretic framework is proposed in [[82\]](#page-21-0) to model behavior of large-scale wind power producers in electricity market considering transmission constraints. The behavior of each player is modeled using a two-stage mathematical problem with equilibrium constraints (MPEC). The profit of the strategic player is maximized in the upper-level problem, and the clearing processes of the day-ahead occur the lower-level problem. The simulation results have shown that wind power producers could benefit in the electricity market if behavior strategically.

A non-cooperative game framework is developed in [[83\]](#page-21-0) to model behavior of the vehicle owners to manage their energy consumption. The vehicle owners are modeled as players that they can schedule efficiently vehicle charging to reduce the users' electricity bill cost. A time-based DR program is developed to schedule the PHEVs charging problem as well as dynamic electricity pricing. The results indicate the potentials and benefits of the proposed model to manage load in power markets.

4.2.9 The Attacker and Defender Strategic Behavior in Power System

A two-person zero-sum strategic game is investigated in [\[84](#page-21-0)] to model the attacker and defender behavior that likes to attacks and defends different measurements of electricity market price, respectively. The attacker changes the prices and the estimated transmitted power in the electricity market to achieve the congestion level and profit. On the other hand, the defender tries to defend the accuracy of network measurements. Simulation result shows that how an attacker can change the prices in the desired direction.

4.2.10 Distributed Energy Resources Participating in Energy Market

A non-cooperative game-theoretic methodology is represented in [\[85](#page-21-0)] to model participation of large distributed energy resources (DERs) in the electricity market. Game theory based on the Nikaido–Isoda function with n-person is applied to each player that maximizes its profit through a distributed decision-making process. The electricity customers can participate in the market as producer and consumer, simultaneously.

4.3 Dynamic Game

The studied subjects in power system problems using dynamic game theory have been shown in Fig. [6.](#page-12-0)

4.3.1 Integration of Distributed Energy Resources in Distribution System

A dynamic game theorem framework based on the Stackelberg game is applied in [[86\]](#page-21-0) to find optimal contract prices between DGs' owners and distribution company (DisCo). The optimal location of DGs is determined using a multi-objective optimization. In this two agent's problem, DGs' owners as a leader offer their contract prices and DisCo as a follower selects its desire offer to minimize its power payment to DGs' owners. Simulation results show that competition between DGs' owners enables the DisCo to buy energy with a lower price.

A nonzero-sum, dynamic game theoretic approach represented in [\[87](#page-21-0)] to control and operation of the individual sources and power electronic loads decision in DC power systems. These loads are modeled as variable resistances. The proposed model improve the reliability and robustness of the system by avoiding the need for central control. Nash equilibrium point maximizes the payoff of both controllable sources and loads.

A three-level dynamic game theoretic is proposed in [\[88](#page-21-0)] to analyze the interactions between the electricity supplier, the charging operator, and crowdfunders. Crowdfunding is efficient way to promote EV charging construction expansion using the proposed game model. In the EV charging construction process, the electricity supplier will be as the leader player, and the operator and crowd funders are modeled as the first and second follower player, respectively.

A hierarchical Stackelberg game is proposed in [[89\]](#page-21-0) to model competition in integrated energy systems with considering multiple distributed energy stations (DESs) and multiple end users (EUs). In this problem, DESs are as leaders that decide for the generated electricity and cooling energy prices. EUs are as followers that determine the consumed energies. The objective is maximizing the profit of each distributed energy stations. Simulation results show the effects of coupling generation of electricity and cooling energies as well as the effects of market scale and the exogenous parameter on the Stackelberg equilibrium.

4.3.2 Demand Side Management and Demand Response

A game-theoretical decision-making scheme based on a four-stage Stackelberg game is proposed in [\[90](#page-21-0)] to model the interactions between the retailer and electricity customers in real-time pricing demand-side management. In the first three stages, retailer maximizes its profit by decision making on the electricity sources procurement at the optimal retail price and its value. The customers as the followers maximize their utility by adjusting their electricity demand. Because each retailer needs to learn game theory

electricity consumption patterns of customers, data is exchanged between the retailers and the customers.

A dynamic game-theoretic approach is applied in [[91\]](#page-21-0) to optimize user behavior through TOU pricing strategies. Utility functions are designed for the company and the users, and backward induction approach is used to obtain Nash equilibrium. Utility companies will to maximize profits while end users seek to minimize their costs. It is considered a game between utility companies and customers using a multi-stage game model. Simulation results show that the proposed strategy will increase the profits of the utility companies and will reduce energy prices for electricity users.

A dynamic game formulation is described in [\[92](#page-21-0)] to model electricity trading between the utility company and multiple end users. The model follows the supply and demand balancing as well as the aggregated load flatting in the power system. The utility company acts as leader and the end users as followers. Therefore, a one-leader, multifollower Stackelberg game is formulated to optimizes the optimal strategy selection. A real time pricing-based demand-response model is also proposed to balance supply and demand. An iterative algorithm is proposed between the utility company and the end users to extract the Nash equilibrium.

The full-competitive of community energy storage (CES) operation is modeled in [[93\]](#page-21-0) by a non-cooperative Stackelberg game in a residential neighborhood area network. For implementation of demand-side management, three potential energy trading framework investigated for integration feasibility of CES device with consumer-owned photovoltaic systems. The CES operator as leader moves to maximize payoff while users as followers follow the CES's actions to determine optimal energy trading strategies. The simulation results show which both CES operator and users benefit in the full-competitive system, simultaneously.

4.3.3 Power Transactions of Generating Entities in Electricity Market

A dynamic non-cooperative game with complete information is proposed in [[94\]](#page-21-0) to schedule generating units in electricity markets. The player corresponds to generating companies (GENCOs), the payoff of each player is the individual profit maximizing, and the optimal strategy profile is extracted by the backward induction concept based the sub-game perfect Nash equilibrium. GENCOs compete to each other to maximize their expected profit. The obtained profit from the auction in the energy markets affects the maintenance strategies.

4.3.4 Supplying and Scheduling of Micro-grids Demand

A dynamic game theory based on backward induction is represented in [\[95](#page-21-0)] to model the end users behavior as independent decision makers on the isolated micro-grid. The interaction among users is analyzed to minimize the total energy cost in the micro-grid by optimal using the available renewable energy resources and backup conventional unit. The user's payoff function will depend on his own strategy as well as other users' strategies. The

simulation results show that the generation costs reduce efficiently compared to the benchmark method.

A two-level continuous-kernel Stackelberg game proposed in [[96\]](#page-21-0) to model multiple interconnected micro-grids behaviors. Some micro-grids compete to decide for their extra energy to sale or storage facilities. Also some other would like to buy additional energy to storage or local demands requirements. Some micro-grids as leaders present their price strategies so that the followers can observe the price strategies of the leaders and decide their own strategies accordingly.

4.3.5 The System Reliability Improvement

A Stackelberg dynamic game theatric is applied in [[97\]](#page-21-0) between utility companies and end-users. The objective is maximizing the payoff function of the utility company and each user. A distributed algorithm is developed to converge the Stackelberg equilibrium with only local information of the utility companies and the end users. The proposed scheme is based on the shared reserve power concept to improve the system reliability. Reliability improvement that is due to physical disturbance of an attacker by manipulating the price data from the utility companies.

4.3.6 The Attacker and Defender Strategic Behavior in Power System

The interaction between the components of the power system is modeled in [\[98](#page-21-0)] to identify the vulnerable and critical components of the system. A dynamic game theory framework is used to analyze the vulnerability of the power grid components, and linear programming method is applied to solve this problem. An instability index is used to measure the negative impact of power outage on the system. When instability is low, power oscillation due to the load disturbance can be quickly stabilized, and no damage will occur. An attacker and defender strategy is proposed to improve the system instability and protect the most critical components.

4.3.7 Power Transactions of Generating Entities in an Electricity Market

A dynamic Bayesian game approach is addressed in [\[99](#page-21-0)] to the model strategic bidding of generation companies (GENCOs) in a power market. GENCOs are modeled in a multi-agent framework to predict and adapt to Nash equilibrium of the electricity market. At the top level of the hierarchical multi-agent system, GENCOs compete with together to supply the demand by determining optimal bidding strategy functions, whereas at the bottom level, the ISO clears day-ahead market with receiving the bidding strategy functions. Simulation results show that the agents can predict the market equilibrium with acceptable errors.

A game theory framework is presented in [\[61](#page-21-0)] to analyze power transactions in a deregulated power market. The proposed model considers a noncooperative game to model two-player game with complete information. The Nash equilibriums of game obtained by the optimal bidding price and bidding generation. A new hybrid solution method is based on 2-dimensional graphical approach employed to obtain Nash equilibriums.

4.4 Evolutionary Game

The investigated challenges in power system problems utilizing evolutionary game theory have been demonstrated in Fig. [7](#page-14-0).

4.4.1 Decentralized Load Management of Plug-In Electric Vehicles

Decentralized load management of plug-in electric vehicles (PEV) is studied by employing an evolutionary game dynamics in [\[100](#page-22-0)]. The authors have proposed a scheme for PEVs working together to participate in ancillary services of the network. In this reference, evolutionary game dynamics is utilized for the interaction of PEVs, where multi-population scenario is introduced for the solution of problem. The proposed model for contribution of PEVs in the grid aims to shift the load and partially supply reactive load of the distribution transformer.

4.4.2 Power Transactions of Generating Entities in an Electricity Market

A study on generator bidding strategy is accomplished in [\[101](#page-22-0)] by using a novel hybrid evolutionary game theory and differential evolution (DE) optimization technique. This study aims to maximize the profit of generating companies (GENCOs) with incomplete information, where the evolutionary game is used to in the first stage to calculate Nash Equilibrium points, and DE method is applied for obtaining optimal bidding strategies. In other words, the proposed model in this reference provides unit commitment and bidding strategies for GENCOs as an optimization problem.

In [\[102](#page-22-0)], the bidding strategies of GENCOs with imperfect information is studied by employing an evolutionary game model. The capability of GENCOs in learning and modifying their estimation of rivals dynamically and retrospectively is considered in this reference. Accordingly, GENCOs can decide on the best bidding strategies for the next round. Such study proves that the capability of learnings of GENCOs dominates the

information scarcity in imperfect information condition of the game.

4.4.3 Demand Side Management and Demand Response

In [\[59](#page-20-0)], the authors have employed a two-level game theory method for studying demand side management with several utility companies. The proposed two-level method models the interaction between companies and residential consumers as non-cooperative and evolutionary games, respectively. Converge to the equilibriums for both games can be attained by utilization of such method, which is effective in reducing peak load and the variation of power demand.

A new demand-side management framework is introduced in [\[103](#page-22-0)] by implementing a game theory concept, where the study aims to minimize total cost of the smart grid. The individual communities decide whether to select between grid power and local power taking into account the strategies of neighbors. It is considered that a small share of the communities is subsidized, which is cooperative with the grid provider.

4.4.4 Supplying and Scheduling of Micro-grids Demand

An evolutionary game theory concept is introduced in [\[104](#page-22-0)] to accomplish micro-grid operation by distributed intelligent systems. The proposed model aims to solve the load sharing problem and dynamic maximization of the microgrid utility by the implementation of the evolutionary game. The system reliability maximization has been concentrated in this study for managing the time-dependence of variable conditions.

4.4.5 Waste Cooking Oil-to-Energy Supply Chain Policy

Waste cooking oil-to-energy supply chain policy is studied by an evolutionary game in $[105]$ $[105]$, where three parties are considered containing the government, biofuel enterprises, and restaurants. Information asymmetry is taken into account among the three players in this model.

5 Comparison of Game Theory Approaches with Different Points of Views

In this section, the application of game theory approaches to the solution of power system problems is compared with different points of view concentrating on researches studied recently. Accordingly the summery of implementation of game theory to power system problem considering the objective of studies, main contributions, case studies and the classification of game theory is prepared in Table [1.](#page-15-0)

The studied decision-making issues in the area of electric power systems by implementing different classifications of game theory approach is provided in Table [2.](#page-18-0)

6 Future Trends

In this section, future trends in the area of power system problems modeling based on game theoretic frameworks are provided. Fairness aspects between participation and non-participation in the decentralized fully-competitive energy trading system from different distributed generation sources can be investigated as one of future trend. Energy trading flexibility could also be extended to achieve socially optimal behavior of the systems.

References	Published year	Objective function	Main contribution	Game theory approach	Case study
[47]	2017	Minimization of the annual cost of building distributed heating energy network	Four benefit allocation schemes based on Shapely, the Nucleolus, DP equivalent method and Nash-Harsanyi game theory are employed to obtain the annual cost assignment among the building clusters	Cooperative game theory	A local area including three buildings located in Shanghai, China
$\left[50\right]$	2017	Profit maximization of distributed energy resources (DER) in electrical energy markets	A profit allocation framework is proposed to encourage each player to participate in coalition formation, where Shapley, Nucleolus, and Merge and Split are evaluated considering profit allocation viewpoint	Cooperative game theory	A power system containing four neighbor grids
[56]	2017	Adopting two low-pass filters for wind power generation and storage system	A game theory-based control strategy is proposed for coordinating and optimizing two low-pass filters to smooth the fluctuation of wind power output	Cooperative game theory	A 99 MW Chinese wind farm
[82]	2017	The optimal bidding strategy for a single strategic producer i is formulated as the bilevel stochastic optimization model	A stochastic EPEC model to obtain the equilibria of the electricity market with strategic wind power producers which consider also transmission constraints	Static game theory	A six-bus system with eight generators P1-P8 and four demands D1-D4, where P2 and P4 are wind power units and the rest are conventional units
[88]	2017	Profit maximization of electricity supplier as the leader and the operator as the follower	A novel crowdfunding financing method for EV charging infrastructures is proposed considering risk preference. The interaction between electricity suppliers, infrastructure operators, and crowd funders is formulated into a Stackelberg game	Dynamic game theory	A 40 kW fast electric vehicle charging pile
[89]	2017	Profit maximization of each DESs as the leader and each energy users as the follower	The strategic substitutive behaviors of distributed energy stations in The multiple energies trading problems is modeled A multi- leader multi-follower (MLMF) Stackelberg game model	Hierarchical dynamic game theory	An integrated energy system (IES) with 4 distributed energy stations and 8 energy users
$[93]$	2017	Maximizing economic benefit of both community energy storage operator and users in the fully-competitive framework, simultaneously	A non-cooperative Stackelberg game is proposed to model fully-competition between CES operator and users	Hierarchical dynamic game theory	A residential community of 40 people with 30, 40, and 50% participating users
$[100]$	2017	Increasing payoff functions of the utility grid and the PEV owners, simultaneously	An evolutionary Game Dynamics called Escort Dynamics is proposed to the decentralized load management of Plug-in Electric Vehicles	Evolutionary game theory	6 PEVs as players under conditions: connection time at 05 h, departure time at 17 h, the battery capacity of 20kWh

Table 1 Objective functions and the main contributions of studies in the area of application of game theory to power system problems

Table 1 (continued)

Table 1 (continued)

Table 1 (continued)								
References	Published year	Objective function	Main contribution	Game theory approach	Case study			
[92]	2015	Maximizing the payoff for all participating microgrids at the Stackelberg equilibrium.	Multiple interconnected microgrids competition to energy trading is modeled by a multi-leader multi-follower Stackelberg game	A two-level continuous- kernel Stackelberg game	A multiply interconnected microgrids			

Table 2 Classification of the investigated issues in power system problems considering game theory application

Interaction of PEVs together in power system to balance the active power load is another subject that requires more research in a fair scheme. Additionally, the interaction of PEVs with dedicated energy storage devices and renewable energy sources can also be defined as one of future trends. Considering reviewed papers in this paper, optimal strategic bidding of GENCOs in energy and ancillary services markets based on the double sided auctions with mixed strategies is not studied and can be one of future trend. Future studies can also include game

based modeling of the impact of renewable generation sources on DSM considering dynamic pricing technique and investigation of the impact of the dynamic pricing on short-term price elasticity in smart grids studies. Testing the dynamic pricing technique on different satisfaction functions, with multiple user types can be analyzed in such research area. Energy consumption control in a network consisting of electricity users can be counted as another future trend. Analysis of the energy consumption game with the existence of cheaters included the detection of cheaters, the design of penalty or reward algorithms to prevent cheating behaviors are issues which have not been discussed, yet. Another future trend can be concentrated on the application of evolutionary game to solve the demand-side management problem in a smart grid. Adaptive control can be considered to model uncertain/time-varying status in the defined cost functions. A dynamic game based demand response models between one utility company and multiple users is studied in a few number of research papers. As a future trend, renewable power resources (e.g., wind turbines and photovoltaic cells) may be taken into account to make the existing model accommodate dynamic ambient changes.

7 Conclusion

Game theory approach is defined as an effective concept for handling decision making processes of different sciences such as economics, computer science, biology, logic science, political science, and psychology. Such concept is successfully implemented in recent studies for dealing with electric power system problems. In this paper, a survey on the application of game theory approach for decision makings associated with electric power systems is provided. At first, the basic foundation and usage of game theory to the above-mentioned sciences is introduced. Then, the basic concepts of game theory for cooperative/non-cooperative, symmetric/asymmetric, zero-sum/non-zero-sum, simultaneous/sequential, perfect information and the imperfect information is introduced. Moreover, different classifications of game theory approach consisting of strategic game, dynamic game, cooperative game, evolutionary game are studied. Also, the employment of various types of such concept for the accomplishment of decision-making process in electric power system problems are analyzed and discussed in details. The contribution of this paper in the area of application of game theory to power system problems can effectively help the researchers in the area of decision making in such systems.

Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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