Archives of Computational Methods in Engineering (2020) 27:135–170
https://doi.org/10.1007/s11831-018-09303-x

ORIGINAL PAPER

Steady and Transient State Analyses on Conjugate Laminar Forced Convection Heat Transfer

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Received: 24 April 2018 / Accepted: 10 December 2018 / Published online: 1 January 2019

Abstract
The term 'conjugate heat transfer' refers to a heat transfer process involving an interaction of heat conduction within a solid body with either of the free, forced, and mixed convection from its surface to a fluid flowing over it. It finds application in numerous fields starting from thermal interaction between surrounding air and fins to thermal interaction between flowing fluid and turbine blades. In this article, a systematic literature review of studies pertinent to laminar conjugate conduction-forced convection heat transfer analysis subjected to internal and external flow conditions is performed. The review reports both steady and unsteady state analyses related to experimental, analytical and numerical investigations, in both rectangular and cylindrical geometries with an exemption to micro and mini channel related studies. The studies are categorically put forth initially and an overview of these studies is presented in tabular and graphical form for a swift glance later under each section. This paper is concluded highlighting the salient features of the review, with respect to physical and mathematical models, methodology and applications. The challenges and scope for future study reported at the end of this paper gives the reader an insight into the gaps in the area of conjugate heat transfer analysis of steady and transient state under laminar forced convection flow regimes.

List of Symbols

Notation

A	Thermal diffusivity ratio
ADI	Alternating direction implicit
Ar	Aspect ratio
B	Buoyancy parameter, angular frequency, Biot number
Bi	Biot number
Be	Dimensionless Bejan number
Br	Brun number
CHT	Conjugate heat transfer
C_R	Thermal capacity ratio
CFD	Computational fluid dynamics
CLEARER	Coupled and linked equations algorithm revised—ER
E	Elastic number
Ec	Viscous dissipation parameter
FDM	Finite difference method
FEM	Finite element method
FIDAP	Fluid dynamics analysis package
FVM	Finite volume method
GITT	Generalized Integral Transform Technique
H	Height
h	Heat transfer coefficient
k	Thermal conductivity
L, l	Length
M	Mach number
Ncc	Conduction-convection parameter or thermal conductivity ratio (solid to fluid)
Nu	Nusselt number
Nu^+	Nusselt number ratio
Nu_Z	Local Nusselt number
Pe	Peclet number
Pr	Prandtl number
Q_t	Total heat generation parameter
q^+	Heat flux ratio
Re, Re_H	Reynolds number
Ri	Richardson number
SIMPLE	Semi-implicit method for pressure linked equations

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SIMPLER	Semi-implicit method for pressure linked equations revised
T_∞	Ambient temperature
T_w	Wall temperature
UFED	Uniform flow effective diffusivity
u	Axial velocity
u_∞	Free stream velocity
v	Transverse velocity

Subscript

max	Maximum
s	Solid
f	Fluid
T	Thickness
w	Wall
t	Total, time, thermalization
∞	Free stream
R	Ratio
sat	Saturation
min	Minimum

Greek Symbols

ϕ	Conductance parameter
ξ	Channel length
ζ, α	Conjugate parameter
δT	Thermal boundary layer thickness
θ_{max}	Dimensionless maximum temperature
ϕ	Porosity
δ	Boundary layer thickness
θ	Dimensionless temperature
β	Radius ratio
η_{tt}	Thermalization time

1 Introduction

Heat transfer study is of major importance in all fields of engineering, research and industry. Consider the heat flow condition in edge emitting semiconductor laser chip. This laser chip, which is used in electronic boards is mounted on a heat sink through which heat is transferred to the surrounding air or liquid coolant to maintain its temperature [1–3]. This shows the involvement of heat transfer in electronics engineering and hence demands to perform thorough heat transfer analysis to obtain better thermal performance of the electronic boards. Similarly consider the heat transfer process in laser heating and melting used for modifying the surface of different materials. During the laser irradiation of metals for femtoseconds, demands accountability of heat transfer which occurs due to non-equilibrium between lattice and electron during initial laser heating duration [3–5]. Various essential areas exists which show the association of heat transfer and its effects on the performance of the related devices, as mentioned below:

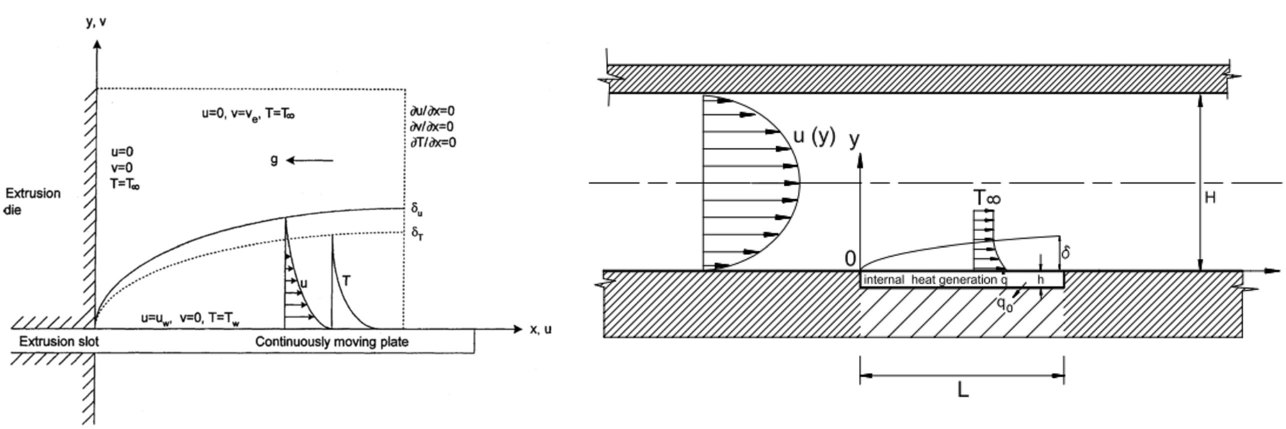
- Heat transfer behavior on the surface of materials during the modification and formation of optical patterns with plasma flux processing [6–8].
- Steered fiber technology used in composite lamination lead to change in thermal behavior of the composites [9–11].
- For efficient performance of rubber, a rubber curing process is used in which non-uniform distribution of temperature arises [10, 12, 13].
- Heat transfer analysis of power cables used in power transmission and network distribution to estimate current carrying capacity of cables [14–18].
- In underground mining, the usage of mine hoist with friction lining and disc brakes develop very high friction on hoist and lining at the time of emergency which generates huge amount of heat leading to affect the tribological properties of the hoist, lining and brake materials [19–21].
- Heat transfer in porous medium effects the thermal performance of devices like biomedical implant, heat shield devices for space vehicles, compact heat exchangers and heat sinks [22–28].
- Heat transfer behavior in design and development of superheater and economizer for better power production in thermal power station [29–31].
- Heat transfer phenomenon considered in design and operation of internal combustion engines [32, 33].
- Heat transfer during the processing of ultra-high strength steel. During processing stage, the steel is subjected to tempering and quenching operation in which the accurate tempering temperature and cooling temperature are considered for improving the mechanical strength of the steel [34–36].
- Air cooling module used in consumer electronic devices such as personal computer, mobile phone, graphic processing unit, central processing unit, electric lamp etc. make use of convection mode of heat transfer. Most of the electronic equipment's casing design is based on natural convection phenomenon for its better thermal performance and improved life [26, 37–39].
- Complex heat transfer phenomenon in nuclear power plants for thermal design of reactor core which plays very crucial role in the power generation [40–42], [43, 44].
- Human comfort has a direct link with heat transfer phenomena. Air-conditioners, water coolers, fans etc. all involve exchange of heat affecting the human comfort [45–47].
- In turbines, the entire physics involved is in heat transfer between the fluids and solids [48–51].

All these application areas mentioned above demand proper analysis of heat transfer phenomena, which are few in number. However, one can hardly provide a comprehensive list of areas where heat transfer application and its study is required, as the scope is so wide. Abundant literature is available related to study of heat transfer in journal articles, electronic notes, books etc., and is being presented since many centuries. In short to say, whenever a temperature difference exists there occurs heat transfer. Heat transfer due to temperature difference occurs mainly due to three mechanisms viz., conduction, convection and radiation. These mechanisms can either happen separately or coupled [52–56]. Coupled heat conduction in solids and heat convection in fluids is the subject of major interest of this article.

Conjugate heat transfer (CHT) is the phenomena of combined heat transfer in solids by conduction and heat transfer in fluids by convection. Heat transfer by conduction and convection is purely due to thermal interaction of solid and fluid domain. This thermal interaction occurs due to conduction of heat in solid part and convection from its surface by any form i.e. free, mixed or forced to the surrounding fluid [57–60]. In many cases, heat transfer also occurs from the fluid by convection to the solid domain. The heat flux and temperature field are continuous at the solid-fluid interface. The temperature of fluid in motion close to the solid can vary rapidly. Near the interface the temperature of fluid and solid are very close whereas at the far distance from the interface the temperature of fluid is either close to ambient or inlet fluid temperature as shown in Fig. 1a. Hence, in CHT analysis the heat conduction in solid is coupled with convection heat transfer in fluid domain by satisfying the prime boundary condition of continuity of heat flux and temperature field at the solid-fluid interface. Forced convection is the common method to get a high rate of heat transfer from a body or to a body which represents flow of energy in both situations. In forced convection, the flow over solid body can be either laminar or turbulent [61–66].

The coupled/combined/conjugate heat transfer in solids and fluids occur in majority of practical and engineering applications. Wherever the heating/cooling mechanism between solid and fluid are observed, their CHT study plays a key role in effectively analyzing and designing of the devices. Various application areas where heat transfer in solids and fluids are combined are listed below [67–76]:

- Fins
- Heat sinks
- Coolers
- Heaters
- Heat exchangers
- Rocket nozzles
- Re-entry vehicles
- Heat shields
- Electronic devices
- Electronic packaging's
- Chemical devices
- Nuclear energy devices
- Turbines blades
- Solar installations
- Ventilation systems
- Drying and food processing chambers
- Thermal goods
- Wood and brick industries
- Twin-screw extruders
- Continuous wire casting
- Optical fiber coating devices
- Building ventilation systems
- Blast furnace device



(a) Flow over continuously moving plate [90] **(b)** Flow in a internally heat generating parallel plate [91]

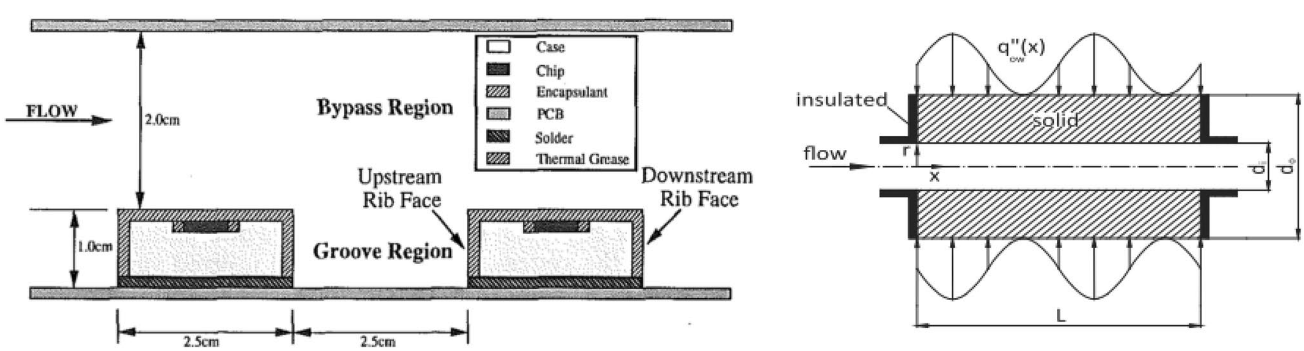
Fig. 1 Different geometries and boundary conditions used in CHT analysis

From last few decades this field of heat transfer has received a great importance in various areas some of which are mentioned above. The best common example for CHT is series of fins, which is attached to a surface and is cooled by convective flow of fluid [77–79]. Another widely analyzed application area is printed circuit boards (PCB) generating electric heat (heat source or obstacles) as shown in Fig. 2a. And these electronic equipment's are cooled in a passage by dissipating the heat from the chips and other surrounding devices in order to make them workable within safe limits. The thorough investigation of thermal performance during combined conduction and convection situations in devices/ components is crucially important in proper design and in obtaining highly efficient devices [80–89].

CHT analysis is reported in literature for different geometries like flat plate, plate fin, cylinders, infinite and semi-infinite plate, parallel plate channel, stack of parallel plates, tubes and thick walled pipes. Heat generation considered is of uniform, non-uniform, periodical/ sinusoidal type. In Fig. 1b parallel plate channel with internal heat generation along one side and fluid flowing internally is demonstrated. Periodically generating heat on the outer surface of a parallel plate channel is shown in Fig. 2b. Different analytical and numerical methods are used to solve the coupled heat transfer equations satisfying the boundary condition of continuity of heat flux and temperature at the solid-fluid interface. Steady and transient heat transfer analysis are reported by many investigators. This review article mainly focuses on the studies carried out on CHT processes under laminar forced convection condition. Review of research works on CHT analysis under laminar forced convection with all types of circumstances investigated since 1961 is presented systematically.

Research analysis on CHT related to minichannels, microchannels, free/mixed and turbulent forced convection are not included in this article. The CHT analysis done by investigators, reviewed in this article under laminar forced convection condition have all satisfied the prime boundary condition of continuity of heat flux and temperature at the solid-fluid interface. This article provides a strong overview of different types of analysis carried out using various methods in CHT in laminar forced convection. A full-scale and meticulous literature study was performed for proper division of works reported, to get better understanding of CHT analysis along with parameters considered by the researchers. Details provided in tabular form in each section give an outright idea of CHT problems addressed in a quick view. Challenges faced in CHT analysis are judiciously predicted and relevant future directions to be carried out are furnished.

The rest of the article is organized as follows. In Sect. 2, analytical studies done on CHT analysis in steady and transient state are discussed. Section 3 gives a detailed view of numerical works carried on all types of CHT situations. Only few experimental investigations are reported in literature which are reviewed and presented in Sect. 4. Further, in all these sections, systematic subsections are provided for steady state and transient state studies which are again classified according to type of flow i.e. internal and external flow. In internal and external flows, work carried on flat plates, parallel plates, tubes/pipes, channels are reviewed. In each section, Tables and Figures are provided to give an overview of the study and understanding of analysis carried out. Finally in Sect. 5, conclusion of entire literature review, challenges existing and faced by researchers in obtaining solution for CHT analysis in laminar forced convection condition and future directions to carry out further analysis, gaps are provided.



(a) Flow in an electronic channel having discrete heat source [92]

(b) Flow inside a channel with periodically carrying heat flux at outer surface [93]

Fig. 2 Heat generating sources in a channel and externally applied heat flux outside the channel surface

2 Conjugate Heat Transfer Analyses Using Analytical Method

In this section the CHT problems solved using various analytical methods are reviewed. Both steady and unsteady problems covering internal and external flows are presented. The study includes geometric details of the problem, dimension and the method used to solve both the solid and fluid flow domain governing equations. All studies related to, from flow over a flat plate to flow over complex geometries, channels, pipes etc. are covered in this review. A separate subsection is provided which covers investigations carried by James Sucec.

2.1 Investigations on Conjugate Heat Transfer Analysis of Steady State Process

2.1.1 Steady State Processes with External Fluid Flow

Perelman analytically studied the conjugate heat transfer (CHT) problem pertaining to 2-D laminar flow over thin plate with heat source. The governing equations for the heat generating plate and boundary layer equations of the flow were solved using the asymptotic solution of integral equation (Cauchy Kernel). The solid-fluid interface boundary condition of continuity of temperature and heat flux was satisfied. In his study he presented closed-form expressions for surface temperature and local Nusselt number (*Nu*) [94]. Luikov et al., presented the analytical solution for internal and external conjugate convective heat transfer problems. A general method of solution based on series of expansion with respect to powers of parameter of generalized Fourier transformation of temperature profile was discussed in detail. From their analysis they proposed a new criterion to characterize the wall thermophysical properties effect on heat transfer [95]. Analytical study of conjugate problem in laminar incompressible flow for convective heat transfer around a flat plate of finite thickness was also done by Luikov. He suggested the design formulas for local *Nu* for the considered study. The solutions were obtained from differential and integral relation of boundary layer [96].

Payvar solved the problem of heat transfer of laminar incompressible flow over a flat plate of finite thickness with lower surface held at constant temperature by using Light hill's method. The solution shows that the temperature of upper surface is found to depend only on Local *Br* and independent of *Pr* [97]. Pozzi and Lupo analytically studied the dynamic field of thermo-fluid obtained by coupling of conduction within a heated plate and forced laminar convection using two expansions. The asymptotic expansions includes Eigen solutions and the other one is expansion describing fluid field at the leading edge of the plate. They studied the Mach number (*M*) influence on the temperature of interface, velocity field and *Nu*. Using these expansions results were obtained utilizing the Pade approximation technique. Velocity profiles were obtained using Stewartson-Doronitzin transformation after estimating temperature distribution [98]. Ma et al., used Fourier series method to inspect a rectangular fin of two dimension with variable heat transfer coefficient (random) on the fin surface. The analytical solution has been described for the basic governing equation with heat dissipation by convection from the tip of the fin to the surrounding. Solution for the other two boundaries of the fin tip in which the tip temperature is equal to that of surrounding and the tip is insulated is derived. They found 8.7% difference in temperature of fin base predicted by one dimensional and two dimensional analysis [99].

Rizk et al., established an analytical method of solution for CHT analysis of laminar and steady flow in a two dimensional rectangular block. Forced convection was subjected on the top of block, at the sides having unlike thermal boundary conditions and considered the bottom of the block as adiabatic wall condition. On the upper surface of block, heat flux distribution and temperature profiles were found to be affected by *Pe*, condition of thermal boundary and conductivity ratio [100]. Prasad and Sarkar carried out an analysis of conjugate laminar forced convection incompressible flow past a flat plate of finite thickness with and without externally imposed pressure gradient, while the bottom surface of the plate was maintained at a uniform temperature. The results are presented in the form of a relationship between the conjugate *Nu* and the *Ncc* between the thermal conductivities of the solid and the fluid, plate characteristics and *Re* [101]. Mossad analyzed coupled conduction and convection heat transfer (CHT) analysis inside a flat plate and of laminar forced flow of fluid respectively. Using the integral technique the convective laminar boundary layer was analyzed. The energy equations for the fluid and plate wall were combined under the condition of continuity in the temperature and heat flux at the fluid-solid interface. His results indicated that for $Br > 0.15$ the problem should be considered as conjugate as the error in local *Nu* obtained were more than 5% (refer Fig. 3a). Results obtained for change in *Br* numbers were in good agreement with previous results by Payvar's solution [97] as shown in Fig. 3b [102].

Harman and Cole presented a CHT analysis applied to the electronic cooling problems. An air cooled electronic device (simple model) was considered for study mounted over a circuit board. To represent the simplified model it comprised of a flush mounted heater, to represent the cooling fluid a steady shear flow was considered and for the circuit board a two layer substrate was included. Fourier transformation

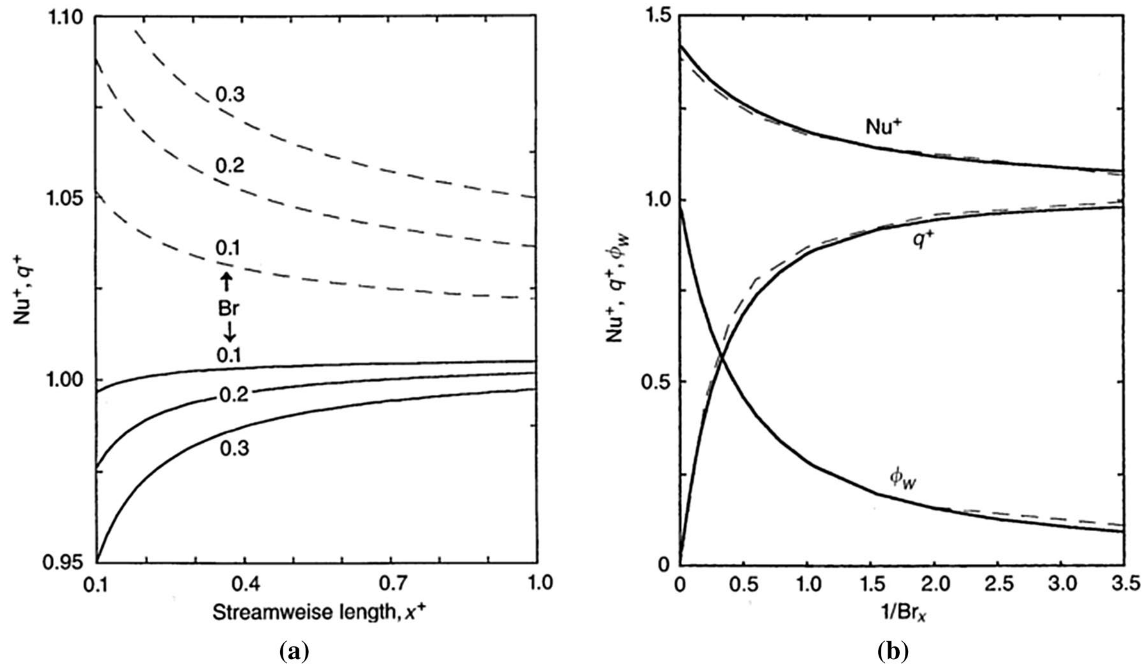


Fig. 3 **a** Nusselt number ratio Nu^+ (dashed lines) and heat flux ratio q^+ (continuous lines) variations for different average Br, **b** Results of Mosaad (continuous lines) in agreement with Payvar's solution (dashed lines) [102]

was used for reducing non-dimensional governing equations into ordinary differential equations and the real space solution are given as integral with the inverse Fourier transform. For the two layer substrate model many numerical results were presented for various values of flow parameter like conjugate Peclet number $((k_f/k_s)Pe^{1/3}$, substrate thickness and velocity gradient [103]. Stein et al., developed an asymptotic solution for CHT problem of a flush mounted heat source placed at the interface of solid and fluid with insulated bottom part of the solid. The results obtained were in agreement with numerical results [104]. Lindstedt and Karvinen did an analytical study of CHT in plate fin with uniform heat flux at base. Validation of results were done with experimental, numerical and existing analytical methods. Fin temperate obtained by conventional fin problem lead to 20% low compared to the proposed uniform heat flux condition as proved by the authors. The analysis was done for natural and all types of flow conditions [105]. In Table 1 overview of analytical works carried out related to steady state external flows are mentioned. All works cited

Table 1 Few details of steady state analysis carried analytically for external flows

Dimension (D)	Geometry	Orientation	Type of thermal load	References
2D	Rectangular	Horizontal	Internal heat source of plate	[94]
2D	Plane, circular tubes and flat plate	Horizontal	Constant temperature of solid	[95]
2D	Plate	Horizontal	Internal heat source of plate	[96]
2D	Flat plate	Horizontal	Constant temperature of plate	[97]
2D	Flat plate	Horizontal	Constant temperature of plate	[98]
2D	Rectangular fin	Horizontal	Constant temperature of fin	[99]
2D	Rectangular block	Horizontal	Uniform temperature at one side of block	[100]
2D	Flat plate	Horizontal	Constant temperature of plate	[101]
2D	Rectangular fin	Horizontal	Constant temperature of plate	[102]
2D	Rectangular plate	Horizontal	Constant heat flux of plate	[103]
2D	Plate	Horizontal	Heat generation of heat source	[104]
2D	Plate fin	Vertical	Uniform heat flux at base	[105]

are related to two dimensional and external flow, type of heat input and geometry of solid domain with reference are provided in below Table 1.

2.1.2 Internal Flow in a Channel or Pipe

Faghri and Sparrow investigated laminar flow in a pipe whose upstream is insulated and downstream is heated uniformly. Wall conductance parameter and *Pe* (5–50) were the dimensionless parameter considered. The tube upstream, due to insulation helps in convection from fluid and causes preheating. They also found that the wall conduction effect can overcome the effect of conduction in fluid [106]. Wijeysundera studied conjugate study in circular and flat ducts under laminar forced convection. Solution of the isothermal boundary condition was used to determine the eigenvalues of the problem. The wall conductance parameter was found to influence fluid and wall temperatures, whereas the enhancement in heat flux by conduction in wall was found to be huge nearer to the duct inlet. The change in *Nu* for change in *Bi* and conductance parameter was found to be significant as shown in Fig. 4 [107]. Thermal analysis of electronic component cooled by forced air was carried out by Zebib and WO. The problem was considered to be a conjugate two dimensional heat transfer problem. After calculating velocity field in the channel, energy equation was solved considering non-uniformities in thermal conductivity. They presented the maximum temperature variation along with air velocity in the duct considered [108].

Luna et al., studied steady state conjugated heat transfer process in the entrance thermal region in a circular tube. The power law fluid considered was flowing by forced convection and the governing equations were solved analytically using integral boundary layer approximation. Heat generation term by axial heat conduction in the fluid and by viscous dissipation was neglected. The solution for solid is coupled with the Laplace equation considering heat conduction in axial direction. Results were presented for different *Ar*, conduction parameter and power law index of fluid [109]. Nield introduced a Nusselt number *Nu* suitable for forced convection in parallel plate heated asymmetrically. This introduced *Nu* was verified for different velocity profiles and for either uniform temperature or heat flux boundary conditions. It was observed that if the velocity profiles are symmetric, then this *Nu* is independent of asymmetry [110]. To use Galerkin method, in order to solve the CHT related problems Horvat and Mavko presented their research work carried analytically. The solid phase of conjugate problem has volumetric heat generation. They employed volume averaging technique to obtain simulation of flow of water across internally heat generating aluminium rods as shown in Fig. 5. Figure 6 shows that the results of variation in *Nu* for change in *Re* obtained by Galerkin approach is suitable to present the considered conjugate problem which is found to be in complete agreement with finite volume method. Even though the method is fast, it is also able to predict in detail the fluid flow and solid temperature fields [111].

Bautista and Mendez studied analytically CHT in channel embedded with a heat source and fluid with fully developed laminar flow. The governing equations for the flow were reduced to integro-differential equation which were used to estimate the temperature changes in the heat source [91]. Another analytical study on channel with outer surface subjected to sinusoidal temperature variation is reported by Barletta in [112]. Bula and Vasquez studied CHT process of laminar fluid flow over heated

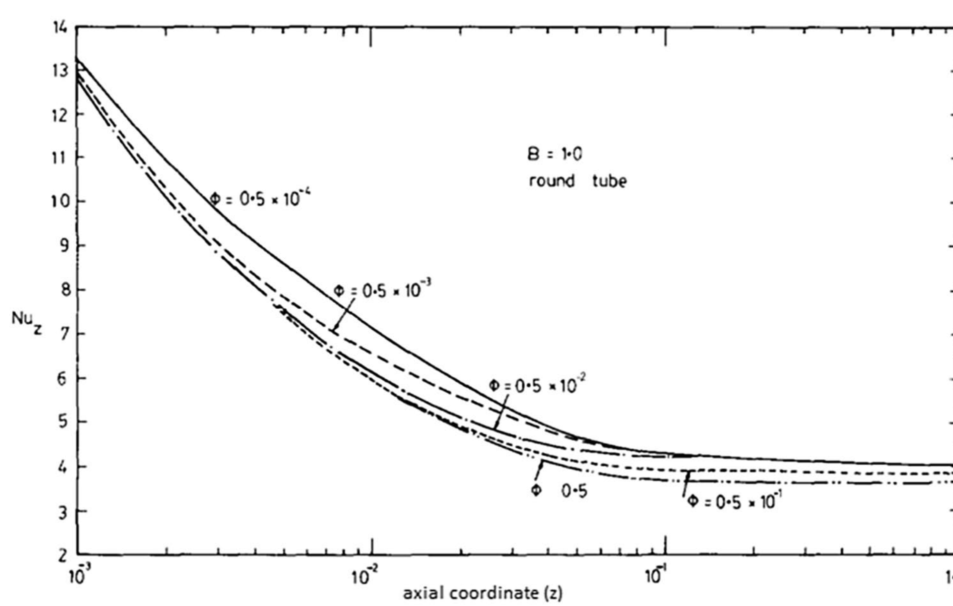


Fig. 4 Variation in local Nusselt number (Nu_Z) for different conductance parameter (ϕ) at Biot number (B) = 1.0 [107]

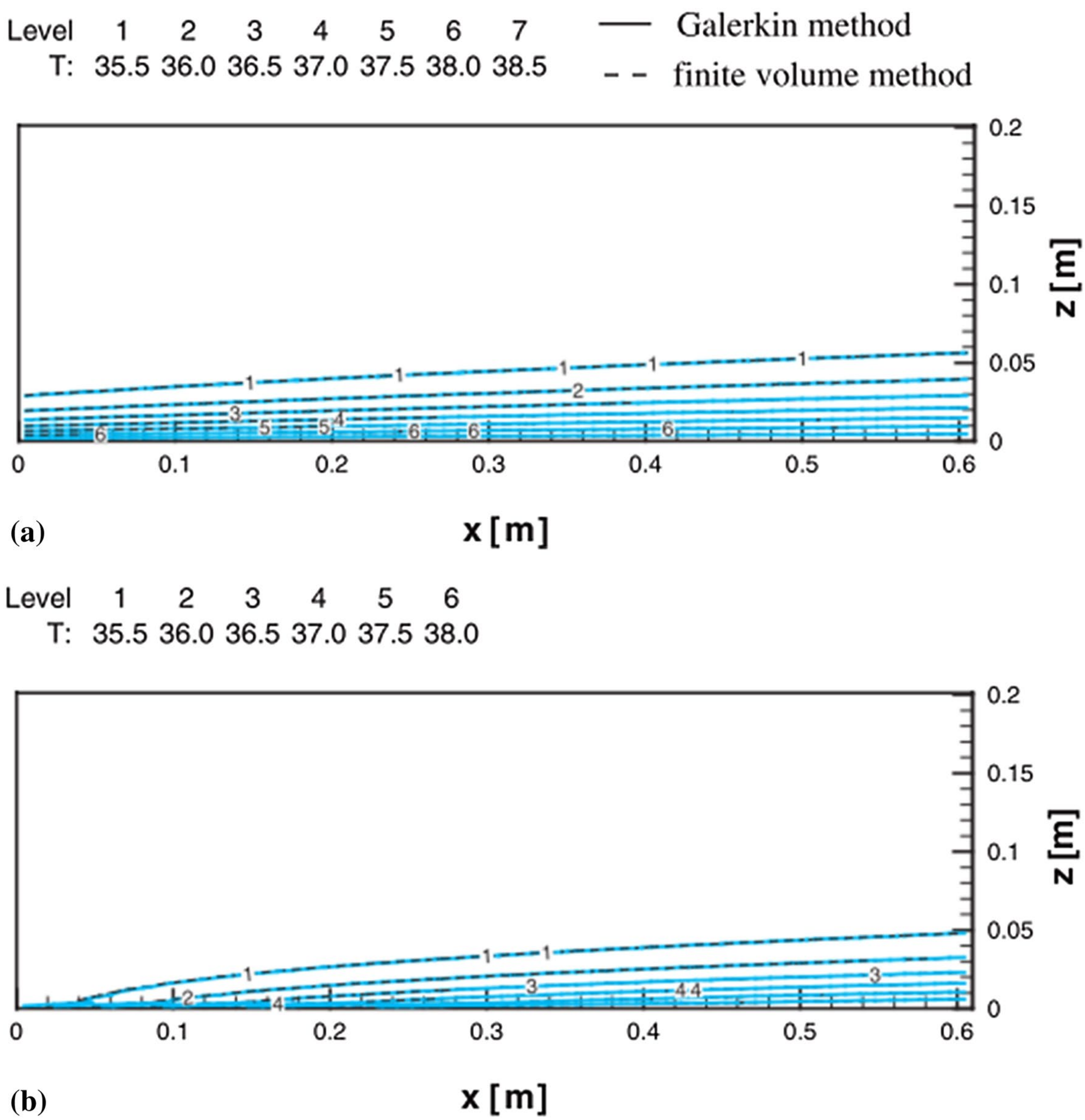


Fig. 5 Prediction of temperature fields using volume averaging technique and comparison with FVM in **a** solid domain and **b** fluid domain [111]

plate channel with laminar fluid flow. Using Chebyshev polynomial approximation the interface temperature was obtained and then they compared the obtained results with FIDAP commercial CFD code for verification. Local and average *h*, *Nu*, *Bi* and interface temperature distributions were estimated. The polynomial was used to uncouple the conjugate condition at the interface [113]. In Table 2 overview of analytical works carried out related to steady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

2.2 Transient State Conjugate Heat Transfer Analysis

2.2.1 Analytical Work Carried by Sucec for Transient State Situations

James Sucec was the first to conduct conjugate heat transfer analysis analytically using Laplace transformation for transient temperature and heat flux distribution in the fluid moving over a plate cooled from below. The temperature of the fluid was varied arbitrarily with time and CHT conditions were coupled with the plate [114]. Sucec conducted another analytical study on transient CHT between fluid and plate.

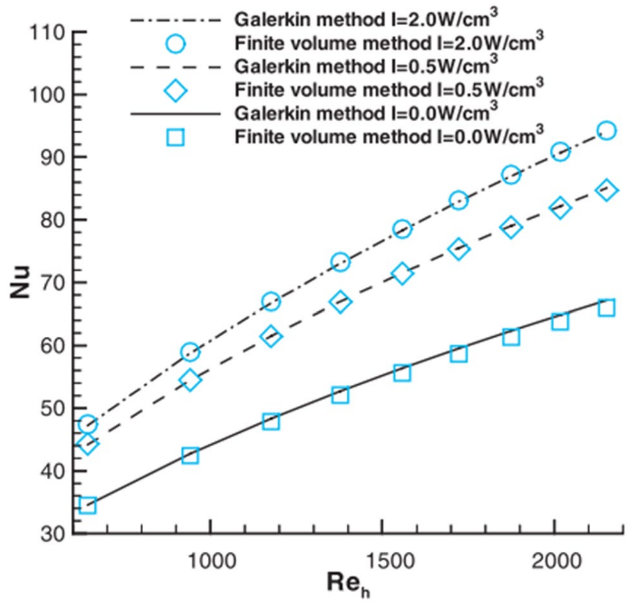


Fig. 6 *Nu* variation for different heat flux as a function of *Re* and comparison with FVM [111]

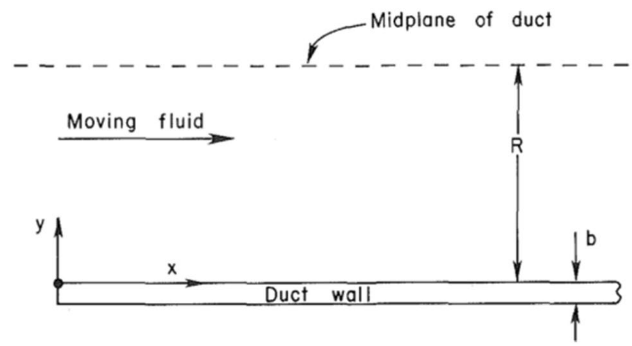


Fig. 7 Lower half of symmetric parallel plate (*b* is thickness of duct wall and *R* is duct half height) [118]

In this study the fluid temperature was varied sinusoidally with time and the plate interacting with fluid was insulated at the base. The fluid was moved in a steady slug fashion and its temperature was varied far from the plate. Method of complex temperature was used in association with Laplace transformation to find an exact solution for this problem. He also described a method to find the conditions (qualitative) to validate the quasi-steady analysis [115]. In his next work, he presented a better quasi-steady method which considers the thermal history effects and thermal storage capacity of the flowing fluid. Using transformation of variables improvement in quasi-steady method was achieved that gives an exact solution for both transient slug and non-slug flows. The results were also compared with benchmark problems of finite difference method [116]. His further study along with Ajay Sawant, was again analytical, which dealt with the unsteady conjugated heat transfer problem of parallel plate duct (refer Fig. 7) with periodically varying inlet fluid temperature. The fully developed flow characteristics, velocity profile of the improved quasi-steady approach considers the effect of finite thermal capacity and thermal history of fluid [117]. Using Laplace transformation, Sucec again established an exact analytical solution pertaining to thermal entrance region in a parallel plate duct considering conjugate transient heat transfer process. Unsteadiness was induced due to rapid change of fluid temperature. Here also comparison was made with FDM solution and established standard quasi-steady approach [118]. In Table 3 overview of analytical works carried out related to unsteady state internal and external flows are mentioned. All works cited are related to two dimensional and by Sucec. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

He finally presented a heat transfer solution for sinusoidal axial heat generation in the parallel plate duct wall in which fluid was considered flowing. The primary solution method

Table 2 Few details of steady state analysis carried analytically for internal flows

Dimension (D)	Geometry	Orientation	Type of thermal load	Optimal spacing study	References
2D	Pipe	Horizontal	Uniform temperature of downstream portion	✗	[106]
2D	Circular and flat ducts	Horizontal	Internal heat generation at the walls	✗	[107]
2D	Duct	Horizontal	Heat generation in chip	✗	[108]
2D	Circular tube	Horizontal	Uniform flux on tube external surface	✗	[109]
2D	Parallel plate	Horizontal	uniform heat flux or temperature of channel	✗	[110]
2D	Nuclear rods	Vertical	Uniform heat generation	✗	[111]
2D	Channel with heat source embedded	Horizontal	Uniform volumetric heat generation by the chip	✗	[91]
2D	Parallel plane channel	Horizontal	Constant heat flux on plate	✗	[113]

Table 3 Few details of unsteady state analysis carried analytically by Sucec

Dimension (D)	Geometry	Orientation	Type of flow	Type of thermal load	Optimal spacing study	References
2D	Flat plate	Horizontal	External	Temperature variation with time of fluid	–	[114]
2D	Parallel plates	Horizontal	Internal	Sinusoidal variation of temperature of fluid	✗	[115]
2D	Parallel plates	Horizontal	Internal	Step change and sinusoidal variation of temperature of fluid	✗	[116]
2D	Parallel plates	Horizontal	Internal	Sinusoidal variation of temperature of fluid	✗	[117]
2D	Parallel plates	Horizontal	Internal	Step change of temperature of fluid	✗	[118]
2D	Parallel plates	Horizontal	Internal	Sinusoidal internal heat generation of duct wall	✗	[119]

was the finite difference solution of partial differential energy equation for the flowing fluid and the duct wall material. He also solved the problem using quasi-steady method and two integral approximate method. The finite difference solutions were compared with the results of quasi-steady method which is shown in Fig. 8, and also to a two integral approximate surface heat flux expression. The overall conclusion that was drawn from his work on CHT analysis was as follows [119]:

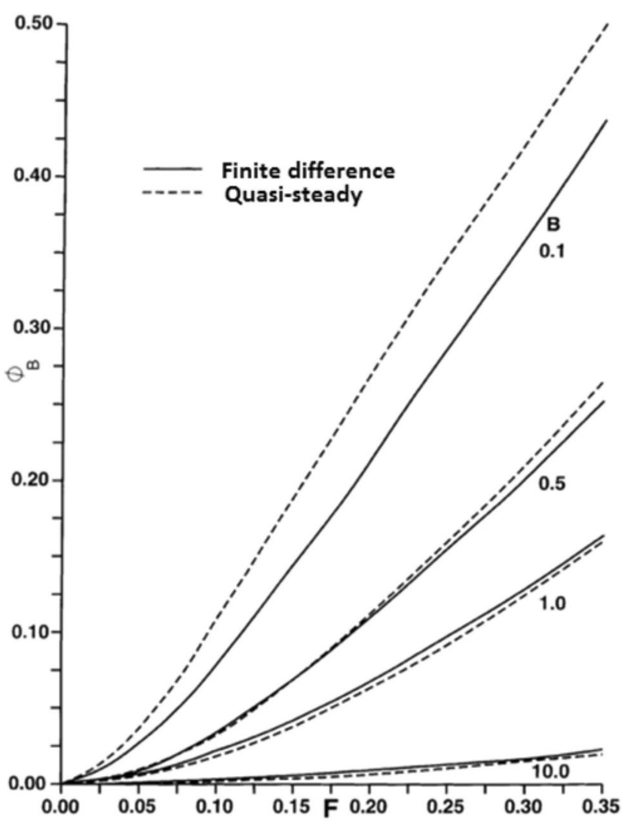


Fig. 8 Fluid temperature variation (ϕ_B) for different locations and different time (F) at different bulk mean fluid value (B) predicted using FDM and quasi-steady method [119]

- The quasi-steady method gives enough accurate result for the duct wall temperature predictions. But this is not applicable for the prediction of bulk mean temperature of fluid.
- The two integral expressions give significant accurate results for surface heat flux and found to be in line with FDM results.
- The flux model gives good accuracy related to time domain and the same is true with quasi-steady method.
- *Nu* was seen to be lightly reliant upon thermal capacity ratio whereas strongly dependent on sudden time change and heat generation.

2.2.2 Analyses of Transient State Process of External Flow

Trevino and Linan used Perturbation method to study the steady and transient process of the external heating of flat plate under a convective flow condition with axial heat conduction through the plate. They analyzed the boundary layers structure and showed the presence of two boundary layers at both edges of considered flat plate [120]. Pozzi and Tognaccini presented an analysis of the conjugate heat transfer in a fluid field flowing over semi-infinite plate. They considered the fluid to be impulsively accelerated taking the thickness effect of the plate along with its thermal conductivity. For solid-fluid thermal coupling and boundary layer, integral formulation was used. The equations were integrated in the form of quasi-analytical by using the Lagrange's method. They found that the energy and velocity field depends on time in an asymptotic region and it is Rayleigh type [121]. Luna et al., investigated CHT of fully developed flow of power law fluids in circular ducts. Quasi-steady approximations were applied for the fluid and the integral boundary layer technique to solve energy equation of fluid coupled with axial conduction in solid. Conduction parameter, *Ar* and power law index were the parameters varied to obtain average temperature variations [122]. Meng et al., developed a new algorithm for long term CHT problem of transient quasi-steady flow. This algorithm increased the efficiency of determining frequency of flow filed computationally [123].

Table 4 Few details of unsteady state analysis carried analytically for external flows

Dimension (D)	Geometry	Orientation	Type of thermal load	References
2D	Flat plate	Horizontal	Constant heat flux on plate	[120]
2D	Semi-infinite flat plate	Horizontal	Constant temperature of plate	[121]
2D	Pipe	Horizontal	Uniform heat flux on tube	[122]

In Table 4 overview of analytical works carried out related to unsteady state external flows are mentioned. All works cited are related to two dimensional and external flows.

2.2.3 Conjugate Heat Transfer Problems of Internal Flow

Karvinen presented quasi-static method, an analytical approach for transient heat transfer in a tube or channel by assuming fully developed velocity profile. Effect of varying wall temperature was taken into account and the fluid heat capacity effect was ignored. The results obtained from this analytical method were verified by comparing with the previous results and the method was also found to be very simple as well [124]. Santos et al., provided an approximate solution by analytical method for thermal entrance region. Transient forced and laminar convection in the parallel plate duct and the interaction of ambient medium (outer of the duct) with the walls were considered for the study. Laplace transform was employed for the periodic solutions and results were discussed for duct wall temperature, heat flux of the wall, bulk temperature of fluid along the axial position. Influence of thermal capacity of wall and external thermal resistance were studied in detail. In the thermal entrance region, damping of amplitudes was greatly influenced by thermal capacity of wall when the external thermal resistance is high. Whereas for low external thermal resistance, no effect was seen on wave propagation speed in the entrance region [125].

Al-Nimr and Abu-Hijleh investigated the validity of assumption of local thermal equilibrium in unsteady conjugate convection in a channel flow. They presented closed form expressions for the fluid and solid domains. It was found that the dimensionless parameters like Bi, channel length (ξ_{max}) and thermal capacity ratio (C_R) of solid to fluid, control the assumption to be validated. During the investigation is was understood that the thermalization time (η_{tt}) is inversely proportional to Bi and directly proportional to ξ_{max} [126]. CHT between plates with fluid flow was modelled using quadrupole method as reported by Maranzana. This quasi analytical procedure is meshless and dependent on integral transform technique [127]. Pozzi and Tognaccini develpoed an exact analysis method for CHT of thermo fluid dynamic field developing in a channel with thicker walls. The results were discussed for interfacial heat flux and temperature distribution at different times at different thermal activity ratio and Eckert number [128]. Another similar study of thick walls of tubes was investigated by Adelaja et al., considering axial wall conduction and fluid conduction. Effect of wall thickness (0.1–2), Bi (0.–10) and Ncc (3–100) were observed to be significant of heat transfer behavior (heat flux, wall temperature and bulk fluid temperature) of fluid at the thermal entrance region [129]. Knupp et al., provided hybrid analytical and numerical solutions to CHT situations in randomly shaped channels using Generalized Integral Transform Technique (GITT) combined with single domain reformulation strategy. Different channels were investigated with the proposed solution technique for heat and fluid flow behavior. Good agreement between results of CFD software COMSOL and proposed solution method were obtained [130]. In Table 5 overview of analytical works carried out related to unsteady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing

Table 5 Few details of unsteady state analysis carried analytically for internal flows

Dimension (D)	Geometry	Orientation	Type of thermal load	Optimal spacing study	References
2D	Tube	Horizontal	Step change of temperature of wall	✗	[124]
2D	Parallel plate duct	Horizontal	Periodically variation of fluid inlet temperature	✗	[125]
2D	Channel	Horizontal	Inlet fluid temperature	✗	[126]
2D	Parallel plate	Horizontal	Uniform outside temperature	✗	[127]
2D	Thick walled tube	Horizontal	Heat flux of fluid at entrance region	✗	[129]
2D	Channel with thick walls	Horizontal	Constant temperature and constant heat flux	✗	[128]
2D	Arbitrarily shaped channel, tube	Horizontal	Heat source of the substrate	✗	[130]

between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

3 Numerical Analysis of Conjugate Steady and Transient State Phenomena's

The CHT studies pertaining to the solution obtained using numerical methods are reviewed in this section. CHT analysis carried out for flow over flat plates, fins, cylinders etc. are all covered. Internal flows in channels, tubes, thick walled pipes are systematically reviewed. FVM and FDM were the common numerical methods employed in studying steady state and transient state external and internal heat transfer processes. Separate subsections are provided for studies carried out on CHT analysis of drying of porous bodies and electronics cooling.

3.1 Steady State Studies on External and Internal Flows

3.1.1 External Flow Over Plates

Sohal and Howell investigated the importance of conduction in a highly thermal conductive plate and convection and radiation in laminar flow. They showed the solution for the plate temperature, where conduction heat transfer along the length of the plate becomes important. Singular integro-differential equation was used related to plate temperature having internal heat generation. Fluid was considered flowing along one side of plates face. Method of iteration was used to solve the differential equation [131]. Karvinen studied numerically the problem of CHT for a flat plate with uniform internal heat generation under forced convection cooling condition. The conduction is taken in only one dimension. The results are compared with the experimental data and were found to be in good agreement. The study was also done in transient flow and turbulent flow as a separate case. Variations in plate temperature were completely different from the free stream temperature [132]. Brosh et al., presented CHT analysis of laminar flow and solid plate in which the line heat source was located at the solid-fluid interface but normal to the flow direction. Temperature profiles were obtained as a function of properties of solid and fluid and of source strength using numerical scheme. Temperature rise considered was moderate assuming constant properties. *Ncc*, intensity of heat source and upstream velocity were the factors investigated. A wide range of results were discussed on the effect of variation in parameters selected as variables [133].

Char et al., determined heat transfer in a laminar boundary layer on a moving plate continuously. They used cubic collocation FDM technique to solve the governing equations. Numerical results are presented for the effect of *Pr*, *Ncc* and *Pe* on CHT of the considered continuous moving plate surface. At constant *Pe*, reduction in thermal conductivity of plate gave reduced axial temperature and at constant *Ncc* due to increase in plate speed lead to low energy flow into the plate by conduction [134]. Yu et al., proposed a solution method for heat conduction and laminar boundary layer in a solid wall pertaining to conjugate problems. Keller box FDM solution for the fluid passing over the wedge and flat plate was used. Heat transfer rate and temperature at the interface of solid and fluid domain with *Pr* varying from 0.001-∞ is analyzed. Local *Nu* correlation was also suggested which was found to be in agreement with numerically obtained data. The research methodology adopted was found to be applicable to free convection problems with plates either horizontal or vertical [135]. Culham et al., choose arbitrarily located heat source on the plate and studied CHT for air flow past the plate. Radiation was also considered from the plate. Using iterative approach and package modeler (META) the temperature profiles were obtained and the results were compared with experimental data. Axial wall temperature and thermal resistance variation was investigated [136].

Lim et al., studied CHT through a wall of variable thickness lined by a boundary layer along one side. Heat transfer rate is reduced by reducing wall thickness in optimal manner along the flow direction. Total heat transfer rate are reduced significantly for $Bi<1$. The laminar boundary layer in conjunction with conduction through the non-uniform wall thickness was solved numerically. Further it was found that for tapered wall the heat transfer rate decreases. Heat transfer and internal resistance of the wall for $Bi<1$ and $Bi>1$ was also investigated [137]. Pop and Ingham examined their method of solution for forced convection over vertical plate and concluded that the solution depends purely on *Pr* [138]. Pop et al., used numerical method to study CHT of a plate whose lower surface is held for constant temperature and leading edge insulated. Pop et al., also used Keller box method to solve the related governing equations. Equations are solved numerically for *Pr* of 0.7 and 7.02 using Runge Kutta Merson method. Temperature profiles for different parameters are obtained for the horizontal flat plate considered [139]. Cole investigated the electronic cooling process in view of scaling laws useful for simple conjugate geometry and shear flow past a flat plate. Numerical results in terms of *Nu* and conjugate *Pe* for fluid flow, solid thickness and thermal conductivity ratio were reported. The limiting behavior of conjugate *Pe* for small and large values was also investigated. He concluded that conjugate heat transfer can be neglected if conjugate $Pe > 18$ [140].

Wang studied CHT problem in non-Newtonian fluid with laminar forced convection. He introduced a conjugate parameter (ζ) related to conjugate problems. $\zeta=0$ for constant wall heat flux and $\zeta=1$ for constant wall temperature as boundary conditions was identified. The influence of power law viscosity index, *Pr* and *ζ* on the behavior of heat transfer rate and temperature profiles were studied [141]. Vynnycky provided CHT account of two dimensional slab over which forced laminar convective fluid flow was considered. Using FDM the governing equations were solved and sweeping was done through Gauss-Seidel iteration. *Re*, *Pr*, *Ncc* and *Ar* were the four non-dimensional parameters considered for study. External and internal thermal conductivities were also considered for the conjugate model [142]. Shu and Pop studied both analytically and numerically the steady forced convection thermal boundary layer past a flat plate with prescribed heat flux. The governing equations are solved by Keller box scheme in combination with the continuous transformation method. Good agreement for asymptotic analytical and numerical results were obtained [143]. Hribersek and Kuhn developed a boundary domain integral method for solving conjugate problem in which the flow field is calculated by velocity-vorticity method. Fluid flow was basically discretized by Weighted Residuals Technique, special fundamental solutions and Gauss divergence theorem [144].

Chida studied the CHT problem pertaining to laminar flow over the upper surface of a finite thickness flat plate with its trailing and leading edges considered to be adiabatic whereas the lower surface was held at constant temperature. Numerically, the associated boundary layer equations of the fluid domain for the flow and thermal fields were solved along with the steady two-dimensional energy equation

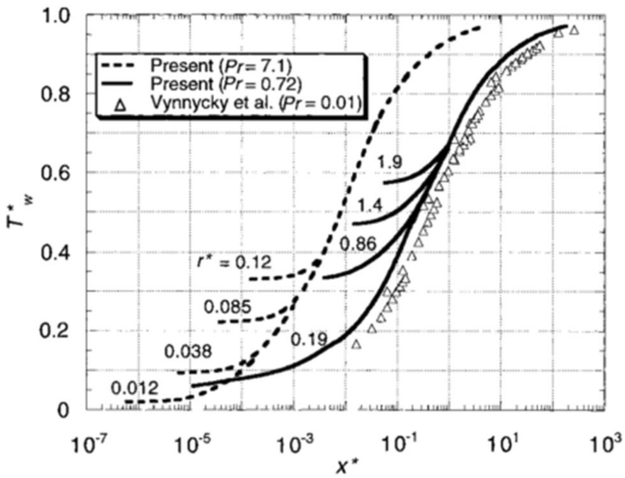


Fig. 9 Flat plate temperature measured by Chida [145] and comparison with Vynnycky et al. [142]

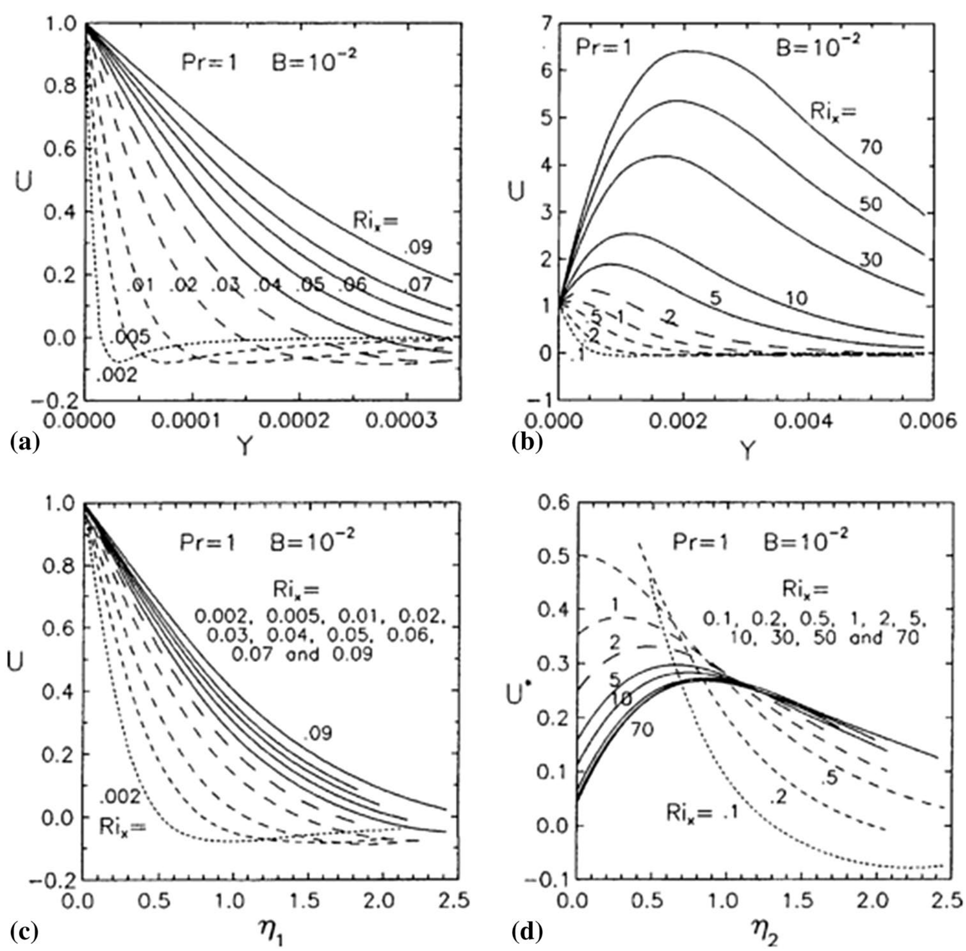


Fig. 10 Velocity distributions for *Ri* along transverse coordinate *Y* (**a** and **b**), along similarity variables (**c** and **d**) at $Pr = 1$ and $B = 10^{-2}$ [90]

for the flat plate considering continuity in temperature and heat flux at the interface. The temperatures at the solid-fluid interface were computed and compared with those available in the literature as shown in Fig. 9 in order to get a clear view of the studied problem of CHT [145]. The thermal behavior and fluid flow characteristics of moving plate (continuously) was studied by Al-Sanea. The plate was of extruded material and the investigation was carried out near to and far downstream from the extrusion slot. The variations in temperature and velocity profiles (Refer Fig. 10) were obtained numerically to map all types of convective flow regimes. Effect of *B* (Buoyancy parameter) and *Pr* on heat transfer and friction coefficients were analyzed. Figure 10 shows the velocity profiles of the fluid domain for different *Ri* (Richardson number). Comparison with experimental and FDM method shows significant agreement with the obtained data. Al-Sanea also found the critical values of *Ri* to define natural, mixed and forced convection regimes [90].

Hajmohammadi and Nourazar numerically studied CHT behavior of plate with variable thermal conductivity. Using differential transform method the non-linear integro-differential equation was solved accurately. Temperature distribution in conjugate analysis was found to be flatter on the plate surface when compared to non-conjugate analysis [146]. Ortega and Ramanathan conducted numerical study on thin plate exposed to forced convection attached with a heat source. Employing Green functions the conjugate problem was solved and developed a *Nu* correlation as function of *k* and *Pe*. For high *Pe*, the source heat transfer behaves like infinite heat source in two-dimension. For different *Pe* and *Ar* influence of *Nu* and free stream temperature were investigated [147]. Krikkis performed numerical study of CHT for convection on one side of flat plate and boiling on other side as shown in Fig. 11 [77]. They found that the heat flux and temperature variations are affected by *Br* and *Ncc* [77].

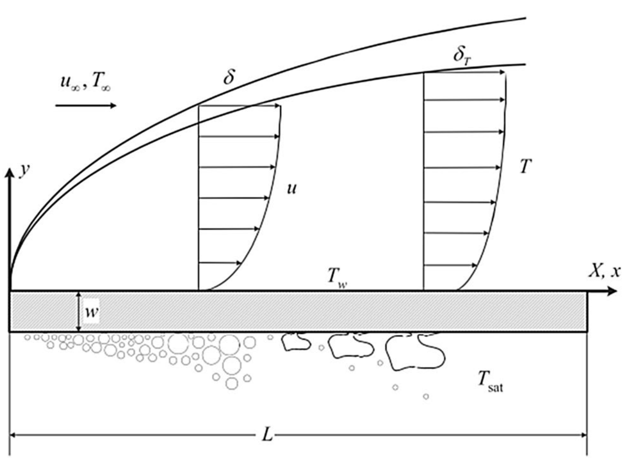


Fig. 11 Physical model showing convection and boiling along alternate sides of plate [77]

In Table 6 overview of numerical works carried out related to steady state external flows are mentioned. All works cited are related to two dimensional and external flows.

3.1.1.1 Conjugate Conduction-Convection Analysis of Plate Fins Sparrow and Chyu conducted numerical conjugate analysis of a plate fin cooled by air. Boundary layer equations were solved using Patankar-Spalding method and results were obtained for different *Ncc* while fixing *Pr* value. They found considerable error using conventional method of analysis for temperature and heat flux [148]. Sunden carried CHT analysis of rectangular fin with laminar boundary layer flow around it. For three different *Pr* (0.021, 0.7 and 5.0) values and *Ncc* (0 to 5) numerical results were obtained. Fin efficiency was found to be comparable with conventional fin theory, but incorrect results were obtained for temperature and heat flux [149]. Sunden did another similar study on plate fin for laminar as well as turbulent flow using FDM. For different *Ncc* and *Re,* results were obtained for local heat flux and temperature distribution. Sunden also found that substantial error exists using conventional method of finding temperature and local heat flux distribution [150]. Huang and Chen performed analysis of conduction in plate fin coupled with convection of power law fluids. Apart from heat flux and temperature, local and overall heat transfer coefficients were estimated for different *Ncc*, *Pr* and flow index. Similar results were obtained for non-Newtonian and Newtonian fluids [151]. Hsiao and Hsu studied heat transfer problem in visco-elastic fluid flow past a plate fin. Using Falkner-Skan flow derivation solutions for flow at the fin tip ($\beta = 1.0$) and the flat plate ($\beta = 0.0$) were obtained. *Pr*, *E* (Elastic number), *Ec* (viscous dissipation parameter) and *Ncc* were the parameters considered for analysis. Local heat transfer coefficient and heat transfer from the plate increased due to elastic effect of flow. Improved heat transfer was also seen considering large values of *Pr*, *E*, *Ec* and *Ncc* [152]. In Table 7 overview of numerical works carried out related to steady state flows over fins are mentioned. All works cited are related to two dimensional and external flows.

3.1.1.2 Work on Heat Generating Vertical Nuclear Fuel Cylinder/Element Jilani et al., analyzed CHT in heat generating vertical cylinder and for specific fluid coolant with $Pr = 0.005$. Analysis was carried for different heat generation parameter (*Q*) and conduction-convection parameter *Ncc*. The results presented show that in the fluid boundary layer and cylinder, radial temperature distribution is significant [153]. Numerical investigation of CHT situation in a vertical plate generating uniform volumetric energy was carried by Jahangeer, Ramis and Jilani. The generating heat dissipating to the coolant moving in upward direction was considered for analysis. *Pr* was held constant at 0.005 (liquid sodium) and numerical results were presented

Table 6 Few details of steady state analysis carried numerically for external flows

Dimension	Geometry	Orientation	Numerical method for solid	Fluid discretization scheme/ technique	Type of thermal load	References
2D	Flat plate	Horizontal	Integrating by parts	Integrating by parts	Uniform heat generation of plate	[131]
2D	Flat plate	Horizontal	Integrating by parts	Integrating by parts	Uniform heat generation of plate	[132]
2D	Wall	Horizontal	FDM	FDM	Heat source at the solid-fluid interface	[133]
2D	Moving sheet	Horizontal	FDM	False transient technique	Uniform heat flux on sheet	[134]
2D	Flat and wedge plate	Horizontal	FDM	Separation of variables	Constant wall temperature and heat flux	[135]
2D	Plate	Horizontal	FVM	FVM	Heat source of circuit board	[136]
2D	Wall with variable thickness	Horizontal	FDM	FDM	Constant temperature of wall	[137]
2D	Plate	Vertical	FDM	Stream function	Constant temperature of plate	[138]
2D	Flat plate	Horizontal	Keller box method	Keller box method	Constant temperature of plate	[139]
2D	Flat plate	Horizontal	FEM	FEM	Constant heat flux from heated strip on flat plate	[140]
2D	Flat plate	Horizontal	FDM	ADI scheme	Constant temperature of plate	[141]
2D	Slab	Horizontal	FDM	ADI scheme	Constant temperature of slab	[142]
2D	Flat plate	Horizontal	Keller box method	ADI scheme	Constant wall heat flux	[143]
2D	Flat plate	Horizontal	Analytically	SIMPLE algorithm	Constant temperature of plate	[145]
2D	Moving plate	Vertical	FVM	SIMPLE algorithm	Constant temperature of plate	[90]
2D	Plate	Horizontal	FVM	UFED model	Heat source on plate	[147]
2D	Plate	Horizontal	FVM	Differential transform method	Uniform heat flux at the base	[146]
2D	Flat plate	Horizontal	Integral technique	Integral technique	Constant wall temperature	[77]

Table 7 Few details of steady state analysis carried numerically for fins

Dimension	Geometry	Orientation	Numerical method for solid	Fluid discretization scheme/ technique	Type of thermal load	References
2D	Plate fin	Vertical	FDM	Patankar-Spalding method	Constant base temperature of fin	[148].
2D	Rectangular fin	Horizontal	FDM	FDM	Constant base temperature of fin	[149]
2D	Rectangular fin	Horizontal	FDM	FDM	Constant base temperature of fin	[150]
2D	Rectangular fin	Vertical	Perturbation technique	Implicit finite technique for boundary layer	Constant base temperature of fin	[151]
2D	Plate fin	Vertical	FDM	FDM	Constant temperature of fin/ fluid	[152]

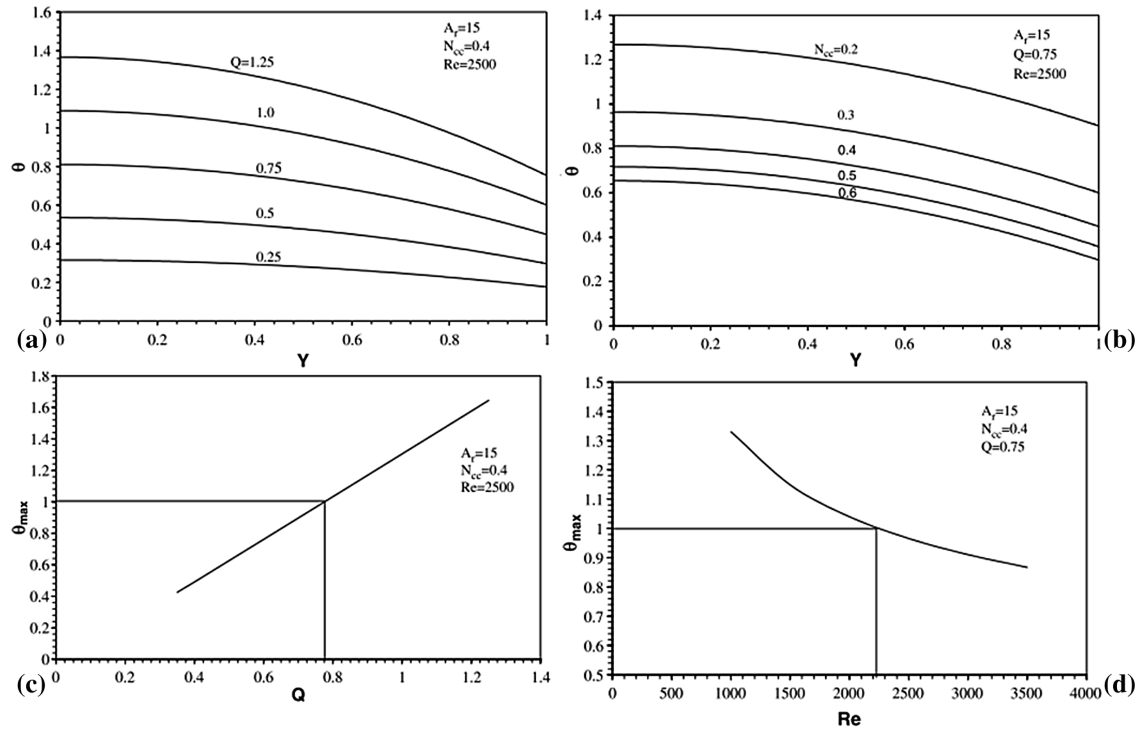


Fig. 12 Effect on dimensionless axial temperature profiles in nuclear fuel element with uniform heat generation for different **a** heat generation parameter *Q*, **b** *Ncc* and maximum temperature for different **c** *Q*, **d** *Re* [158]

for a wide range of plate *Ar*, *Q* and *Ncc*. It was concluded that, upper and lower value of considered parameters exists above and below which the nuclear fuel elements temperature exceeds its allowable limit, and its related results are shown in Fig. 12. It was quite obvious to find that increasing *Re* the heat transfer rate enhanced. A similar study was conducted by Ramis and Jilani to compare uniform and non-uniform volumetric energy generation. The parameters considered were *Re*, *Ar*, *Q* and *Ncc* to analyze the thermal distribution in the rectangular nuclear fuel element and fluid domain. Figure 13 shows comparison between uniform and non-uniform energy generation study for the parameters considered [154]. Ramis and Jilani reported another work of establishing criterion for boundary layer solution of conjugate study related to fuel element. Critical analysis of maximum dimensionless temperature (θ_{max}) for *Re*, *Ar*, *Q* and *Ncc* were in detail investigated. The conclusion drawn was that, upper limit of *Ar* exists above which θ_{max} is negligible. The pumping power required by upward moving coolant can be reduced by increasing value of *Ncc* [155]. Another study by the same authors was on investigation of heat transfer and fluid flow characteristics of liquid sodium ($Pr=0.005$) flowing over the rectangular fuel element considered as solid domain generating volumetric energy non-uniformly. They found that the fuel element heat dissipation rate to the coolant is independent of *Ar*, *Q* and *Ncc* but increases proportionally with increase in *Re*. Upper value of *Re* and *Ncc* exists above which reduction in coolant temperature is negligible [156]. Razak et al., applied the same method to study the heat transfer and fluid flow characteristics of liquid sodium as a coolant flowing over a non-uniform heat generating rectangular plate. Their special focus was on exit temperature of the coolant. They found that exit temperature distribution of the coolant is independent of *Ar* while other parameters were kept constant [40, 157]. In Table 8 overview of numerical works carried out related to steady state flows over cylindrical/ rectangular plates by Jilani, Ramis and Co are mentioned. All works cited are related to two dimensional and external flows.

3.1.2 Conjugate Heat Transfer Analysis of Parallel Plate Channels

Barozzi and Pagliarini proposed a general procedure which is fast and simple to deal CHT problems. Effect of wall conduction on heat flux, *Nu*, interface temperature and bulk temperature of fluid flowing inside the pipe was focused. *Pe*, wall to fluid *Ncc*, length and thickness of the pipe were analyzed to check the axial conduction significance. Wall conduction reduced the *Nu*, when it was compared with the analysis carried for *Nu* without wall conduction effect [159]. Bilir analyzed CHT for a flow (thermally developed) inside

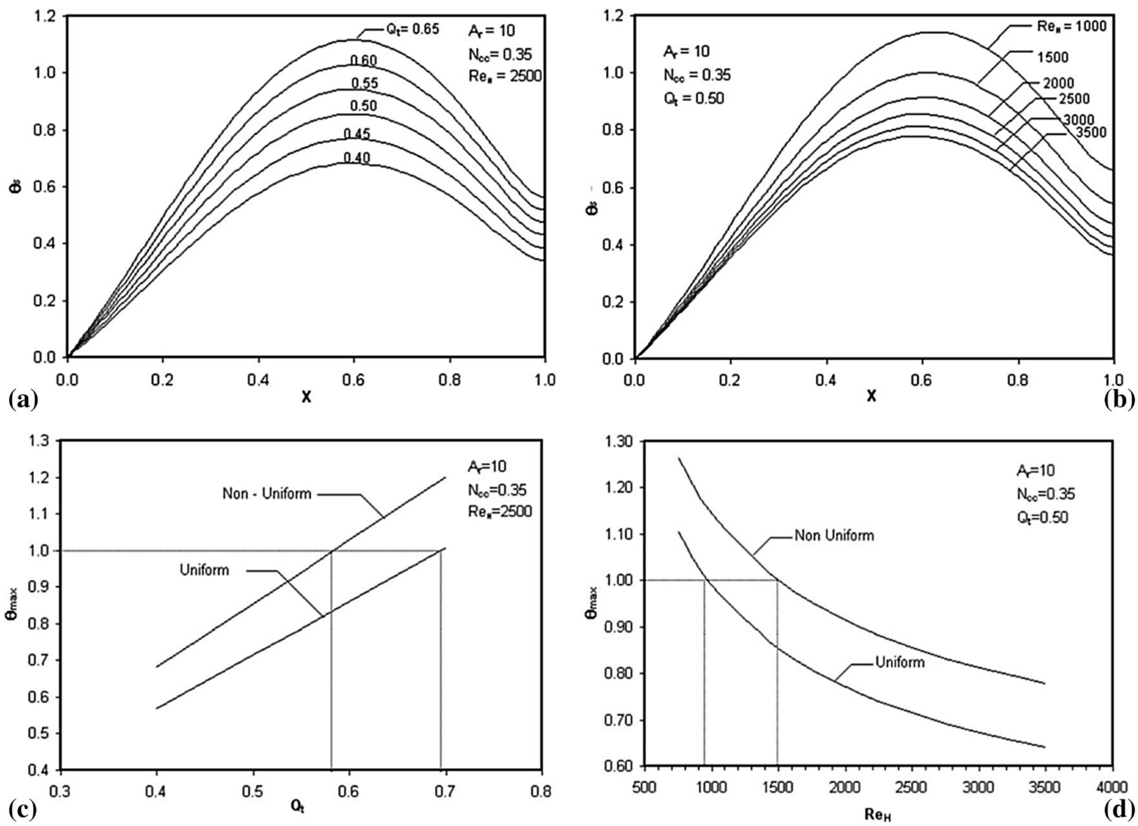


Fig. 13 Effect on dimensionless axial temperature profiles in nuclear fuel element with non-uniform heat generation for different **a** heat generation parameter Q_t, **b** Reynolds number Re_H and maximum temperature for different **c** Q_t, **d** Re_H [154]

Table 8 Few details of steady state analysis carried numerically for fins

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/ technique	Type of thermal load	References
2D	Nuclear cylinder	Vertical	FDM	FDM with marching technique	Uniform heat generation of cylinder	[153]
2D	Nuclear fuel plate	Vertical	FDM	FDM with marching technique	Uniform heat generation of plate	[158]
2D	Nuclear fuel plate	Vertical	FDM	ADI scheme	Uniform and non-uniform heat generation of plate	[154]
2D	Nuclear fuel plate	Vertical	FDM	ADI scheme	Uniform heat generation of plate	[155]
2D	Nuclear fuel plate	Vertical	FDM	ADI scheme	Non-uniform heat generation of plate	[156]
2D	Nuclear fuel plate	Vertical	FDM	ADI scheme	Non-uniform heat generation of plate	[40]

the pipe considering axial fluid and wall conduction. In this study, Bilir investigated the effect of *Pe*, wall to fluid *Ncc* and thickness ratio and observed that *Pe* is quite effective when it is ≤ 20 [160]. Wang and Georgiadis have shown numerical simulation of CHT in a laminar cross flow over an array of cylinders arranged with pitch to diameter ratio of two. Heat transfer between solid and fluid represented by *Nu* is analyzed for *Re* ranging from 100 to 400, $Pr = 0.71$ and fluid to solid thermal conductivity ratio (*Ncc*) from 0.01 to 100. They concluded that uniform heat generation case considered in this study is completely different when compared to constant temperature cylindrical array if $Ncc > 1$ [161]. Al-Zaharnah et al., analyzed thermal stresses induced due to CHT in pipes in which the flow is fully developed. Thermal stresses due to *Ncc* and *Pr* were investigated and found that low radial stresses are induced when *Ncc* and *Pr* and also low. Variations in stresses according to locations were found at different values of variables [162].

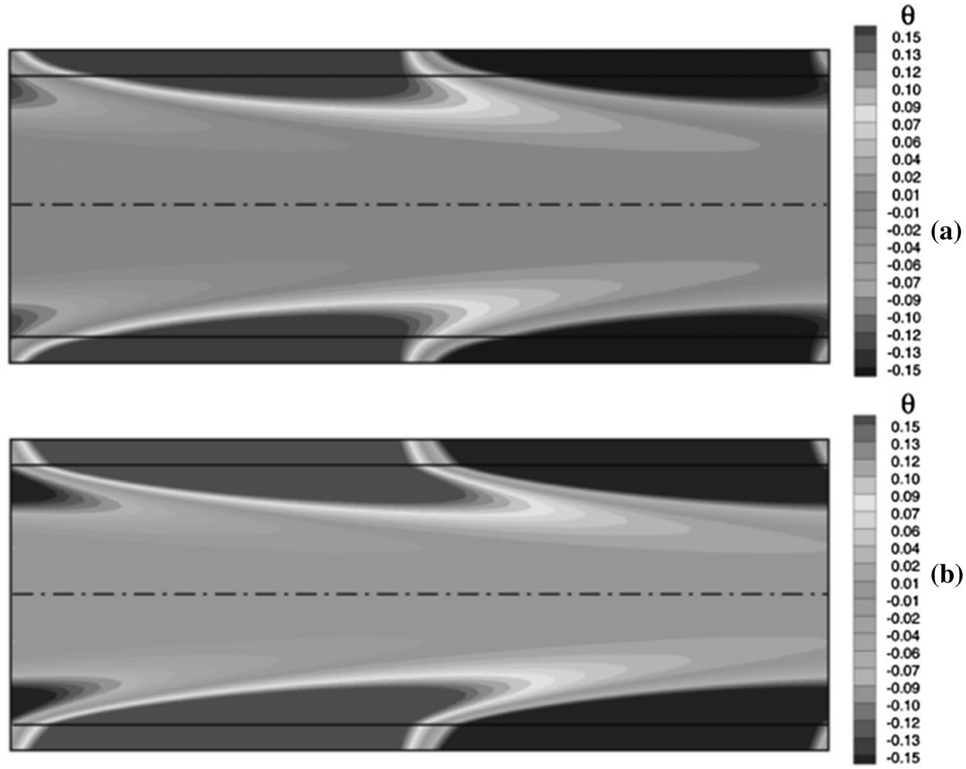


Fig. 14 Temperature distribution in solid and fluid domain for **a** γ $(Ncc)=0.5$, **b** $\gamma=3$ for fixed values of Pe, $B=100$ (angular frequency) and $\sigma=1.2$ (channel half width) [112]

Barletta et al., investigated the CHT problem related to forced laminar flow through a parallel plate channel with its external wall having temperature distribution undergoing longitudinal change sinusoidally. Assuming fully developed flow, the temperature profile in fluid and solid part, along with average and local *Nu* were computed numerically. In Fig. 14, temperature distributions obtained for parallel plate numerically are presented. A comparison between the results obtained analytically and numerically reveled excellent agreement between them [112].

Alves and Altemani conducted a numerical analysis of conjugated cooling of heater placed in a channel wall. For specific value of pressure drop, mass flow rate, pumping power the investigation was done. They found decrease in temperature of heater for increase in flow rate of air and thermal conductivity of the substrate [163]. Jagad et al., developed a numerical iterative method to analyze the interface condition between solid and fluid. Algorithms were discussed for the related problem in case of unstructured and structured mesh. Correlation of local *Nu* was developed for uniform heat flux considering structured mesh. The conclusion drawn from their study was mainly that the unstructured mesh are stable for complex domain mapping [164]. Olakoyejo performed a numerical CHT study on square and circular channels with internal heat generation. Optimal channel diameter, elemental volume and optimal channel spacing were found for widespread choice of parameters [165]. Similarly, Bello-Ochende et al., conducted

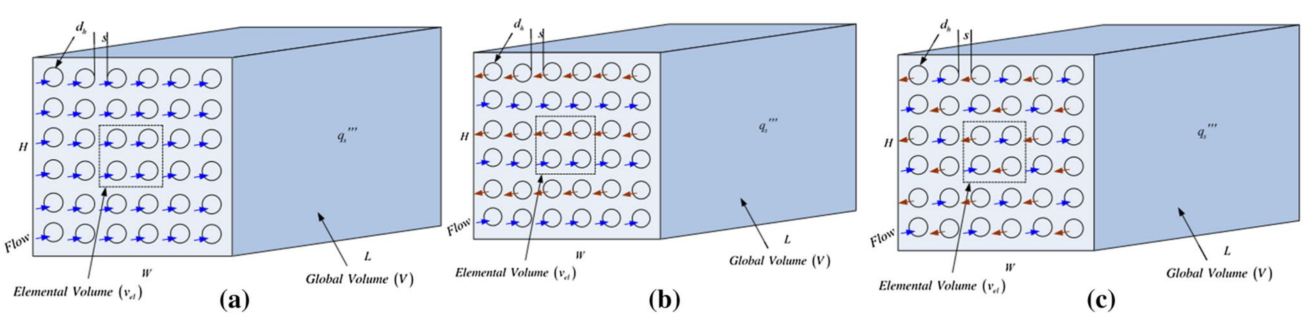


Fig. 15 Array of cylindrical channels arranged for **a** parallel flow (PF-1), **b** counter flow in every second row (CF-2) and **c** counter flow relative to each cylinder (CF-3) [166]

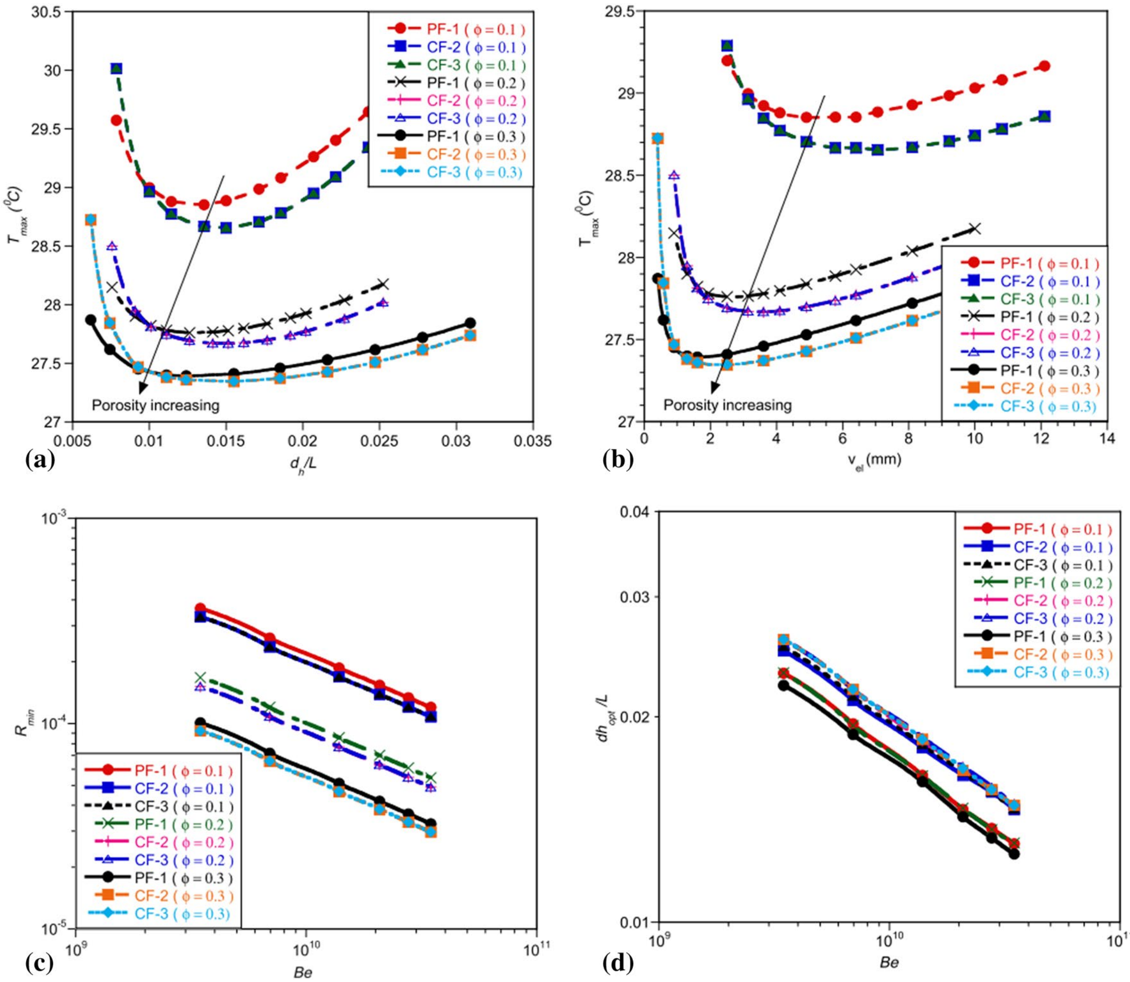


Fig. 16 Peak temperature variations due to change in **a** optimized hydraulic diameter (d_h), **b** optimized elemental volume (V_{el}). **c** Variations in minimized global thermal resistance R_{min} (dimensionless) for change in *Be* and **d** variations in optimized d_h for change in *Be* [166]

three-dimensional CHT analysis of circular channels with internally generating heat placed like an array system having dissimilar flow orientations viz., parallel flow, counter flow relative to every other cylinder and counter flow in every second row compared to its neighboring rows which is shown in Fig. 15. Channel width and diameter of the cylinders was the variables for the design of geometric configuration. Analysis was performed for different heat generation rate, porosity (ϕ), flow orientation and pressure difference to obtain the optimal diameter and channel width. Optimization algorithm was applied to obtain optimal spacing at which increase in thermal performance with reduced thermal resistance was observed for wide range of pressure difference *Be* (dimensionless Bejan number). In Fig. 16, numerical results are presented for different flow orientations and elemental volume to obtain optimal spacing as obtained by the authors. When results were compared with studies in literature, agreement was found for influence of pressure difference on thermal resistance [166].

Li et al., studied the CHT behavior of depositing layer on the walls of channels using level set method. Effect of *Re*, *Ncc* of depositing layer and local *Nu* of fluid was investigated [167]. Huang numerically studied conjugate heat and mass transfer in membrane of parallel plates. Friction factor, *Nu* and Sherwood number were obtained along the duct length. Effect of entrance on fundamental data were investigated and experimental validation was performed [168]. CHT analysis of laminar forced flow in a pipe with periodic varying heat flux axially was done by Aydin et al. Effect of *Ncc* (k_s/k_f), dimensionless amplitude of varying heat flux and diameter ratio on average and local *Nu*, interface temperature and heat flux distribution was analyzed. Few results of downstream variations of temperature along the length of tube for different parameters are depicted in Fig. 17. Based on uniform and periodic heating, the change in temperature distribution is also seen to be linear or periodic (Fig. 17b) whereas with change in k_s/k_f ratio, there is no significant change in wall temperature as shown in Fig. 17a. Results

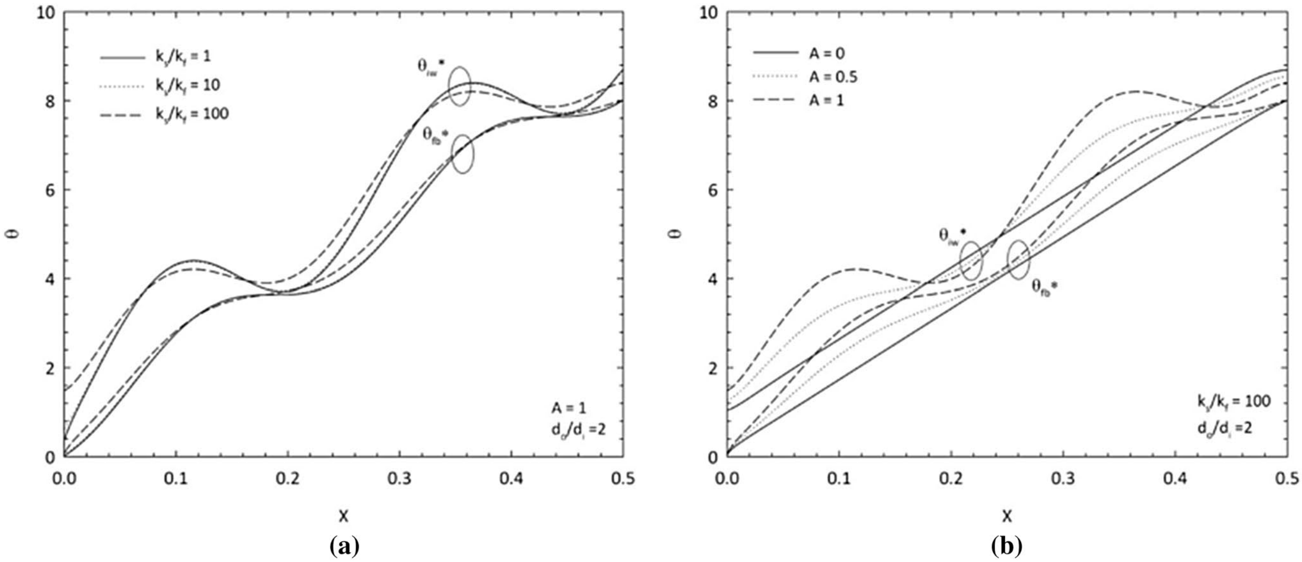


Fig. 17 Variations in dimensionless temperature (θ) along tube length **a** wall temperature for various thermal conductivity ratio and **b** bulk fluid temperature for different amplitude of varying heat flux [93]

were in agreement with data published in literature [93]. Samee et al. analyzed the effect of *Pr* on average exit temperature of coolants flowing between heat generating plates. Their study was at optimum spacing between the plates. For non-uniform heat generation the average exit temperature is higher than uniform heat generation in plates [169]. In Table 9 overview of numerical works carried out related to steady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

3.1.2.1 Steady State Investigations Considering Heat Source in Electronics Cooling Ramadhyani et al., considered CHT from heat source of channel which was fixed on its one wall. Single and two heat sources embedded in a large substrate were selected in this study. It was shown that for low *Pe* and high substrate to fluid *Ncc,* heat transfer from source to substrate makes a major influence. No effect on heat transfer for side by side heat source was found for the considered parameters [172]. Sugavanam et al., did numerical study on heat transfer analysis of uniformly powered strip heat source located on conducting substrate. This study was carried in detail and presented results for different *Ncc*, *Re* for parallel plate and fully developed flow [173]. Bautista and Mendez also conducted numerical CHT analysis in rectangular channel with discrete source of heat generating uniform volumetric energy due to electric current. Neglecting longitudinal heat conduction effects of the fluid and considering uniform heat flux to the substrate, the governing equations of the thermal fields in the coolant fluid and the heat source chip were solved numerically. They presented that CHT process is controlled by a dimensionless conjugate parameter (α) [91]. Habeeb also worked on electronic cooling of chip considering three dimensional situation for laminar and turbulent flow [174]. Cheng et al.[175] also carried a much similar work on electronics cooling based on synergy principle in a channel with four discrete heat sources. Durgam et al., analyzed CHT laminar forced and natural convection cooling of heat sources made of aluminium. In Table 10 overview of numerical works carried out related to steady state internal flows are mentioned. Optimum distribution of heat sources was identified based on geometric parameter $\lambda = 1.3$ [70]. All works cited are related to two or three dimensional, internal flows and for electronics cooling purpose. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

3.2 Transient State Analysis of External Flows Over Plates and Internal Flows in Channels

3.2.1 Conjugate Analysis of Flow Over Plates

Juncu investigated heat/mass transfer of forced convection from a plate of finite thickness to a viscous steady stream. The governing transient conditions were studied and parameters like *Re* and *Pr* was varied. In this study the prime factor studied was the effect of product of *Ar* and thermodynamic ratio on heat/mass transfer rate. The results showed that the heat transfer from plate, sphere and cylinder are of similar

Table 9 Few details of steady state analysis carried numerically for internal flows

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/ technique	Type of thermal load	Optimal spacing study	References
2D	Pipe	Horizontal	FEM	FEM	Uniform heat flux on pipe	✗	[159]
2D	Two regional pipe	Horizontal	FDM	Exponential scheme by Patankar	Constant temperature of wall	✗	[160]
2D	Cylinder	Vertical	Transformation technique	Stream function	Internal heat source on cylinder	✗	[161]
2D	Pipe	Horizontal	FVM	FVM	Uniform heat flux on pipe	✗	[162]
2D	Channel	Horizontal	FVM	SIMPLER algorithm	Constant heat flux at channel surface	✗	[170]
2D	Backward facing step	Horizontal	Taylors series expansion	ADI scheme	Constant wall temperature	✗	[171]
2D	Parallel plane channel	Horizontal	Bubnov–Galerkin FEM	Bubnov–Galerkin FEM	Sinusoidal external temperature wall	✗	[112]
2D	Rectangular	Horizontal	FVM	SIMPLE algorithm	Temperature of heater block	✗	[163]
2D	Pipe channel	Horizontal	FVM	FVM	Uniform base heat flux on pipe	✗	[164]
3D	Square and circular channels	Horizontal	FLUENT CFD software	FLUENT CFD software	Internal heat generation of solid	✓	[165]
3D	Array of cylinders	Vertical	FLUENT CFD software	FLUENT CFD software	Internal heat generation of solid	✓	[166]
2D	Channel	Horizontal	FVM	FVM	Constant temperature of channel	✗	[167]
2D	Parallel plate	Horizontal	FVM	FVM	Solution temperature in membrane	✗	[168]
2D	Pipe	Horizontal	FVM	ADI scheme	Periodically varying heat flux on tube	✗	[93]
2D	Parallel plate	Vertical	FDM	Stream function vorticity	Heat generation of plate	✓	[169]

Table 10 Few details of steady state analysis carried numerically for internal flows

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/ technique	Type of thermal load	Optimal spacing study	References
2D	Channel	Horizontal	Control volume finite difference	Control volume finite difference	Discrete het source on one channel	✗	[172]
2D	Parallel plate	Horizontal	FVM	SIMPLER	Uniform heat flux of strip on substrate	✗	[173]
2D	Channel with heat source embedded	Horizontal	FVM	Lighthill's approximation	Uniform volumetric heat generation by the chip	✗	[91]
3D	Channel	Horizontal	FVM	SIMPLE algorithm	Electronic chip as heat source	✗	[174]
2D	Channel	Horizontal	Full field computational method	CLEARER algorithm	Four discrete heat source in the channel	✗	[175]
2D	Channel	Horizontal or vertical	COMSOL multiphysics		Heat sources in the channel	Optimal distribution of heat sources	[70]

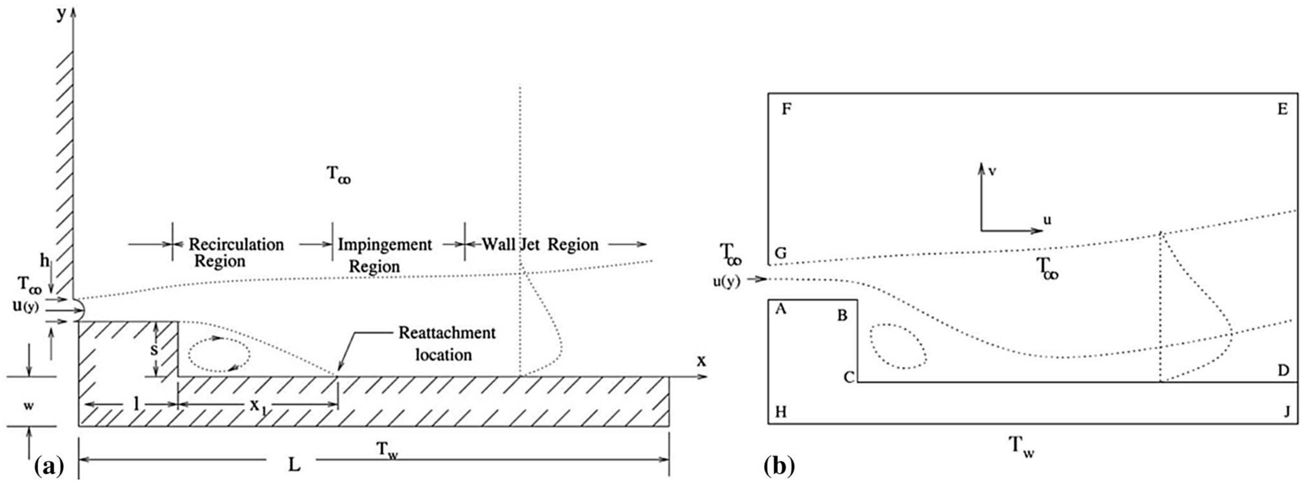


Fig. 18 **a** backward facing step with wall jet and **b** boundary conditions [179]

nature for uniform heat condition except wake phenomenon [176]. Another similar unsteady analysis for conjugate heat/mass transfer from a plate was reported by Juncu in [177].

Kanna and Das investigated numerically and analytically conjugate heat transfer from plane wall jet. *Re*, *Pr*, *Ar* and thermal conductivity ratio were the factors analyzed. Closed form solutions were established for local and average *Nu* and conjugate temperature at the boundary interface [178]. They performed another similar CHT study for a backward facing step. For much of the geometries the analysis was done to measure junction temperature, average and local *Nu*. In down-stream direction (along AB, refer Fig. 18) the conjugate interface temperature decreases and again with increasing wall thickness (*w*) it further reduces. And along CD, the interface temperature reduces and then increases further in downstream direction. But overall for increasing *w* the interface temperature reduced due to more slab thickness as shown in Fig. 19 [179]. Kanna and Das performed CHT study on backward facing step flow problem which is considered as a benchmark problem. He was the first to investigate CHT analysis in this benchmark problem. Influence of *Re*, *Pr*, *Ncc* and thickness of slab on local and average *Nu* and interface temperature was investigated [171].

He and Oldfield analyzed CHT in high pressure turbine blades subjected to hot streaks of unsteady behavior by a new approach. Mismatch analysis was carried out between convection and conduction in the fluid and solid domain respectively. A hybrid approach was then applied to match the time domain fluid field and frequency oriented conduction. This was achieved to reduce error in FDM by introducing semi-analytical interface condition which is harmonic in nature. Comparison between experimental and analytical

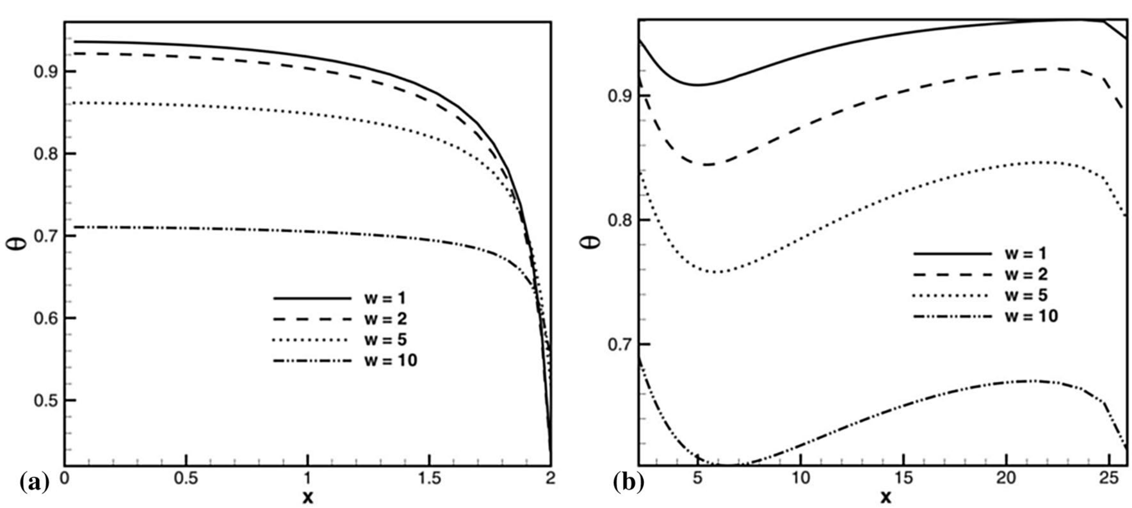


Fig. 19 Conjugate heat transfer at different location and effect of w (wall thickness) [179]

Table 11 Few details of unsteady state analysis carried numerically for external flows

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/technique	Type of thermal load	References
2D	Flat plate	Horizontal	FDM	Nested defect correct iteration algorithm	Uniform temperature of plate	[176], [177]
2D	Plane wall	Horizontal	FDM	ADI scheme	Constant temperature of wall	[178]
2D	Plane wall with backward facing step	Horizontal	FDM	ADI scheme	Constant temperature of wall	[179]
2D	Turbine blade		FVM	FDM	Hot streak of fluid	[180]

data was done and obtained good agreement. The outcome of this study was related to heat transfer influenced by transient non-linearity. They found a major disparity in time scale between solid conduction and fluid convection [180]. In Table 11 overview of numerical works carried out related to steady state internal flows are mentioned. All works cited are related to two dimensional and external flows.

3.2.2 Transient Analysis of Internal Flow in Channels/Parallel Plates

Lin and Kuo investigated wall conduction effects on transient forced flow in a long pipe when the heat flux was changed in step of the pipe. *Pe*, *Ncc*, radius ratio (β) and thermal diffusivity ratio were found to effect on unsteady characteristics of heat transfer. The capacity ratio was having a critical impact on unsteady heat transfer. System with large *Ncc*, β and length took longer time to reach steady state and also for system with smaller values of *Pe* and thermal diffusivity ratio [181]. Considering convection from ambient in a channel flow, unsteady CHT was studied numerically by Yan. Heat capacity and wall conduction effects were considered in the numerical solution. In this study Yan selected various dimensionless parameters like wall thickness, *Ncc*, *A* (thermal diffusivity ratio) and *Nu* and analyzed there effect on temperatures at interface, bulk temperature and interfacial heat flux. The results presented show the wall effect has an important role in CHT problem. To reach steady state, time required is more if wall thickness and *Ncc* are larger and if *A* and *Nu* are smaller [182]. Lee and Yan carried transient CHT analysis of long pipe with outer surface at constant temperature. *Pe*, *Ncc*, radius ratio (β) and thermal diffusivity ratio were the parameters chosen as variables. Local *Nu*, interface temperature and bulk temperature of fluid were analyzed. The transient heat transfer behavior was found to depend on β, *Ncc* and thermal diffusivity ratio [183].

Nigen and Amon performed CHT analysis of grooved channels with different materials. Coefficients of local skin friction and stream function contours were shown for steady and transient conditions. *Nu* and surface temperature are analyzed as a function of *Re*. Apart from the importance of CHT analysis, importance of transient super-critical flow regime and its mechanism of heat transport was identified [92]. In their next study they performed numerical simulation for influence of flow regime, heat generation and material composition on spatial distribution of heat flux, temperature and *Nu* at the interface. In this study they found that the internal heat generation strongly affects *Re* and convective heat transport phenomena [184].

Moraga and Medina applied FVM to obtain solution for transient CHT problem in a food of plate shape at the time of freezing. Diffusion equation was employed in the food to simplify the solution instead of using energy equation. Results for air temperature distribution, food temperature variation, heat transfer coefficient and velocity vectors were presented. To validate the results, experimental data was used [185]. Numerical study of conjugate heat transfer in horizontal channel was done by Chiu et al. Experimental results were used to validate their numerical models result and found to be in excellent agreement, which is depicted in Fig. 20. Characterization of effects of conjugate heat transfer was achieved using the numerical study for design variables and operating conditions. Rate of heat transfer was found to be impacted from uniformity and temperature level of the channel walls, heated sections surface and gas phase. It was also found that appropriate handling of conjugate heat transfer is required for channel wall design and thermal process modelling (with increased temperature sensitivity) [170].

Bilir performed two different studies on CHT in pipes. Both the study was considering two dimensional thick wall with axial conduction in wall and fluid. Focus was on thermally developing entrance regime with step change in temperature of downstream heated surface. Effect of *Pe*, *Ncc*, thermal diffusivity ratio and thickness ratio (wall to inner radius) were analyzed on heat transfer behavior at different time durations. In the first study the downstream surface was provided with step change in temperature whereas in the second study step change in fluid downstream section was introduced. Convection from the outer surface was considered in the second study and the associated parameter *Bi*

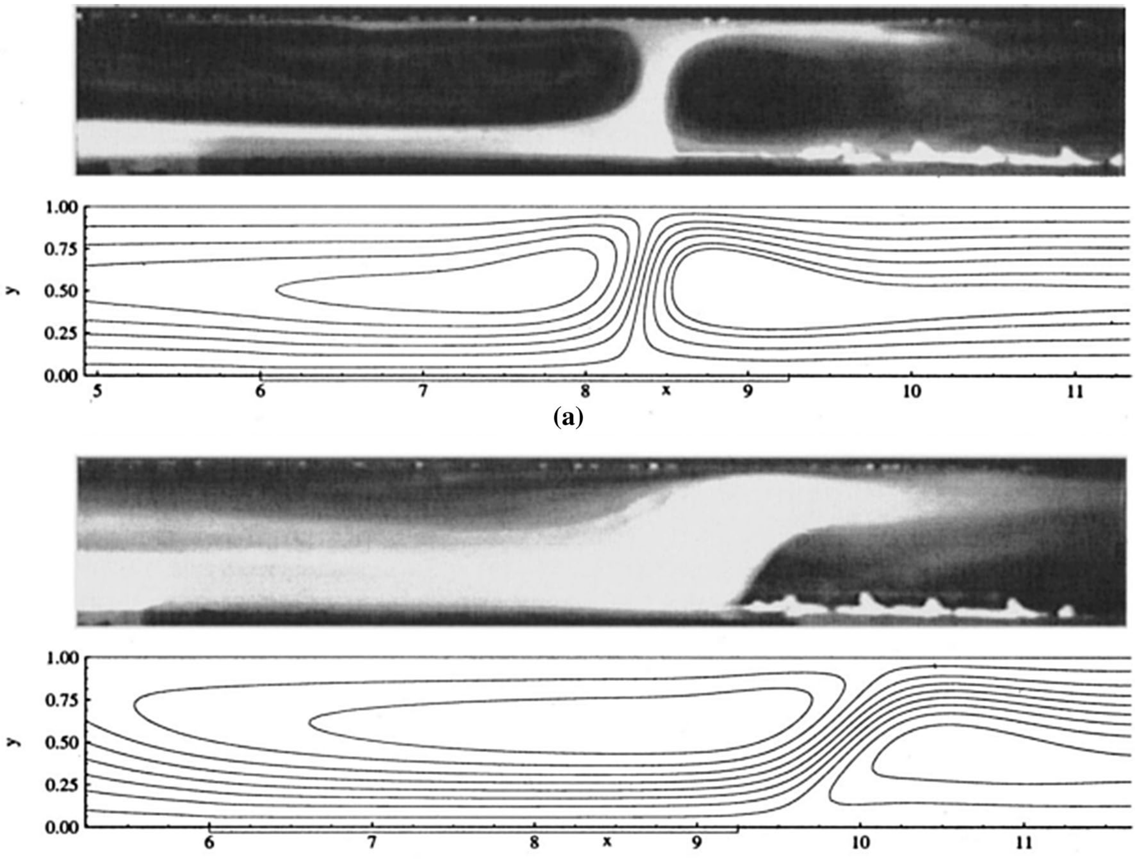


Fig. 20 Validation of numerical results with experimental results for streamlines in a ceramic heated region for Re at **a** 9.48 and **b** 29.7 [170]

was analyzed [186, 187]. Ates and Bilir performed their next study in 2010 which was again on thick walled pipes with axial conduction in wall and fluid and similar boundary conditions. The above mentioned parameters were investigated and found to effect the heat flux at the interface for entire time and for steady state process [188].

Abu-Hijleh et al., studied the transient conjugate thermal behavior of forced convection flow in a finite wall thickness channel. Axial conduction in solid and fluid is considered. They found that Bi, Pe, ξ_{max} (channel length), C_R (thermal capacity ratio) and α_R (thermal diffusivity ratio) control the local thermal equilibrium assumption. In channel with small C_R the influence of Bi on thermalization time (η_{tt}) is negligible and η_{tt} is proportional to Bi. The effect of the five parameters considered were separately analyzed for the effect on η_{tt} [189]. Unsteady CHT analysis was reported for parallel plates by Lin et al., considering the effect of wall conduction. They proposed an inverse method to know the boundary conditions and found to be helpful in predicting the boundary conditions [190]. Zueco performed CHT analysis of parallel plate channel subjected to periodic variations of surface temperature. Channel thickness, *Pe, Ncc, A*, viscous dissipation parameter and angular frequency were determined [191]. Arici and Aydin conducted CHT analysis of a forced laminar flow in a pipe with wall conduction and viscous dissipation. *Nu* and distribution of temperature at the developing entrance region was analyzed. The results indicated that the viscous dissipation and heating effect depend upon Brinkman number and thermal conductivity [192]. Pirouz et al., used Lattice Boltzmann Method (LBM) to solve CHT analysis in a channel with upper and lower wall mounted with small obstacles as shown in Fig. 21. At $Pr = 0.72$, Re from 200 to 1000 and $A = 10$, 100, 1000 and considering the obstacle distance the numerical simulations were performed (See Fig. 21). Removal of heat from the base of channel is more if A is more whereas reducing the obstacle distance increases h from obstacles [193]. In Table 12 overview of numerical works carried out related to unsteady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

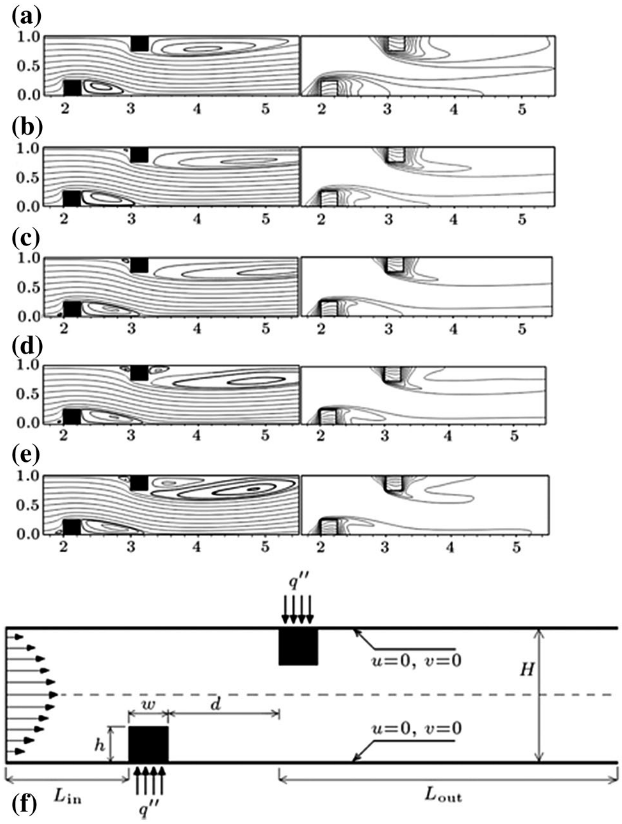


Fig. 21 Variations in streamlines (left half) and isotherms (right half) for $A = 10$ for *Re* **a** 200, **b** 400, **c** 600, **d** 800, **e** 100 and **f** geometrical configurations of channel with small obstacles [193]

3.2.2.1 Conjugate Drying of Porous Bodies in a Channel Oliveria and Haghighi studied conjugate convective drying of wood medium in a channel using Luikov's equation for porous bodies. Influence on heat and mass transfer of convective fluid due to variations of same in wood is thoroughly investigated employing commercial code FIDAP [194]. Murugesan et al., performed conjugate analysis of drying brick placed in a channel. Moisture content and temperature were predicted, and this result show faster drying of leading edge than other locations. They found different results compared to one dimensional results of boundary layer approximations. *Nu* and Sherwood number were also investigated to see analogy between heat and mass transfer. Free convection study was also done for low Re [195]. Bonis and Ruocco studied conjugate drying of food in a channel with prime focus of effect of air temperature on food. Time dependent heat and mass transfer study was based on finite-element formulation. Drying kinetics of the substrate (food) was also proposed and removal of moisture content was included. The conjugated drying nature of food was presented in details for the selected variables [196]. Lamnatou et al., performed CHT analysis of air flow in a channel having a porous body. FVM analysis was done using luikov's equation to find variations in temperature, moisture content, flow parameters and concentration for different flow rates. This study can be used in designing of drying chambers of agricultural products [197]. Lamnatou et al., investigated heat/mass transfer situation by varying relative position of pair of blunt plates. FVM was employed to analyze the flow parameters and temperature for various arrangements and moisture content. Tandem, staggered and side-by-side arrangements of plates (Fig. 22) were tried to obtain optimum arrangement in order to get maximum reduction in plate's moisture content and better drying. Effect of blunt geometry of plates on blockage between the channel walls and indirectly on heat/mass transport was studied in detail. Results obtained were verified experimentally. The fluid flow behavior was observed in detail for different arrangements of plates as shown in Fig. 23 [198]. Another very similar study using FVM approach on conjugate drying of porous medium was done by Lamnatou et al., for single and double plate configurations [199]. In Table 13 overview of numerical works carried out related to unsteady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

4 Experimental Studies on Conjugate Heat Transfer Analysis of Laminar Flow

Chiu et al., studied conjugate heat transfer experimentally (setup is shown in Fig. 24) of a heated section in a horizontal channel. Experimental data obtained was used to understand basic features of fluid flow and its related thermal transport and also to validate their numerical model of conjugate heat transfer. Experimental results offered physical understanding of effects of conjugate heat transfer [170]. Malik et al., experimentally investigated conjugate heat transfer in concentric cylindrical enclosure heated from bottom. By changing the disc temperature, outer diameter of the cylinder and material (Mild steel, aluminum and stainless steel) of the inner cylinder at the bottom, radial and axial temperature behavior of the enclosure was analyzed. The result for mild steel at different axial locations is shown in Fig. 25. Maximum temperature was found at the enclosure as least for aluminum at the inner cylinder and highest in case of stainless steel [200]. Cukurel et al., performed an experimental study on CHT analysis in small scale serpentine passage. Flat and ribbed slab shapes were analyzed using infrared thermography. Local temperature, *Nu* distributions were checked and found that their experimental procedure is well suited for CHT analysis when it was compared with previous studies [201]. Yusoff et al., performed experimental CHT

Table 12 Few details of steady state analysis carried numerically for internal flows

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/ technique	Type of thermal load	Optimal spacing study	References
2D	Long pipe	Horizontal	FDM	Fully implicit scheme	Step change in uniform wall heat flux	✗	[181]
2D	Long pipe	Horizontal	FDM	Fully implicit scheme	Constant temperature of pipe	✗	[183]
2D	Channel	Horizontal	FDM	Fully implicit scheme	Temperature of fluid	✗	[182]
2D	Grooved channel	Horizontal	Spectral element technique	Semi implicit three step temporal	Concentrated heat generation of channel	✗	[92]
2D	Grooved channel	Horizontal	Spectral element technique	Semi implicit time splitting scheme	Local and Uniform heat generation of channel material	✗	[184]
2D	Thick pipe	Horizontal	FDM	FDM	Step change in temperature of downstream outer section	✗	[186]
2D	Thick pipe	Horizontal	FDM	FDM	Step change in temperature of fluid downstream section	✗	[187]
2D	Rectangular	Horizontal	FVM	SIMPLE algorithm	Food temperature	✗	[185]
2D	Parallel plate channel	Horizontal	FDM	Fully implicit scheme	Inlet fluid temperature	✗	[189]
2D	Parallel plate	Horizontal	FDM	Full implicit scheme	Wall heat flux	✗	[190]
2D	Parallel plate	Horizontal	FDM	Implicit scheme	Periodical change of surface temperature	✗	[191]
2D	Pipe	Horizontal	FVM	ADI scheme	Uniform heat flux on outer pipe	✗	[192]
2D	Thick pipe	Horizontal	FDM	FDM	Step change in base heat flux	✗	[188]
2D	Channel	Horizontal	LBM	LBM	Uniform flux on channel	Performed optimum study of distance between obstacle	[193]

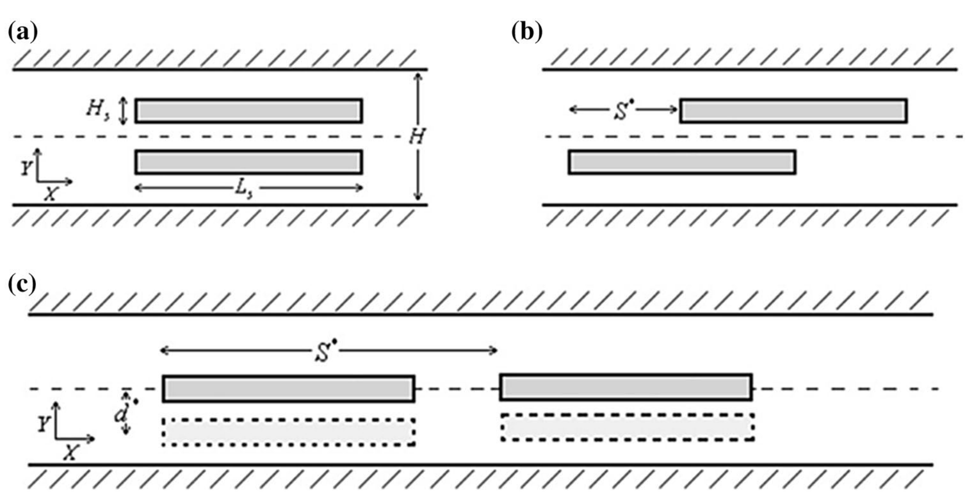


Fig. 22 **a** side-by-side, **b** staggered and **c** Tandem arrangements of plates [198]

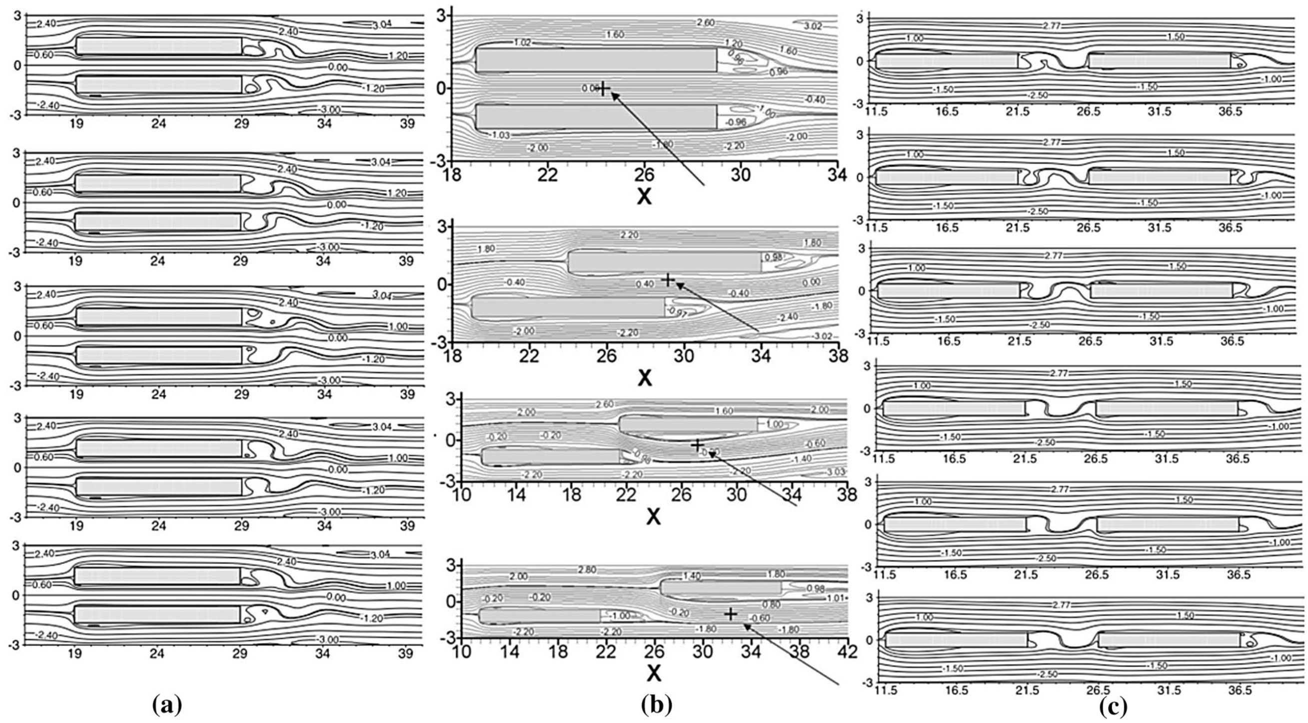


Fig. 23 Contours of flow and streamlines for different arrangements of plates at Re = 926 for **a** side-by-side, **b** staggered (arrow indicating monitoring point) and **c** tandem [198]

Table 13 Few details of steady state analysis carried numerically for drying of porous bodies

Dimension	Geometry	Orientation	Numerical method	Fluid discretization scheme/ technique	Type of thermal load	Optimal spacing study	Reference s
2D	Wood block	Horizontal	FEM	FEM	Temperature of wood	✗	[194]
2D	Brick	Horizontal	FEM	Galerkin's weighted residual method	Temperature of brick	✗	[195].
2D	Rectangular food plate	Horizontal	FEM	FEM	Air temperature	✗	[196].
2D	Rectangular	Horizontal	FVM	ADI scheme	Food temperature	✗	[197]
2D	Rectangular plate (3 configurations)	Horizontal	FVM	Stream function	Uniform heat flux on plate	✓	[198]
2D	Rectangular plate (2 configurations)	Horizontal	FVM	Stream function	Uniform heat flux on plate	✓	[199]

analysis for PCBs in laminar flow regime. Air velocities were considered in the range 1.51 to 5.12 m/s. Simulation was also performed using FLUENT software. Thermal performance was analyzed for the selected velocities of air and packaging (Plastic Leaded Chip Carrier (PLCC)) space [202]. Numerical CHT analysis of optimum spacing for heat sources was performed using numerical and experimental approach by Durgam et al., reported in [70]. In Table 14 overview of numerical works carried out related to unsteady state internal flows are mentioned. All works cited are related to two dimensional and internal flows. Work related to optimal spacing between the plates or channels is mentioned with tick mark (✓) if attempted else cross mark (✗) if not attempted.

5 Concluding Remarks, Challenges and Future Directions

The conclusions made from the above exhaustive literature review of conjugate heat transfer analysis of steady and transient state under laminar forced convection flow condition is divided into following heads.

Geometry/Physical model:

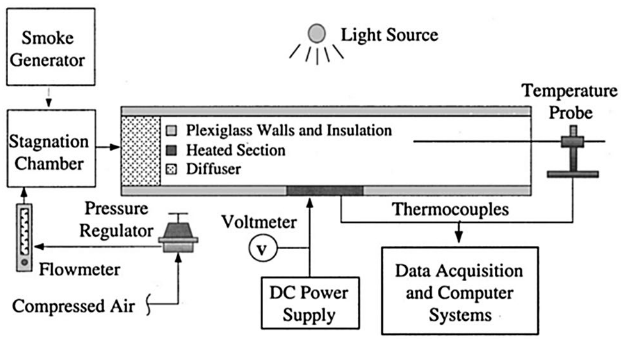


Fig. 24 Experimental setup used by Chiu et al., to study conjugate heat transfer in horizontal channel [170]

- In the above reported literature review, maximum number of researchers have used a two dimensional physical model for their investigation with more number of models positioned in horizontal direction and only few models were arranged in vertical and inclined position.
- In some of the limited findings, the three dimensional geometry is used and in some special cases the length of the plate is considered as infinite or assumed as continuously moving.
- Under transient state condition, large numbers of studies were related to pipe or grooved channel type of geometries.

Boundary conditions and other parameters:

- Boundary conditions pertaining to the conjugate heat transfer research works included in the present literature review have used constant/uniform heat flux and temperature conditions on the surface of the solid domain along with symmetric and asymmetric heating of plate/plates/channel/pipes. While no slip boundary condition is assumed at solid fluid interface.
- Only limited works have used the concept of thermal and hydrodynamic developing region. And few of the studies

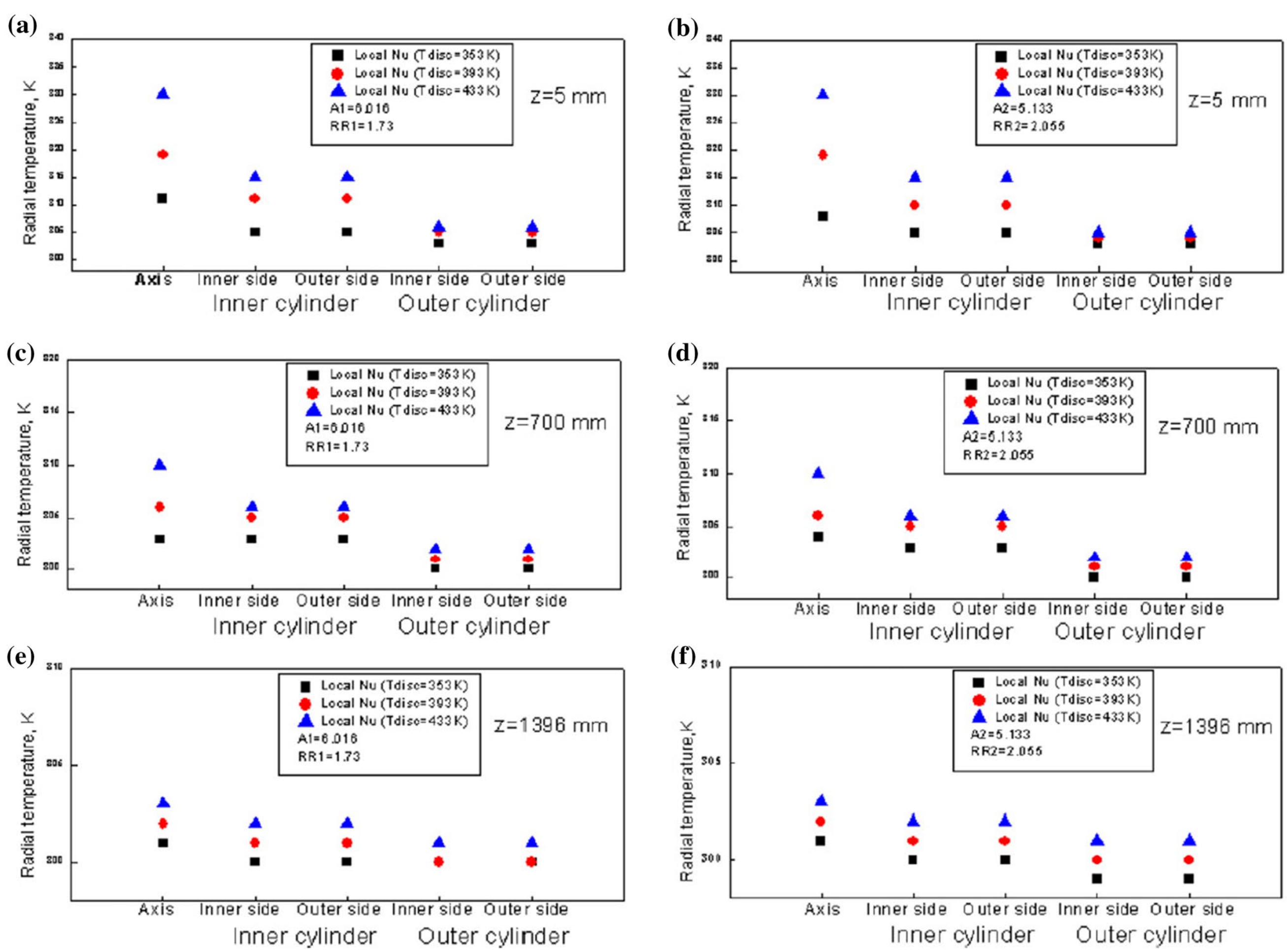


Fig. 25 Inner cylinder of mild steel having radial temperature distributions at a distance (axial) of **a** 5 mm, **b** 5 mm, **c** 700 mm, **d** 700 mm, **e** 1396 mm and **f** 1396 mm [200]

Table 14 Few details of experimental analysis of conjugate heat transfer

Type of study	Dimension	Geometry	Orientation	Type of flow	Type of thermal load	Optimal spacing study	Reference s
Steady	2D	Channel	Horizontal	Internal	Constant heat flux on channel	✗	[170]
Steady	2D	Concentric cylinders	Vertical	Internal	Constant heat flux at bottom of cylinder	✗	[200]
Steady	2D	Flat and ribbed channel	Horizontal	Internal	Uniform heat flux on channel	✗	[201]
Steady	3D	Channel	Horizontal	Internal	Heat source of PCB	Optimum packaging space study	[202]

have considered the effect of wall thickness on conduction in axial and transverse directions.

- Prandtl number, Nusselt number, Reynolds number, Biot number, Peclet number, Conduction and Convection parameter, thermal diffusivity ratio, Aspect ratio, wall thickness and wall conductivity ratio were mostly used non-dimensional entities by the investigators for steady and transient conjugate heat transfer analysis.
- Heat generation parameter was considered in heat sources, obstacles or protruding sources mounted over or under the plates/channel.

Methodology:

- Maximum numbers of studies reported in the present literature review have used Numerical approach to get the solution of the conjugate problem. While very less number of studies were illustrated in which either analytical or experimental approach is performed.
- Finite difference numerical formulation is widely used method by most of the researchers compared to the finite volume and finite element methods.
- Less number of authors have used the Boundary layer approximation equation while, maximum investigations made were based on full Navier Stokes equation.
- Stream function and vorticity formulation technique, and commercially available CFD code were used by few researchers and large numbers of researchers have used SIMPLE algorithm/ ADI technique for fluid domain.

Applications:

- The electronics cooling application area is the prime focus of most of the investigations, as seen in the above literature survey in which the problem of optimal spacing between the heat sources mounted on electronics circuit board or optimal spacing between the stacks/plates is generally not addressed.
- Less numbers of problems were addressed on conjugate heat transfer associated with application areas like nuclear fuel elements, solar absorber plates, Heat exchangers, Fins and Turbo machines.

Special cases:

- In only few studies the consideration is given to viscous effects, Non Newtonian fluid, and Power law fluid along with slug flow conditions irrespective of steady state or transient conjugate heat transfer analysis.
- Areas like porous media and pump work needed for forced convection are rarely reported in the literature.

Challenges:

- Satisfying the boundary conditions like slip flow, continuity of heat flux and temperature at solid fluid interface is difficult to handle.
- Difficulty in dealing with complex and irregular geometries subjected to Non Newtonian fluid flow conditions.
- Achieving the complete thermal and fluid flow analysis in analytical and experimental methods is a tough task.
- Non uniform heat generation along with entropy affects in electronics devices is difficult to investigate under conjugate conduction-convection condition.
- Handling non-linear boundary conditions and dealing with temperature dependent solid and fluid properties like thermal conductivity, heat transfer coefficient, velocity, pressure etc. is an open challenge.
- Developing an indigenous code for analysis requires its validation. And making it more compatible and computationally fast is a real test of programming skill one has to develop.
- Availability of Numerical schemes to deal with the more numerical accuracy and complex geometries and flow condition is still a challenge.

Future scope:

- More focus may be given to the conjugate problems where the effect of radiation heat transfer plays a vital role.
- Biomedical application areas need much attention under conjugate heat transfer investigation such as blood flow in arteries and veins, pumping of blood by the heart, functioning of lungs and many more.
- Manufacturing process like casting, rolling, extrusion, blanking and stamping may be focused and investigated with conjugate condition.
- In high temperature application areas like boilers, nuclear reactors, manufacturing processes, turbines and many more should be analyzed with temperature dependent properties of materials under consideration to obtain better accuracy.
- Fluid-structure interaction is least paid attention in conjugate problems, hence demand researchers to work in this field using Finite Element Method.
- Optimization is an important feature to be considered ahead. Soft computing techniques like Genetic Algorithm, Fuzzy logic and others can be easily adopted to achieve optimization.
- Entropy generation minimization from thermodynamics aspect is rarely reported in conjugate studies. Future works pertaining to entropy generation in electronic devices will be useful for industrial applications.
- Conjugate phenomena occurring in batteries, solar absorber plate, fuel cells, dielectric plates is an important area where studies are not reported.
- More realistic transient conjugate heat transfer problems occurring in nuclear reactors (fuel rod and coolant), heat exchangers (tube wall and flowing fluid) and turbo machines (turbine blade surface and impinging fluid jet) are to be addressed.
- In case of internal flow problems discussed in literature, very few works are concerned with optimal spacing but none of the work is reported on critical spacing. Hence researchers may consider the problem of identifying the critical spacing in internal flows for better and efficient way of design and development of different devices in related areas of conjugate heat transfer.
- Experimental works reported in this field can be repeated for detailed analysis using available commercial software like COMSOL, Ansys Fluent, Star CCM, Star-CD, FloTHERM, LS Dyna (multiphysics), Flow-3D etc.
- Wide scope is available to study conjugate heat transfer process which play a significant role in many cases such as: during flow of molten metal in a mould during casting, flow of fluid over subsonic and supersonic bodies, fluid flow through isotropic and anisotropic porous media, during hot metal rolling and extrusion, in internal combustion engines, in battery panels used in electric car and many more.
- Coupled heat transfer problems need more attention in the field of turbo machines as the flow may be either single phase or multiphase with slug conditions.
- Complex and highly nonlinear transient heat transfer situations and fluid flow problems occurring in superheater and economizer can be addressed.
- Conjugate heat transfer phenomenon occurring in some devices with two-phase flow condition as in heat pipes, thermo syphon and vapour chamber can be considered by the researchers in future.
- Parallelization of FVM codes can be achieved as they consume more time to obtain faster results.

Compliance with Ethical Standards

Conflict of interest The authors declare that there is no conflict of interest.

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