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ORIGINAL PAPER

Structural Dynamic Model Updating Techniques: A State of the Art Review

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Abstract This paper presents a review of structural dynamic model updating techniques. Starting with a tutorial introduction of basic concepts of model updating, the paper reviews direct and iterative techniques of model updating along with their applications to real life systems. The main objective of this paper is to review the most widely applied model updating techniques so that beginners as well as practising engineers can appreciate, choose and then utilize the most suitable model updating technique for their customized application. Another objective is to highlight the current issues, applications and observations for further advancements in the field of model updating.

1 Introduction

Use of thin and light-weight products in modern day machines and structures is increasing day by day. Therefore better dynamic testing and analysis tools are becoming the urgent need of hour. In the automotive, aircraft and spaceship engines, there is an ever existing demand of attaining better fuel economy; which can be met to a good extent by using thin products as well as with the use of light weight materials such as aluminium and plastics composites instead of the conventionally used heavy weight materials such as steels. Particularly in the case of satellites, some parts are so thin that they can get collapsed just due to their own weight if tested under the effect of gravity. Thin and light weight products have lot more tendencies to vibrate than their thick and heavy weight counterparts. Excessive vibrations can even result in pre-mature failure of products, for example, whether it is the suspension of an automobile, wing of an aircraft, the printed-circuit-board installed in a spaceship, blades of an air-cooler, or the compact-disc of a computer etc. On the other hand, consumers of today's world desire for non-vibrating and silent functioning of such products. Thus it becomes very important for engineers to understand the vibration behavior of structures through their dynamic analysis. Dynamic analysis aims at understanding, evaluating, analyzing and modifying (if required) the structural dynamic behavior which can be represented by many terms such as natural frequencies, eigenvalues, eigenvectors, damping ratios, Frequency Response Functions (FRFs) etc. The dynamic analysis of structures can be done through either experimental route or by using theoretical approach [77, 33].

The theoretical route involves the formation of an analytical model of the system either using a classical method [46] or through Finite Element (FE) method [91]. The application of classical method is generally limited to simple systems only, while FE method is preferred for real life complex systems. However FE method is not able to predict the dynamic responses of structures with complete accuracy due to the presence of certain errors in the FE model. Such errors are inherent in an FE model due to following reasons:

1. Faulty boundary conditions
2. Incorrect values of material properties
3. Discretization of continuum or a poor quality mesh
4. Difficulty in modeling complex real life shapes

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5. Assumptions for simplification purpose such as considering damping on a linear basis instead of non-linear
6. Incorrect modeling of joints
7. Use of rounding off methods in computations

Thus there is a need to correct an FE model so that its vibration behavior matches with the actual dynamic response obtained experimentally as shown in Fig. 1. The procedure used to update the model is called FE model updating (FEMU) [39]. In FEMU techniques, the experimental and FE responses are first compared and correlated so as to ensure that the FE model under consideration qualifies for further updating procedure and a conceptually new FE model is not required. A number of graphical as well as numerical comparison and correlation techniques are used for this purpose. Further to avoid the incompatibility in the sizes of experimental and FE data sets the use of size compatibility techniques is required. In FEMU, experimental results are considered as targets and the inputs of FE model are adjusted in such a way that the outputs of FE model have a better match with their experimental counterparts.

A number of researchers have proposed the various structural model updating techniques both by classical approach and FE route. While going through the available literature on this area, a need is felt to summarize all the results and conclusions made by different researchers. Therefore, this paper is an attempt to provide a review of major research activities carried out in the model updating field.

In this paper, first the basic theoretical issues related to FEMU procedure have been discussed. After which, a state of the art review of direct and iterative techniques of FEMU has been presented. Application areas of FEMU have also been discussed. The final part of the paper discusses the current problems and future directions for FEMU related research.

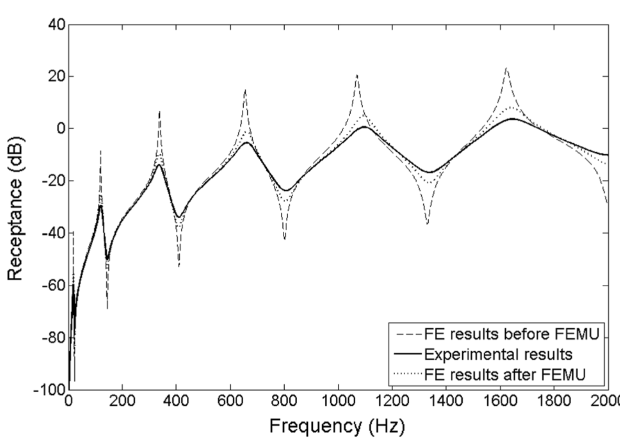


Fig. 1 Comparison of experimental, initial analytical and updated analytical results

2 Finite Element Model Updating (FEMU) Procedure

2.1 Finite Element Method

In FE method, a complex continuous region of a structure is discretized into simple geometric shapes called finite elements. Figure 2 shows a cantilever beam of length 910 mm, width 50 mm and thickness 5 mm. The cantilever beam is divided into 60 finite elements. Each element has two nodes. At each node, two degrees of freedom are measured, out of which one is the displacement in y-direction and the other is the rotation about z-axis. Both the degrees of freedom of node number '1' are fully constrained.

The finite elements can be axial elements, torque elements, beam bending elements, thin plate bending elements, thick plate bending elements, etc. Approximate results of displacement shapes and stress fields can be obtained for such finite elements using shape functions. Continuity across element boundaries can be maintained using either displacement or energy approaches. The displacement approach makes use of equilibrium, compatibility and the constitutive laws; while the energy approach is based on the principal of virtual work. In energy approach, the internal work is equated to external work. For dynamic analysis purpose, each element needs to be expressed in form of elemental mass, stiffness and damping matrices. Equations (1) and (2) represent the elemental mass and stiffness matrices for a beam element.

$$[m_e] = \frac{\rho A a}{105}\begin{bmatrix} 78 & 22a & 27 & -13 \\ 22a & 8a^2 & 13a & -6a^2 \\ 27 & 13a & 78 & -22a \\ -13a & -6a^2 & -22a & 8a^2 \end{bmatrix} \quad (1)$$

$$[k_e] = \frac{EI}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ 3a & 4a^2 & -3a & 2a^2 \\ -3 & -3a & 3 & -3a \\ 3a & 2a^2 & -3a & 4a^2 \end{bmatrix} \quad (2)$$

where m_e, ρ, A, a, k_e, E and I are respectively the element mass matrix, density, area of cross-section of element, half-length of element, element stiffness matrix, Young's modulus of elasticity and moment of inertia of cross-section of beam element. Subsequently, the individual elements are assembled to form their global counterparts, which are also jointly known as system matrices. These system matrices along with certain boundary conditions are used to formulate a set of governing equations, which are then processed on a computer to evaluate dynamic characteristics of the system.

From above discussion it can be concluded that the dynamic response of the structure depends upon a number

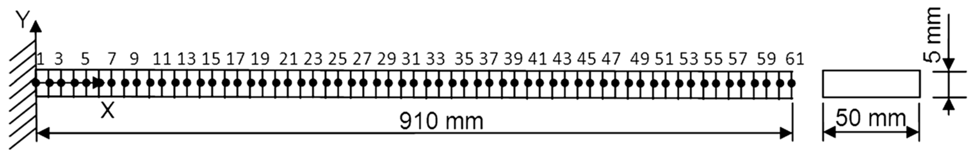


Fig. 2 FE model of a cantilever beam structure

of parameters. These parameters are called input parameters which affect the dynamic response parameters. These input parameters may be belonging to material, structural, finite element or computational categories. The various input parameters are assembled in the form of cause and effect diagram and are shown in Fig. 3.

Dynamic response is generally measured in terms of eigenvalues, eigenvectors, and FRFs. The frequency at which a structure naturally vibrates once it is set into motion is called its natural frequency. Square of the natural frequency is known as eigenvalue. Eigenvector or mode shape of a particular mode defines the displacement configuration of structure at corresponding natural frequency. First six modeshapes of a cantilever beam structure are as drawn in Fig. 4. Extreme and intermediate positions of different points of a cantilever beam are drawn in Fig. 5.

FRF (*i*, *j*) is defined as ratio of harmonic response at location '*i*' and the harmonic excitation force at location '*j*'. If the measured response is in the form displacement, then corresponding FRF is called as receptance (or admittance or dynamic compliance or dynamic flexibility) FRF. Otherwise, velocity or acceleration response signals can be used to produce mobility or accelerance FRF respectively. Accelerance FRF is also sometimes written as inertance FRF. Further any FRF is called as a point or transient FRF depending upon whether the response and force are measured at same or different locations respectively. Point and transient Receptance FRF have been drawn in Fig. 6a, b respectively.

2.2 Experimental Method

In FEMU, the experimental results can be obtained by two ways: simulated experimental results and real life

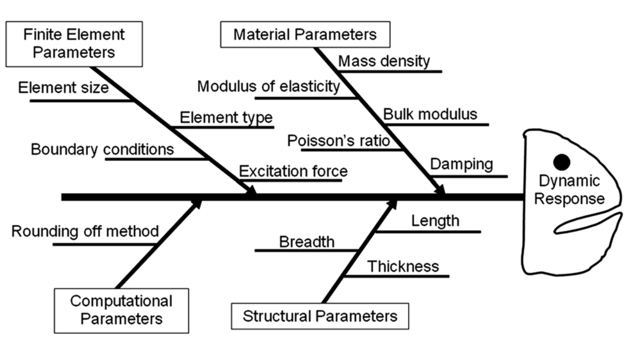


Fig. 3 Cause and effect diagram

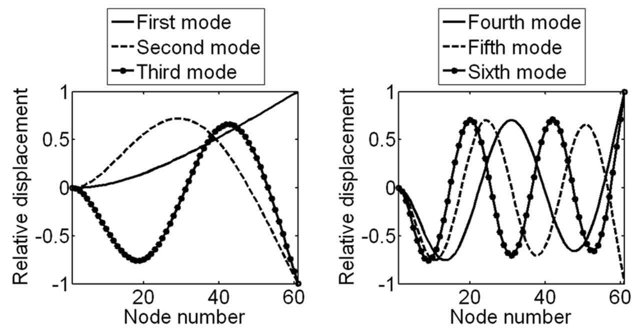


Fig. 4 Modeshapes of cantilever beam structure. **a** First, second and third modeshapes, **b** Fourth, fifth and sixth modeshapes

experimental results. Former type of results are generated by varying some parameters of the FE model thereby producing a perturbed FE model, whose simulation results are assumed as experimental results and are surely different from the dynamic results of original FE model. Real life experimental results can be generated through modal testing.

A representative experimental set-up for finite element model updating of a structure is shown in Fig. 7. One end of the test structure can be fixed to a non-vibrating base through welding, bolting or riveting etc. thereby resulting in grounded support condition. Test structure can also be supported on very soft springs or light elastic bands, which leads to a free support condition. In such situation, it is important to attach the suspension as close as possible to nodal points of the particular mode. Another method of supporting the structure is in situ type, where the test structure is connected to some other component which presents a non-rigid attachment. Input to the test set-up generally includes a short impulse or, a sine wave excitation over a frequency range of interest, or a white noise. This force input is provided by either a shaker, or an impulse hammer thereby resulting in forced vibration and free vibration cases respectively. Structure can also be excited by suddenly releasing it from a deformed position. Selection of any particular type of shaker depends upon its frequency range, acceleration, rms (root-mean-square) velocity, rms displacement, maximum load, operating temperature range, power supply, overall dimensions and mass. Impulse hammer is selected based upon its force range, frequency range, sensitivity, time constant, operating temperature range, power supply, mass, diameter and length of head. Output of the experimental set-up is

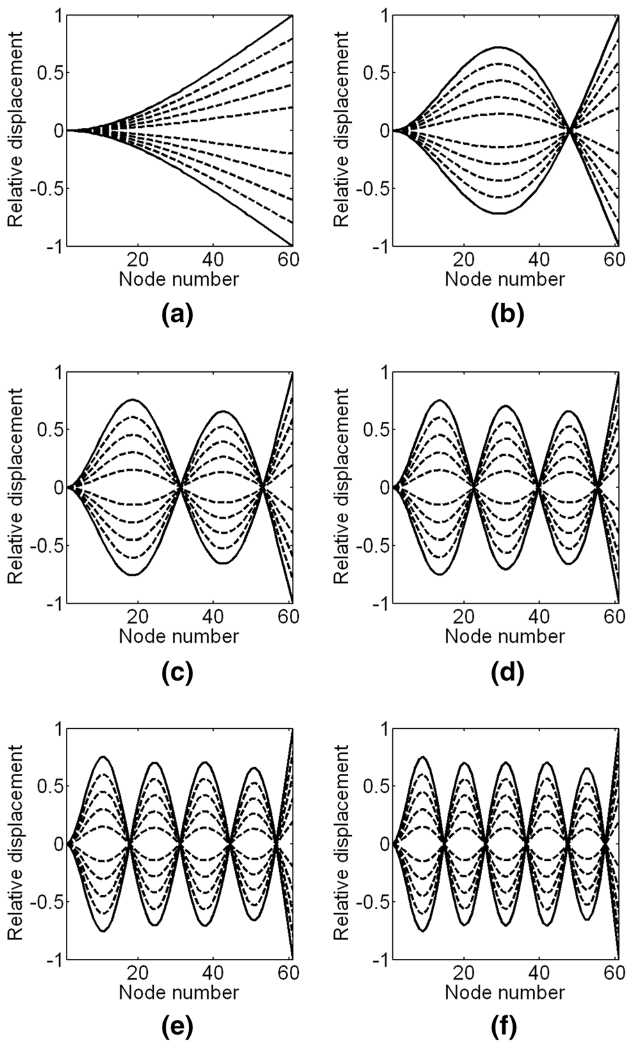


Fig. 5 Extreme and intermediate positions drawn using solid and dashed lines respectively. **a** First mode, **b** second mode, **c** third mode, **d** fourth mode, **e** fifth mode, **f** sixth mode

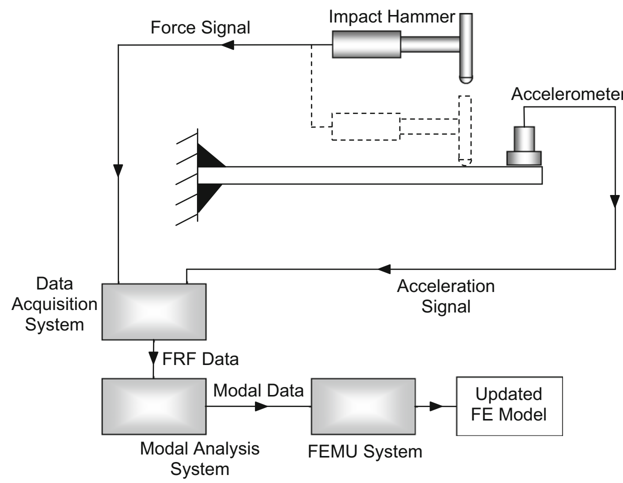


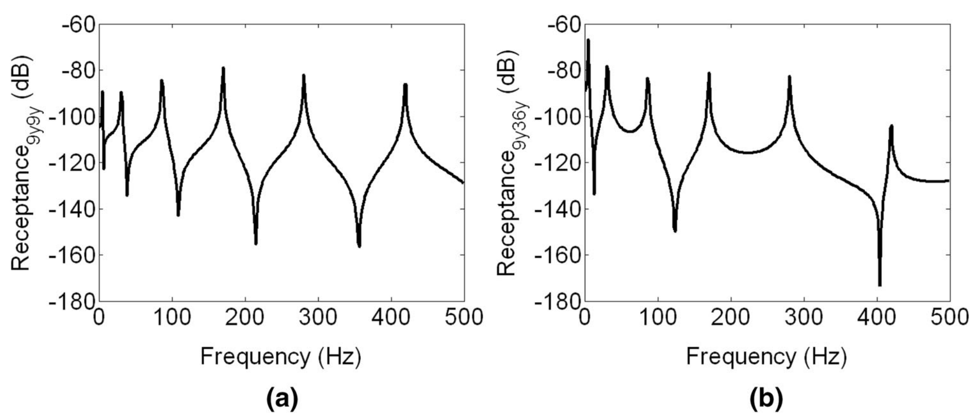
Fig. 7 Representative experimental set-up for FEMU

generally in the form of a function of acceleration, which is measured at a number of points using sensors like accelerometers etc. Selection of such sensors is generally based upon their characteristics such as mass, acceleration range, threshold acceleration, sensitivity, frequency range, and temperature sensitivity, operating temperature range and power supply. Signal from accelerometer is then conditioned using a signal conditioner, which is selected depending upon its sensor excitation current, sensor excitation voltage, frequency range, output voltage, gain, power supply, operating temperature range, overall dimensions and mass. Conditioned signal is then passed on to Fast Fourier Transform analyzers so that the time domain signal can be converted to frequency domain. Such frequency domain data is then analyzed on a computer to identify the experimental dynamic response of the test structure. Experimental dynamic response is then compared with its FE counterpart to examine any errors and also to ascertain sufficient degree of correlation between the two as discussed in next section.

2.3 Comparison and Correlation Techniques

Before applying any FEMU technique, the experimental and FE data sets need to be compared so as to ensure

Fig. 6 Receptance FRF. **a** Point FRF, **b** transient FRF



existence of some correlation between experimental and FE responses and also to determine that whether it is worth to update the proposed FE model or a completely new model is required. These techniques include comparison of FRFs, natural frequencies and mode shapes (Modal Scale Factor (MSF), Modal Assurance Criterion (MAC), Normalised MAC and Coordinate MAC (COMAC) etc.). These techniques are discussed as:-

2.3.1 *Comparison of FRFs*

A simple method of comparing experimental and FE results is to plot experimental and FE FRFs on a single graph as shown in Fig. 8. A visual comparison of FRFs is done to find out presence of any correlation between the experimental and FE results. For example, Fig. 8a shows a clear correlation between the experimental and FE results. This is the case where FE model updating is required to increase the correlation further, while Fig. 8b shows the case of total mismatch between the measured and predicted results. Under such conditions, instead of FE model updating, a conceptually new FE model is required.

2.3.2 *Comparison of Natural Frequencies*

It is important to compare the natural frequencies obtained from FE analysis of the structure with the ones acquired through modal testing as shown in Table 1. If the percentage errors are small, one can update the FE model of the structure in order to minimize such errors. But if the errors are very large, say, more than 200 %, then it is not advised to update the model rather a new formulation of the FE model is required.

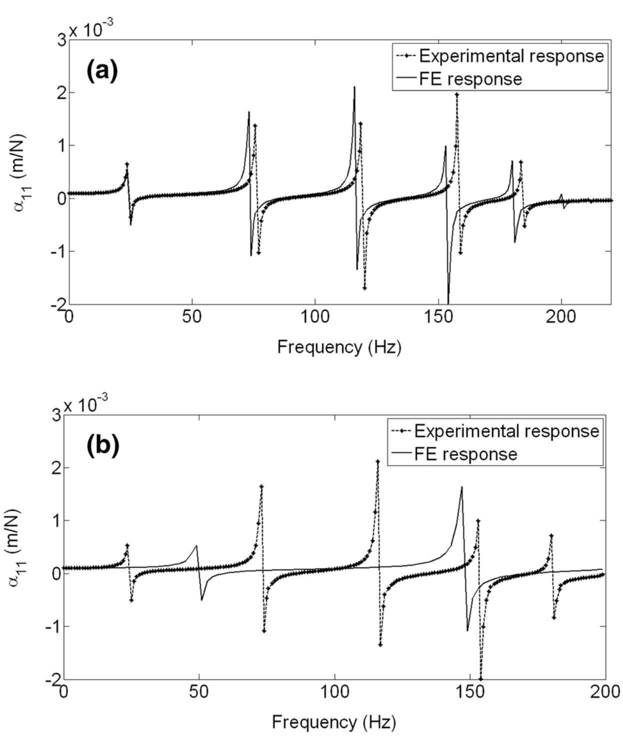


Fig. 8 Comparison of FRF plots. **a** Good correlation, **b** poor correlation

2.3.3 *Comparison of Modeshapes*

Mode shape portrays the pattern or configuration in which the structure vibrates at any particular natural frequency. Mode shapes are inherent properties of a structure. These do not depend on the forces or loads acting on the structure. These will change if the material properties (mass, stiffness, damping properties), or boundary conditions (supports) of the structure change. A simple method for comparing mode shapes is through overlaying technique as shown in Fig. 8. By this method, the difference between experimental and FE mode shape is drawn on a graph. In this method the difference between two corresponding mode shapes should approach zero. Main limitation of such graphical methods is that they are not so supportive if the correlation process is to be automated. For automatic correlation of modes, we need some quantitative (numerical) measure of correlation, which can be easily implemented through a computer program. One such measure is MSF, which is slope of the best straight line through the modes plotted on an x–y graph with experimental mode on one axis and analytical on other [2]. Desired value of MSF is unity for good correlation. MSF between experimental and analytical mode shapes can be obtained using Eqs. (3) and (4); where $\{\emptyset_X\}_i$ and $\{\emptyset_A\}_j$ represent the ith experimental and jth analytical mode shape respectively; while superscript 'T' denotes the transpose of the corresponding vector.

$$\mathrm{MSF}\left(\{\emptyset_X\}_i, \{\emptyset_A\}_j\right) = \frac{\{\emptyset_X\}_i^T\{\emptyset_A\}_j}{\{\emptyset_A\}_j^T\{\emptyset_A\}_j} \tag{3}$$

$$\mathrm{MSF}\left(\{\emptyset_A\}_j, \{\emptyset_X\}_i\right) = \frac{\{\emptyset_A\}_j^T\{\emptyset_X\}_i}{\{\emptyset_X\}_i^T\{\emptyset_X\}_i} \tag{4}$$

Main drawback of MSF is that it does not provide any information regarding the scatter of the x–y plots. To avoid this problem, it is recommended to use MAC. This criterion is also known as mode shape correlation coefficient [72]. It can be calculated using Eq. (5).

$$\mathrm{MAC}\left(\{\emptyset_X\}_i, \{\emptyset_A\}_j\right) = \frac{\left|\{\emptyset_X\}_i^{\mathrm{T}}\{\emptyset_A\}_j\right|^2}{\left(\{\emptyset_X\}_i^{\mathrm{T}}\{\emptyset_X\}_i\right)\left(\{\emptyset_A\}_j^{\mathrm{T}}\{\emptyset_A\}_j\right)} \tag{5}$$

MAC is a measure of scatter of points from the straight line correlation. MAC value equal to unity means perfect

Table 1 Comparison of natural frequencies

Mode no.	Experimental (Hz)	FE (Hz)	Difference in frequencies (%)
1	24.1	24.3	0.8
2	76.1	73.1	3.9
3	119.3	116.1	2.7
4	158.0	153.9	2.6
5	184.1	180.7	1.8
6	211.6	200.5	5.2

correlation, while a MAC value equal to zero means no correlation between two modes. Figure 9a shows a typical MAC plot between perfectly correlated experimental and FE modes. It is seen that all the diagonal elements are equal to unity and off-diagonal elements are nil, which means perfect correlation between experimental and FE modes. A poor correlation case is shown in Fig. 9b, where none of the diagonal element is unity; and off-diagonal elements are also not zero.

One major limitation of MAC is that it cannot identify the systematic deviations, which can be overcome to some extent by combining MAC with the best fit straight line plots described earlier. Another limitation of MAC is that it is not a true orthogonality check because of absence of mass or stiffness matrix in the formula of MAC. This limitation can also be avoided by using normalized MAC (NMAC) [72]. The NMAC is also called as normalized cross orthogonality as represented by Eq. (6).

$$\mathrm{NMAC}\left(\{\emptyset_\mathrm{X}\}_\mathrm{i},\{\emptyset_\mathrm{A}\}_\mathrm{j}\right) = \frac{\left|\{\emptyset_X\}_i^\mathrm{T}[\mathbf{W}]\{\emptyset_A\}_j\right|^2}{\left(\{\emptyset_X\}_i^\mathrm{T}[\mathbf{W}]\{\emptyset_X\}_i\right)\left(\{\emptyset_A\}_j^\mathrm{T}[\mathbf{W}]\{\emptyset_A\}_j\right)} \tag{6}$$

NMAC includes a weighting matrix [**W**], which can be replaced by either mass matrix or stiffness matrix. NMAC has a limitation of not describing the spatial distribution of correlation, for which purpose, one can use Coordinate MAC (COMAC) as given in Eq. (7) [70] as:-

$$\mathrm{COMAC}(k) = \frac{\sum_{l=1}^{L}\left|(\emptyset_X)_{kl}(\emptyset_A)_{kl}\right|^2}{\sum_{l=1}^{L}(\emptyset_X)_{kl}^2\sum_{l=1}^{\mathrm{L}}(\emptyset_A)_{kl}^2} \tag{7}$$

COMAC should be calculated only after finding '*L*' number of the correlated mode pairs through MAC or NMAC. COMAC value close to '1' indicates good correlation at a particular coordinate say '*k*'. The COMAC plots showing good and poor correlation are drawn in Fig. 10a, b respectively.

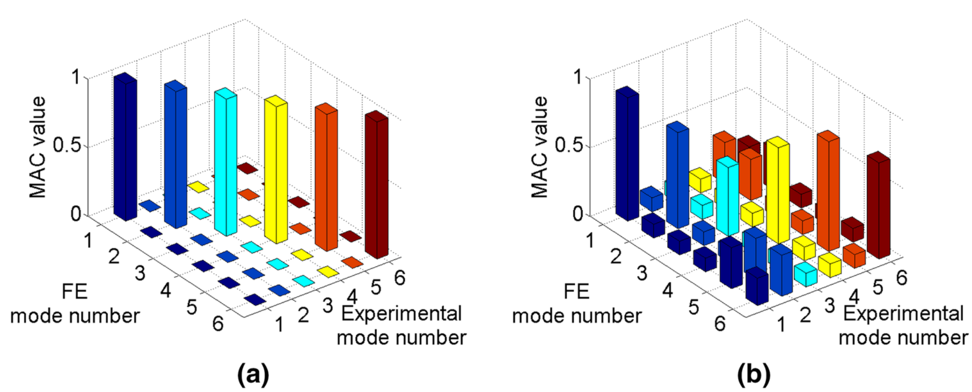
MAC or its variants consider mode shape matching at only discrete coordinates. If one wants to consider the mode shape correlation on a continuous basis, then image processing and pattern recognition based Zernike moment descriptor can be used [107]. Further, a scanning laser vibrometer can also be used for 3D digital image correlation [48].

After comparison and correlating the experimental and FE data sets, next step is to make these compatible with each other so that FEMU can be performed. Compatibility techniques have been explained in next sub section.

2.4 Size Compatibility Techniques

Measured degrees of freedom are generally far lesser than their FE counterparts due to limited number of sensors used in experimental set-up. Moreover some degrees of freedoms are very difficult to be measured (e.g. rotational degrees of freedom or those corresponding to the points which are physically inaccessible). Thus the size of FE

Fig. 9 Modes correlation using MAC. **a** Good correlation, **b** poor correlation



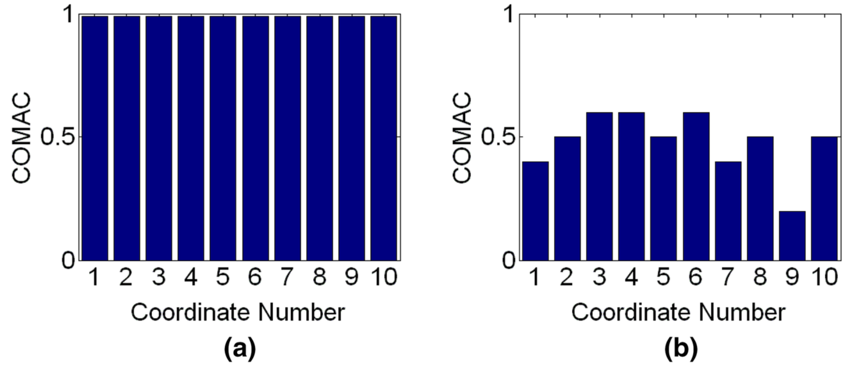


Fig. 10 Modes correlation using COMAC. **a** Good correlation, **b** poor correlation

modal matrices is generally not compatible with their experimental counterparts, while in FEMU one-to-one correspondence between the two results sets is required; which can be achieved by either expanding the experimental results or by reducing the FE model as explained in following sections.

2.4.1 Coordinate Expansion

Measured data sets can be expanded so that they are of the same size as their FE counterparts. Such a process is called as coordinate expansion. A simple method of coordinate expansion is to substitute the unmeasured coordinates by their FE counterparts. This method is computationally very efficient. Main disadvantage of this method is that sometimes it leads to erroneous solutions during FEMU. To avoid this problem, some transformation matrix based coordinate expansion should be done as used in Eq. (8); where 'N' and 'n' are the sizes of expanded and measured eigen-vectors respectively.

$$\{\emptyset_{\text{expanded}}\}_{N\times 1} = [T]_{N\times n}\{\emptyset_{\text{measured}}\}_{n\times 1} \tag{8}$$

For an undamped system, the transformation matrix [T] can be obtained by writing the governing equations of the system in partitioned matrices form using Kidder's method [60]. In Kidder's method, sub-matrices are related to measured and unmeasured eigenvectors. For a damped system, partitioned matrices based method can be extended to deal with complex measured modes [47]. Another method of coordinate expansion is based on the concept that unmeasured eigenvectors can be expressed as a linear combination of measured eigenvectors [90]. System equivalent reduction and expansion processes (SEREP) [89], Curve fitting [110] and FE eigenvectors in conjunction with MAC matrix based methods are also used in coordinate expansion [71].

2.4.2 Model Reduction

Model reduction process is basically the inverse of the expansion process. This process is generally applied to an analytical model so that the size of the FE model matrices can be reduced and brought closer to their experimental counterparts. A simple method of model reduction is to eliminate those degrees of freedom (DOF) which are not available in experimental data. Model thus obtained is known as reduced model. Main drawback of this method is that mass and stiffness terms related to the eliminated DOF are completely lost and nowhere compensated [33].

Another method of model reduction is to produce a condensed model which will represent the entire structure completely but approximately. In this method a condensed model is obtained by transforming the original model as given in Eq. (9); where 'n' and 'N' are the sizes of reduced and finite element model based eigen-vectors respectively.

$$\{\emptyset_{\text{reduced}}\}_{n\times 1} = [\mathrm{T}]_{n\times N}\{\emptyset_{FE}\}_{N\times 1} \tag{9}$$

One method of obtaining condensed model is through static reduction [45], in which size of system matrices is reduced by neglecting inertia and stiffness terms associated with unmeasured DOFs. This method is generally known as Guyan reduction method. Main drawback of static reduction method is that it does not reproduce any of the eigenvalues or eigenvectors of the original full FE model. Another method of model reduction is improved reduced system (IRS) method which can closely reproduce the eigenvalues and eigenvectors of the original FE model [88, 40]. If reduced model is required to have eigenvalues and eigenvectors exactly same as that of original FE model then SEREP should be used [89, 97].

3 Review of FEMU Techniques

The FEMU techniques can be broadly classified into two categories:-

- direct (non-iterative) techniques and
- iterative techniques

Direct techniques can provide the solution to model updating problem in just a single step; hence these are computationally very efficient, and divergence related problems do not occur. Another important feature of direct techniques is that they reproduce the measured data exactly. Therefore measurement noise and spurious modes are also reproduced by such techniques. Thereby, these techniques required very good quality modal testing and analysis procedures. Direct techniques compute a closed-form solution for system matrices using the structural equations of motion and orthogonality properties of modes. These techniques are also called matrix techniques, because such techniques found the solution in the form of updated system matrices by solving a set of matrix equations. The main drawback of direct techniques is that updated mass and stiffness matrices may not be symmetric and positive definite. It becomes very difficult to understand such kind of system matrices on a physical basis.

Dynamic response of FE model of any structure is a function of a number of parameters, as explained in Sect. 2.1. The iterative techniques compute updated values of material and structural parameters of FE model in such a way that during each iteration mismatch between experimental and FE response is reduced. These techniques are also called as gradient based techniques. Iterations are stopped when the values of updating parameters stop converging or the error function is reduced to tolerable level. Error function is generally a non-linear function of experimental and FE responses such as eigenvalues, eigenvectors or FRFs. Iterative techniques result in only symmetric and positive definite updated system matrices, which can be easily understood on a physical basis. But these techniques require a number of iterations before arriving at the final result. Thus iterative techniques are computationally less efficient and divergence related problems can also arise during iterations. The technology and research developments reported for both direct and iterative techniques are discussed in following subsections.

3.1 Direct Techniques of FEMU

Some researchers have developed direct techniques which consider adjustment of elements of system matrices on a mathematical basis rather than a physical basis. A few researchers have also worked for development of such direct techniques, which can consider adjustment of physical properties of system matrices. The direct techniques are further classified into two categories, viz. matrix elements adjustment based direct techniques and physical property adjustment based direct techniques.

Foremost, in 1978, Baruch and Bar-Itzhack developed a direct method of FEMU by assuming FE mass matrix to be correct [11]. They updated the FE eigenvectors and FE stiffness matrix by minimizing the weighted norm of difference between measured and FE eigenvectors subject to orthogonality constraints. During same time, Baruch also developed a direct method in which the FE stiffness matrix and FE eigenvectors are updated in such a manner that some weighted norm of the difference between the updated and analytical stiffness matrices is minimized using Lagrange multipliers [10]. In this method, the FE mass matrix is assumed correct and hence not updated. Later in 1979, Berman adopted the mathematical approach of Baruch [10] and proposed a direct method for updating the mass matrix [14]. This method also included an additional constraint to preserve the symmetry of updated matrices. These techniques [11, 10, 14] are also called as techniques of reference because one of the three quantities (eigenvectors, mass and stiffness matrices) is assumed to be reference and the other two are updated.

In 1983, Berman and Nagy proposed another method in which the basic approach of the method of Baruch [10] and the method of Berman [14] were combined to update both mass and stiffness matrices of the system in a sequential manner [15]. Method of Berman and Nagy has been used by various researchers in a number of applications. Application of this method was also studied by Modak et al. [83] for dynamic design of a fixed–fixed beam and an F-structure. Bais et al. [9] applied the direct techniques of Baruch [10] and also of Berman and Nagy [15] for dynamic design of a drilling machine. Dhandole and Modak [30] performed the FE model updating of vibro-acoustic cavities using the direct method of Berman and Nagy [15]. They investigated a simulated example of a two dimensional rectangular cavity with a flexible surface having structural modeling errors related to the material property, geometry and boundary conditions; under incomplete and noisy simulated experimental data. Effectiveness of the method was compared with an iterative method on the basis of accuracy of prediction of vibro-acoustic natural frequencies and the frequency responses both inside as well as outside the frequency range taken during updating. It was concluded in the study that the direct technique resulted in an accurate prediction of the vibro-acoustic natural frequencies and the response inside the frequency range considered during model updating. However, beyond the updating frequency range, the predictions based on the direct updated vibro-acoustic models are not as accurate as given by the iterative technique. Further the direct technique seems to be quite versatile compared to its iterative counterpart for dealing with complex cavities. This is due to the reason that for complex cavities, complete knowledge of modeling inaccuracies and selection of updating parameters becomes very difficult. Recently, Modak [81] also developed a direct vibro-acoustic model updating technique using modal test data. This technique can be used for updating the vibro-acoustic FE model of such systems that involve an elastic structure enclosing a medium, like air. By using this technique, mass and stiffness matrix of both the structural as well as the acoustic parts of the model can be updated by preserving their symmetry.

The method of Berman and Nagy [15] was further extended by Ceasar [19] by including additional constraints related to preservation of the total mass of system and the interface forces. Matrix perturbation concept was used by Chen et al. [26] to simultaneously update mass and stiffness matrices of the system. Based upon first order approximation Sidhu and Ewins [99] developed a direct method using error matrix approach. This method was useful for only small modeling errors. Kabe [57] suggested a direct method which could retain the bandwidth of system matrices by identifying the null elements of original system matrices and constraining them to remain zero during FEMU.

Further, a range of direct updating techniques were formulated by Caesar [20] considering different objective functions, constraints and whether mass matrix or stiffness matrix is updated first. Later, Lim [73] proposed a FEMU method in which sub-matrix-scaling factors are used as updating parameters. Due to the use of sub-matrix-scaling factors number of unknowns is reduced appreciably. Smith and Beattie suggested a quasi-Newton method for stiffness updating which preserve structural connectivity and can also handle noisy modal data [101, 13]. Application of this method was studied later by Ramamurti and Rao [95] by using different finite elements. They updated the FE models of a rectangular plate with holes using 3-D plate elements; a crane using 3-D beam elements and a rectangular plate using brick elements. Bucher and Barun [17] developed modal data based direct method having the capability to deal with partially known experimental eigensolutions. The method was verified by applying it to numerical examples of a clamped beam and a serial spring-mass system. Farhat and Hemez [37] proposed a FEMU method using an element-by-element sensitivity methodology and demonstrated the potential of the method by using several simulation examples. Later, Friswell et al. [41] developed a method for simultaneous updating of damping and stiffness matrices assuming the mass matrix to be correct. Singular value decomposition and matrix approximation technique based method was proposed by Xiamin [111]. This method is particularly useful if the mismatch between experimental and FE responses is very large. Recently, Carvalho et al. [21] developed a method which can identify and prevent the reproduction of spurious modes into updated results. The direct techniques discussed so far mainly aim at removing the mismatch between experimental and FE responses, without bothering for any adjustment of physical properties of system matrices. Presently research is also oriented towards development of direct techniques which aim at reducing the mismatch between experimental and FE responses by adjusting the physical properties of the FE model.

In 2007, Hu et al.[49] proposed the cross-model-cross-mode method (CMCM) in which adjustment of physical properties of the system matrices is carried out to update the mass and stiffness matrices simultaneously. Satisfactory performance of the method was demonstrated by applying it for simulated examples of a shear building model and a three-dimensional frame structural model. During the demonstration, some preset error coefficients were introduced into the FE model of the structures. Then the CMCM method was used to estimate the correction factors of FEMU. Main disadvantage of the method is that it requires the measurement of spatially complete modal data, which is generally not possible for large-scale real life structures.

Further, in 2011, substructure energy approach (SEA) based physical property adjustment type direct method was developed by Fang et al. [34]. Main benefit of this method is that it can handle spatially incomplete experimental modal data also as opposed to the CMCM method. In this method the complete system is divided into several sub systems. Instead of updating the complete system, only the critical sub systems are identified and updated using a set of linear simultaneous equations deduced from the energy functional of substructure models and substructure modes. For validation purpose, they applied the method for updating the models of a mass-spring system, a two-dimensional and a three-dimensional lattice structure using simulated experimental data. Results obtained by Fang et al., for numerical example of a three-dimensional lattice structure showed that the method worked satisfactorily for FEMU purpose. Jacquelin et al. [53] developed a direct probabilistic model updating technique that takes into account the uncertainties related to experimental results by using the random matrix approach. Recently, Jiang et al. [56] reduced the model updating process to the problem of the best approximation type. This technique is successfully used for numerical examples of undamped systems. However the technique needs to be tested further for damped simulated systems as well as for actual experimental results.

Major contributions related to direct techniques of FEMU have been presented in a brief chronological format in Table 2.

3.2 Iterative Techniques of FEMU

Algorithm for iterative techniques of FEMU is drawn in Fig. 11. In such techniques an error function is minimized iteratively to find the updating parameters or correction factors. These techniques originated in 1974, when Collins et al. proposed the eigendata sensitivity based iterative

Table 2 Major contributions in direct techniques of FEMU

References	Major Contribution
Baruch and Bar-Itzhack [11]	Updating of stiffness matrix
Berman [14]	Updating of mass matrix
Berman and Nagy [15]	Updating stiffness, mass matrices in sequential manner
Ceasar [19]	Improvement of Berman and Nagy method by preservation of total mass and interface forces
Chen et al. [26]	Use of matrix perturbation concept for simultaneous updating of mass and stiffness matrices
Sidhu and Ewins [99]	Error matrix approach based direct updating technique to handle small modeling errors
Kabe [57]	Constraint on retaining bandwidth of system matrices
Caesar [20]	Formulation of a number of direct techniques
Lim [73]	Sub-matrix scaling factors based updating technique
Smith and Beattie [101]	Updating stiffness matrix with fix structural connectivity
Bucher and Braun [17]	Updating for incomplete experimental modal data
Farhat and Hemez [37]	Element-by-element sensitivity methodology based updating technique
Friswell et al. [41]	Simultaneous updating of damping and stiffness matrices
Ramamurti and Rao [95]	FEMU using different finite elements
Xiamin [111]	Matrix approximation technique based FEMU method
Modak et al. [83]	Dynamic design of fixed–fixed beam and F-structure using the method of Berman and Nagy
Bais et al. [9]	Dynamic design of drilling machine using method of Baruch and also Berman and Nagy
Carvalho [21]	FEMU method which can identify and prevent the reproduction of spurious modes into updated results
Hu et al. [49]	Physical property adjustment based cross-model-cross-mode method
Dhandole and Modak [30]	FEMU of vibro-acoustic cavities using the method of Berman and Nagy
Dhandole and Modak [31]	Application of method of Berman and Nagy in vibro-acustics field
Fang et al. [34]	Substructure energy approach based physical property adjustment type direct method
Jacquelin et al. [53]	Development of probabilistic model updating
Jiang et al. [56]	Development of direct updating technique using best approximation method
Modak [81]	Updating of vibroacoustic finite element models using modal test data

method also known as inverse eigensensitivity method (IESM) [27]. Later, Chen and Garba [25] used matrix perturbation technique for iteratively computing the eigensolution and eigendata sensitivities. This method was further improved by Kim et al. [62] by including second order sensitivities. Further the convergence of this method was improved by Lin et al. [75] by employing both the FE and the experimental modal data for evaluating sensitivity coefficients. This also helped in application of the method for cases of large errors.

In IESM, modal data such as eigenvalues, eigenvectors and damping ratios are used to form an error function. Modal data is obtained through modal analysis of measured FRFs. If modal analysis is not done carefully then the extracted modal data may contain certain errors, which is further transmitted to the results of FEMU. To avoid this problem one can use the measured FRFs directly for model updating using Response Function Method (RFM) as proposed by Lin and Ewins [76]. RFM does not require any modal extraction to be performed on measured data thereby eliminating the chances of errors due to modal analysis being transmitted to updated results. Application of RFM for dynamic design of a fixed–fixed beam and an F-structure was discussed by Modak et al. [83]. Effectiveness of IESM and RFM has been compared by Imregun et al. [52], Modak et al. [84]. For incomplete experimental data case (where experimental eigenvector is not completely known) with no noise, RFM works better than IESM, while latter performs better in the presence of noise particularly when the updating range covers a greater number of modes. Moreover, in RFM, if the number and location of testing frequencies are not selected properly then the method may not be able to converge as reported by Modak et al. [84] based upon their comparative study using simulated experimental data.

While formulating the basic RFM, Lin and Ewins [76] had not taken into account damping. Later, Arora et al. [3, 4] proposed two techniques of extending the basic RFM in

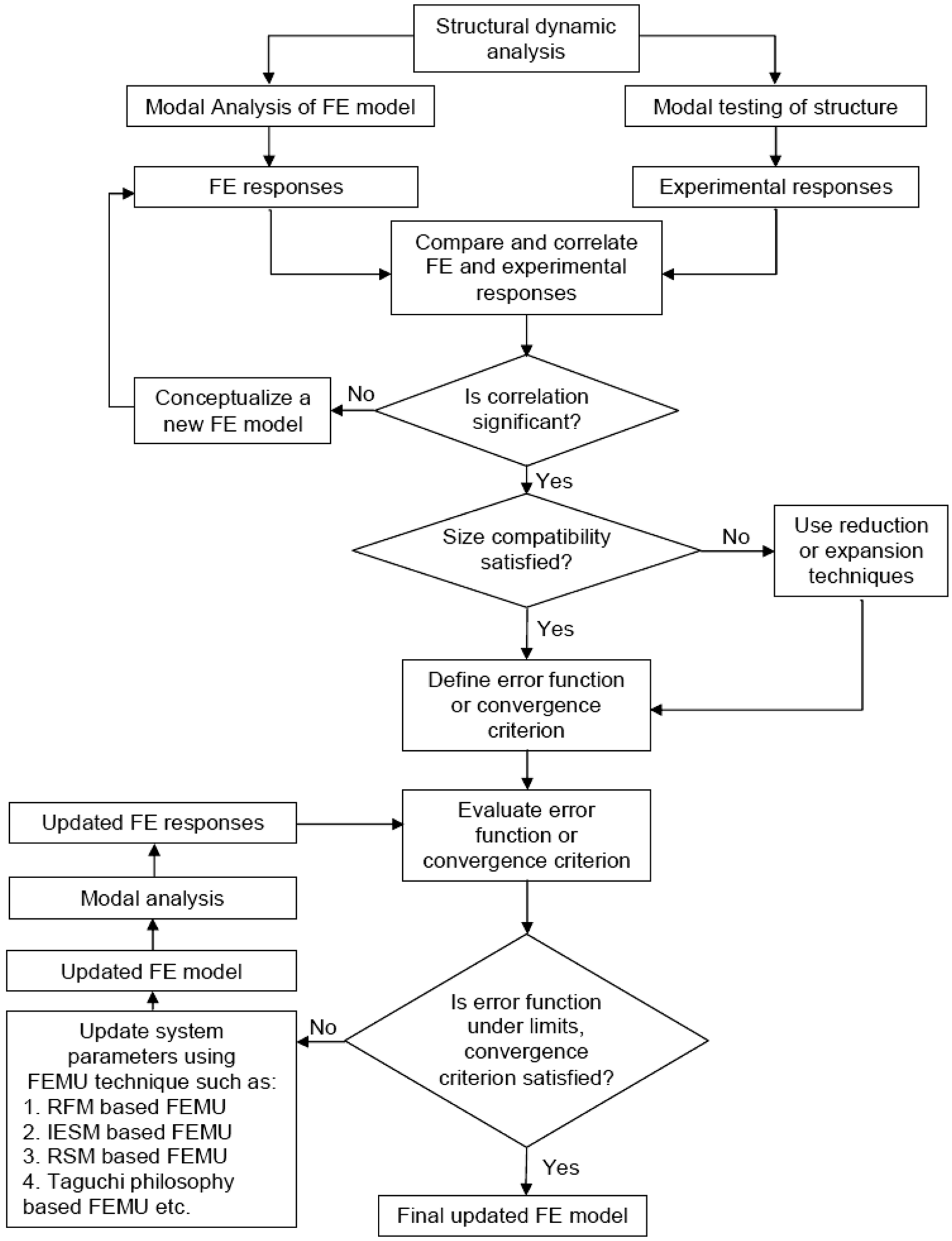


Fig. 11 Algorithm for iterative techniques of FEMU

order to widen the applicability of the method by considering damping. First proposed technique was to combine RFM with damping identification method of Pilkey [92]. This technique involved a two step procedure in which first step involved updating of only mass and stiffness matrices while damping matrix was obtained in second step by using the mass and stiffness matrices found in first step.

Second proposed technique by Arora et al. [4] was to consider the model parameters and system matrices in a complex form so as to deal with complex modes of damped structures. Later they compared these two techniques and found that complex parameter based FEMU technique gives better results than the one with damping identification [5]. Arora et al. [6] also worked on the use of complex parameters based RFM of FEMU for dynamic design purposes. Arora [7] also compared the accuracy of basic RFM [76] against the direct method Bernam and Nagy [15] and found that the performance of basic RFM was superior to the direct method in terms of the accuracy in prediction of FRFs. Recently, in 2012, the basic RFM [76] was further extended by Pradhan and Modak [93] to develop a normal RFM that was based on the estimates of normal FRFs computed by using only stiffness and mass matrices. Normal RFM was later used by Pradhan and Modak [94] for damping matrix identification using FRF experimental data.

In 2000, Modak et al. [82] developed a constrained optimization based FEMU technique. They evaluated the performance of the said technique by applying it to a fixed–fixed beam structure. This technique was computationally more intensive than IESM but was able to address the difficulty that might arise due to large difference between the sensitivities of natural frequencies and mode shapes. Later, they also used the constrained optimization based FEMU for dynamic design of fixed–fixed beam and F-structure [85]. Use of this technique for better FE formulation of acoustic cavities was done by Dhandole and Modak [32].

Neural network (NN) method has also been applied for FEMU by Atalla and Inman [8] for FE model updating of a flexible frame. In this method, a NN is trained and validated using FE responses. Thereafter this NN can be employed to obtain updated physical parameters by taking experimental responses as inputs. Once the NN model is properly trained, the NN based calculations are relatively fast compared to conventional optimization techniques regardless of the complexity of the real life structure. NN based model updating method is also robust to the noise present there [67]. Main limitation of this method is that it requires a large number of training data ‘qp’ (where ‘p’ is the number of updating parameters and ‘q’ is the number of levels or values which any updating parameter can take). However this problem can be circumvented by using an orthogonal array method in which the number of training data sets are reduced to $p(q-1)+1$ only [23, 16].

Because of the experimental limitations, if only natural frequencies (or eigenvalues) are measured then FEMU can also be performed by using reduced order characteristic polynomial (ROCP) based method as proposed by Li [69]. Li applied the ROCP based method for FE model updating of a beam structure elastically constrained at one end. In this method a polynomial is defined in terms of the measured eigenvalues, FE eigenvalues, FE eigenvectors and updating parameters. Assuming the measured natural frequencies to be the roots of the polynomial, a set of non-linear equations are derived. This set of non-linear equations is further solved for calculating the values of the updating parameters.

Sometimes, it becomes difficult to measure FRFs accurately due to small size or delicate nature of test structure [like a hard disk drive (HDD)]. Under such conditions Model Updating using Base Excitation (MUBE) method can be applied as proposed by Lin and Zhu [74]. They used MUBE method for FEMU of a cantilever beam and a truss structure, and showed that the method works satisfactorily. In this method, base of the structure is excited with an unknown force input using an electric shaker and the displacement output measured from the test set-up is used for model updating. This method is of great use when excitation force is not known or difficult to measure. Jamshidi and Ashory [55] compared and found that MUBE gives better updating results than RFM for incomplete experimental data with measurement noise.

Further the problem of model updating for a real life structure is quite complex due to presence of non-linearity, damping, measurement errors and large number of updating parameters. Under such conditions, the traditional model updating techniques may fail to converge or may get stuck in local minima rather than finding a global minimum of the error function. In that kind of situations, it is advisable to use such optimization techniques which are capable of finding the global optimum results even for a complicated optimization problem. Few such promising techniques are Simulated Annealing (SA), Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). Levin and Lieven [68] used and compared SA and GA for FEMU and observed that SA based FEMU method performs better than its GA based counterpart. Success of both SA and GA based FEMU techniques depends upon appropriate choice of updating parameters. If updating parameters are not selected properly, then both these techniques give unsatisfactory results. Marwala [79] applied the PSO method in the field of FEMU. PSO is a population based stochastic search algorithm derived from social-psychological behavior of biological entities (birds, fish, ants, etc.) when they are foraging for resources (food). It is particularly useful if the number of updating parameters is very large. Further, Mthembu et al. [87] used PSO for selecting the best model from amongst a number of updated structural models.

Kwon and Lin [64] suggested a robust FEMU technique using the concept of Taguchi method and were able to obtain good FEMU results. They used both frequency as well as modal data for formulation of the objective function. This technique was reported to be robust against various noises because the parameters were updated in such a way that signal to noise ratio is maximized.

It is to be mentioned here that due to their iterative nature, the iterative techniques seem to be computationally inefficient, particularly, if applied to FE models of large size structures. Working on these directions Guo and Zhang [44] suggested the use of response surface methodology (RSM) based FEMU. In this technique, FE calculations are not required at each iteration step, thereby making it computationally efficient. In this method, n-dimensional response surfaces are created by taking updating parameters as inputs and dynamic FE responses as outputs. Updated values of model parameters are found by using the response surfaces and the measured responses by minimizing the error function. RSM based FEMU is reported to be as accurate as the sensitivity based method, and, moreover it is more robust and computationally efficient than the sensitivity based FEMU.

Later, Ren and Chen [96] applied RSM as well as sensitivity based FEMU techniques to a precast concrete bridge and compared the convergence of objective function by both the techniques. Their study showed that for a given number of iterations, the objective function of FEMU is reduced to a lower level by using the RSM based FEMU technique than its sensitivity based counterpart. In traditional RSM based FEMU technique, the experimentally measured signals were first transformed to one or more response parameters such as natural frequencies. This reduced the information in the training data used for developing response surface models. In order to circumvent this problem, time domain based results were used by Shahidi and Pakzad [98] for RSM based FEMU technique. Time domain based technique was helpful in extracting more information from measured signals and compensate for the error present in the meta-models. Efficiency of RSM based FEMU technique was further increased by Chakraborty and Sen [22] whilst developing an adaptive RSM based FEMU technique by replacing the least square method with the moving least square method.

It is important to mention that FE model of a real life structure contains a large number of parameters. If all the parameters are taken as updating parameters, then the model updating problem becomes too complex and time consuming. A large number of updating parameters also result in ill-conditioning of the problem or trapping in many local minima [61, 39]. To identify and select only a few important updating parameters, Fissette et al. [38] proposed a force balance method. This method can be used for error location and then updating parameters are selected from only the regions of modeling errors. Waters [108] proposed a modified force balance method in which it is assumed that the regions of response errors are not necessarily an indication of modeling error. Recently, Kim and Park [61] developed an automated parameter selection procedure. This is a two phase method, wherein, during first phase an updating parameter is assigned to each erroneous FE. Then two neighboring updating parameters are merged if their sensitivity (of response w.r.t. parameters) is of same sign thereby reducing the number of updating parameters. This is repeated until all the neighboring updating parameters have opposite sign of sensitivity. If the number of updating parameters is acceptable then the procedure can be stopped; otherwise one can move on to the second phase. In second phase those two neighboring parameters of opposite sign are found and grouped together which will result in least reduction in total sensitivity. This method was validated by applying it to the FE model of cover of a HDD having 1115 FEs and 6732 dofs. Kim and Park used the force balance method of Fissette and Ibrahim for error localization purpose and found that out of a total of 1115 FEs only 628 FEs contained the modeling errors. After the application of first stage of automatic parameter selection procedure, Kim and Park successfully reduced the number of updating parameters to 150; which was further drastically reduced to 20 using the second phase of their suggested procedure.

Moreover the updating parameters are generally the physical parameters such as thickness, density, modulus of elasticity, damping, Poisson's ratio etc. of structure. These parameters do not have a single discrete value throughout the body of a real life object. Spatial distribution of such parameters was considered by Adhikari and Friswell [1] by expressing the updating parameters as spatially correlated random fields.

If a number of identical test structures are taken, all may not have same value of a particular material parameter viz. modulus of elasticity, density etc. This variability in seemingly identical test pieces may arise due to many sources such as geometric tolerances, manufacturing processes etc. Thus each material parameter has a stochastic nature with a mean value and a variance. Such stochastic nature of material parameters was taken into account during FEMU by Mares et al. [78] using a simulated example and obtained quite satisfactory results. In this method, the mean value was represented by the centre of the scatter ellipse while the size and orientation of the ellipse was determined by variance of test data. The updated analytical scatter ellipse overlays quite closely the experimental scatter ellipse. Same work was then extended by Mottershead et al. [86] by carrying out stochastic FEMU for a set of physical structures.

The stochastic FEMU based approach is based on probabilistic models, which require large number of tests to be conducted and also the large volumes of test data; which demands for high experimental as well as computational efforts. In order to get rid of the large quantities of test data, one can opt for the approach that was used by Khodaparast et al. [59] for performing interval FEMU in which ranges or distributions of updating parameters are found rather than just one 'true' value of any updating parameter. Table 3 presents the major technological developments and advancements in the field of iterative techniques of FEMU.

4 Applications of FEMU Techniques

FEMU is widely applicable in the field of better FE model formulations, damage analysis of structures, non-destructive characterization of material properties and also for dynamic design purposes. With the aim of better FE model formulations, FEMU has been used in a number of applications such as aircrafts, satellites, automobiles, nuclear power plants, rotor bearing systems, laser spot welds, bridges, dams, multi-storey buildings, steel frames, truss structures, acoustics, etc. It also finds applications in

Table 3 Major contributions in iterative techniques of FEMU

References	Contribution
Collins et al. [27]	Development of inverse eigensensitivity method (IESM)
Chen and Garba [25]	Use of matrix perturbation technique for improved computation of sensitivities of IESM
Kim et al. [62]	Improvement of IESM by Including second order sensitivities
Fissette et al. [38]	Selection of updating parameters using force balance method for FEMU
Lin and Ewins [76]	Development of response function method (RFM)
Lammens et al. [65]	Application of FEMU in structural optimization of a tennis racket
Imregun et al. [52]	Comparison of IESM and RFM based FEMU techniques
Lin et al. [75]	Improvement of convergence of IESM
Waters [108]	Development of modified force balance method for error localization in FEMU
Levin and Lieven [68]	Development and comparison of simulated annealing and genetic algorithm based FEMU techniques
Atalla and Inman [8]	Development of neural networks based FEMU technique
Levin and Lieven [67]	Robust FEMU technique using neural networks
Cunha and Piranda [28]	Application of FEMU in identification of elastic constants of composite materials
Wahab et al. [106]	Application of FEMU in damage analysis of reinforced concrete beams
Modak et al. [82]	Development of constrained optimization based FEMU technique
Chang et al. [23]	Improvement of neural network based FEMU technique using orthogonal array method
Li [69]	Reduced order characteristic polynomial (ROCP) method
Sinha et al. [100]	Application of FEMU in dynamic design of spacer locations of tubes of nuclear power plants
Modak et al. [83]	Application of RFM for dynamic design of a fixed–fixed beam and an F-structure
Modak et al. [84]	Comparative study of RFM and IESM
Butkewitsch and Steffen [18]	Application of FEMU in design synthesis of a heavy truck side guard
Bais et al. [9]	Application of iterative method for dynamic design of a drilling machine
Guo and Zhang [44]	Development of response surface method (RSM) based FEMU technique
Kwon and Lin [63]	Development of frequency selection method for FRF based FEMU
Hwang and Kim [51]	Improvement of computational aspect of FEMU based damage detection technique
Teughels and Roeck [103]	Application of FEMU in damage identification of highway bridge
Modak et al. [85]	Application of constrained optimization based FEMU technique for dynamic design of fixed–fixed beam and F-structure
Marwala [79]	Development of particle swarm optimization based FEMU technique
Kwon and Lin [64]	Development of Taguchi based FEMU technique
Vepsa et al. [105]	Application of FEMU to a feed water pipeline of a nuclear power plant
Zhou and Farquhar [113]	Application of FEMU in finding the in vivo material property of wheat stem
Mottershead et al. [86]	Application of stochastic FEMU technique to a set of physical structures
Decouvreur et al. [29]	Study of effect of dispersion error on acoustic FEMU
Jaishi and Ren [54]	Modal flexibility residual based FEMU technique
Lin and Zhu [74]	Development of model updating using base excitation (MUBE) method
Gondhalekar et al. [43]	Application of FEMU for dynamic design of a complex automobile structure
Kim and Park [61]	Development of automated parameter selection procedure for FEMU
Zapico et al. [112]	Application of neural networks based FEMU to a steel frame
Jamshidi and Ashory [55]	Comparative study of MUBE and RFM based FEMU techniques
Arora et al. [3]	Development of damped FEMU technique by combining RFM with a damping identification method
Arora et al. [4]	Development of damped FEMU technique by implementing RFM using complex parameters technique
Arora et al. [5]	Comparative study of RFM based damped FEMU techniques
Arora et al. [6]	Dynamic design of F-structure using complex parameters technique based RFM
Husain et al. [50]	Application of FEMU to laser spot welds
Fang and Perera [35]	Application of RSM based FEMU in damage analysis of a bridge
Dhandole and Modak [30]	Application of direct method of Berman and Nagy for vibro-acoustic FEMU
Weber et al. [109]	Consistent regularization of nonlinear FEMU in damage identification

Table 3 continued

References	Contribution
Mthembu et al. [87]	Application of particle swarm optimization in selection of updated FE models
Ren and Chen [96]	Application of RSM based FEMU technique to a concrete bridge
Adhikari and Friswell [1]	Development of spatially distributed FEMU technique using spatially correlated random fields
Dhandole and Modak ([31]	Comparative study of vibro-acoustic FEMU techniques
Chellini et al. [24]	Application of FEMU in damage analysis of a steel–concrete composite frame
Arora [7]	Comparative study of FEMU techniques
Lepoittevin and Kress [66]	Use of FEMU for modal damping prediction
Khodaparast et al. [59]	Development of interval FEMU technique using the Kriging predictor
Mottershead et al. [86]	Application of sensitivity method for FEMU of helicopter airframe
Goller et al. [42]	Application of stochastic FEMU for complex aerospace structures
Bayraktar et al. [12]	Application of FEMU to arch dam systems
Khanmirza et al. [58]	Application of FEMU to multi-storey shear buildings
Fang and Perera [36]	Application of RSM based FEMU in damage identification of reinforced concrete frame
Dhandole and Modak [32]	Acoustic FEMU using constrained optimization technique
Pradhan and Modak [93]	Development of normal RFM
Pradhan and Modak [94]	Application of RFM in damping matrix identification
Shahidi and Pakzad [98]	Development of time domain results based model updating
Chakraborty and Sen [22]	Development of moving least square method based model updating

damage analysis of rotor blades of helicopters, bridges, multi-storey framed concrete buildings, reinforced concrete frames, reinforced concrete beams, etc. It has also been used successfully for non-destructive characterization of elastic constants of composites, longitudinal moduli of living wheat, structural damping, etc. FEMU has also been used for dynamic design of automobiles, drilling machines, spacers of nuclear power plants, F-structures, beams and tennis racket etc. The flow chart shown in Fig. 12 also represents the applications of FEMU in a number of disciplines.

5 Research Issues in FEMU

There are conflicting opinions of various researchers regarding use of objective functions and test structure used in FEMU. Future research needs to be directed towards standardizing a benchmark objective function and test structure, so that these can be used worldwide by different researchers for comparing the effectiveness of different FEMU techniques.

It is also a well known fact that damping is present in all real life structures. But still there are many model updating techniques in which damping effects of a real life structure have been neglected [8],69, 68, 96, 104, 80, 102]. Such techniques need to be investigated so as to increase their ability to deal with damped real life systems.

Existing FEMU techniques aim towards mapping only the dynamic behavior of the structure. More efforts are required for developing such FEMU techniques which can predict accurately the dynamic as well as static behavior of structures.

More research is required in the development of realistic updated mass matrices by combining the FEMU techniques with digital image processing and x-ray/ultrasonic images. 3-D digital image processing has been used for mode-correlation purpose, but its use for objective function formulation still awaits more efforts.

Fuzzy logic, neural networks and their combination have been used, for model formulation purpose, in many such fields where system parameters are not understood properly. FEMU is a similar problem where the elements of large scale system matrices are not known exactly. Thus research can also be directed towards the application of such black-box based techniques in FEMU.

FEMU has been used to infer the in vivo material properties of crops. Still, more research efforts are required in the direction of application of FEMU for better modeling and characterization of bones, flesh and plant leaves etc.

6 Conclusions

In this paper a review of a number of direct and iterative techniques of FEMU has been presented. Direct techniques find little applications in industry due to the fact that

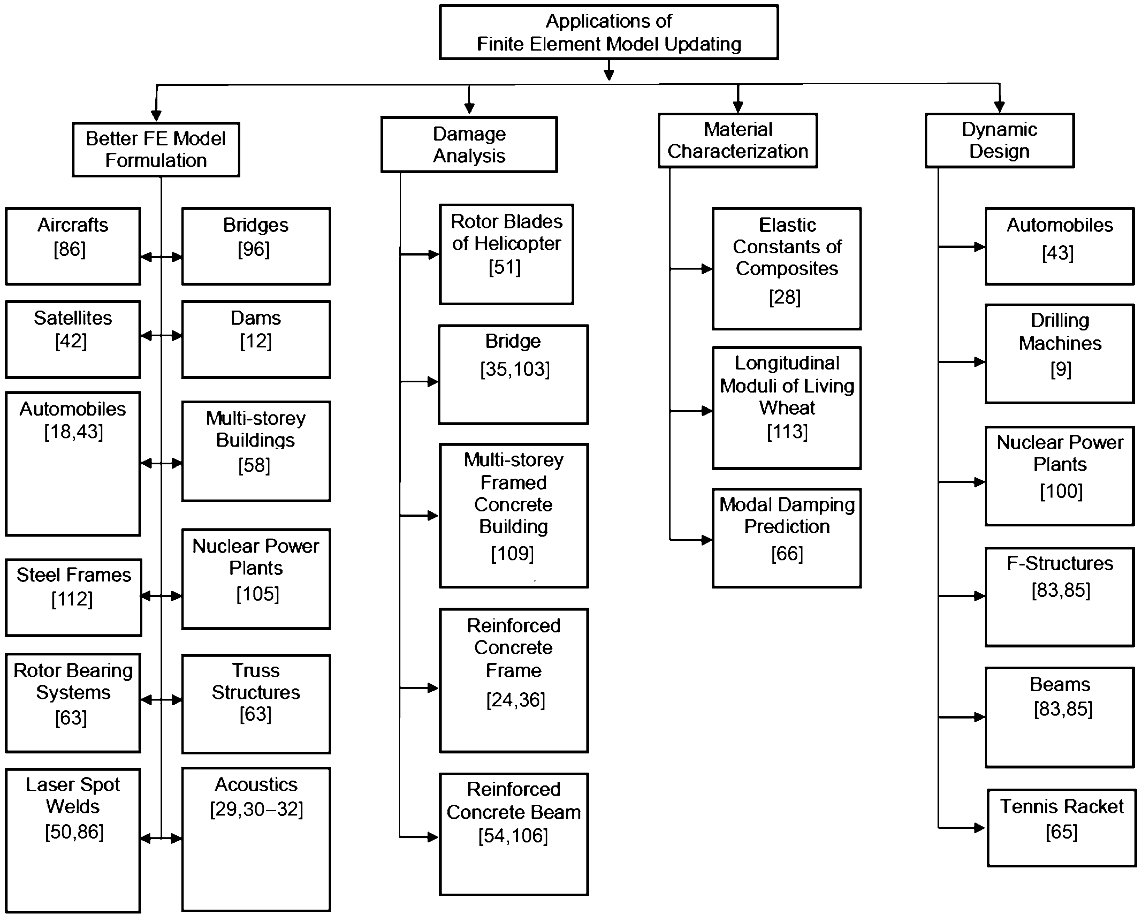


Fig. 12 Applications of FEMU

updated FE models of most of the direct techniques are difficult to be understood on a physical basis. While a few physical property adjustments based direct techniques are also now available in literature; their application to real life systems needs to be explored further. On the other hand, iterative techniques have received considerable attention from industrial application point of view and have been applied successfully in a number of fields such as aircrafts, automobiles, satellites, machine tools and civil structures etc. Further a number of future research directions have been highlighted which can be used for further advancements in the field of model updating.

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