# Agglomeration characteristics of nano-size TiO<sub>2</sub> particles using analytical solution

Jong-Sang Youn<sup>\*,\*</sup>, SeJoon Park<sup>\*\*,\*</sup>, Hyunwook Cho<sup>\*</sup>, Yong-Won Jung<sup>\*</sup>, and Ki-Joon Jeon<sup>\*,†</sup>

\*Department of Environmental Engineering, Inha University, 100 Inha-ro, Nam-gu, Incheon 22212, Korea \*\*Department of Industrial and Management Engineering, Myongji University, Myongji-ro 116, Cheoin-gu, Yongin-si, Gyeonggi-do 17058, Korea

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**Abstract**–We developed equations for nano-sized titanium dioxide ( $\text{TiO}_2$ ) particles self preserving time (SPT) lag that combines with agglomerate key parameters such as primary particle size (PPS), geometric standard deviation (GSD) and mass fractal dimension (MFD). A statistical formula has been developed that relies on SPT lag as the key parameter of agglomerates. Finally, this research presents the first analytical solution by integrating these key parameters into one formula, which can be utilized as a handy tool to calculate the time for reaching the asymptotic state.

Keywords: Mass Fractal Dimension, Self Preserving Time, Agglomerate, Nanoparticle, Primary Particle Size, Geometric Standard Deviation

### INTRODUCTION

Coagulation is one of the common processes that can be easily found in many advanced materials in diverse industry fields [1-4]. Particles are formed by gas-to-particle conversion under high temperature condition followed by coagulation which is composed of nanoparticle chains agglomerates [1,5]. Many studies have been conducted to figure out the synthesis process and the properties of particles, and many applications of nanoparticles are pouring out [2-4].

Experiments have been attempted to grasp Brownian coagulation with agglomerations. For instance, Matsoukas and Friedlander [6] showed that the rate of coagulation in the production of metal oxide particles increased with decreasing mass primary particle size (PPS) and fractal dimension. One example is that the PPS of commercially available TiO<sub>2</sub> (Degussa AG, TI 1234) agglomerates is between few nm to 20 nm in flame and diffusion reactors [1]. In a recent study, non-spherical titania agglomerates enlarged much faster than spherical agglomerates [7]. Then, non-spherical titania agglomerates moved to an asymptotic mass fractal dimension (MFD) while oxidation proceeded in high temperature [7]. Furthermore, the morphological transition of agglomerates was evaluated using Monte-Carlo simulations in many processes including diffusion [8], sintering and coagulation [9]. One study performed a stochastic simulation algorithm wherein the primary particle was observed in order to find its movement and determine the role of deterministic and stochastic forces on the particle [10]. Another study provided simulation results of agglomeration using computational algorithm [11]. Morphology of agglomerates was classified as reaction-limited and diffusion-limited with the proposed mechanism [12].

Aerosol dynamics models are another ones developed in order to figure out how the morphological structure affects the behavior of aerosol agglomerates. A variety of aerosol models are accessible to explain aerosol dynamics on the basis of mathematical delineation of particle size distribution, such as moment [13-15] and sectional models [16,17]. The evolving agglomerates were simulated starting from individual spherical particles, which assumed power law distribution and evaluated the influences of fractal dimension and PPS [18]. Self-preserving time (SPT) lag was provided by Vemury et al. [19] to calculate the time to reach the asymptotic size distribution. A correlation equation of the two was devised for SPT in the free molecular as well as continuum regimes in terms of initial geometric standard deviation (GSD). The simulations were conducted with a discrete sectional model. In a later study, MFD was integrated into the model and its effects were investigated [20].

The first analytical solution was proposed in order to trace the evolving size distribution of spherical aerosols regarding lognormal distribution and undergoing Brownian coagulation in the free molecular regime [21]. The analytical solution was extended later to include fractal agglomerates with a focus on changes of the SPT lag as a function of MFD. In addition, the sectional model [17] and analytical solution [22] were compared to predict the shape of the asymptotic agglomerate size distribution. The sectional method was used to calculate the asymptotic values of 1.46, 1.52, and 1.61 for D<sub>f</sub> of 3.0, 2.5, and 2.0, respectively. In contrast, Park and Lee [22] obtained the asymptotic values by analytical solutions, resulting in 1.36, 1.39, and 1.48, respectively. However, what was overlooked in both studies is that they did not address the effect of PPS. It has been identified that the size of primary particles is an important variable from previous experimental studies. Ulrich and Subramanian [23] also emphasize the importance of PPS by describing the physical properties and behavior of agglomerates generated from the gas phase at high temperature. The properties immensely rely on

<sup>&</sup>lt;sup>†</sup>To whom correspondence should be addressed.

E-mail: kjjeon@inha.ac.kr

<sup>&</sup>lt;sup>\*</sup>Both authors contributed equally to this work.

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the agglomerate size and the number and size of primary particles.

The time to reach asymptotic distribution has much to do with nanoparticle synthesis as noted in previous studies. Thus, the aim of this study was to develop equations for nano-sized TiO<sub>2</sub> particles SPT lag incorporating all important parameters of agglomerates such as PPS, GSD, and fractal dimension. We especially focused on the effect of primary particle diameter (PPD), which has been shown to influence coagulation rate. Moreover, we demonstrated the effectiveness of the other two parameters on PPS.

## METHOD

#### 1. Model Development

Eq. (1) describes the particle size distribution by Brownian coagulation using an integro-differential equation.

$$\frac{\partial \mathbf{n}(\mathbf{v}_{j}, \mathbf{t})}{\partial \mathbf{t}} = \frac{1}{2} \int_{0}^{v_{j}} \beta(\mathbf{v}_{i}, \mathbf{v}_{j} - \mathbf{v}_{i}) \times \mathbf{n}(\mathbf{v}_{i}, \mathbf{t}) \times \mathbf{n}(\mathbf{v}_{j} - \mathbf{v}_{i}, \mathbf{t}) d\mathbf{v}_{i}$$
(1)  
$$-\mathbf{n}(\mathbf{v}_{j}, \mathbf{t}) \int_{0}^{\infty} \beta(\mathbf{v}_{i}, \mathbf{v}_{j}) \times \mathbf{n}(\mathbf{v}_{i}, \mathbf{t}) d\mathbf{v}_{i}$$

where  $n(v_{j_i}, t)$  refers the particle size distribution as a function of time t and the two particles collision kernel which is for two particles having volume  $v_i$  and  $v_{j_i}$  respectively [24]. The collision kernel in the regime of free molecular is expressed in Eq. (2).

$$\beta(\mathbf{v}_{i},\mathbf{v}_{j}) = \pi (\mathbf{d}_{ai} + \mathbf{d}_{aj})^{2} \left(\frac{\mathbf{K}_{b} \mathbf{T}}{2 \pi \rho}\right)^{\frac{1}{2}} \left(\frac{1}{\mathbf{v}_{i}} + \frac{1}{\mathbf{v}_{j}}\right)^{\frac{1}{2}}$$
(2)

where  $\pi (d_{ai}+d_{aj})^2$  refers the agglomerates of collision cross section

and  $\left(\frac{K_b T}{2\pi\rho}\right)^{\frac{1}{2}} \left(\frac{1}{v_i} + \frac{1}{v_f}\right)^{\frac{1}{2}}$  refers the average relative velocity between

colliding agglomerates. Eq. (3) expresses the number of primary particles in the agglomerate  $N_{pp}$  expressed by the relationship between PPD ( $d_p$ ) and the radius of gyration of the agglomerate ( $R_p$ ).

$$N_{pp} = \frac{v}{v_0} = k \left(\frac{2R_g}{d_p}\right)^{D_f}$$
(3)

where  $v_0$  refers the primary particle volume, k refers the structure prefactor, and  $D_f$  refers the fractal aggregate with a fractal dimension [18,25].

If  $D_f$  is between 2 and 3, the agglomerate cross sections can be expressed by

$$\operatorname{cross\,section} \propto \left( \mathbf{R}_{gi} + \mathbf{R}_{gj} \right)^2 \tag{4}$$

The cross section of the new agglomerate without the limitation of  $D_f$  may have a result greater than sum of the individual agglomerates' counterparts. Hence,  $D_f$  larger than was recommended. Furthermore, the assumption of computer simulation provided  $D_f$  is a constant value [24]. k is assigned to 1 in the incorporation of Eqs. (2) and (3). The collision kernel of agglomerates is described in Eq. (5). The kinetic gas theory for rigid spherical particles with  $D_f$ larger than 2 in the free molecular regime is described as

$$\boldsymbol{\beta}(\mathbf{v}_i, \mathbf{v}_j) = \mathbf{K} \left( \frac{1}{\mathbf{v}_i} + \frac{1}{\mathbf{v}_j} \right)^{\frac{1}{2}} \left( \mathbf{v}_i^{\frac{1}{D_j}} + \mathbf{v}_j^{\frac{1}{D_j}} \right)^2$$
(5)

where, 
$$K = \left(\frac{6k_bT}{\rho}\right)^{\frac{1}{2}} \left(\frac{3}{4\pi}\right)^{\frac{2}{D}-\frac{1}{2}} \left(\frac{d_p}{2}\right)^{2-\frac{\delta}{D_f}}$$
,  $k_b$  refers to the Boltzmann

constant,  $\rho$  means the particle density, and T is the absolute temperature [6].

The coagulation phenomena have been explained by the moment method because of the efficient structure and low computational [26,27]. Eq. (6) explains the particle size distribution at the  $k^{th}$  moment.

$$\mathbf{M}_{k} = \int_{-\infty}^{\infty} \boldsymbol{v}^{k} \cdot \mathbf{n}(\boldsymbol{v}, \mathbf{t}) \mathrm{d}\boldsymbol{v}$$
(6)

In this research, we assumed that the moment model is lognormal size distribution. The function of lognormal size distribution is provided as

$$n(v, t) = \frac{1}{3 \nu \sqrt{2\pi} \ln \sigma_g(t)} \exp \left[ \frac{-\ln^2(v/v_g(t))}{18 \ln^2 \sigma_g(t)} \right]$$
(7)

where  $\sigma_g(t)$  refers the GSD, N(t) refers the total number concentration, and  $v_g(t)$  refers the geometric number mean volume of particles. Various moment equations can be derived by incorporating Eq. (1) with (6) and (7). The specific details of the moment equation development are provided elsewhere [28]. For the moments, we used the differential equations calculated by a Runge-Kutta 4<sup>th</sup> order method and for comparison purposes only (Table S1). The key parameters of size distribution including GSD and geometric mean diameter can be decided for each time step. Park and Lee [22] provided various assumptions to derive the analytical solution. Once these equations are numerically solved, the assumptions are no longer needed.

#### 2. Simulation Conditions

We estimated the impact of agglomerate size distribution on coagulation rate by altering PPS, MFD, and initial GSD, which parameters conditions are provided in Table 1. We determined the time to reach the asymptotic state, i.e., SPT lag of fractal agglomerates,  $\tau_f$  by tracking the development of  $\sigma_g$ . Landgrebe and Pratsinis [17] and Vermury and Pratsinis [19] defined that  $\tau_f$  is the time takes for  $\sigma_g$  close to the asymptotic value within 1%. The simulation conditions of set 1 were equal to those in prior studies [19, 22], which indicate our results are comparable to results in prior studies.  $d_p$  and  $\sigma_{g0}$  in sets 2 and 3 were varied to evaluate the dependence of  $\tau_f$ .

#### **RESULTS AND DISCUSSION**

#### 1. Effects of $d_p$ and $D_f$ to Coagulation Rate

The effects of  $d_p$  and  $D_f$  to the growth of agglomerates were evalu-

Table 1.	Simulation	conditions
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Set	$\sigma_{\!_0}$	MFD	Particle number concentration (# cm <sup>-3</sup> )	PPS (nm)
1	1			2
2	1	2.0/2.5/3.0	$10^{16}$	1-20
3	1-3			1-20

ated before performing the simulation. It was carried out by investigating the relationship between spherical particles ( $\beta_s$ ) and the collision kernels of agglomerates ( $\beta_a$ ) in the same particle volume. In contrast to non-spherical particle  $\beta_a$  in Eq. (5), the spherical particles  $\beta_s$  is expressed in Eq. (8) by setting  $D_f=3$ .

$$\beta_{s} = \left(\frac{6k_{b}T}{\rho}\right)^{\frac{1}{2}} \left(\frac{3}{4\pi}\right)^{\frac{1}{6}} \left(\frac{1}{u} + \frac{1}{v}\right)^{\frac{1}{2}} \left(u^{\frac{1}{3}} + v^{\frac{1}{3}}\right)^{2}$$
(8)

Dividing Eq. (5) by Eq. (8), the ratio can be derived as

$$\frac{\beta_a}{\beta_s} = \left(\frac{3}{4\pi}\right)^{\frac{2}{D_f} - \frac{2}{3}} \left(\frac{\mathbf{d}_p}{2}\right)^{2 - \frac{6}{D_f}} \left(\frac{\mathbf{v}^{1/D_f} + \mathbf{u}^{1/D_f}}{\left(\mathbf{v}^{1/3} + \mathbf{u}^{1/3}\right)^2}\right)$$
(9)

The ratio of the collision kernels is described in Eq. (10). We assume monodisperse (u = v) and extreme size ratio (v >> u).

$$\frac{\beta_a}{\beta_s} = \left(\frac{3}{4\pi}\right)^{\left(\frac{2}{D_j}-\frac{2}{3}\right)} \left(\frac{d_p}{2}\right)^{2-6/D_j} v^{\frac{2}{D_j}-\frac{2}{3}} = \left(\frac{2R_g}{d_p}\right)^{\left(2-\frac{2}{3}D_j\right)}$$
(10)

where  $v = \left(\frac{2R_g}{d_p}\right)^{D_f} \left(\frac{\pi}{6}d_p^3\right)$  following Eq. (3) where k is set to 1. As

shown in Eq. (10), higher collision rates were achieved from agglomerates with a smaller PPS and a smaller MFD.

### 2. Effect of $D_f$ to the Asymptotic GSD ( $\sigma_g$ )

Several researchers reported that  $\sigma_g$  applies to asymptotic value if coagulation is the main mechanism [18,20,22]. We used the analytical calculation provided by Park and Lee [22] to evaluate the asymptotic  $\sigma_g$  (Fig. 1). The fractal agglomerates are almost the same as the analytical results (symbols overlapped) reported by previous research [22]. The self-preserved GSD for  $D_f$ =3.0, 2.5 and 2.0 was 1.36, 1.40 and 1.48 respectively. The influences of assumptions used in the previous study [22] are insignificant by confirming the similarity. Similar results were published using the discrete-sectional methods elsewhere [20,18]. Differences between the sectional and moment models result from the presumed size distribution. Thus, those asymptotic values must differ accordingly. Differences between the number and volume based model originate from the finite sectional spacing, which leads to numerical diffusion [17,18].



Fig. 1. Asymptotic as a function of  $D_f$  [18,20,29].



Fig. 2.  $d_p$  Effect as a function of  $D_f$  on: (a) GSD; (b) SPT lag [20].

#### 3. Effect of $d_p$ to $\tau_f$ and $\sigma_g$ as a Function of $D_f$

We conducted simulation for  $d_p$  to evaluate the influence of  $d_p$  on  $\tau_f$  and  $\sigma_g$ . With all the other parameters remaining constant, we applied 1 nm, 10 nm, and 20 nm of  $d_p$  values.  $\sigma_g$  as a function of  $D_f$  is shown in Fig. 2(a) for 3 PPS and the effect of  $d_p$  is insignificant. On the other hand,  $d_p$  has a large effect to  $\tau_f$  as shown in Fig. 2(b).  $\tau_f$  was calculated using Eq. (11).

$$\tau_f = \left(\frac{3k_b T d_p}{\rho_p}\right)^{\frac{1}{2}} \left(\frac{3}{4\pi}\right)^{\frac{1}{6}} N_0 t \tag{11}$$

where t was obtained by the simulation runs [20].

When  $d_p=2 \text{ nm}$ ,  $\tau_f$  does not change much along different  $D_f$ . In addition, they are very similar to those of the sectional method even though  $\sigma_g$  are different [20]. However, the impact on  $\tau_f$  becomes dominant as  $d_p$  and  $D_f$  increase.  $\tau_{20 \text{ nm}}$  is around 15 times larger than  $\tau_{2 \text{ nm}}$  ( $D_f=3$ ) despite the same asymptotic  $\sigma_g$  value for both (Fig. 2(a)).

Small primary particles in the agglomerates are likely to have larger drag forces so that their coagulation speed is faster than large particles. Therefore, smaller primary particles can reach the asymptotic value faster. Then, we investigated a statistical formula for  $\tau_f$  as a function of D<sub>f</sub> where the PPS was 2 nm. Eq. (12) shows a one-way non-linear regression model with an exponential relationship:

$$\tau_{2 \text{ nm}} = \frac{1}{0.2734 + \exp\left(\frac{42.1}{D_f^2} - 11.8\right)} \quad (D_f \ge 2.0) \tag{12}$$

We adopted the exponent following the ones by Park et al. [30]. When we compared Eq. (12) to prior research [20], the results between two methods (discrete-sectional and moment) were very similar for  $d_p=2 \text{ nm}$  (Fig. 2(b)).

# 4. Combined Effect; $d_p$ and $D_f$ on $\tau_f$

We statistically evaluated the dependence of  $\tau_f$  on both  $d_p$  and  $D_f$  using nonlinear regression 2-way ANOVA test ( $\alpha$ =5%). Normalized SPT lag ( $\tau_n$ ) was used in this study to facilitate the analysis as the ratio of SPT lag of agglomerates consist of any PPS larger than 2 nm with the same  $D_f$  of 2 nm (Eq. (13)).

$$\tau_n = \frac{\tau_f}{\tau_{2 \text{ nm}}} \tag{13}$$

Then, Eq. (14) shows the dependence of  $\tau_n$  on  $d_p$  and  $D_f$ .

$$\tau_n = 1.079394 + 1.04 \times 10^{-5} e^{(0.268549a_p + 2.855175D_j)} (R^2 = 0.994)$$
(14)

The real time lag of agglomerates comprised of given  $d_p$  and  $D_f$  can be obtained by incorporating Eqs. (13) and (14), and it becomes Eq. (15).

$$\tau_{f} = [1.079394 + 1.04 \times 10^{-5} e^{(0.268549d_{p} + 2.835173D_{j})}]$$

$$\left[ 0.2734 + e^{\left(\frac{42.1}{D_{f}^{2}} - 11.8\right)} \right]$$
(15)

Fig. 3 shows the SPT lag for diverse  $D_f$  as a function of  $d_p$  and it shows that  $d_p$  does not seem to have significant effect with a small  $D_f$  (e.g. 2). There was no big change in the SPT lag over the entire range of PPS. In contrast, the influence of  $d_p$  on  $\tau_n$  becomes obvious with increases of  $D_f$ .

### 5. Integrated Effect; $d_p$ , $D_f$ , and $\sigma_{g0}$ on $\tau_f$

We investigated the reliance of  $\tau_f$  as a function of  $d_p$ ,  $\sigma_{g0}$  and  $D_f$  using sequential statistical method least nonlinear regression 3-way ANOVA test ( $\alpha$ =5%). Fig. 4 describes  $\tau_f$  of 2 nm agglomerates as a function of  $\sigma_{g0}$  using previous studies [19,21] and this study results.



Fig. 3. MFD and PPS effects on  $\tau_i$  in the free molecular regime.



Fig. 4. Comparison of  $\tau_{f}$  to obtain the self-preserving distribution [19,21].

To predict  $\tau_{\beta}$  Vemury et al. [19] applied two correlation equations based on  $\sigma_{e0}$  which was smaller than the asymptotic value,

$$\tau_f = A \sigma_{g0} - B (\sigma_{g0} - 1)^2 \exp[-C(\sigma_{g0} - 1)^3]$$
(16)

and for  $\sigma_{a0}$  larger than the asymptotic value,

$$\log_{10}\tau_f = D + E\sigma_{g0} \tag{17}$$

Note that  $d_p=2$  nm and  $D_f=3$  were applied for Eqs. (16) and (17). We statistically obtained that the time lag depends on parameters for the moment model. The format of Eq. (16) was adopted for  $\sigma_{g0}$  smaller than the asymptotic value even though the parameters A-C are not constants anymore. Eq. (18) expresses a new formula for  $\sigma_0$  larger than the asymptotic value.

$$\tau_f = \exp(D + E\sigma_0) + F(R^2 = 0.992)$$
(18)

While variable C is only a function of MFD, it was found that variables A to F in Eqs. (16) and (18) significantly depend on PPS and MFD. We evaluated the dependence of  $\tau_{f}$  on all these parameters with 3-way ANOVA test ( $\alpha$ =95%). The 3-way ANOVA test includes the A-F factors which are listed in supporting information (SI. 2). The influence of  $D_f$  and  $\sigma_{g0}$  on  $\tau_f$  is described in Fig. 5. As the  $\sigma_{g0}$  approaches to the asymptotic value, the SPT lag becomes shorter. The relationship between  $\sigma_{g0}$  and  $\tau$  shows an exponential trend. Moreover, it needs more time for reaching the asymptotic value once the agglomerates become bigger.  $\tau_f$  becomes A if the initial aerosols start from monodisperse ( $\sigma_{g0}$ =1) shape. Through these processes, the newly developed equations will play a role in estimating the time for reaching the asymptotic value.

# 6. Implication to Nanoparticle Synthesis

The synthesized PPS is usually larger than 2 nm in many industrial applications. For instance, TiO<sub>2</sub> is synthesized via flame, and the primary particle size is roughly 20 nm [31], and the primary particle size of anatase TiO<sub>2</sub> nanoparticles synthesized via chemical vapor deposition (CVD) is between 12 and 23 nm [32]. Prior models used 2 nm for PPS of 2 nm [19,22]. Although the influence of d<sub>p</sub> for a small D<sub>f</sub> (e.g. 2) is negligible,  $\tau_f$  increases exponentially as d<sub>p</sub> increases when D<sub>f</sub> is 3 (Fig. 4). Based upon Eq. (17), 20 nm primary



Fig. 5.  $D_f$  and  $\sigma_{g0}$  effects on the SPT lag: (a)  $d_p=2$  nm; (b)  $d_p=20$  nm.

particles take more time than 2 nm primary particles for reaching a self-preserving distribution. This is modifiable by the new formula which provides a more accurate estimation in terms of the effects of important parameters.

## CONCLUSION

We investigated the influences of initial GSD and PPS on the change in SPT lag and GSD of fractal agglomerates with regards to the free molecular regime. The moment method presents very similar results to those of the analytical solution. On the other hand, the values obtained by sectional methods were not sufficiently close to the asymptotic values because of the assumptions on the size distribution applied to these models.

The reliance between  $\sigma_{g0}$  and  $\tau_f$  had an exponential relationship with the PPS, initial GSD, and MFD. The asymptotic GSD had nothing to do with PPS. The influence of PPS to the SPT lag was insignificant in the case of small the MFD (D<sub>f</sub>=2.0). SPT lag is reduced as the initial GSD approaches the asymptotic value. A statistical formula was developed with high R<sup>2</sup> values (>0.99) for SPT lag on initial GSD, PPS, and MFD of agglomerates. The developed formulae in this study will be used for a convenient method to evaluate the SPT lag of fractal agglomerates.

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# SUPPORTING INFORMATION

Additional information as noted in the text. This information is available via the Internet at http://www.springer.com/chemistry/journal/11814.

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