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Experimental and numerical study on the effects of surfactant as drag reducing agent in paraffin wax oil pipeline

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Abstract−This study presents an experimental and simulated approach to investigate the drag-reducing phenomenon of surfactants. The experiments to examine the effects of surfactant as a drag reducing agent (DRA) were conducted using the Fann VG viscometer. The paraffin wax oil samples were treated with different temperatures, surfactant concentrations, and surfactant compositions. We used two discrete surfactant compositions, measured apparent viscosities of paraffin wax oil samples for different conditions, and analyzed the effectiveness of DRA based on apparent viscosity results. To examine the effects of surfactant as DRA, under a pipeline transport situation, we used the results from the experiments for a simulation with assumed field operational data. The simulation results are consistent with experimental data. We have observed that, on paraffin wax oil, pressure loss inside the pipeline is inversely related to surfactant concentration and temperature. We verified, experimentally and numerically, that surfactant can act as DRA and can be a possible solution to pipeline friction-related problems in an oil field. However, with paraffin wax oil, the change of temperature has a greater effect than does change of surfactant concentration or composition.

Keywords: Surfactant, Drag Reducing Agent (DRA), Pressure Loss, Viscosity, Pipeline

INTRODUCTION

For many years, surfactants have been used in the petroleum industry as an injection fluid in one of the enhanced oil recovery (EOR) methods. Surfactants improve the recovery factor value by reducing residual oil saturation value due to reduction in surface tension between oil and rock matrix. Another use is as a drag reducing agent (DRA). A DRA is a particular chemical compound with the ability to reduce friction between two adjacent surfaces [1]. In the pipeline transportation process, fluid is continuously in contact with the inner side of pipeline, causing friction. This friction is another contributing factor to pressure loss in the pipeline. Thus, in some oil fields, it is not uncommon to see booster pumps in the pipeline system. The basic purpose of DRA is to help minimize pressure loss due to friction. The use of friction-reducing DRA can reduce the number of booster pumps, or the electric power needed by booster pumps, thus cutting down operational costs for that field.

Other use of DRA involves increasing the flow capacity of the pipeline [2]. If the desired flow rate is greater than the flow capacity of the current pipeline, it is necessary to install a new pipeline with greater flow capacity. DRA injection into the pipeline may act as an alternative solution for this flow capacity problem. DRA reduces friction, which decreases pressure loss along the pipeline; therefore, the flow capacity of the pipeline increases with the same operating pressure as before the injection [3].

The surfactant mechanism in decreasing friction inside the pipeline is not fully understood. However, it is said to have the characteristic of finding surfaces to attach to. Surfactant has a molecular

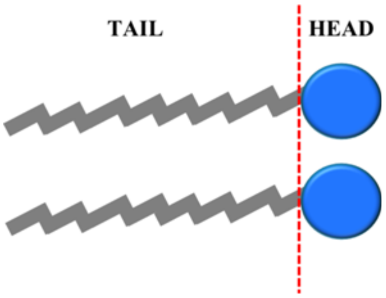

Fig. 1. Molecular form of surfactant.

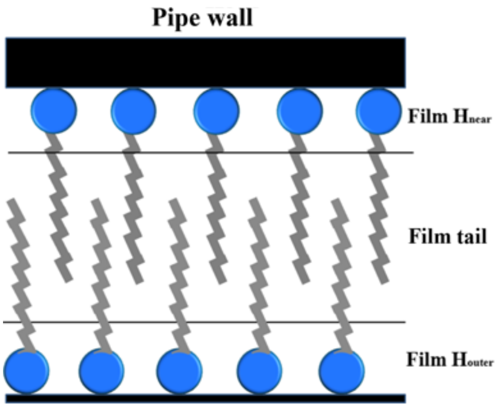

Fig. 2. Thin layer formed by surfactant inside pipe.

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form of "head" and "tail" [4], as shown in Fig. 1. Fig. 2 shows an example of a surfactant forming a thin layer, which prevents the fluid from coming directly into contact with the inside wall of the pipeline, decreasing friction force between fluid and pipeline.

Experimental investigations of DRA effects under various conditions in pipelines have been reported. Lescarboura and Wahl [5] investigated a polymer-based DRA which performed better in viscous oils than in low viscosity oils at low Reynolds numbers (2,100 to 10,000) and performed equally well in all viscosity levels at Reynolds numbers above 10,000. Wahl et al. [2] investigated the use of polymeric DRA to increase the flow capacity of the Trans Alaska Pipeline System (TAPS). They found that the DRA was effective at low concentrations (10-20 ppm), and the use of it accounted for approximately 200,000 bbl/day increments in TAPS throughput. Hamouda et al. [6] investigated DRA performance with a scaled-up laboratory test. The result was compared with field trials. The flow-loop tests confirmed earlier findings that wax deposition and its rate reduced the performance of DRA. However, the field trial did not show a measurable effect of wax on DRA performance. This was thought to be attributed to the much lower axial cooling gradient, resulting in lower wax deposition rate, in the case of the field trial. Prasetyo [3] investigated DRA effects with field application in KS field, South Sumatra, Indonesia. The result was accelerated production of KS field production with an increased throughput, up to 90% of baseline pipeline capacity. Additional advantage of DRA application in KS field was a cut on operating cost from shutting down one of three booster pump stations while maintaining the throughput.

Few reports are found about experiments involving surfactant as DRA and the result of its use in a pipeline. Campbell and Jovancicevic [7] carried out laboratory and field tests with surfactants to investigate their effects on drag reduction and corrosion in a pipeline. The result showed that the use of surfactants yielded increased production from drag reduction mechanisms, as well as providing enhanced corrosion inhibition. Jovancicevic et al. [8] investigated the performance of DRAs which were chemically modified from surfactant corrosion inhibitors. The result was well performing DRAs which promoted enhanced corrosion inhibition, but were strongly dependent on temperature. Kamel and Shah [9] also investigated flow behavior of surfactant-based (SB) fluids, albeit in straight tubing. The SB fluids in their experiment exhibited a significant drag reduction, which increased with concentration. These SB fluids also showed better drag reduction than conventional polymers. Sletfjerding et al. [10] studied the use of surfactant as DRA in Gullfaks Satellites project, North Sea. The surfactant of choice was zwitterionic, N-alkylbetaine type. The surfactant was put on a field test and resulted in the increase of flow rate. Note that the surfactant in Sletfjerding et al. [10] was used in heating medium pipe bundle, with only the heating medium (water with 17% Monoethylene Glycol) and no oil involved in the process. Al-Roomi et al. [11] also analyzed the addition of surfactant in heavy oil. The study also used two different surfactant types in a laboratory scale experiment. The result shows that the relationship between viscosity and the dispersed phase concentration will vary with variation of emulsion characteristics such as the type of surfactant, surfactant concentrations, and temperature.

This paper demonstrates the application of N-stearylbetaine and N-behenylbetaine types of surfactants, as DRA in a laboratory scale. The laboratory work explained in this paper analyzes how temperature and concentration of surfactants impact the effectiveness of surfactant as DRA. This paper further illustrates the results of a one-phase pipeline flow simulation, in order to explore DRA effect on pressure loss.

FLUID FLOW INSIDE A PIPELINE

1. Bernoulli's Equation

Considering there is no heat influx and no work is undertaken on the flow system, with incompressible and frictionless fluid, Bernoulli's equation for ideal fluids, as shown below, can be used,

$$0.3048z_1+430.54\frac{p_1}{\rho g}+0.093\frac{v_1^2}{2g}=0.3048z_2+430.54\frac{p_2}{\rho g}+0.093\frac{v_2^2}{2g}$$

where z is the elevation of pipe from the ground (ft), p is pressure (psi), ρ is density (lbm/ft^3), v is fluid velocity (ft/s), and g is gravitational acceleration (32.185 ft/s^2).

Pressure loss may occur inside the pipeline, for several reasons. One reason is the friction between the flowing fluid and the inside wall of the pipeline. Therefore, Bernoulli's equation for real fluids, the friction is added as shown in the following equation:

$$\begin{aligned}0.3048z_1+430.54\frac{p_1}{\rho g}+0.093\frac{v_1^2}{2g}&=0.3048h_f\\&=0.3048z_2+430.54\frac{p_2}{\rho g}+0.093\frac{v_2^2}{2g}\end{aligned}$$

where h_f is the head loss (ft) due to friction.

Friction is influenced by several variables, such as the roughness factor of the inside wall of a pipeline and the flow regime of the fluid. Information about friction factor value is important because that value can determine the magnitude of pressure loss along the pipeline.

2. Reynolds Number

The Reynolds number is used to determine whether a flow is categorized as a laminar, turbulent, or transition flow (between laminar and turbulent). The following equation shows the general equation for a Reynolds Number calculation, as well as the equation expressed in field units.

$$N_{Re}=92.1\frac{SG\cdot Q}{\mu d}$$

where N_{Re} is the value of Reynolds number, SG is the fluid specific gravity, μ is fluid viscosity (centipoise), Q is fluid flow rate (bbl/day), and d is the inside diameter pipeline (inch).

The Reynolds number range of values is used to determine the flow regimes of a fluid [12]. If the Reynolds number is less than 2,000, the flow is considered laminar. Above 4,000, the flow is considered a turbulent. Therefore, a Reynolds number value between 2,000 and 4,000 is determined to be a transition zone. In this zone, the flow can be either laminar or turbulent, or it can alternate between these patterns. DRA works in turbulent flow zone, reducing production rate of turbulent energy. At high Reynolds number, the velocity profile in the core of the flow is flatter under drag-reducing conditions [13].

The friction factor refers to the value of hindrance given by a particular surface, which is caused by the friction between flowing fluid and the surface. The equation for friction factor is different for each flow regime, whether laminar or turbulent by the Darcy-Weisbach friction factor equation:

$$f=\frac{64}{N_{Re}}$$

where f is the friction factor. The friction factor for a turbulent flow regime is affected by the roughness factor of the pipeline. For example, the Blausius correlation can be used for smooth pipelines with N_{Re} is less than 10^5, as shown in the following equation [14]:

$$f=\frac{0.0791}{N_{Re}^{0.25}}$$

With friction factor known, the magnitude of pressure loss inside the pipeline can be calculated using a re-arranged Darcy-Weisbach equation:

$$\Delta p=0.145f\cdot\frac{L}{D}\cdot\frac{\rho v^2}{2}$$

where Δp is the pressure loss (psi) and L is the length of the pipe (km).

EXPERIMENTS AND SIMULATION

1. Experimental Apparatus and Setup

An experimental apparatus was set up to analyze the effects of temperature and concentration of surfactants on the effectiveness of surfactants as DRA (Fig. 3). The main parts of the apparatus consist of a Fann VG Viscometer and a Fann thermo cup. Other utilities are as follows: thermometer, beakers, electronic scale, stirring rod, and pipette.

The Fann VG viscometer measures the apparent viscosity of fluid. This apparatus can provide rotation velocities of 3 RPM, 6 RPM, 100 RPM, 200 RPM, 300 RPM, and 600 RPM. The velocity of rotation is transmitted into the fluid through a rotor. The fluid in the cup will follow the rotation and turn the bob inside the rotor. The rotation of the bob will result in deviation, which can be viewed as a dial reading (θ). In this experiment, a rotor with a diameter of 3.87 inch was used.

In the Fann VG viscometer, flow rate is caused by the rotation of the rotor. Thus, Reynolds numbers can be determined with the following equation:

$$N_{Re}=\frac{\Omega\kappa R^2}{\mu}$$

where Ω is the value of rotor rotation per minute (RPM), κ is the diameter ratio between the cup and rotor, and R is the diameter of Fann thermo cup (inch).

Apparent viscosity refers to the ratio of shear stress to rate of shear of a non-Newtonian fluid such as lubricating grease, or waxy crude oil, calculated from Hagen-Poiseuille equation and measured in poises. The value of shear stress at a given shear rate value is needed in order to measure the apparent viscosity of fluid (μ_a). In the Fann VG viscometer, the value of shear stress and shear rate can be determined from dial reading and value of rotation per minute. The equations to determine shear stress, shear rate, and apparent viscosity are as follows:

$$\tau=0.0106\times\theta$$

$$\gamma=1.704\times N$$

$$\mu_a=100\frac{\tau}{\gamma}$$

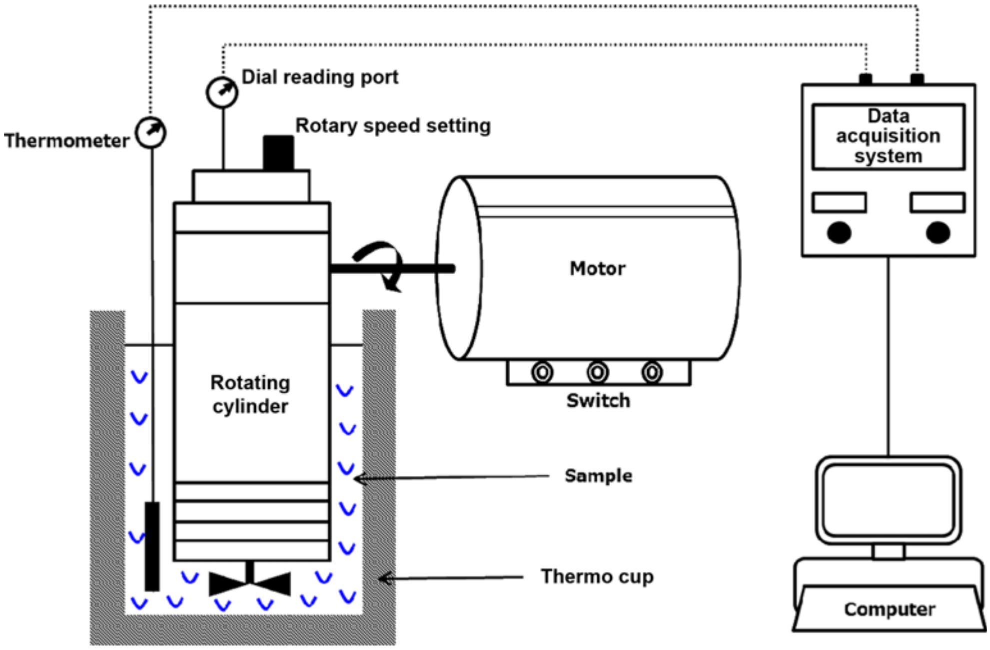

Fig. 3. Schematic diagram of experimental apparatus.

Table 1. The properties of waxy crude oil

Density at 68 °F, lbm/ft^3	Content, wt%		Viscosity at 68 °F, cp	Pour point, °F
	Wax	Asphaltene-resins		
54	14	10	94	78.8

Table 2. Conditions for pressure loss simulation

Variables	Case 1	Case 2	Case 3
Pressure at the end of pipeline (psi)	60	60	60
Fluid flow rate (bbl/day)	21,000	21,000	21,000
Pipeline length (km)	70	70	70
Outside diameter (inch)	24	24	24
Inside diameter (inch)	19.31	19.31	19.31
Submerged pipeline length (km)	-	0.2	0.2
Outside diameter (inch)	-	24	8
Inside diameter (inch)	-	19.31	6.8
Roughness (inch)	0.001	0.001	0.001
Air temperature (°F)	68	68	68
Water temperature (°F)	-	57.2	57.2

where τ is the value of shear stress (lbm/ft^2), γ is the value of shear rate (s^{-1}), θ is the dial reading, and N is the rotation per minute (RPM).

2. Experimental Procedure

To begin the experiment, N-stearylbetaine and N-behenylbetaine types of surfactants, coded as Surfactant A and Surfactant B, and paraffin wax oil sample were prepared (Table 1). In this experiment, the test concentrations of surfactants were 100 ppm, 200 ppm, and 300 ppm. The temperatures used were 100 °F, 110 °F, 120 °F, 130 °F, and 140 °F.

After the paraffin wax oil sample was prepared, it was poured into a Fann thermo cup, which was installed on a Fann VG viscometer. The Fann thermo cup was then set to the lowest temperature included in the experiment. After the paraffin wax oil sample reached this temperature, the Fann VG viscometer was turned on at the lowest RPM. If the pointer in the Fann VG viscometer was stable, the pointed value was taken as the dial reading for the RPM.

After the dial reading at the first test condition had been taken, the experiment continued with the next value of RPM. Then, the temperature could be set to the next temperature included in the experiment. The procedure was repeated until the final temperature condition and RPM (140 °F and 600 RPM) were reached. Fig. 4 depicts the flowchart for the experiment procedure.

3. Pressure Loss Simulation

The pressure loss simulation was performed using PipeSim 2000 with the assumption of one-phase fluid flow (black oil). To examine the effects of surfactant as DRA, under a pipeline transport situation, results from the experiments were used for a simulation with assumed field operational data (Table 2). The input from the laboratory experiment was waxy crude oil's °API and viscosity range with temperature, surfactant concentration, and type of surfactants. Other assumed data are as follows: pressure at the end of pipeline (P_{out}) of 60 psia, fluid flow rate of 21,000 bbl/day, ambient air temperature of 68 °F, and ambient water temperature of 57.2 °F. These input data were then used for the simulation with the Moody (one-phase) flow equation, and the result was the pressure at the inlet of the pipeline (P_{in}). With the data results from laboratory experiments, pressure loss inside the pipeline for a given temperature, surfactant concentration, and type of surfactants can all be determined.

There are three cases for the simulation. Case 1 is a straight pipeline with a length of 70 km, an elevation of 0 m, an outside diameter of 24 inches, an inside diameter of 19.31 inches, and a roughness of 0.001 inch. Case 2 is similar to case 1, except that it contains a segment of the pipeline that is submerged under water. The length of the submerged pipeline segment is 0.2 km and starts at 45 km from the inlet point. Case 3 is similar to case 2, except that the submerged pipeline segment has an outside diameter of 8 inches and an inside diameter of 6.815 inches. Fig. 5 shows the simulation model for case 1, and Fig. 6 shows the simulation model for cases 2 and 3.

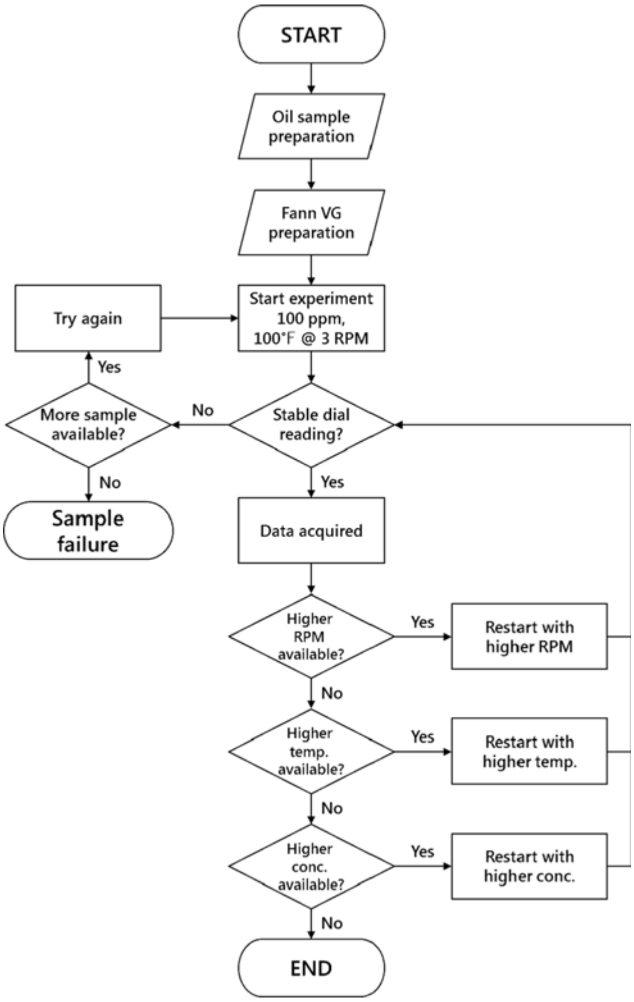

Fig. 4. Flowchart for the experimental procedure.

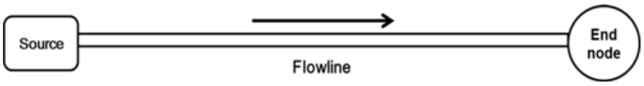

Fig. 5. Schematic model of simulation model for case 1.

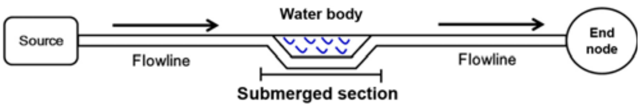

Fig. 6. Schematic model of simulation model for case 2 and 3.

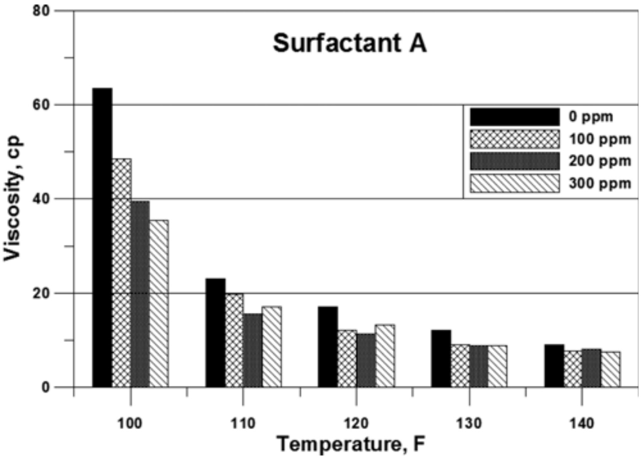

Fig. 7. Apparent viscosity with surfactant A at different test temperatures and concentrations.

RESULTS AND DISCUSSION

1. Experiments for Viscosity Reduction

The experimental results show relationships between surfactant concentrations and test temperatures with the apparent viscosity of the oil sample. An increase in temperature yielded lower apparent viscosity. Likewise, an increase in surfactant concentration also tended to yield lower apparent viscosity of the oil sample. Anomalous results can be observed at surfactant concentrations of 300 ppm, where at a temperature of 110 °F and 120 °F, the apparent viscosity value is shown to be greater than the value at a concentration of 200 ppm.

As observed in Fig. 7, the change of concentration of surfactant A yielded significant viscosity reduction at 100 °F. By adding 100 ppm of surfactant A to the initial sample condition, the apparent viscosity was reduced from 63 cp to 48 cp. By increasing the surfactant concentration to 200 ppm, the apparent viscosity was reduced to 39 cp. Another concentration increment, to 300 ppm, only reduced the apparent viscosity value to 35 cp.

Experimental results show the lowest apparent viscosity value of the paraffin wax oil sample to be 140 °F with concentration of 300 ppm. However, surfactant A is most effective at 100 °F and a concentration of 300 ppm. At this condition, the reduction of viscosity from the initial condition was as much as 28 cp. Temperature also had a positive effect in reducing the apparent viscosity of the oil sample. The greatest reduction occurred when the temperature was increased from 100 °F to 110 °F; viscosity reduction was as much as 40 cp.

The apparent viscosity value with surfactant B generally decreased with the increase of temperature and surfactant concentration, as observed in Fig. 8. However, for each test temperature (except the highest, 140 °F), the increase of concentration from 200 ppm to 300 ppm increased the apparent viscosity value. One possible explanation is that, at a concentration of 300 ppm, surfactant B formed an emulsion with the oil sample, causing the oil sample to be thicker than the oil sample condition at a concentration of 200 ppm. This behavior may also be observed in a study by Ashrafizadeh and Kamran [15], where the increase of surfactant concentration generated an increase in viscosity.

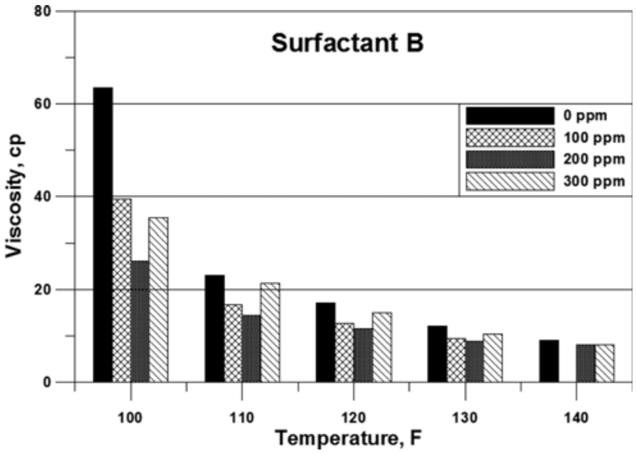

Fig. 8. Apparent viscosity with surfactant B at different test temperatures and concentrations.

The apparent viscosity did not increase with the increase of surfactant concentration at 140 °F. At this temperature, the apparent viscosity value at a concentration of 300 ppm was the same as the value at a concentration of 200 ppm. It is suspected that the high surfactant concentration reduced emulsion thickness.

For a laboratory test with surfactant B, results for apparent viscosity, at surfactant concentration of 100 ppm and test temperature of 140 °F, could not be obtained. This is the case because the pointing needle in the Fann VG Viscometer could not stabilize on the dial reading. The test at this specific condition could not be repeated because of the limited amount of surfactant B and laboratory test time.

With surfactant B, the lowest average value of apparent viscosity was 7.9 cp. This is shown at 140 °F and a surfactant concentration of 300 ppm. However, the most effective work of surfactant B occurred at 100 °F and a surfactant concentration of 200 ppm. At this condition, the apparent viscosity value of the oil sample was reduced from 63 cp to 26 cp. This result is close to the best work given by an increase in temperature from 100 °F (apparent viscosity of 63 cp) to 110 °F (apparent viscosity of 23 cp). From these results, one can observe that temperature has an obvious effect of reducing the viscosity and it is also safe to say that it may have a detrimental effect for the surfactant. Though the surfactant in this study still provided drag reduction with increased temperature, the increase of temperature is decreasing the effectiveness of its drag reducing ability.

2. Pressure Loss Simulation

For case 1 with the straight pipeline and no elevation changes, the pipeline pressure profile observed was linearly declining (Fig. 9). The profile is inversely proportional, as predicted before the simulation. The simulation for the initial condition shows the highest value of pressure loss. The pressure loss for this "no surfactant" con-

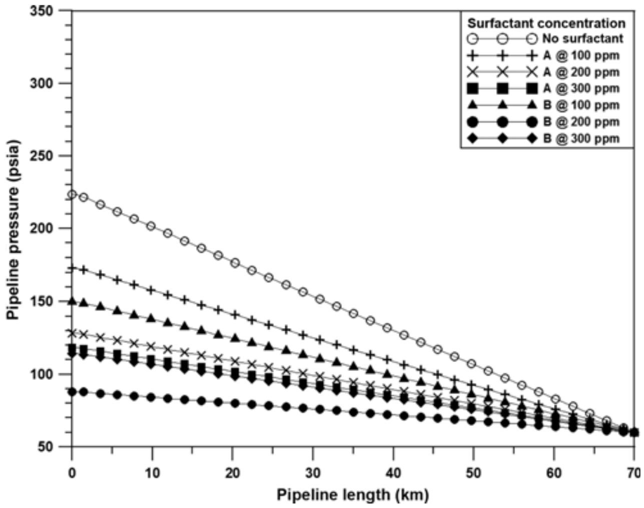

Fig. 9. Simulation results of pipeline pressure profile for case 1.

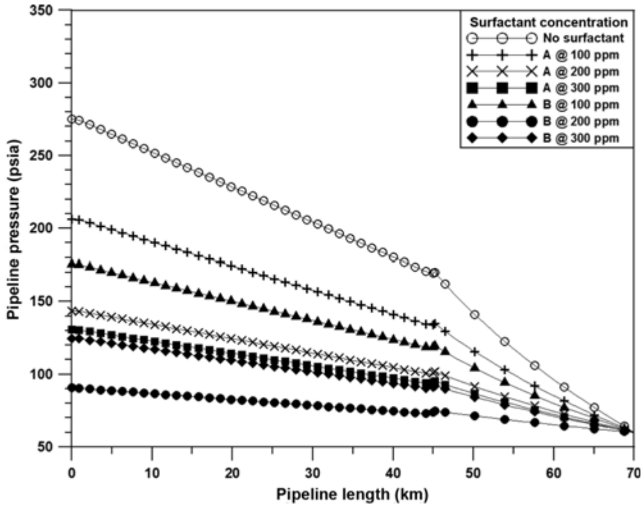

Fig. 10. Simulation results of pipeline pressure profile for case 2.

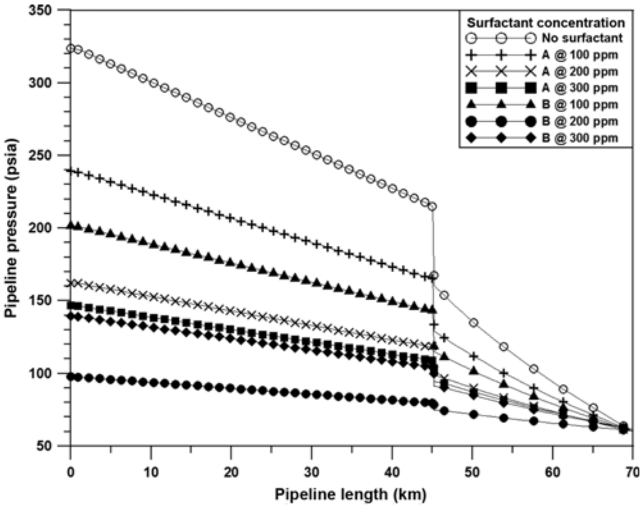

Fig. 11. Simulation results of pipeline pressure profile for case 3.

dition is approximately 163 psi. In line with the experiment results, the lowest value of pressure loss is shown at condition of surfactant B with the added concentration of 200 ppm. This condition shows a pressure loss of approximately 28 psi.

The pipeline pressure profile trend for case 2, as shown in Fig. 10, is slightly different from that of case 1. At the section where the pipeline is supposed to be submerged in the water for 0.2 km, notches of increased pressure can be observed on each condition. We also observed that the initial or inlet pressure of case 2 was generally higher than that of Case 1. On the initial condition, with no surfactant, the inlet pressure is registered at approximately 275 psi, whereas the inlet pressure of case 1 is approximately 223 psi. However, for the condition of 200 ppm of surfactant B, there is only approximately 10 psi difference in inlet pressure between cases 1 and 2. The average increase of inlet pressure for case 2 (compared to case 1) is 25.5%. The lowest value of pressure loss is shown by the use of surfactant B at a concentration of 200 ppm, which is 37.69 psi. The pressure increase at the section where the pipeline is supposed to be submerged is thought to occur because of a sudden drop in temperature. The temperature drop increased the viscosity of the fluid (in this case, black oil). The increased viscosity then increased the pressure needed to force the oil to move forward.

The pipeline pressure profile for case 3 shows the most interesting pattern of all the cases. As observed in Fig. 11, pressure profiles for each concentration of surfactant have a significant drop in pressure. In line with the Darcy-Weisbach equation, the plunging pressure phenomena are thought to occur because of a sudden decrease in pipeline diameter, from 19.31 inches to 6.815 inches.

CONCLUSIONS

Two types of surfactants were tested for their performance as DRA. The results of laboratory experiment and the pipeline simulation indicated that both types of surfactants can be used as DRA. The effect of temperature and concentration was also observed during the experiment and simulation, and the optimum working conditions for surfactant A and surfactant B are different.

Based on the laboratory experiments, the optimum working condition for surfactant A is 100 °F and a concentration of 300 ppm. Meanwhile, the optimum working condition of surfactant B is 100 °F with a concentration of 200 ppm. In the laboratory experiment, the optimum working condition is defined as the condition with the largest value of viscosity reduction relative to the initial condition (with no surfactant).

In general, surfactant B exhibited superior performance on a paraffin wax type of oil than did surfactant A. It is possible that the use of surfactant B at certain concentrations may render the pipeline system independent of temperature changes and pipeline diameter reduction.

A paraffin wax type of oil has higher sensitivity to changes in temperature, when compared with changes in types and concentrations of surfactants used. In the initial condition, without surfactant, increasing the test temperature from 100 °F to 110 °F resulted in viscosity reduction of approximately 40 cp. However, this value of viscosity reduction is relatively close to the work of surfactant B at 100 °F and a concentration of 200 ppm, which yielded a viscosity reduction of 37 cp.

The simulation results are consistent with experimental data. The lowest value of pressure loss is shown at condition of surfactant B with the added concentration of 200 ppm. This condition shows a pressure loss of approximately 30-40 psi from each case. Change of pressure loss inside pipeline was also observed with additional operation parameter, such as inside diameter, submerged section. In case of simulation including submerged section, the increase of pressure loss at the section where the pipeline is supposed to be submerged is thought to occur because of a sudden drop in temperature or decrease in pipeline diameter.

For future work, it would be useful to perform an economic analysis of surfactant injection into a pipeline, compared with the use of insulation methods or a heater for the same type of oil. A test on different oil samples with the same type of surfactants (surfactant A and B) is also encouraged, as is the use of real field data. The use of real field data will ensure that the experiment and simulation results have more practical applicability to the petroleum industry.

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