Korean J. Chem. Eng., 31(8), 1321-1329 (2014)
DOI: 10.1007/s11814-014-0076-5

pISSN: 0256-1115
eISSN: 1975-7220

Smith predictor based fractional-order PI control for time-delay processes

Truong Nguyen Luan Vu* and Moonyong Lee,†**

*Faculty of High Quality Training, University of Technical Education Ho Chi Minh City, Vietnam
**School of Chemical Engineering, Yeungnam University, Gyeongsan 712-749, Korea
(*Received 30 June 2013 • accepted 2 March 2014*)

Abstract−A new fractional-order proportional-integral controller embedded in a Smith predictor is systematically proposed based on fractional calculus and Bode's ideal transfer function. The analytical tuning rules are first derived by using the frequency domain for a first-order-plus-dead-time process model, and then are easily applied to various dynamics, including both the integer-order and fractional-order dynamic processes. The proposed method consistently affords superior closed-loop performance for both servo and regulatory problems, since the design scheme is simple, straightforward, and can be easily implemented in the process industry. A variety of examples are employed to illustrate the simplicity, flexibility, and effectiveness of the proposed SP-FOPI controller in comparison with other reported controllers in terms of minimum the integral absolute error with a constraint on the maximum sensitivity value.

Keywords: Smith Predictor, Fractional-order Proportional-integral (FOPI) Controller, Bode's Ideal Transfer Function, Fractional Calculus

INTRODUCTION

Time delays are commonly encountered in process control problems arising from distance velocity lags, recycle, composition analysis loops, as well as the time required to transport mass and energy. Processes with large time delays are not easily controlled using a PID controller due to additional phase lag caused by time delay that tends to destabilize closed-loop control systems. To compensate for the negative effects of time delays, the Smith predictor [1] is the one of the most effective control techniques that accounts for time delay, and is well known as a dead-time compensator for stable processes with large time delays. Although the Smith predictor offers potential improvement by ignoring the delay term from the closed-loop characteristic equation and significantly facilitating controller design in comparison with conventional feedback systems, the closed-loop performance can be poor in the face of inevitable mismatch between the model and actual process. To overcome this limitation, many approaches have been suggested to improve the performance of the Smith predictor control system [2-7]. These approaches basically fall under two frameworks: determining and tuning the parameters of an appropriate controller (i.e., PI or PIP) [2-5]; and modified Smith predictors [6,7].

Recently, fractional calculus [8] (i.e., fractional integro-differential operators) has been successfully used with satisfactory results to model and control processes with complicated dynamical behavior. Fractional calculus extends ordinary differential equations (ODE) to fractional-order differential equations (FODE) as a generalization of integration and differentiation to non-integer orders. Correspondingly, a fractional-order proportional-integral-derivative (FOPID) controller is a generalization of a standard (integer) PID controller, but affords more flexibility in PID controller design due to its five controller parameters (in contrast to the standard three): proportional gain, integral gain, derivative gain, integral order, and derivative order. Different works have been introduced to facilitate their use in the literature [9-13]. Oustaloup [9] suggested fractional-order algorithms for the control of dynamic systems based on non-integer derivatives. Podlubny [10] introduced the generalized PID controller, $PI^{\lambda}D^{\mu}$, which involves a fractional-order integrator (λ) and a fractional-order differentiator (μ). The two extra parameters (λ and μ) give this type of controller improved flexibility over integer PID controllers, giving it wide industrial applicability [11-13]. In addition, due to the advantages of fractional calculus, the design of FOPI controllers combined with Smith predictors (SP-FOPI) has also become an attractive research issue. Although the applicable method has been reported in the literature [14,15], the design and tuning of the SP-FOPI controller is still difficult and challenging because of its complex iterative steps with non-analytical forms.

We focused on a simple and efficient analytical design method for the FOPI controller embedded in a Smith predictor structure (SP-FOPI), which can provide enhanced performance for both set-point tracking and disturbance rejection problems. Based on fractional calculus and Bode's ideal transfer function [11,15-17], the proposed SP-FOPI tuning rules can be analytically derived for FOPDT models and can be easily applied to various process models by using the frequency domain.

THEORY DEVELOPMENT

1. Fractional Calculus

Various mathematical definitions and concepts of fractional calculus have arisen, all of which affect potential results due to different initial conditions. Therefore, a unique definition of fractional differentiation should be considered to avoid ambiguity by restriction to null values. The most commonly used method is the Rie-

†To whom correspondence should be addressed.
E-mail: mynlee@yu.ac.kr
Copyright by The Korean Institute of Chemical Engineers.

mann-Liouville (RL) definition [8], which is generalized by two equalities that are easily proved for integer orders:

$$ {}_aD_t^v f(t)=\frac{1}{\Gamma(n-v)}\frac{d^n}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{v-n+1}}dt,\quad n-1<v<n \tag{1}$$

where $\Gamma(\bullet)$ denotes Euler's gamma function. The values a and t are the limits. Note that fractional calculus Eq. (1) develops a non-integer (fractional) order functional operator, ${}_aD_t^v$, wherein $v(v\in\Re)$ represents the order of the operator that generalizes usual derivatives (for positive v) and integrals (for negative v).

The Laplace transform of the RL fractional derivative/integral in Eq. (1) under zero initial condition for order v ($0<v<1$) is given as follows:

$$ L\{{}_aD_t^{\pm v}f(t)\}=s^{\pm v}F(s) \tag{2}$$

A fractional order system can be represented by a typical FODE diagram in the time domain. A single-input, single-output (SISO) linear time invariant (LTI) system, relaxed at t=0, are can be expressed as:

$$ \sum_{i=0}^{n} a_i D_0^{v_i} y(t)=\sum_{j=0}^{m} b_j D_0^{w_j} u(t) \tag{3}$$

where u(t) and y(t) are the input and output of the system, respectively.

As a result, system Eq. (3) can be described in the Laplace domain by the following transfer function:

$$ G(s)=\frac{Y(s)}{U(s)}=\frac{b_m s^{w_m}+b_{m-1}s^{w_{m-1}}+\cdots+b_0 s^{w_0}}{a_n s^{v_n}+a_{n-1}s^{v_{n-1}}+\cdots+a_0 s^{v_0}} \tag{4}$$

where v_i and w_j are arbitrary, real, and positive.

For both simulations and hardware implementations, the transfer function including the fractional order of s is commonly approximated by an integer order transfer function with similar behavior, which includes an infinite number of poles and zeros. Nevertheless, reasonable approximations can be obtained with finite numbers of poles and zeros by using the Oustaloup continuous integer-order approximation [9], which is based on a recursive distribution of poles and zeros:

$$ s^v \cong k\prod_{n=1}^{N}\frac{1+\left(\frac{s}{\omega_{z,n}}\right)}{1+\left(\frac{s}{\omega_{p,n}}\right)} \tag{5}$$

Eq. (5) is legitimate in the frequency range $[\omega_l, \omega_h]$, where the gain, k, should be regulated so that both sides of (5) have unit gain at the gain crossover frequency, s^v (i.e. ω_c=1 rad/s). The number of poles and zeros (i.e. N=8) is chosen, since ω_l and ω_h are, respectively, $0.001\omega_c$ and $1000\omega_c$. Moreover, low values of N result can give simpler approximations, but may cause ripples in both gain and phase behaviors. The ripples can be functionally removed by increasing N, but computation costs will be increased.

2. Design of FOPI Controller in Frequency Domain

The fractional integro-differential equation of a FOPI controller is described by:

$$ u(t)=K_C e(t)+K_I D_t^{-\lambda}e(t),\ (\lambda>0) \tag{6}$$

where K_C, K_I and λ denote the proportional term, integral term, and fractional order in the FOPI controller, respectively.

A continuous transfer function of the FOPI controller can be obtained through Laplace transformation as in the following:

$$ G_C(s)=K_C+\frac{K_I}{s^\lambda} \tag{7}$$

From Eq. (7), it is clear that the FOPI controller involves three parameters (K_C, K_I, and λ) that require tuning, since the fractional order λ is not necessarily an integer. This expansion provides more flexibility in achieving control objectives, since the FOPI generalizes the conventional integer PI controller.

The FOPI controller is represented in the frequency domain by substituting $s=j\omega$ into Eq. (7):

$$ G_C(j\omega)=K_C+\frac{K_I}{(j\omega)^\lambda} \tag{8}$$

The fractional power of $j\omega$ is written as

$$ (j\omega)^\lambda=\omega^\lambda j^\lambda=\omega^\lambda\left[e^{j\left[\frac{\pi}{2}+2n\pi\right]}\right]^\lambda=\omega^\lambda\left[e^{j\left[\frac{\pi}{2}\lambda+2n\lambda\pi\right]}\right] \tag{9}$$

where $n=0, \pm\frac{1}{\lambda}, \pm\frac{2}{\lambda}, \ldots, \pm\frac{m}{\lambda}$. Hence, the convenient form is given as follows:

$$ (j\omega)^\lambda=\omega^\lambda[\cos(\gamma_I)+j\sin(\gamma_I)],\quad \gamma_I=\frac{\pi\lambda}{2} \tag{10}$$

The FOPI controller in terms of the complex equation is established by substituting Eq. (10) into Eq. (8):

$$ G_C(j\omega)=\left(K_C+\frac{K_I\cos(\gamma_I)}{\omega^\lambda}\right)-j\frac{K_I\sin(\gamma_I)}{\omega^\lambda} \tag{11}$$

3. SP-FOPI Controller Design Procedure

Consider a block diagram of a Smith predictor control scheme as shown in Fig. 1, where G(s) and $\tilde{G}(s)$ are the transfer function process and process model, respectively. $G_m(s)$ and θ_m are the model transfer function and model delay time, respectively. $G_C(s)$ denotes the controllers. y(s), r(s), d(s), and u(s) correspond to the controlled output, the set-point input, the disturbance input, and the manipulated variables, respectively.

The closed-loop transfer function for set-point changes is given by:

$$ \frac{y(s)}{r(s)}=\frac{G_C(s)G(s)}{1+G_C(s)[G_m(s)+G(s)-\tilde{G}(s)]} \tag{12}$$

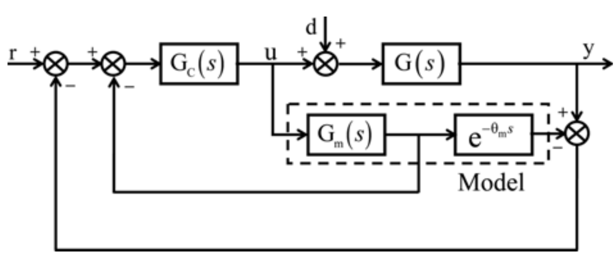

Fig. 1. Block diagram of a Smith predictor control structure.

The closed-loop transfer function of Eq. (12) is commonly reduced to Eq. (13) by assuming the perfect model condition, $\tilde{G}(s)=G(s)$.

$$\frac{y(s)}{r(s)}=\frac{G_C(s)G_m(s)e^{-\theta_m s}}{1+G_C(s)G_m(s)} \tag{13}$$

Hence, the controller for a Smith predictor control scheme can be directly obtained by using a direct synthesis (DS) approach:

$$G_C(s)=\frac{1}{G_m(s)}\left[\frac{\frac{y(s)}{r(s)}}{e^{-\theta_m s}-\frac{y(s)}{r(s)}}\right] \tag{14}$$

Based on the DS approach, the controller is derived by employing two key assumptions: the process model $\tilde{G}(s)$ is available, and the desired closed-loop response, $[y(s)/r(s)]_d$, can be expressed as a closed-loop transfer function for set-point changes. Accordingly, the ideal controller, $G_C(s)$, can be derived by replacing the unknown $y(s)/r(s)$ by $[y(s)/r(s)]_d$.

$$G_C(s)=\frac{1}{G_m(s)}\left[\frac{\left(\frac{y(s)}{r(s)}\right)_d}{e^{-\theta_m s}-\left(\frac{y(s)}{r(s)}\right)_d}\right] \tag{15}$$

The controller given by Eq. (15) does not have the standard form of an FOPI controller. Therefore, it is necessary to find an FOPI controller that closely matches the ideal Smith predictor controller by considering the desired closed-loop transfer function of the fractional-order system given by the unit feedback system in Fig. 2.

$$\left[\frac{y(s)}{r(s)}\right]_d=\frac{L(s)}{1+L(s)}=\frac{1}{1+\left(\frac{s}{\omega_{cg}}\right)^{\gamma}} \tag{16}$$

where L(s) is the ideal open-loop transfer function given by Bode feedback amplifiers [16], which is defined as $L(s)=(\omega_{cg}/s)^{\gamma}$, $\gamma\in\Re^{+}$. The value ω_{cg} is the gain crossover frequency of L(s). The value denotes the slope of the magnitude curve, which may assume non-integer values. L(s) is a fractional-order differentiator for $\gamma<0$ and a fractional-order integrator for $\gamma>0$.

In accordance with Eq. (16), the dynamic behavior of this transfer function corresponds to the first- and the second-order systems, so it is often used as the reference system. Moreover, this transfer function can be considered as the result of closed-loop connection between the fractional-order integrator with gain and order of system. Bode [16] calls this transfer function the ideal open-loop transfer function. Motivated by these results, we developed the tuning of the SP-FOPI controller based on this fractional reference model in this paper.

To account for the time delay, the closed-loop transfer function

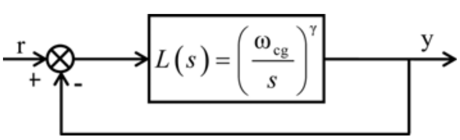

Fig. 2. A fractional-order control system based on Bode's ideal transfer function.

of the fractional reference model becomes [11]:

$$\left[\frac{y(s)}{r(s)}\right]_d=\frac{e^{-\theta s}}{1+\left(\frac{s}{\omega_{cg}}\right)^{\gamma}} \tag{17}$$

By substituting Eq. (17) into Eq. (14), the controller for the desired set-point changes is generally obtained by:

$$G_C(s)=\frac{1}{G_m(s)\left(\frac{s}{\omega_{cg}}\right)^{\gamma}} \tag{18}$$

Since a first-order-plus-dead-time (FOPDT) process model is commonly used in the process industry, it is assumed that the process has FOPDT dynamics, namely,

$$G(s)=\tilde{G}(s)=\frac{Ke^{-\theta s}}{\tau s+1} \tag{19}$$

where K, τ, and θ denote the process's gain, time constant, and dead time, respectively.

Substituting Eq. (19) into Eq. (18), gives the ideal controller for FOPDT processes as:

$$G_C(s)=\frac{1}{K}\left[\frac{\tau s+1}{\left(\frac{s}{\omega_{cg}}\right)^{\gamma}}\right] \tag{20}$$

By substituting $s=j\omega$ into Eq. (20), the resulting FOPI controller is established in the frequency domain as:

$$G_C(j\omega)=\frac{\cos(\varphi)+\tau\omega\sin(\varphi)}{K\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}}+\frac{\tau\omega\cos(\varphi)-\sin(\varphi)}{K\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}}j \tag{21}$$

where,

$$e^{-j\theta\omega}=\cos(\phi)-j\sin(\phi),\ \phi=\theta\omega \tag{22}$$

$$\left(j\frac{\omega}{\omega_{cg}}\right)^{\gamma}=e^{j\pi\gamma/2}\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}=\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}\cos(\varphi)+j\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}\sin(\varphi),\quad \varphi=\frac{\pi\gamma}{2} \tag{23}$$

By comparing Eq. (21) and Eq. (11), analytical tuning rules of the integral and proportional terms of the proposed FOPI controller are obtained, respectively:

$$K_I=\frac{[\sin(\varphi)-\tau\omega\cos(\varphi)]\omega^{\lambda}}{K\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}\sin(\gamma_I)} \tag{24}$$

$$K_C=\frac{[\cos(\varphi)+\tau\omega\sin(\varphi)]}{K\left(\frac{\omega}{\omega_{cg}}\right)^{\gamma}}-\frac{K_I\cos(\gamma_I)}{\omega^{\lambda}} \tag{25}$$

These tuning rules can be applied for various dynamic models, since many systems can be approximated as an FOPDT model. Note that the ideal FOPI controller to give a desired closed-loop response given in Eq. (17) perfectly must have frequency-dependent controller parameters as seen in Eqs. (24) and (25). However, the controller we aim to design is an FOPI controller with constant controller parameters that approximates the ideal FOPI controller closely. For this purpose, the frequency variables ω and ω_{cg} will be considered as a decision variable in the optimization problem for finding optimal controller parameters. Eqs. (24) and (25) will then be used for estimating the corresponding optimal K_C and K_I values.

4. Selection of Tuning Parameters

The tuning parameters have been selected in such a way that the resulting controller can be obtained in terms of the tradeoffs between performance and robustness for both servo and regulatory problems. Accordingly, an integral absolute error (IAE) criterion is considered for the evaluation of the closed-loop performance:

$$\mathrm{IAE} = \int_0^\infty |e(t)|dt \tag{26}$$

To evaluate the robustness of the control system, the maximum sensitivity (Ms) criterion is commonly considered in the literature for its many useful physical interpretations. In particular, the Ms value is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point (−1, 0), which indicates a correlation between the Ms value and the stability margin of the control system.

The Ms value is defined as the maximum value of the sensitivity function over frequency, namely,

$$\mathrm{Ms} = \max_{\substack{0<\omega<\infty \\ 0<\omega_{cg}<\infty \\ 1<\gamma<2}} \left| \frac{1}{1+G_C(j\omega)G_m(j\omega)} \right| \tag{27}$$

To ensure fair comparison, the proposed SP-FOPI controllers are tuned by adjusting the ratio ω/ω_{cg} and γ so that Ms values are identical, i.e., each controller has the same or higher robustness compared with the other controllers in terms of the maximum magnitude of the sensitivity function.

In general, the procedure of selection of tuning parameters can be given as follows: the fractional-order of integral (λ) is first selected on the basis of process dynamics. Second, the suitable Ms value (robustness level) for the specified control system is selected. Finally, the parameters of proportional and integral gains are obtained by adjusting the values of ω, ω_{cg} and γ to satisfy the given Ms value. Note that the set of (ω, ω_{cg}, γ) is not unique to a given Ms value. The remaining degree of freedom is used to find the optimal set of (ω, ω_{cg}, γ) for the best IAE value while it satisfies the Ms equality constraint. In the case of the implementation in practice, it also follows the above-mentioned procedure; the resulting controller parameters and the corresponding values of optimal set of (ω, ω_{cg}, γ) are used in Eqs. (24) and (25) as a fixed value.

SIMULATION STUDY

To demonstrate the simplicity and effectiveness of the proposed SP-FOPI tuning rules, five examples are given in this section. Moreover, the integral order λ is one of the most important factors and significantly affects the performance and robustness of the closed-loop system. Therefore, the effects of λ are also analyzed in this section. The fractional-order is scanned in the range of λ; the best proposed controller parameters are then picked based on the IAE criterion for a given desired Ms value.

1. Example 1

Consider the FODT process model studied by Padula et al. [18]:

Table 1. Controller parameters and performance index for various values of (example 1)

λ	ω/ω_{cg}	γ	K_C	τ_I	Ms	IAE_{sp}	IAE_d
0.7	3.700/2.920	1.370	0.783	0.135	1.20	2.070	1.93
0.8	3.980/2.995	1.381	1.167	0.177	1.20	1.450	1.24
0.9	3.985/3.390	1.390	1.751	0.201	1.20	1.144	0.89
1.0	4.300/3.120	1.425	1.746	0.186	1.20	1.135	0.77
1.1	4.500/2.345	1.301	1.720	0.258	1.20	1.280	0.83
1.2	4.700/2.154	1.234	1.696	0.305	1.20	1.430	0.92
1.3	4.800/1.963	1.168	1.663	0.375	1.20	1.610	1.07
1.4	4.900/1.775	1.100	1.624	0.490	1.20	1.840	1.31

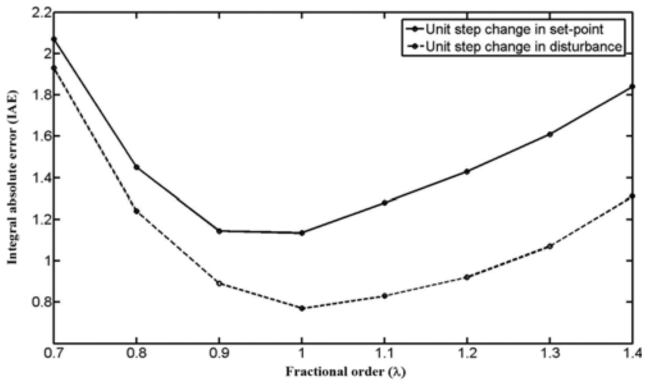

Fig. 3. Effects of fractional order λ on IAE for example 1.

$$G(s) = \tilde{G}(s) = \frac{e^{-0.67s}}{(s+1)} \tag{28}$$

The fractional order λ ranges from $0.7 \le \lambda \le 1.4$. The proposed SP-FOPI controller parameters were calculated for different λ values with a desirable Ms value of 1.20 as shown in Table 1. Fig. 3 shows the effects of λ values on the closed-loop performance. The best fractional order λ that provides the smallest values of IAE for both set-point tracking and disturbance rejection is 1.0. Accordingly, the proposed SP-FOPI controller becomes an integer Smith predictor PI (SP-PI) controller.

Since no valid design methods for SP-FOPI and/or SP-FOPID controllers exist in the literature, the FOPI controller tuning rules suggested by Padula et al. [18], Chen et al. [19], and Gude et al. [20] are used to design the controllers, which are compared with the proposed SP-FOPI controller. To achieve more robustness, the proposed controller was tuned to have Ms=1.20, since the fractional order λ=1.0 was selected. The resulting controller parameters and the calculated performance and robustness indices are summarized in Table 2.

Unit step changes in the set-point and load disturbance were introduced at t=0 (sec) and t=15 (sec), respectively. The corresponding simulation results are shown in Fig. 4. It is evident from Table 2 and Fig. 4 that the proposed controller affords improved closed-loop performance with fast and well-balanced responses over those of the other controllers for both the set-point and disturbance rejection problems. The controller output (manipulated variable) responses shown in Fig. 5 demonstrate that the control efforts of all comparative controllers are sufficiently smooth for successful operation.

Table 2. Controller parameters and performance index for example 1

Tuning methods	K_C	τ_I	λ	Ms	Set-point			Disturbance		
					IAE	Overshoot	TV	IAE	Overshoot	TV
Proposed	1.746	0.186	1.0	1.20	1.135	0.251	3.373	0.770	0.545	1.648
Padula et al.	0.61	1.10	1.0	1.42	1.803	0.000	0.483	1.803	0.631	1.000
Chen et al.	0.74	0.71	1.0	1.89	1.731	0.231	1.596	1.267	0.607	1.556
Gude et al.	0.47	1.19	1.12	1.26	3.186	0.069	0.688	3.100	0.661	1.150

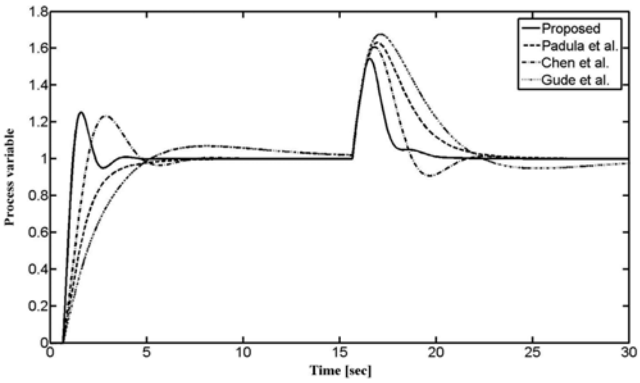

Fig. 4. Simulation results of various controllers for example 1.

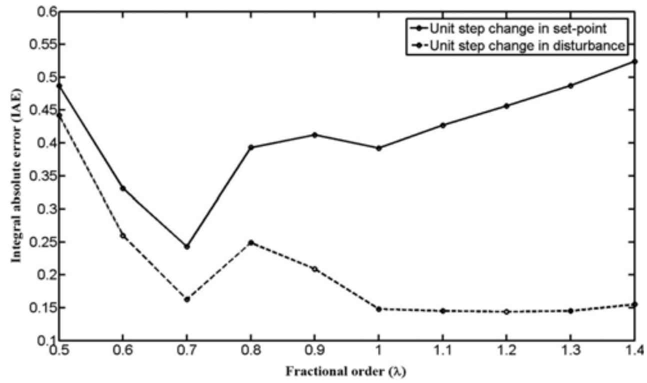

Fig. 6. Effects of fractional order λ on IAE for example 2.

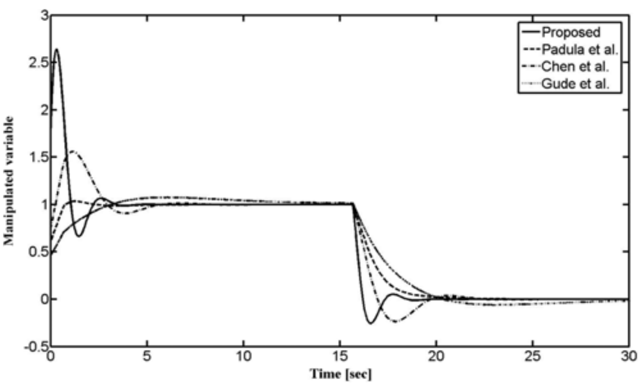

Fig. 5. Controller output of various controllers for example 1.

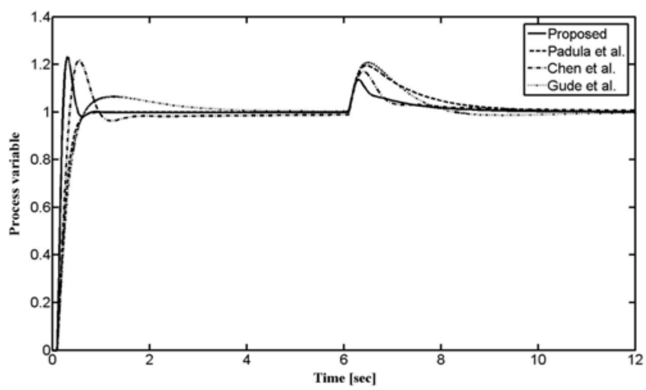

Fig. 7. Simulation results of various controllers for example 2.

Table 3. Controller parameters and performance index for various values of (example 2)

λ	ω/ω_{cg}	γ	K_C	τ_I	Ms	IAE_{sp}	IAE_d
0.5	23.000/21.885	1.3885	3.990	0.045	1.20	0.487	0.442
0.6	21.558/19.041	1.3982	5.995	0.068	1.20	0.331	0.260
0.7	17.150/14.186	1.3970	6.00	0.088	1.20	0.243	0.163
0.8	11.523/6.975	1.3592	3.500	0.133	1.20	0.393	0.249
0.9	8.000/5.0530	1.3597	2.896	0.160	1.20	0.412	0.209
1.0	7.0000/5.000	1.3670	3.361	0.163	1.20	0.392	0.148
1.1	6.1140/4.953	1.3880	3.826	0.160	1.20	0.427	0.145
1.2	5.173/4.8650	1.4310	4.316	0.155	1.20	0.456	0.144
1.3	4.958/4.7093	1.4279	4.858	0.150	1.20	0.487	0.145
1.4	4.750/4.5400	1.4074	5.496	0.147	1.20	0.524	0.155

2. Example 2

The following process model is considered as follows [18]:

$$G(s)=\tilde{G}(s)=\frac{e^{-0.1s}}{s+1} \qquad (29)$$

The fractional order λ is scanned in the range of $0.5\le\lambda\le1.4$. Table 3 summarizes the simulation results with different values of λ and a given Ms=1.20. Fig. 6 shows the effects of λ on the IAE values for this process. It is clear that the best value of λ is 0.70, which produces the smallest value of IAE for set-point tracking and acceptable value of IAE for disturbance rejection. Accordingly, the proposed SP-FOPI control system consistently affords prominent performance with λ=0.7 in comparison with those of the other order counterparts.

The set-point and disturbance output responses of the various controllers are shown in Fig. 7, where unit step changes in the set-point and load disturbance were introduced at t=0 (sec) and t=6 (sec), respectively. The performance improvement of the proposed method is apparent. The results shown in Fig. 8 imply that all the control efforts of all controllers are smooth enough for successful

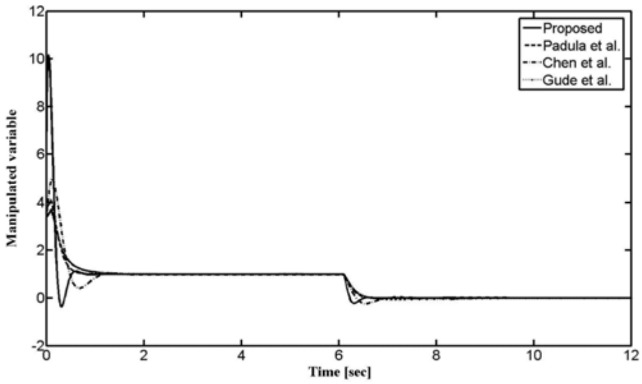

Fig. 8. Controller output of various controllers for example 2.

operation. The resulting controller parameters, together with the performance and robustness indices, are listed in Table 4, which indicates that the proposed method provides better performance for both set-point tracking and disturbance rejection with smaller IAE values.

3. Example 3

The following process model is considered as follows [15]:

$$G(s)=\tilde{G}(s)=\frac{e^{-s}}{0.09s+1} \tag{30}$$

Table 5 shows the simulation results via the proposed method, since the fractional order λ ranges from of $0.8\le\lambda\le1.4$ and the desired Ms is given by 1.20. Fig. 9 implies that the best fractional order λ is 1.1, which provides the best proposed SP-FOPI controller based on the least IAE values for both set-point tracking and disturbance rejection problems.

Accordingly, the F-MIGO controller (i.e., peak sensitivity constrained integral gain optimization for the fractional-order PI control system) introduced by Monje et al. [15] has demonstrated superiority over integer PI controllers, such as the AMIGO controller [15]. In simulation studies, the proposed SP-FOPI controller is compared with those of the F-MIGO and AMIGO design methods, as well as the SP-PI controller. For F-MIGO method, λ is also chosen as 1.1. For the SP-PI method, the controller parameters are designed based on the direct synthesis for a first-order delay-free process model in following forms: $K_c=\tau/K\tau_c$ and $\tau_I=\tau$, where τ_c is a tuning parameter that corresponds to the close-loop time constant.

To provide a fair comparison, the proposed SP-FOPI controller was tuned to have the same robustness level as that of the SP-PI controller (τ_c=1.914) for Ms=1.00, since and were selected.

Fig. 10 and Fig. 11 show the closed-loop time responses and controller output signals to both set-point and disturbances changes, respectively. The resulting controller characteristics are listed in Table 6. These tables and figures imply that the proposed SP-FOPI controller has the fastest and best-balanced responses with the lowest settling time and IAE values compared with other schemes.

4. Example 4

The following high-order and non-minimum phase process model

Table 5. Controller parameters and performance index for various values of (example 3)

λ	ω/ω_{cg}	γ	K_C	τ_I	Ms	IAE_{sp}	IAE_d
0.8	17.00/12.260	1.3438	0.170	0.0158	1.20	1.272	1.241
0.9	14.00/9.0000	1.3652	0.150	0.0165	1.20	1.223	1.208
1.0	7.000/6.0000	1.2850	0.110	0.0160	1.20	1.170	1.157
1.1	20.000/9.090	1.3550	0.441	0.0260	1.20	1.138	1.108
1.2	13.000/9.115	1.3760	0.550	0.0265	1.20	1.158	1.123
1.3	9.870/8.6600	1.4090	0.658	0.0270	1.20	1.171	1.138
1.4	8.585/7.7260	1.4180	0.792	0.0290	1.20	1.199	1.156

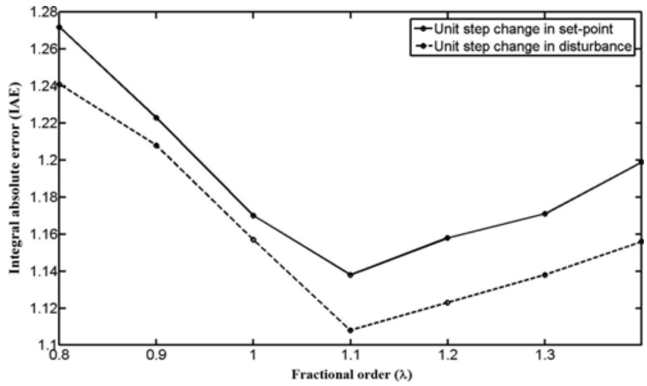

Fig. 9. Effects of fractional order λ on IAE for example 3.

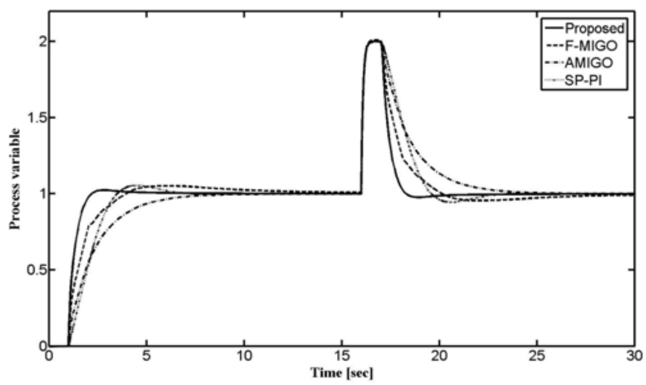

Fig. 10. Simulation results of various controllers for example 3.

Table 4. Controller parameters and performance index for example 2

Tuning methods	K_C	τ_I	λ	Ms	Set-point			Disturbance		
					IAE	Overshoot	TV	IAE	Overshoot	TV
Proposed	6.00	0.088	0.70	1.20	0.243	0.231	16.35	0.163	0.138	1.557
Padula et al.	3.75	1.00	1.00	1.40	0.267	0.00	3.481	0.267	0.195	1.007
Chen et al.	3.26	0.41	0.70	1.71	0.690	0.217	8.617	0.540	0.183	1.707
Gude et al.	3.39	0.74	1.12	1.35	0.460	0.065	3.047	0.310	0.206	1.134

Table 6. Controller parameters and performance index for example 3

Tuning methods	K_C	τ_I	λ	Ms	Set-point IAE	Set-point Overshoot	Set-point TV	Disturbance IAE	Disturbance Overshoot	Disturbance TV
Proposed	0.487	0.148	1.1	1.00	1.402	0.017	0.563	1.401	1.000	1.034
F-MIGO	0.320	0.604	1.1	1.56	2.262	0.054	0.830	2.257	1.000	1.107
AMIGO	0.160	0.381	1.0	1.48	2.381	0.000	0.840	2.381	1.000	1.000
SP-PI	0.047	0.09	1.0	1.00	2.140	0.055	1.072	2.141	1.000	1.119

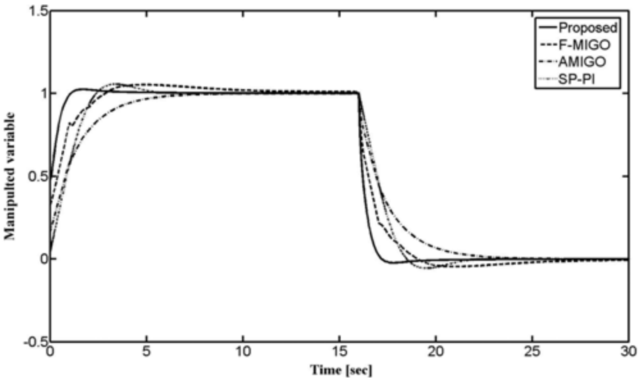

Fig. 11. Controller output of various controllers for example 3.

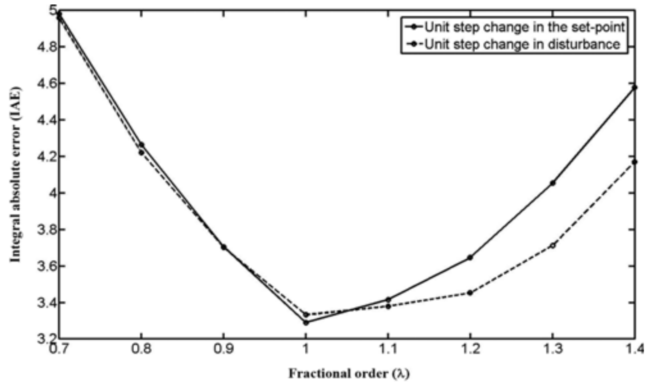

Fig. 12. Effects of fractional order λ on IAE for example 4.

Table 7. Controller parameters and performance index for various values of (example 4)

λ	ω/ω_{cg}	γ	K_C	τ_I	Ms	IAE_{sp}	IAE_d
0.7	1.629/1.099	1.2245	0.734	0.407	1.39	4.982	4.959
0.8	1.542/1.042	1.2280	0.865	0.520	1.39	4.265	4.222
0.9	1.520/0.963	1.2360	0.935	0.611	1.39	3.706	3.703
1.0	1.500/0.900	1.2248	1.035	0.725	1.39	3.290	3.333
1.1	1.450/0.844	1.2138	1.120	0.825	1.39	3.417	3.380
1.2	1.350/0.764	1.2230	1.127	0.890	1.39	3.647	3.454
1.3	1.278/0.6755	1.2115	1.135	0.982	1.39	4.055	3.713
1.4	1.250/0.588	1.1625	1.158	1.113	1.39	4.578	4.172

is considered as follows [18]:

$$G(s)=\tilde{G}(s)=\frac{1-s}{(s+1)^3} \tag{31}$$

It can be approximated as an FOPDT by [18]:

$$G(s)=\tilde{G}(s)=\frac{e^{-2.39s}}{1.62s+1} \tag{32}$$

The fractional order λ ranges from of $0.7\le\lambda\le1.4$, since Ms=1.39.

It is evident from Table 7 and Fig. 12 that the best proposed controller is obtained, while the fractional-order λ is chosen as 1.0.

To obtain the same robustness level for other comparative controllers, such as the FOPID controller suggested by Padula et al. [18] and integer PID obtained by AMIGO [18], the ratio ω/ω_{cg}=1.5/0.9 and parameter γ=1.2248 were selected for the proposed method, and the closed-loop time constant was adjusted to be τ_c=6.625 for the SP-PI method in terms of Ms=1.39. The simulation results are summarized in Table 8, while the closed-loop time responses and controller output signals are plotted in Fig. 13 and Fig. 14, respectively. It is evident from the tables and figures that the proposed SP-FOPI controller provides a performance improvement for both set-point tracking and disturbance rejection.

5. Example 5

Consider the previously reported fractional-order plant model for a heating furnace [21,22]:

$$G(s)=\tilde{G}(s)=\frac{1}{14994s^{1.31}+6009.5s^{0.97}+1.69} \tag{33}$$

To apply the proposed SP-FOPI tuning rules, Eq. (33) has to be approximated as an FOPDT using Oustaloup's recursive approximation [9,23], which is widely used in fractional calculus:

Table 8. Controller parameters and performance index for example 4

Tuning methods	K_C	τ_I	τ_D	λ	μ	Ms	Set-point IAE	Set-point Overshoot	Set-point TV	Disturbance IAE	Disturbance Overshoot	Disturbance TV
Proposed	1.035	0.725	0.00	1.0	1.0	1.39	3.29	0.029	1.61	3.33	0.75	1.40
Padula et al.	0.37	1.57	1.01	1.0	1.2	1.39	5.20	0.082	0.86	5.24	0.74	1.18
AMIGO	0.48	2.12	0.69	1.0	1.0	1.39	4.52	0.043	1.61	4.66	0.78	1.64
SP-PI	0.24	1.62	0.00	1.0	1.0	1.39	9.03	0.000	0.76	9.24	0.84	1.73

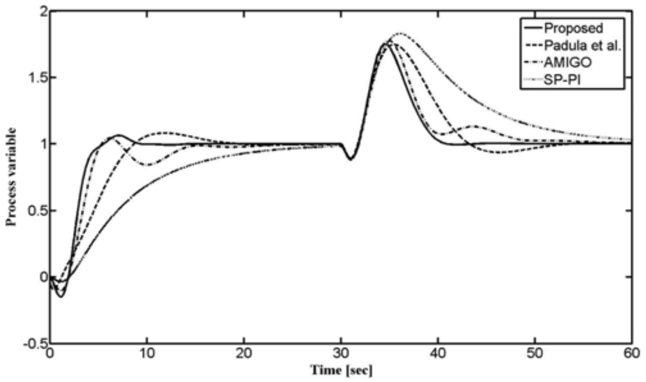

Fig. 13. Simulation results of various controllers for example 4.

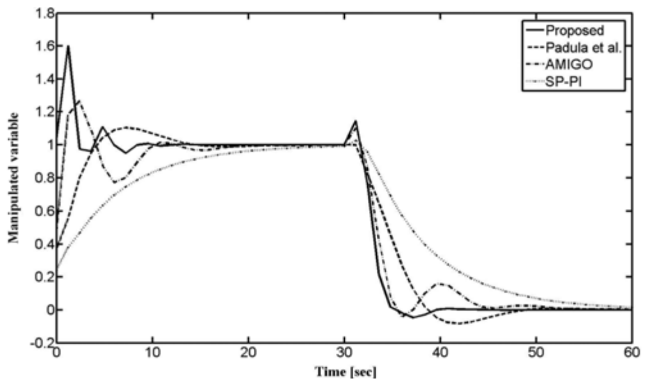

Fig. 14. Controller output of various controllers for example 4.

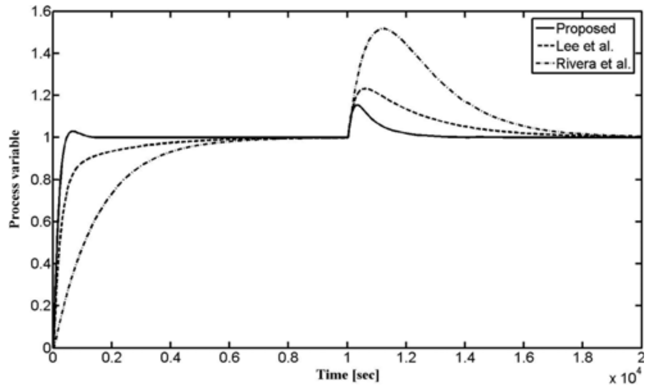

Fig. 15. Simulation results of various controllers for example 5.

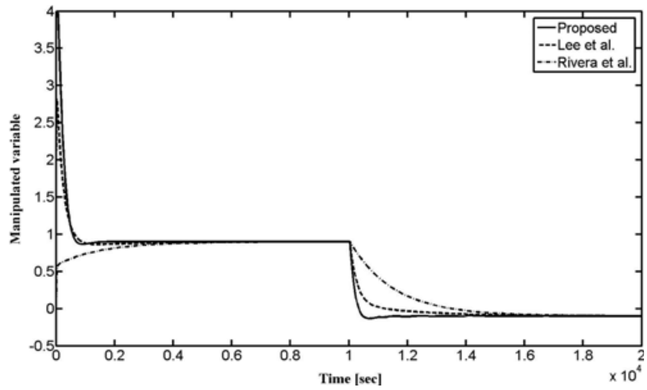

Fig. 16. Simulation results of various controllers for example 5.

$$G(s)=\tilde{G}(s)=\frac{1.110e^{-32.1s}}{953.289s+1} \tag{34}$$

Following a similar approach, the best fractional-order λ is 0.7 for obtaining the best SP-FOPI controller via the proposed method. To quantitatively evaluate the performance enhancement of the proposed controller with some typical kinds of PID controllers, such as the PID controller obtained by Lee et al. [24] and the PID cascaded with a lag filter provided by Rivera et al. [25], the proposed controller was tuned to have more robustness than others, since the ratio ω/ω_{cg}=0.129/0.00562 and parameter γ=1.0252 were selected to satisfy Ms=1.0. All controller characteristics are listed in Table 9. The set-point and load disturbance responses are shown in Fig. 15, where unit step changes in the set-point and load disturbance were introduced at t=0 (sec) and t=10000 (sec), respectively. Clearly, the proposed SP-FOPI controller provides superior closed-loop responses, the smallest settling time and IAE values for both servo and regulatory problems, since the controller efforts are sufficiently smooth for successful operation as shown in Fig. 16.

CONCLUSIONS

A new methodology for designing a fractional-order proportional-integral controller embedded in the Smith predictor is systematically proposed on the basis of fractional calculus and Bode's ideal transfer function. The analytical tuning rules of the proposed SP-FOPI controller are simply derived by considering the frequency domain for achieving enhanced performance for both servo and regulatory problems.

Many illustrative examples have been used to demonstrate the flexibility and effectiveness of the proposed method. The simulation results demonstrate that the proposed SP-FOPI controller consistently affords improved performance with fast and well-balanced

Table 9. Controller parameters and performance index for example 5

						Set-point			Disturbance		
Tuning methods	K_C	τ_I	τ_D	λ	Ms	IAE	Overshoot	TV	IAE	Overshoot	TV
Proposed	4.35	76.32	0.00	0.7	1.00	185	0.029	3.69	164.9	0.154	1.09
Lee et al.	1.48	954.17	0.88	1.0	1.30	581.2	0.000	3.33	645.1	0.234	1.00
Rivera et al.[a]	0.57	969.34	15.71	1.0	1.30	1532	0.000	0.90	1701	0.515	1.00

[a] $G_C(s)=\left(K_C+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{\tau_F s+1}$, where τ_c=1500, τ_F=15.714

closed-loop responses for various process models.

ACKNOWLEDGEMENTS

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012012532) as well as that of the University of Technical Education Ho Chi Minh City (UTE), Vietnam.

REFERENCES

1. O. J. Smith, *ISA J.*, **6**, 28 (1959).
2. K. J. Åström, C. C. Hang and B. C. Lim, *IEEE Transactions on Automatic Control*, **39**, 343 (1994).
3. T. Hägglund, *IEEE Control System Magazine*, **12**, 57 (1992).
4. D. Lee, M.-Y. Lee, S. Sung and I. Lee, *J. Process Control*, **9**, 79 (1999).
5. M. Morari and E. Zafiriou, *Robust Process Control*, Prentice-Hall, Englewood Cliffs, NJ (1989).
6. H. Q. Zhou, Q. G. Wang and L. Min, *ISA Transactions*, **46,** 493 (2007).
7. A. S. Rao and M. Chidambaram, *ISA Transactions*, **47,** 407 (2008).
8. K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*, Wiley-Interscience (1993).
9. A. Oustaloup, *La Commande CRONE: Commande Robuste d'Ordre Non Entier*, Hermes, Paris (1991).
10. I. Podlubny, *IEEE Transactions on Automatic Control*, **44,** 208 (1999).
11. R. S. Barbosa, T. Machado and I. M. Ferreira, *Nonlinear Dynamics*, **38**, 305 (2004).
12. C. A. Monje, B. M. Vinagre, V. Feliu and Y.-Q. Chen, *Control Eng. Pract.*, **16**, 798 (2008).
13. T. N. L. Vu and M. Lee, *ISA Transactions*, **52**, 583 (2013).
14. V. Feliu-Batlle, R. Rivas-Perez, F. G. Castillo-Garcia and L. Sanchez-Rodriguez, *J. Process Control*, **19**, 506 (2009).
15. C. A. Monje, Y. Q. Chen, B. M. Vinare, D. Xue and V. Feliu, *Fractional order systems and controls funamental and application*, Spinger-Velag (2010).
16. H. W. Bode, *Bell System Technical J.*, **19**, 421 (1940).
17. H. W. Bode, *Network analysis and feedback amplifier design*, Van Nostrand, New York (1945).
18. F. Padula and A. Visioli, *J. Process Control*, **21**, 69 (2011).
19. Y. Q. Chen, T. Bhaskaran and D. Xue, *ASME J. Computational and Nonlinear Dynamics*, **3**, 0214031 (2008).
20. J. J. Gude and E. Kahoraho, *IEEE International Conference on Emerging Technologies and Factory Automation (ETFA)* (2009).
21. I. Podlubny, L. Dorcak and I. Kostial, *Proceedings of the 36th IEEE CDC*, San Diego (1999).
22. K. B. Mohamed and B. B. Naceur, *Commun Nonlinear SCI Numer Simulat*, **15**, 1267 (2010).
23. A. Oustaloup, F. Levron and F. Nanot, *EEE Trans Systems I: Fundamental Theory and Applications*, **47**, 25 (2000).
24. Y. Lee, S. Part, M. Lee and C. Brosilow, *AIChE J.*, **44**, 106 (1998).
25. D. E. Rivera, M. Morari and S. Skogestad, *Ind. Eng. Chem. Proc. Des. Dev.*, **25**, 252 (1986).