

Structural-acoustic optimization of stiffened panels based on a genetic algorithm

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Abstract: For the structural-acoustic radiation optimization problem under external loading, acoustic radiation power was considered to be an objective function in the optimization method. The finite element method (FEM) and boundary element method (BEM) were adopted in numerical calculations, and structural response and the acoustic response were assumed to be de-coupled in the analysis. A genetic algorithm was used as the strategy in optimization. In order to build the relational expression of the pressure objective function and the power objective function, the enveloping surface model was used to evaluate pressure in the acoustic domain. By taking the stiffened panel structural-acoustic optimization problem as an example, the acoustic power and field pressure after optimized was compared. Optimization results prove that this method is reasonable and effective.

Keywords: structural-acoustic optimization; acoustic radiation power; finite element method; boundary element method; stiffened panel structure; genetic algorithm

CLC number: TB532 **Document code:** A **Article ID:** 1671-9433(2007)04-0055-07

1 Introduction

Recently, lots of attention has been paid to structural vibration and its induced acoustic radiation of complicated structural design. For example, the noise level in the passenger compartment of a vehicle can be greatly reduced by optimizing its structural design parameters. The structural acoustic optimization problem has also become a hot-spot in the multidisciplinary design optimization domain. In most acoustic researches, finite element method (FEM) is used to obtain the structural frequency response and boundary element method (BEM) is adopted to perform low frequency acoustic radiation analysis. In acoustic radiation optimization design analysis, the design objective function may be the acoustic pressure, acoustic intensity and acoustic power, as well as the changeable structural variables or acoustic parameters such as structural dimension, surface velocity and frequency. Wang Semyung and Lee Jeawon^[1] used design sensitivity analysis (DSA) to optimize the exterior acoustic field problem of a shell box. H. W. WODTKE and J. S. LAMANCUSA^[2] have analyzed the acoustic radiation power of the circular sandwich plates with damping layers under exterior excitation. J. Lončarić and S. V. Tsynkov^[3] have performed

analysis of minimum acoustic radiation power of acoustic cavity by using active control technique. J. C. O. NIELSEN^[4] has optimized the acoustic radiation power of railway sleepers and drawn a conclusion that changing the dimensions of the sleepers can lead to a reduction of sound power substantially. Jorge P. Arenas^[5] used matrix model to analyze the acoustic power and differentiated the acoustic power equation to obtain the design sensitivity results, and using it to optimize acoustic. To minimize the acoustic intensity on some contour, Kavch Ghayour^[6] has differentiated the real and imaginary parts of the pressure equation to perform DSA in his dissertation. Steffen Marburg^[7-8] has analyzed structural acoustic radiation of shell box by modifying the finite shell element models, and applied the result to optimize a sedan chassis. D. Duhamel^[9] employed the genetic algorithms to minimize the acoustic pressure level in a domain behind the barrier by shape optimization. Zaiwei Li and Xinhua Liang^[10] have optimized the damping structural acoustic using response surface method. Jeawon Lee, Semyung Wang and Altay Dike^[11] used genetic algorithms to optimize the topology structure of thin plate. ZHANG Jun, ZHAO Wen-zhong, XIE Su-ming and ZHANG Wei-ying^[12] have optimized the acoustic pressure at interested positions and frequencies using the feasible

direction method, in which, the iSIGHT code is applied as optimization platform. H.Denli and J.Q.Sun^[13] have optimized the acoustic problem of sandwich structures with cellular cores, in which, the objective is minimum noise radiation in a wide frequency band, subject to the constraints on the fundamental frequency and weight is presented. The sensitivity functions of the radiated acoustic power are used to improve the computational time and accuracy in near optimization in study.

In this paper, the stiffened panel structure's acoustic radiation optimization problem under external loading is investigated. Structural response and the acoustic response are assumed to be de-coupled in the analysis. In section 2, the frequency response problems of the structure are analyzed. Taken the structural response (harmonic normal velocity) as boundary conditions, the boundary element method is used to calculate the acoustic radiation behavior (acoustic pressure or acoustic power). Thus the problem to reduce the acoustic radiation level of the vibration structure has been converted into directly minimizing the total radiation acoustic power flow from the structural surface. In order to build the relational expressions of pressure objective function and power objective function, the enveloping surface model is conceived to evaluate the pressure in acoustic domain. During the stiffened panel structure's acoustic radiation optimization, the acoustic radiation power is defined as an objective function and the dimension as the design variable, the constraints including the fundamental frequency, maximum stress and total weight and so on.

2 Structural vibration formulations

The structural vibration response analysis is the premise and basis of structural-acoustic optimization. Finite element model application is necessary in the complicated structure dynamic analysis.

Considering a continuous structure under dynamic load $F(x, t)$, if the fluid medium influence can be omitted, in the structure domain Ω^S , the differential equation that governs the behavior of this structure dynamic system can be expressed as:

$$M\ddot{U} + C\dot{U} + KU = F(x, t), \quad x \in \Omega^S, t > 0. \quad (1)$$

In Eq.(1), Ω^S is the structure's domain, U is the nodal

displacement vector matrix, M is the structural mass matrix, K is the structural stiffness matrix and C is the viscous damping matrix. Considering the excitation force $F(x, t)$ is harmonic load, and it can be expressed as:

$$F(x, t) = f(\omega)e^{i\omega t}. \quad (2)$$

In Eq.(2), $f(\omega)$ is the magnitude of the harmonic load, ω is the circular frequency and it is considered as a constant, i is the imaginary unit where $i^2 = -1$. Using the complex variable method, the nodal displacement vector can be expressed as $U(x, t) = u(\omega)e^{i\omega t}$, where $u(\omega)$ is a column matrix of the nodal displacement vectors.

Time dependency of the dynamic problem can be eliminated by substituting displacement vector equation and harmonic load equation into Eq.(1), and then obtain the spatial state operator equation as:

$$-\omega^2 M + i\omega C + Ku(\omega) = f(\omega). \quad (3)$$

The frequency response equation can be written as shorthand

$$A(\omega)u(\omega) = f(\omega),$$

where $A(\omega) = -\omega^2 M + i\omega C + K$, and yield the nodal displacement vector matrix $u(\omega) = A(\omega)^{-1} f(\omega)$.

Defining the nodal velocity vector as $v(\omega)$, and it can be expressed as $v(\omega) = i\omega u(\omega)$. At the interface between the structure and fluid, the nodal particle normal velocity vector can be expressed as:

$$v_n(\omega) = i\omega N A^{-1}(\omega) f(\omega). \quad (4)$$

In Eq. (4), the N is the nodal normal vector matrix, and it correlates with structural surface shape. The nodal particle normal velocity is used as boundary condition in acoustic boundary element method analysis.

3 Acoustic radiation analysis

Since the fluid dynamic loading has been neglected, structural response analysis can be treated independently from the acoustic radiation analysis. The boundary element methods have many advantages in solving the acoustic radiation problem, so it is unnecessary to generate a complicated three-dimensional acoustic model. Considering the

analysis accuracy, the frequency range using BEM follows that the wave length is no less than six element length, so only the low-medium frequency acoustic radiation problem of the continuous structure is analyzed. The standard wave equation is reduced to the Helmholtz equation in the harmonic response problem as:

$$\nabla^2 p + k^2 p = 0. \quad (5)$$

In Eq.(5), p is acoustic pressure, $k(=\omega/c)$ is the wave number, c is the velocity of the wave propagation and ∇^2 is Laplace operator. On the interface of the structural fluid S , the acoustic pressure must satisfy the Neumann boundary condition, i.e. $\frac{\partial p}{\partial n} = -i\omega\rho v_n$,

where v_n is nodal normal velocity of the interface, ρ is density of fluid and \mathbf{n} is the outer-normal units vector of the structure surface. Moreover, the sound pressure p is such that the Sommerfeld condition is satisfied at infinity: $\lim_{r \rightarrow \infty} [r(\frac{\partial p}{\partial r} - ikp)] = 0$. The acoustic pressure at any position within the acoustic domain can be computed from the Helmholtz integral equation. The solution has two steps: first, evaluating the pressure variable on the acoustic boundary using the structural surface normal velocity, and then calculating the pressure variable within the acoustic domain using the boundary pressure information.

4 Acoustic radiations power formulation

The acoustic intensity is defined as the product of pressure and velocity at the time averaged. In the acoustic field, the acoustic intensity can be written as

$$I = \frac{1}{T} \int_0^T \text{Re } p_i(\mathbf{r}) \text{Re } v_n^*(\mathbf{r}) dt. \text{ Account the total cycle}$$

T , and the acoustic intensity in the acoustic field is

$$I(\mathbf{r}) = \frac{1}{2} \text{Re}\{p(\mathbf{r})v_n^*(\mathbf{r})\}. \quad (6)$$

In Eq.(6), $p(\mathbf{r})$ is the acoustic pressure of the field, $v_n^*(\mathbf{r})$ nodal normal complex conjugate velocity.

4.1 Acoustic radiation power formulations

In the acoustic domain, the acoustic power describes the energy flow of an assumed integral surface, and acoustic radiation power can be defined as Π :

$$\Pi = \int_S I(\mathbf{r}) dS = \frac{1}{2} \int_S \text{Re}(p(\mathbf{r})v_n^*(\mathbf{r})) dS. \quad (7)$$

In Eq.(7), S is an integral surface in the acoustic domain. If omitting the acoustic transmission loss and the acoustic absorption of structure surface, the acoustic radiation power of exterior field is equal to the structure surface acoustic radiation power. The structure surface radiation acoustic power can be written as

$$\Pi = \frac{1}{2} \text{Re} \int_S p_f v_n^* dS. \quad (8)$$

In Eq.(8), S is the structure-fluid interface, p_f is acoustic pressure on the structural surface, v_n^* is nodal normal complex conjugate velocity on the structural surface.

4.2 Structure surface acoustic radiation power formulation

For the continuous vibration structure, the total acoustic radiation power at surface can be written as

$$\Pi = \frac{1}{2} \text{Re} \int_S p_f v_n^* dS, \text{ there have } p_f = \rho c v_n, \text{ the phase}$$

angle between acoustic pressure p_f and normal velocity v_n is 0. Substitute the Eq.(4) into it, and then the acoustic pressure at structure surface can be expressed as $p_f = \rho c v_n = i\omega\rho c N A^{-1} f(\omega)$.

Disperse the structural surface with M number of elements and N number of nodes, at the nodes, the acoustic pressure is $p_f = i\omega\rho c N A^{-1} f(\omega)$.

The structural surface acoustic radiation power is equal to the sum of M number elements radiation power, which

$$\text{can be expressed as } \Pi = \frac{1}{2} \text{Re} \sum_{j=1}^M \int_{S_j} p_{fj} v_{nj}^* dS_j. \text{ In the}$$

equation, S_j is integral surface of elements j , p_{fj} is

acoustic pressure of the element j , v_{nj} is normal

velocity of the element j . The normal velocity v_{nj} of

element j is determined by the node's normal velocity

of the element j . Considering the nodal normal vector

matrix N , the equation of structural acoustic radiation

power Π can be written as discrete form:

$$\Pi = \frac{1}{2} \rho c \omega^2 \{[A^{-1} f(\omega)]\}^T S_n [A^{-1} f(\omega)]. \quad (9)$$

In Eq.(9), S_n is normal vector matrix coefficient, and

$$S_n = \sum_{j=1}^M \int_{S_j} N^T N dS_j.$$

On the other hand, the acoustic power radiated from the structure surface is a function of frequency. The external excitation frequency varies over a band, which may include resonant frequencies of the structure. Then in the acoustic radiation power formulation, the objective function is the radiated power over this band. This frequency averaged acoustic radiation power over this band can be obtained by integrating Π_a over the frequency band:

$$\Pi_a = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \Pi(\omega) d\omega \cong \frac{1}{\omega_2 - \omega_1} \sum_{i=1}^m \Pi(\omega_i) \Delta\omega_i. \tag{10}$$

In Eq.(10), ω_2 and ω_1 are the upper and lower bounds of the band, and $\Pi(\omega_i)$ is acoustic radiation power in frequency ω_i .

4.3 Acoustic power optimization formulations

After the structural acoustic radiation power equation is defined, the structure acoustic optimization problem can be converted into searching a group of design variables in the design scope, which makes the structural acoustic radiation power minimum, defining the acoustic power by substituting acoustic pressure for objective function have many advantages in structural acoustic optimization. It turns the vector parameter analysis into scalar quantity parameter analysis; it does not need to analyze the pressure of a defined field point. If define frequency averaged acoustic radiation power as the objective function, then the structural acoustic optimization equation can be stated as:

$$\begin{aligned} \text{Minimize } \Pi_a(b) &= \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \Pi(\omega, b) d\omega \\ \text{Subject to } &b_i^l \leq x_i \leq b_i^u, \\ &\sum_{i=1}^n m_i - m_0 \leq 0, \\ &-\omega_f(b) + \omega_0 \leq 0. \end{aligned} \tag{11}$$

In Eq. (11), b_i^l and b_i^u are respectively the lower and upper bounds of the i th design variable parameter, m_i is the mass of the i th frame element, m_0 is the allowable maximum total mass, $\omega_f(b)$ is the fundamental frequency of the stiffened panel and ω_0 is the allowable minimum fundamental frequency (the

fundamental frequency of initial design). The lower and upper bounds of the design parameters are set to avoid excessive element distortion and inconsistent cell geometry. An optimal solution is a structure that radiates minimum acoustic power in the given frequency band and satisfies the set of constraints. In the acoustic power optimization equation, the constraints include the structural fundamental frequency, structural maximum stress, and structural total weight and so on.

5 Enveloping surface

Nowadays, the pressure of field is used as a criterion to evaluate acoustic performance of exterior acoustic field. The acoustic optimization is to minimize the pressure of the defined field point. In the structural acoustic optimization, the pressure of the acoustic field is a microcosmic parameter and the acoustic power of the structure radicalization is a macroscopic parameter. If the acoustic transmission loss and the acoustic absorption of structure surface are omitted, the total acoustic power of the exterior field is equal to the structure acoustic radiation power.

Since the acoustic radiation power isn't taken as reference standard in acoustic test, it is necessary to build the relational expression of pressure objective function and power objective function. In the acoustic tests, the distance between structure and testing point is often a constant. In the acoustic radiation power analysis, the integral surface of the closure structure is irregular. For evaluating the structural acoustic optimization based on acoustic power, a sphere enveloping surface with the vibration structure inside is considered.

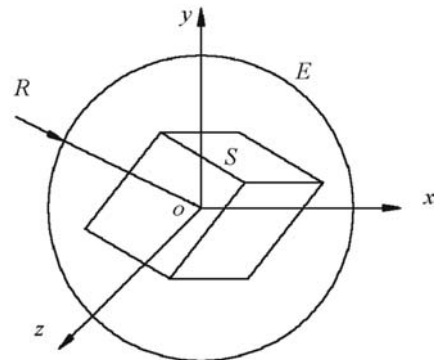


Fig.1 Sphere enveloping surface

In Fig.1, there is a sphere enveloping surface E with radius R , the vibration structure S is completely inside. The nearest distance between the points on the enveloping surface and the structure is a constant, which is determinate by test requirement. The enveloping surface is defined as integral surface, the acoustic power on the integral surface can be written as $W_E = \frac{1}{2} \text{Re} \int_E \frac{p_a^2}{\rho c} dE$, where p_a is equivalent

acoustic pressure amplitude value on the enveloping surface. According to the structural acoustic radiation power, there is $II = W_E$, and the pressure p_a can be obtained, it can be taken as reference value in structural acoustic pressure optimization.

6 Numerical examples

There have an example given to demonstrate the above process. The example contains both the finite element and boundary element methods, and the genetic algorithm is employed for structural acoustic radiation optimization. In the numerical example, the optimization objection is to reduce the sound pressure level while the structural weight not changed.

6.1 The genetic algorithm

The genetic algorithm is applied to optimize the acoustic radiation power. Genetic algorithms are based on Charles Darwin’s Theory of Descent with Modification by Natural Selection, and it has a number of advantages over more traditional optimization techniques in practical design applications. These advantages are as follows: genetic algorithm has strong global optima for complicated optimization problems; genetic algorithm has no need to compute the derivatives of objective functions to optimize them; genetic algorithm can accomplish the optimization of such mixed parameter problems with no need to approximate continuous parameters by discrete parameters; genetic algorithm has ability to be implemented in parallel. The steps of genetic algorithms include encoding scheme, initializing a population, selection, crossover and mutation. Optimization analysis operates selection, crossover and mutation till the global optima can be found.

6.2 The initial design of stiffened panel structure

There have a stiffened panel in the air which is under

external harmonic excitation. The parameters of the stiffened panel are as follows: The length of panel is 1m and the width of panel is 0.6 m. In the panel, there have six T-bar stiffeners (show in the Fig.2). The stiffened panel is discredited with 77 nodes, including 120 triangle elements and 44 beam elements. The material of the stiffened panel is steel with mechanics performance as follows: density $\rho=7\ 800\ \text{kg/m}^3$; modulus of elasticity $E=210\ \text{GPa}$; Poisson ratio $\mu=0.3$. The material property of the fluid is shown as follows: density $\rho=1.225\ \text{kg/m}^3$; sound speed $c=340\ \text{m/s}$.

At the initial design, the thickness of plate is 0.007 m, the dimension of T-bar is shown in Fig.2. The total weight of stiffened panel is 54.15 kg, and the fundamental frequency of stiffened panel is 12.8 Hz.

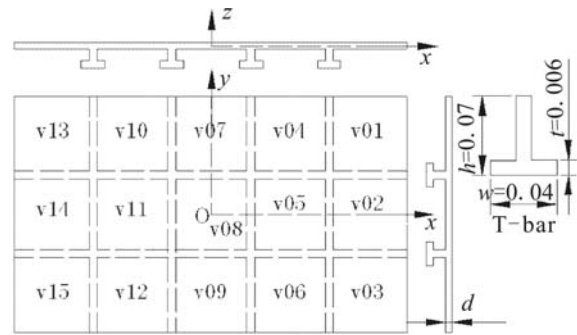


Fig.2 Stiffened panel structure

Dividing the plate into fifteen parts, the plate’s thickness of each part is defined as design variable. As shown in Fig.2, the design variable is named. The frequency of external harmonic excitation force is 20 Hz on the action point at the corner of the plate, the amplitude value of excitation force vector is (-100,150,200) N.

6.3 The constraint of optimization method

In this study, the constraint of design variable includes dimensions of the stiffened panel. The thickness of plate is d_i , there have,

$$0.003\ \text{m} \leq d_i \leq 0.012\ \text{m} \quad i = 1, 2, \dots, 15;$$

The width of T-bar is w_j , there have:

$$0.03\ \text{m} \leq w_j \leq 0.05\ \text{m} \quad j = 1, 2, \dots$$

The height of T-bar is h_k , there have:

$$0.05\ \text{m} \leq h_k \leq 0.1\ \text{m} \quad k = 1, 2, \dots, 6.$$

The weight constraint structure is limited to 54.15 kg, just as shown in the Eq.(11)

$$\sum_{i=1}^n m_i - 54.15 \leq 0.$$

Then the equation

$$\Pi_a(b) = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \Pi(\omega, b) d\omega$$

is defined as objective function.

6.4 The numerical calculation results

The finite element code MSC/Nastran is applied for structural frequency response analysis. The frequency response analysis is evaluated at various frequency ranges between 1 Hz and 601 Hz (the main acoustic energy flow in this range), the frequency step is 5 Hz.

The nodal normal velocity is treated as boundary condition in boundary element method analysis, and then the structural acoustic radiation power can be obtained. The boundary element code LMS-Sysnoise is applied for acoustic radiation power analysis. The acoustic response analysis at various frequency ranges between 1 Hz and 601 Hz, the frequency step is 5 Hz.

Integrating MSC/Nastran code and LMS-Sysnoise code, the genetic algorithm is used as the strategy, there have optimized structural-acoustic of the stiffened panel. Table1 contains partial design variable of the plate thickness and height of T-bar stiffeners about initial design and optimum design. The total weight of stiffened panel at optimum design is 54.1 kg, the fundamental frequency of stiffened panel is 13.1 Hz, and they are satisfying the constraint condition.

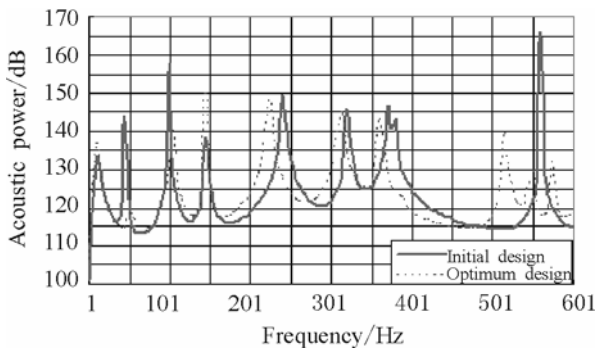


Fig.3 The acoustic power optimization

Fig.3 contains the structure acoustic radiation power of initial design and optimum design. It shows that the peak value of acoustic radiation power has been decreased about 15 dB. Fig.4 contains the equivalent

acoustic pressure of any field point on the enveloping surface of initial design and optimum design, the radius of enveloping surface is 10 m, the center of enveloping surface is the mid point of plate. It shows that the peak value of acoustic pressure at the enveloping surface has been decreased about 16 dB.

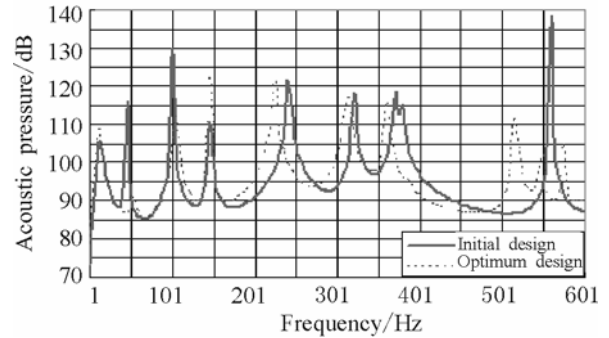


Fig.4 The equivalent acoustic pressure optimization

Fig.5 contains the initial design and optimum design acoustic pressure of field point (0, 0, 10), it shows that the peak of acoustic pressure has been decreased about 7 dB. Fig.6 contains the initial design and optimum design acoustic pressure of field point (0, 0, 20), it shows that the peak of acoustic pressure has been decreased about 6 dB.

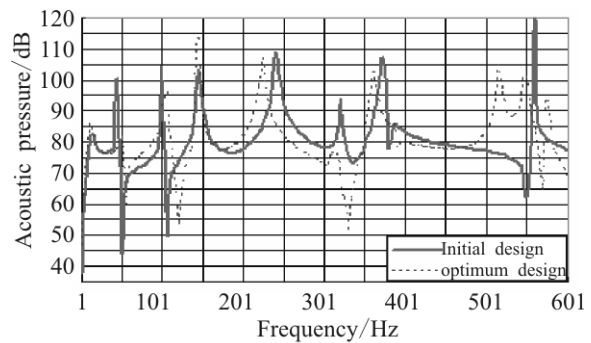


Fig.5 Acoustic pressure of field point

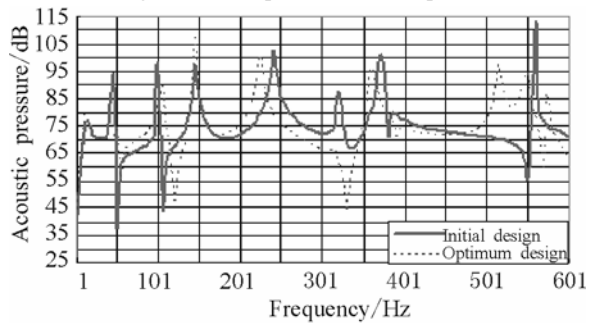


Fig.6 Acoustic pressure of field point

7 Conclusions and discussion

This paper has analyzed the structural-acoustic

optimization problem based on the assumption that structural behavior and acoustic behavior are de-coupled. In the optimization function, the acoustic radiation power is defined as objective function and the constraints are total weight. The numerical examples indicate that the reduction of peak of acoustic radiation power is obvious; the acoustic pressure on the enveloping surface is obvious too. On the other hand, comparing the field point acoustic pressure, the reduction of peak of acoustic pressure is obvious. The numerical results show that using the acoustic radiation power as objective function is feasible.

Only the low-medium frequency acoustic radiation problem of the continuous structure has been analyzed. Considering the accuracy, the high frequency acoustic radiation problem has not been analyzed in this paper.

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