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A frequency and velocity-dependent impedance method for prediction of rail/foundation dynamics

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Abstract: This paper presents an efficient numerical tool for the prediction of railway dynamic response. A behavior calibration of the infinite Euler-Bernoulli beam resting on continuous viscoelastic foundation is proposed. Constitutive laws of the discrete elements are determined for a rectilinear ballasted track. A three-dimensional model coupled with an adaptive meshing scheme is employed to calibrate the beam model impedances by finding the similarity between the output signals using the genetic algorithm. The model shows an important performance with significant reduction in computational effort. This study emphasizes the major impact of the excitation characteristics on the parameters of the discrete models.

Keywords: moving loads; rail vibrations; rail/foundation interaction; dynamic impedances; genetic algorithm

1 Introduction

Running trains on an imperfect wheel/rail interface generates dynamic loads on the railway structure, which leads to vibrations of the track and supporting soil that may seriously influence the living and work environment of people. As pointed out by a number of authors (Madshus and Kaynia, 2000; Auersch, 2008; Alves Costa *et al.*, 2009) the track response is strongly influenced by the soil, in which the normally stiff soils yield completely different levels of ground-born vibration compared to very soft soils.

For soil/structure problems under dynamic loading, the soil can be compared to a frequency-based filter with regard to the response of the structure. In other words, it contributes in the amplification of some components of the excitation signal to the detriment of the attenuation of the other ones. In order to accurately reproduce the structure response, the system parameters should be carefully selected. Several researchers have carried out studies to develop simplified models composed of discrete elements. Mulliken and Karabalis (1998) proposed a frequency-independent discrete model for the analysis of the dynamic interaction between two rigid foundations posed at the top surface of an elastic soil layer. Ju (2003) employed three-dimensional (3D) finite element simulations to determine equivalent matrices of mass, damping and stiffness for an embedded foundation subjected to dynamic loading. Avilés and Suárez (2002) studied the dynamic behavior of a 3D axisymmetric model consisting of a one-story superstructure supported by an embedded rigid foundation. They investigated the effective period and damping of the system by considering the contribution of the soil layer. By analyzing the transfer function of the interacting system, the targeted parameters were associated, respectively, with the resonant period and peak amplification. 3D finite element/infinite element modeling was performed by Zhai *et al.* (2010) to predict the free field vibration generated by high-speed trains. Using a Green's function for multi-layered poroelastic half-space, Wang *et al.* (2017) analyzed the ground vibration due to railway traffic in layered ground with shallow ground water table. They conclude that the wavelength of wheel-rail unevenness has a notable effect on the resulting displacement and pore pressure. A 2.5D finite/infinite element approach presented by Hsiao Hui and Yang (2010) allows the computation efficiency to be enhanced in the analysis of the ground vibrations due to underground trains. Maravas *et al.* (2014) developed a simple oscillator attached on a flexible base to simulate the response of a single or a multi-story structure supported by either surface footing or pile foundation resting on an elastic half-space. They proposed analytical relations for the simplified model

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parameters comprising the damping and the natural period. However, the railway track dynamics problem is considered between the most critical problems due to the related issues comprising the moving loads velocity and the formation of the track structure.

Numerical models intended to predict the track/ground response under high-speed train loading have been widely reported in the literature. They are mainly concerned with the ground borne vibrations in the low frequency range. Despite the recent advances in numerical techniques, the infinite beam resting on a continuous elastic foundation remains a frequent solution (Paolucci *et al.*, 2003; Yang *et al.*, 2003; Koh *et al.*, 2003; Ang and Dai, 2013; Tran *et al.*, 2014; Mezeh *et al.*, 2019) for the assessment of the interaction forces acting on the rail foundation. In this context, Paolucci *et al.* (2003) represented the track embankment by means of an elastic foundation. Yang *et al.* (2003) transmitted the dynamic impact of the moving train into the soil stratum using the response of an infinite beam resting on an elastic foundation. Koh *et al.* (2003) studied four cases of a moving vehicle that traversed a rail-beam supported by a viscoelastic foundation. Later, Ang and Dai (2013) investigated the motion of a high-speed train at constant velocity, crossing a zone with a sudden change of foundation stiffness. They highlighted the problem of a jumping wheel. Tran *et al.* (2014) studied the impact of velocity change (acceleration or deceleration) on the response of the train/track system.

As a matter of fact, the use of continuous support eliminates the dynamic effect of sleepers. However, this hypothesis is considered suitable for slab tracks modeling whereas for ballasted tracks, it neglects the sleeper passing frequency. To overcome this simplification, the rail is modeled with predefined elastic supports. Zhai and Cai (1997), Kouroussis and Verlinden (2015) and Connolly *et al.* (2019) analyzed the track using three layers of spring-damper units representing, respectively, the rail pads, ballast and subgrade. In their studies, the shear coupling effect in the ballast was taken into account by means of shear springs and dampers. Lei and Noda (2002) and Xia *et al.* (2010) employed a computational model consisting of a beam resting on two layers of discrete viscoelastic units for the vehicle and track coupling system. For a more complete discussion of the vehicle and track coupling problem, refer to Connolly *et al.* (2015).

Note that the previously mentioned studies omitted the existence of any correlation between the parameters of the discrete models and the frequency content and/or the velocity of the moving loads. The present study aims at improving and adapting the well-known model, i.e., the infinite Euler-Bernoulli beam that rests on continuous viscoelastic foundation, in order to correctly simulate the dynamic response of the rail under high-speed moving harmonic loads. This goal is achieved by conducting a large number of numerical simulations using a sophisticated 3D finite difference model for a wide range of selected loading frequency that can reach 50 Hz. The 3D model involves a high realistic simulation of force transmission from the rail interface to the ground via an adaptive meshing scheme, i.e., step-by-step procedure in the time domain based upon the creation of load-attached moving nodes on the rail rolling surface. The analysis is performed within the frame of a reference case consisting of a classical rectilinear ballasted track supported by a homogeneous clayey soil layer underlain by a rigid substratum. The obtained results are employed to determine the parameters of the simplified discrete model using the genetic algorithm, which is applied to find the convergence between the responses of the two models. This procedure leads to the proposed frequency and velocity-dependent dynamic impedances for rail vibrations modeling. A major influence of the moving load characteristics on the parameters of the studied model is found.

The paper is organized as follows: Section 2 describes the main components of the reference case as well as the 3D finite difference model used. Section 3 presents the novel predictive tool of track response under dynamic loading. Finally, Section 4 discusses the obtained results.

2 Reference case

The reference case consists of a conventional ballasted railway track. It rests at the top surface of soft clayey soil which extends over a total depth H = 5 m. The standard gauge track is composed of two parallel continuous welded rails discreetly supported by regularly spaced horizontal sleepers. The rail-sleeper interface is considered to be perfectly linear elastic. As shown in Fig. 1, the railway foundation consists of ballast and sub-ballast layers which ensure the loads transmission to the subsoil.

The elastic properties E (Young′s modulus), ν (Poisson′s ratio) and ρ (material density) of the track foundation are given in Table 1.

Table 2 lists the mechanical parameters of the rail-beam.

A finite spatiotemporal domain is considered to numerically simulate the problem of interaction between train loads, track and subsoil. The track formation is represented by 3D solid elements with eight nodes per element whereas two-noded elastic structural elements

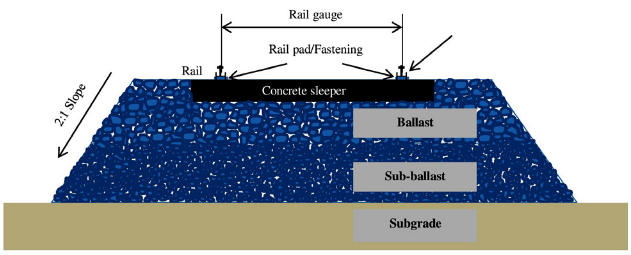

Fig. 1 Cross-section of the rectilinear ballasted track

are used to model the rails and sleepers.

At the model boundaries, viscous elements are employed to prevent spurious reflections that could strongly impact the stability of numerical solving. In this study, it is assumed that there is no interaction between the two rails due to the similarity in load paths along each rail. That is why the symmetry condition is applied along the track centerline. On the other hand, the clayey soil is supposed to overlay a rigid bedrock. Consequently, at the bottom of the model, the nodes are prevented from moving. The top surface is considered to be free. Figure 2 shows an illustrative example of the computational finite difference grid with the adopted boundary conditions.

The Load-Attached Moving Node (L-AMN) scheme which has been developed by Mezeh *et al.* (2018a) is employed to model the track loading via periodic adaptation of the spatial mesh. The proposed approach has been efficiently implemented in the three-dimensional finite difference explicit code "FLAC^{3D}" in which a Matlab subroutine has been implemented in order to allow a rapid development of the generic input files. Further details on the L-AMN approach can be found in Mezeh *et al.* (2018a), including the coupling of the adaptive formulation with the finite difference code for predicting ground-borne vibration from railways.

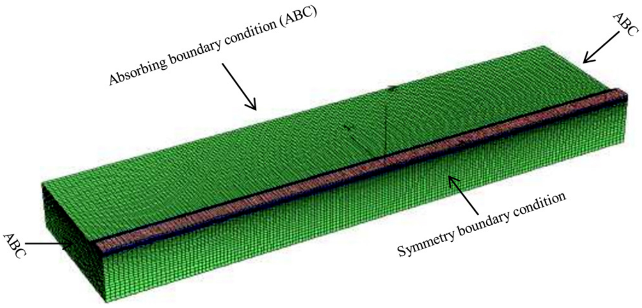

Fig. 2 Computational finite difference grid: 194972 zones and 207972 grid points

Note that the FLAC program has been successfully used in the analysis of the traffic-induced ground vibration (Mhanna *et al.*, 2012) and the efficiency of different techniques for the attenuation of vibrations (Mhanna *et al.*, 2014).

During the numerical simulation, the vertical deflection of the rail, noted by y_r, is recorded at selected predefined points including unloaded moving nodes; they are located at each time step with respect to the load-attached moving axis X_r. Figure 3 shows a typical output of the interaction model comprising the spatial spread of the induced seismic waves and the rail dynamic response obtained under the effect of a high-speed moving load traveling the rail at constant velocity.

3 Predictive model for rail vibrations

Prediction of train-induced vibrations has to cope with the complexity of the mutual dynamic interactions between train loads, track components and the subsoil. A classical solution is based upon the substructuring approach (Paolucci *et al.*, 2003; Yang *et al.*, 2003; Kouroussis, 2012; Kouroussis *et al.*, 2016; Alexandrou *et al.*, 2016; Sun *et al.*, 2016) and it consists of two calculation phases. The first phase aims to study the train/track interaction using a Spring-viscous Damper-lumped Mass system (SDM model) which provides a simplified approach to account for the dynamic behavior of the rail foundation. The second phase simulates the induced seismic waves that propagate in the substructure of the railway by applying the previously obtained reactions at the decoupling interface. In this context, Kouroussis *et al.* (2016) have employed this approach to investigate the impact of foundation and vehicle modeling, local defects, and vehicle speed on the generation of ground-borne vibrations. They have validated the model parameters by means of experimental measurements performed on the L161 line in Brussels (Belgium). Alexandrou *et al.* (2016) have studied the vibrations

Table 1 Mechanical parameters of the track foundation

Layer	*E* (Mpa)	*ν*	*ρ* (kg·m^3)
Ballast	130	0.4	1600
Sub-ballast	80	0.4	1600
Subsoil	25	0.45	1800

Table 2 Mechanical parameters of the rail

Rail parameter	Value
Young's modulus *E*	210 GPa
Poisson's ratio *ν*	0.25
Material density *ρ*	7897 kg/m^3
Area of the rail section *A*	6.5538×10^{-3} m^2
Second moment of inertia I_y	1.2449×10^{-5} m^4

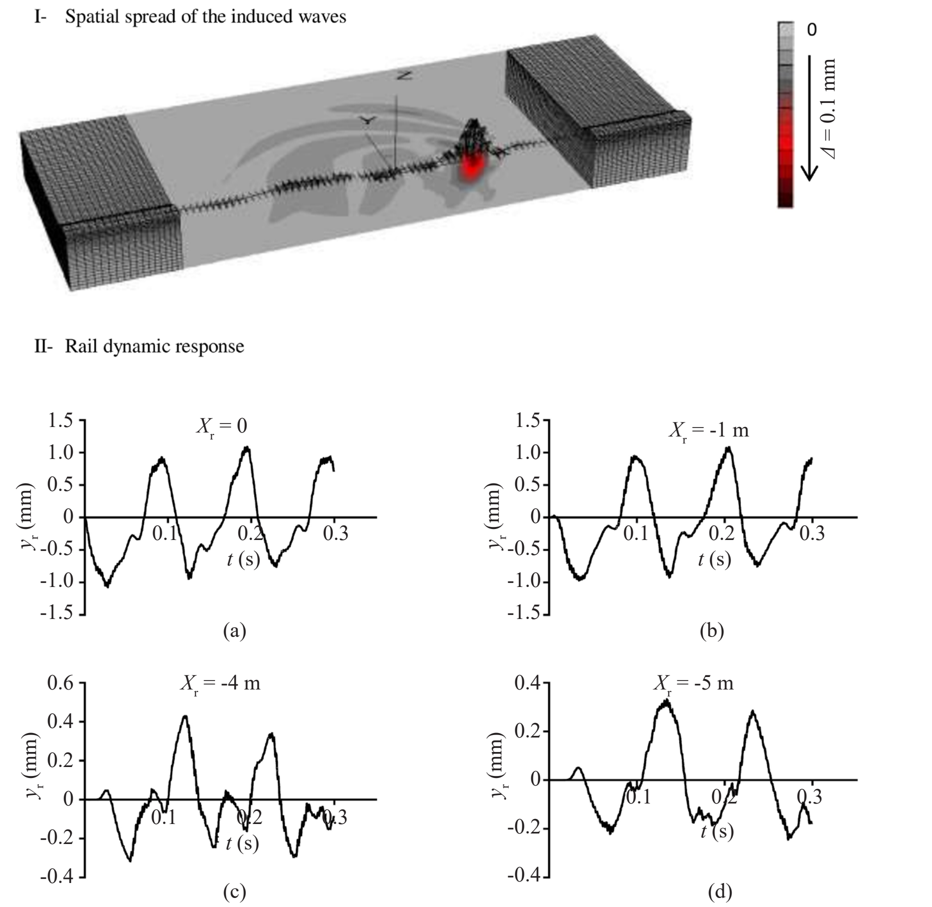

Fig. 3 Dynamic response of the track/ground interaction model due to single moving load

induced by urban tramways in which they have assessed the periodic impact on the rail due to a flat wheel using a flat spot model taking into account Hertz′s contact law.

Figure 4 shows an illustration of the aforementioned modeling methodology of ground vibrations.

By focusing on the SDM models which are reported in the literature review, the effect of the frequency content and traveling velocity of the applied moving loads on the dynamic impedances is generally neglected. Thus, the purpose of the current study is to evaluate the sensitivity to load frequency and velocity of the dynamic impedances. Herein, the infinite Euler-Bernoulli beam with constant mass per unit length m resting on a continuous viscoelastic layer is considered to assess the response of the rail under moving loads. The equivalent foundation is comprised of an infinite series of linear elastic vertical springs with linear stiffness k. The damping of the equivalent layer is integrated through linear viscous dampers η continuously distributed beneath the beam. Note that the dynamic behavior of a single viscous dashpot is similar to that of a semi-infinite bar excited from its end. This shows the potential capacity of a limited number of discrete elements to represent the response of a continuum with an unlimited number of degrees of freedom.

In order to obtain a good agreement between the predictions of the rail response from the simplified and 3D models, the dynamic impedances are calibrated using a curve fitting procedure. Figure 5 shows a schematic illustration of the adopted procedure for the assessment of the frequency and velocity-dependent impedances of the rail foundation.

3.1 Statement of the problem

3.1.1 Beam on viscoelastic foundation

The problem of a harmonic load $P(t)$ that moves at constant velocity V along an infinite beam resting on a continuous viscoelastic foundation is analytically solved by Andersen *et al.* (2001). $P(t)$ is expressed in terms of the amplitude P_0 and angular frequency ω as follows:

$$P(t) = P_0\,\mathrm{e}^{-\mathrm{i}\omega t} \tag{1}$$

where t denotes the time and i the imaginary unit.

The displacement field y_a of the beam is expressed in the moving spatial reference X_r as follows, in which four wave numbers $(\tau X_\mathrm{r} + \mathrm{i}\sigma X_\mathrm{r})$ are highlighted:

$$y_a(X_\mathrm{r}, t) = \begin{cases} A_1\,\mathrm{e}^{-\tau_1 X_\mathrm{r} + \mathrm{i}(\sigma_1 X_\mathrm{r} - \omega t)} + A_2\,\mathrm{e}^{-\tau_2 X_\mathrm{r} + \mathrm{i}(\sigma_2 X_\mathrm{r} - \omega t)}, & X_\mathrm{r} > 0 \\ A_3\,\mathrm{e}^{-\tau_3 X_\mathrm{r} + \mathrm{i}(\sigma_3 X_\mathrm{r} - \omega t)} + A_4\,\mathrm{e}^{-\tau_4 X_\mathrm{r} + \mathrm{i}(\sigma_4 X_\mathrm{r} - \omega t)}, & X_\mathrm{r} \le 0 \end{cases} \tag{2}$$

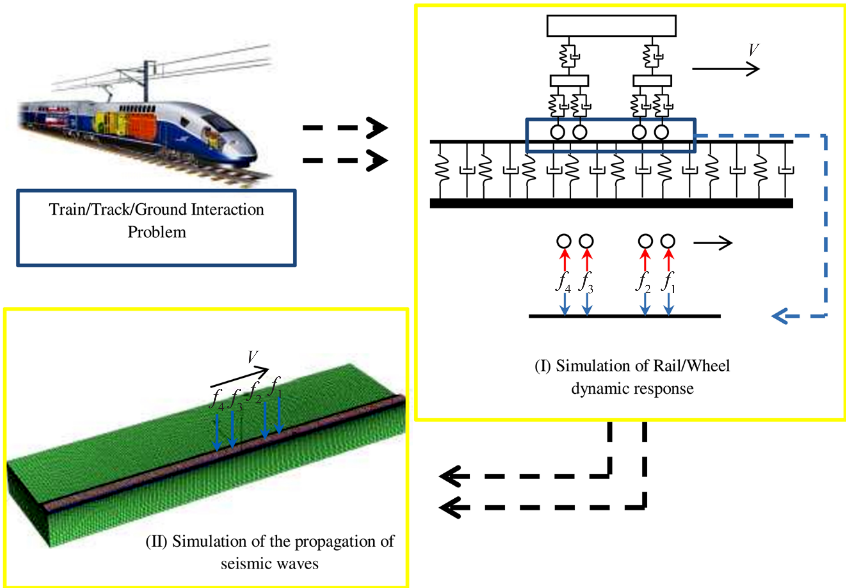

Fig. 4 Substructuring approach for the assessment of ground vibrations

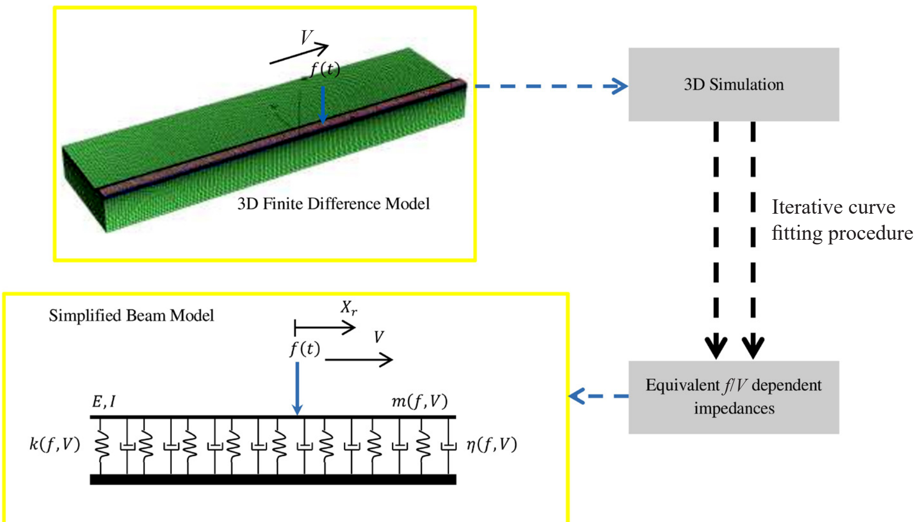

Fig. 5 Calibration process of the simplified predictive model

The four complex numbers $A_j\left(j=1,\ldots,4\right)$ could be determined by imposing the continuity of the displacement, rotation, moment and shear force at the loaded point $\left(X_{\mathrm{r}}=0\right)$.

Note that this analytical solution is proposed for a uniform motion of deterministic load. However, in the case of adopting of nonlinearity contact theory, non-uniform motion, etc., numerical methods must be employed. In this context, Mezeh *et al.* (2018b) have proposed the PCU method, which is based upon a step-by-step solving of the dynamic equation by performing a periodic change of the initial condition at the beginning of each step.

3.1.2 Optimization process

In this work, the dynamic parameters of the simplified model $\left(k,\eta,m\right)$, are calibrated to minimize the divergence with the sophisticated 3D model. Obviously, for each case studied, the two models are subjected to a similar load. The optimization problem consists of:

$$Find\ \boldsymbol{\alpha}\in\Omega / g\left(\alpha\right)=\min_{\Omega}\left[g\right] \tag{3}$$

where $\boldsymbol{\alpha}$ is a vector of the simplified beam parameters, and g is called the objective function which is expressed in the search space $\Omega=\mathbb{R}^3$:

$$g:\Omega\rightarrow\mathbb{R} \tag{4}$$

Herein, the genetic algorithm (GA) is employed to solve Eq. (3). As indicated by its name, the vector $\boldsymbol{\alpha}$ is optimized by a process oriented by genetic operations in which the fittest individuals in the population $N-1$ have more chance to govern the characteristics of the next generation. For more details about genetic algorithms, the reader to the literature (Michalewicz, 1996; Reeves, 2003).

The objective function g which is intended to measure the deviation between the output signals, is determined for each vector $\boldsymbol{\alpha}$ as the square root of the mean squared error (MSE). It is calculated using a set of N_p points (governed by the output of the 3D model):

$$g=\sqrt{\frac{1}{N_\mathrm{p}}\sum_{1}^{N_\mathrm{p}}\left[y_\mathrm{r}-\mathrm{imag}\left(y_a\right)\right]^2} \tag{5}$$

where the applied load is taken as $\mathrm{imag}[P]$; that is why the imaginary part of the analytical solution has appeared in the expression of the objective function.

3.2 Dynamic impedances of the simplified model

To investigate the behavior of the simplified model, 40 loading cases are taken into account. They correspond to four sets of 10 cases of linear frequency f ranging from 5 to 50 Hz that fall within the frequency range, which is usually retained in vehicle/track/soil interaction problems, including building response. Each set is characterized by a uniform traveling velocity V which varies between 50 and 300 km/h. After applying each loading case, characterized by (f,V), to the 3D model, the dynamic response of the rail y_r is obtained.

The stiffness, damping and mass factors representing the dynamic components of the rail foundation are depicted in Fig. 7. Note that each triplet (k,η,m) for a loading scenario (f,V) is obtained after reaching the convergence of the GA. Herein, the evolution of an initial random population of 300 individuals is simulated over 100 generations.

The obtained results show that the trend of the curve representing the linear stiffness k of the equivalent foundation is generally increasing with the frequency and velocity. Conversely, the vibrating mass per unit length m tends to decrease simultaneously with f and V. Therefore, the mobilized frequency f_0 (Eq. (6)) of the simplified model tends to increase with the loading

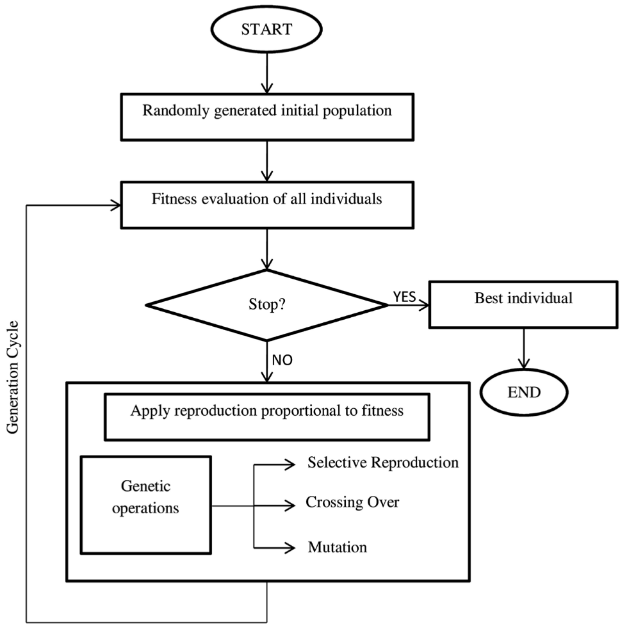

Fig. 6 Flowchart of the Genetic Algorithm

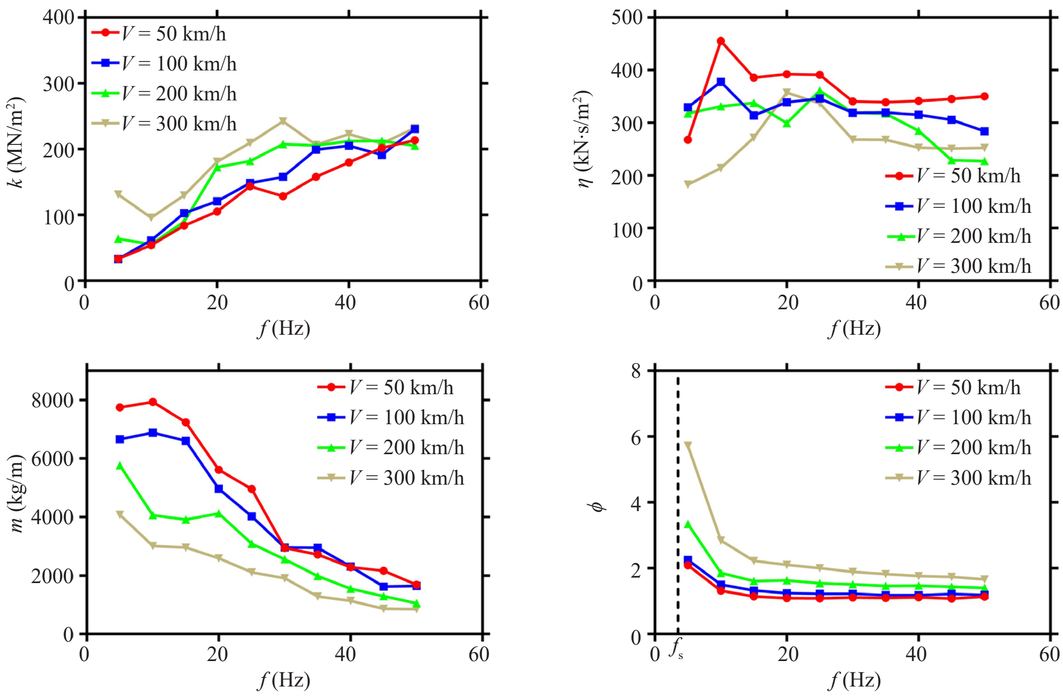

Fig. 7 Dynamic impedances of the viscoelastic foundation for rail vibrations prediction

frequency and traveling velocity. Figure 7 also illustrates the variation of f_0 to load frequency ratio Φ according to f. It reflects a clear tendency of the dimensionless ratio to have a vertical asymptote in the vicinity of the subsoil natural frequency f_s which is equal to 3.5 Hz (shear wave speed divided by the layer depth H). This fact highlights the capacity of the equivalent foundation to rigidify depending on the external loading.

$$f_0(f,V)=(2\pi)^{-1}\sqrt{\frac{k}{m}} \tag{6}$$

Note that the current study aims to reproduce the foundation effect on the rail dynamic response through the use of a calibrated SDM model. Therefore, such frequency and velocity-dependent discrete elements seem to pose non-conceptual problems; they cannot be built in reality.

On the other hand, Fig. 8 depicts the relative divergence between the two output signals. The committed error in the calculated displacement field is expressed in terms of the normalized value of the objective function, Γ (Eq. (7)), which is calculated at the load position $(X_r=0)$ after the convergence of the optimization algorithm.

$$\Gamma=\frac{g}{\max[y_r]} \tag{7}$$

The continuous viscoelastic foundation is found to give acceptable results. The calibration process reveals that the error ranges from 8% to 20%. Note that the numerical modeling gives rise to a stationary response preceded by a transitory part, whereas the analytical solution is stationary from $t=0$. That is why the parameter Γ seems to overestimate the committed error on the displacement field. Figures 9 to 12 present the dynamic response of the simplified model compared to that of the 3D model, at X_r = 0, for the four cases of V. They correspond to a load amplitude $P_0=100$ kN. As stated before, an excellent agreement in terms of amplitude and phase between the two models can be easily noticed. This conclusion remains valid on the whole plan (f,V).

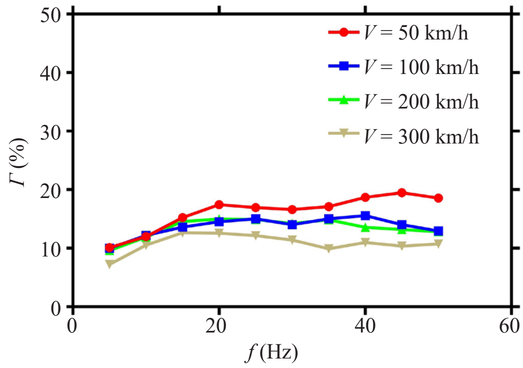

Fig. 8 Optimal capacity of the viscoelastic layer to represent the dynamic behavior of the rail foundation

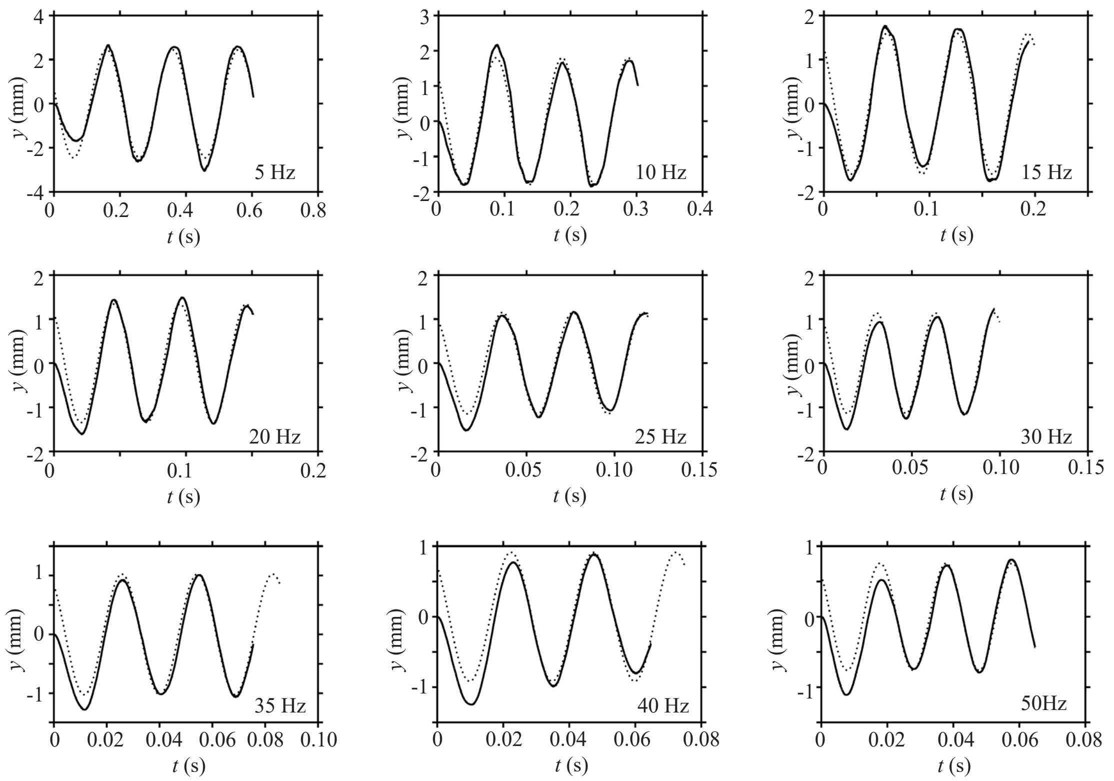

Fig. 9 Dynamic response of the rail at X_r= 0; case of harmonic load moving at V= 50 km/h. solid line: 3D simulation, dotted line: beam model

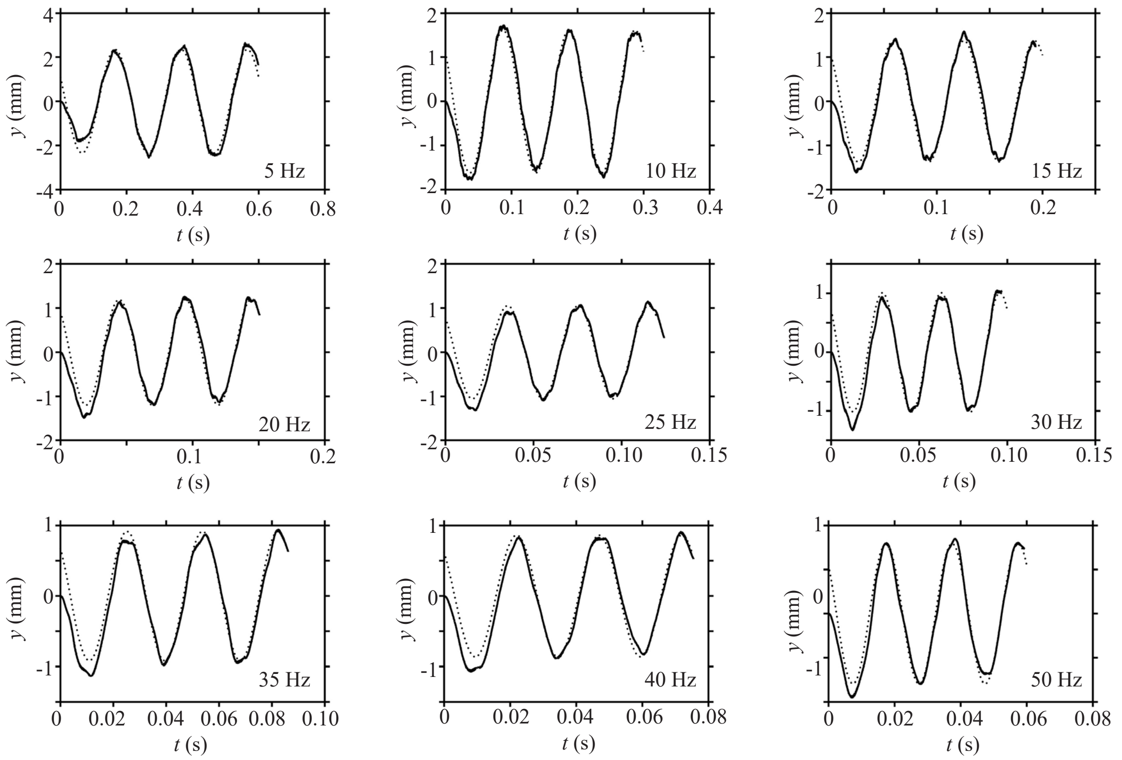

Fig. 10 Dynamic response of the rail at $X_r = 0$; case of harmonic load moving at $V = 100$ km/h. solid line: 3D simulation, dotted line: beam model

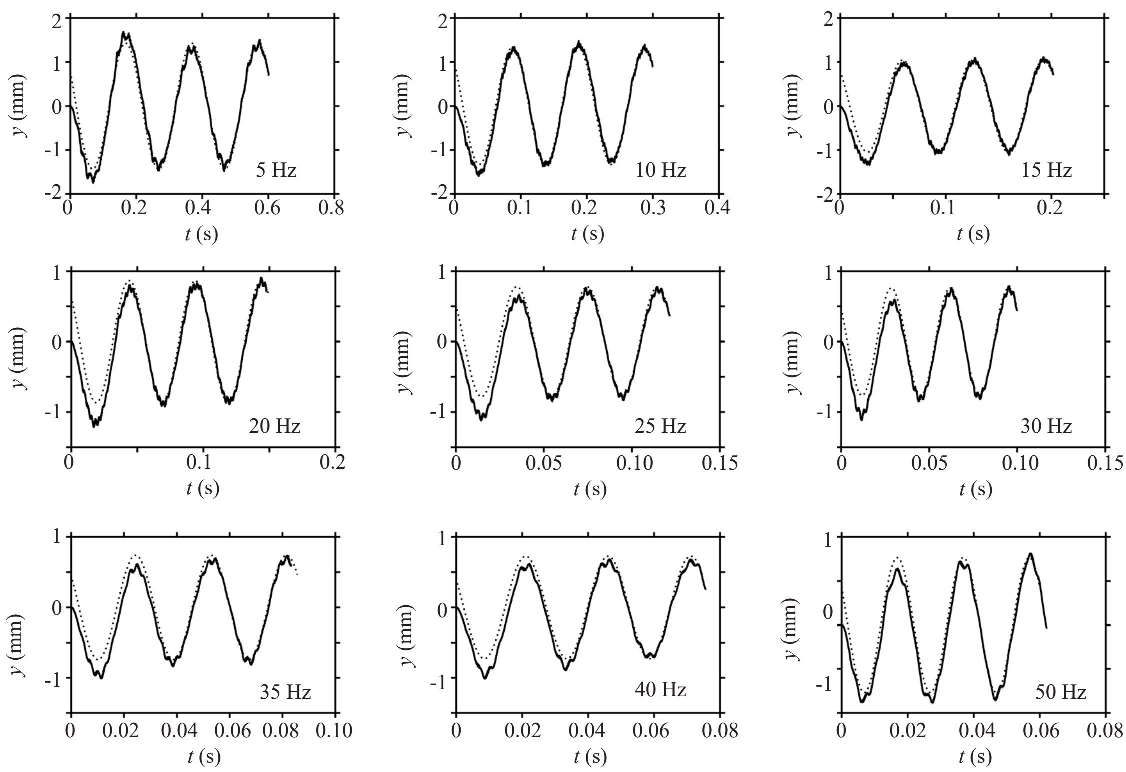

Fig. 11 Dynamic response of the rail at $X_r = 0$; case of harmonic load moving at $V = 200$ km/h. solid line: 3D simulation, dotted line: beam model

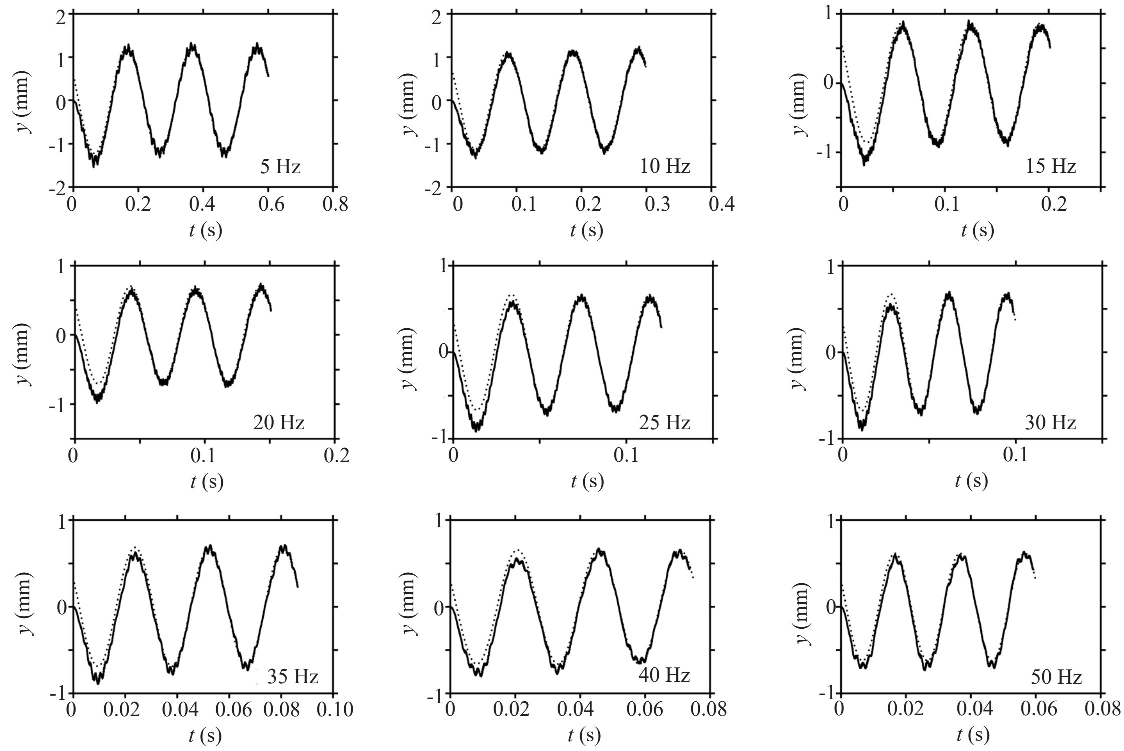

Fig. 12 Dynamic response of the rail at $X_r = 0$; case of harmonic load moving at $V = 300$ km/h. solid line: 3D simulation, dotted line: beam model

The simplified model with improved input parameters is proved to be able to reliably reproduce the dynamic response of the rail. The mutual dynamic interaction is taken into account through the proposed behavior of the discrete elements. Note that the ratio between the required calculation time of the three-dimensional simulation and that of the simplified model exceeds 2000 for several cases. Consequently, it proves the efficiency of the simplified model, which avoids time-consuming 3D simulations that require very complex modeling of the different components of the system.

4 Conclusion

This study shows the significant effect of the track/ground dynamic interaction on the parameters of the discrete models. Generally, it can be concluded that a simplified calculation protocol should be preceded by a reliability study before being used in the vibratory analysis of railways under high speed time-variant moving loads. In this study, the reference site has been chosen as a rectilinear ballasted track resting at the top of a homogeneous clayey soil. Physical decoupling between the rail and the other components of the track has been proposed in which the rail (Euler-Bernoulli beam) has been considered to be supported by a viscoelastic foundation. To integrate the foundation dynamic effect in the response of the beam, the results obtained from three-dimensional simulations have been used. The sensitivity of the dynamic impedances of the simplified model has been investigated with respect to the frequency and traveling velocity of the load. The genetic algorithm has been used for parameter optimization purposes. The continuous viscoelastic foundation has been found to give acceptable results in which the error on the displacement field ranges from 8% to 20%. On the other hand, the developed model allows a large decrease in the calculation time when compared to an equivalent 3D advanced simulation.

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References

Alexandrou G, Kouroussis G and Verlinden O (2016), "A Comprehensive Prediction Model for Vehicle/Track/Soil Dynamic Response due to Wheel Flats," *Proc Inst Mech Eng Part F J Rail Rapid Transit*, **230**: 1088–1104, doi:10.1177/0954409715576015.

Alves Costa P, Calçada R, Silva Cardoso A and Bodare A (2010), "Influence of Soil Non-Linearity on the

Dynamic Response of High-Speed Railway Tracks," *Soil Dyn Earthq Eng*, **30**: 221–235, doi:10.1016/j.soildyn.2009.11.002.

Andersen L, Nielsen SRK and Kirkegaard PH (2001), "Finite Element Modelling of Infinite Euler Beams on Kelvin Foundations Exposed to Moving Loads in Convected Co-Ordinates," *J Sound Vib*, **241**(4): 587–604, doi:10.1006/jsvi.2000.3314.

Ang KK and Dai J (2013), "Response Analysis of High-Speed Rail System Accounting for Abrupt Change of Foundation Stiffness," *J Sound Vib*, **332**(12): 2954–2970, doi:10.1016/j.jsv.2013.01.005.

Auersch L (2008), "The Effect of Critically Moving Loads on the Vibrations of Soft Soils and Isolated Railway Tracks," *J Sound Vib*, **310**(3): 587–607, doi:10.1016/j.jsv.2007.10.013.

Avilés J and Suárez M (2002), "Effective Periods and Dampings of Building-Foundation Systems Including Seismic Wave Effects," *Eng Struct*, **24**(5): 553–562, doi:10.1016/S0141-0296(01)00121-3.

Connolly DP, Galvín P, Olivier B, Romero A and Kouroussis G (2019), "A 2.5D Time-Frequency Domain Model for Railway Induced Soil-Building Vibration due to Railway Defects," *Soil Dyn Earthq Eng*, **120**: 332–344, doi:10.1016/j.soildyn.2019.01.030.

Connolly DP, Kouroussis G, Laghrouche O, Ho CL and Forde MC (2015), "Benchmarking Railway Vibrations Track, Vehicle, Ground and Building Effects," *Constr Build Mater*, **92**: 64–81, doi:10.1016/j.conbuildmat.2014.07.042.

Hsiao Hui H and Yang YB (2010), "Analysis of Ground Vibrations due to Underground Trains by 2.5D Finite/Infinite Element Approach," *Earthquake Engineering and Engineering Vibration*, **9**(3): 327–335, doi:10.1007/s11803-010-0017-1.

Ju SH (2003), "Evaluating Foundation Mass, Damping and Stiffness by the Least-Squares Method," *Earthq Eng Struct Dyn*, **32**: 1431–1442, doi:10.1002/eqe.285.

Koh CG, Ong JSY, Chua DKH and Feng J (2003), "Moving Element Method for Train-Track Dynamics," *Int J Numer Methods Eng*, **56**(11): 1549–1567, doi:10.1002/nme.624.

Kouroussis G, Florentin J and Verlinden O (2016), "Ground Vibrations Induced by InterCity/InterRegion Trains: a Numerical Prediction Based on the Multibody/Finite Element Modeling Approach," *J Vib Control*, **22**(20): 4192–4210, doi:10.1177/1077546315573914.

Kouroussis G and Verlinden O (2015), "Prediction of Railway Ground Vibrations: Accuracy of a Coupled Lumped Mass Model for Representing the Track/Soil Interaction," *Soil Dyn Earthq Eng*, **69**: 220–226, doi:10.1016/j.soildyn.2014.11.007.

Kouroussis G, Verlinden O and Conti C (2012), "A Two-Step Time Simulation of Ground Vibrations Induced by the Railway Traffic," *Proc Inst Mech Eng Part C J Mech Eng Sci*, **226**(2): 454–472, doi:10.1177/0954406211414483.

Lei X and Noda N-A (2002), "Analyses of Dynamic Response of Vhicle and Track Coupling System with Random Irregularity of Track Vertical Profile," *J Sound Vib*, **258**(1): 147–165, doi:10.1006/jsvi.2002.5107.

Madshus C and Kaynia Am (2000), "High-Speed Railway Lines on Soft Ground: Dynamic Behaviour at Critical Train Speed," *J Sound Vib*, **231**(3): 689–701, doi:10.1006/jsvi.1999.2647.

Maravas A, Mylonakis G and Karabalis DL (2014), "Simplified Discrete Systems for Dynamic Analysis of Structures on Footings and Piles," *Soil Dyn Earthq Eng*, **61–62**: 29–39, doi:10.1016/j.soildyn.2014.01.016.

Mezeh R, Mroueh H, Hosseingholian M and Sadek M (2019), "New Approach for the Assessment of Train/Track/Foundation Dynamics Using In-Situ Measurements of High-Speed Train Induced Vibrations," *Soil Dyn Earthq Eng*, **116**: 50–59, doi:10.1016/j.soildyn.2018.10.024.

Mezeh R, Sadek M, Hage Chehade F and Mroueh H (2018a), "Adaptive Meshing Scheme for Prediction of High-Speed Moving Loads Induced Ground Vibrations," *Comput Geotech*, **100**: 188–202, doi:10.1016/j.compgeo.2018.03.014.

Mezeh R, Sadek M, Hage Chehade F and Shahrour I (2018b), "Adaptive Analysis of Infinite Beams Dynamics Problems Using the Periodic Configuration Update Method in the Time Domain," *Int J Numer Anal Methods Geomech*, **42**: 618–635, doi:10.1002/nag.2757.

Mhanna M, Shahrour I, Sadek M and Dunez P (2014), "Efficiency of Heavy Mass Technology in Traffic Vibration Reduction: Experimental and Numerical Investigation," *Computers and Geotechnics*, **55**: 141–149, https://doi.org/10.1016/j.compgeo.2013.08.002

Mhanna M, Sadek M and Shahrour I (2012), "Numerical Modeling of Traffic-Induced Ground Vibration," *Comput Geotech*, **39**: 116–123, doi:10.1016/j.compgeo.2011.07.005.

Michalewicz Z (1996), *Genetic Algorithms + Data Structures = Evolution Programs*, 3rd Ed., London, UK, UK: Springer-Verlag.

Mulliken JS and Karabalis DL (1998), "Discrete Model for Dynamic Through-the-Soil Coupling of 3-D Foundations and Structures," *Earthq Eng Struct Dyn*, **27**(7): 687–710.

Paolucci R, Maffeis A, Scandella L, Stupazzini M and Vanini M (2003), "Numerical Prediction of Low-Frequency Ground Vibrations Induced by High-Speed Trains at Ledsgaard, Sweden," *Soil Dyn Earthq Eng*, **23**(6): 425–433, doi:10.1016/S0267-7261(03)00061-7.

Reeves C (2003), "Genetic Algorithms," In: Glover F, Kochenberger GA, Editors, *Handb. Metaheuristics*, Springer US, p. 55–82, doi:10.1007/0-306-48056-5_3.

Sun L, Xie W, He X and Hayashikawa T (2016), "Prediction and Mitigation Analysis of Ground Vibration Caused by Running High-Speed Trains on Rigid-Frame Viaducts," *Earthquake Engineering and Engineering Vibration*, **15**(1): 31–47, doi:10.1007/s11803-016-0303-7.

Tran MT, Ang KK and Luong VH (2014), "Vertical Dynamic Response of Non-Uniform Motion of High-Speed Rails," *J Sound Vib*, **333**(21): 5427–5442, doi:10.1016/j.jsv.2014.05.053.

Wang F, Tao X, X Lili and Siddharthan R (2017), "Green′s Function of Multi-Layered Poroelastic Half-Space for Models of Ground Vibration due to Railway Traffic," *Earthquake Engineering and Engineering Vibration*, **16**(2): 311–328. https://doi.org/10.1007/s11803-017-0384-y.

Xia H, Cao YM and De Roeck G (2010), "Theoretical Modeling and Characteristic Analysis of Moving-Train Induced Ground Vibrations," *J Sound Vib*, **329**(7): 819–832, doi:10.1016/j.jsv.2009.10.007.

Yang YB, Hung HH and Chang DW (2003), "Train-Induced Wave Propagation in Layered Soils Using Finite/Infinite Element Simulation," *Soil Dyn Earthq Eng*, **23**(4): 263–278, doi:10.1016/S0267-7261(03)00003-4.

Zhai W and Cai Z (1997), "Dynamic Interaction Between a Lumped Mass Vehicle and a Discretely Supported Continuous Rail Track," *Comput Struct*, **63**(5): 987–997, doi:10.1016/S0045-7949(96)00401-4.

Zhai W, He Z and Song X (2010), "Prediction of High-Speed Train Induced Ground Vibration Based on Train-Track-Ground System Model," *Earthquake Engineering and Engineering Vibration*, **9**(4): 545–554. https://doi.org/10.1007/s11803-010-0036-y