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# A new solution to estimate the time delay on the topographic site using time domain 3D boundary element method

Reza Tarinejad<sup>1†</sup>, Mohsen Isari<sup>1‡</sup> and Abdollah Sohrabi-Bidar<sup>2§</sup>

1. Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran

2. School of Geology, College of Science, University of Tehran, Tehran, Iran

**Abstract:** This study focuses on investigating spatial variation of ground motion that has great influence on the dynamic behavior of the large structures located on the surface topography. One of the most effective parameters on the spatial variation of ground motion is the difference between the arrival time of seismic waves in different points located on the abutments. In this research, a three-dimensional model of the Pacoima Dam site is prepared. The time domain 3D boundary element method is used to apply non-uniform excitation at the dam supports. This model is subjected to vertically propagating incident SH and P waves. The time delay can be characterized by calculating the value of the time delay for which the cross-correlation between two records is maximized. Finally, to obtain the time delay in a topographic site, a function considering effective parameters such as the height from the canyon base, wave velocity and predominant frequency, is presented. Furthermore, a code was developed for generating the spatially variation of seismic ground motions. The results show that the proposed functions have an acceptable accuracy in estimating the time delay to generate non-uniform ground motion.

Keywords: time delay; boundary element method; record generation; non-uniform record; Pacoima Dam

#### **1** Introduction

In the past few years, many researchers have shown that earthquake records in different points on the ground surface are non-uniform for many reasons, such as difference in the arrival time of waves at separate stations (known as wave-passage effect), and reflection and refraction of the seismic waves. It has been proven that spatial variation has a great influence on the dynamic behavior of structures. One of the main parameters affecting the spatial variation of ground motion is topography characteristics such as hills and canyons (Wu et al., 2016). Therefore, in the seismic analysis of the large-scale structures with extended contact surface of the ground, consideration of spatial variation in ground motion is important. Due to a lack of records of arrays in different points of a topography such as a canyon, seismic analysis considering nonuniform ground motion is a difficult task. Most of the numerical analyses for investigating the topography effect are limited to two-dimensional or simplified three-

E-mail: r\_tarinejad@tabrizu.ac.ir

dimensional models. In order to investigate the problems caused by the topographic effect, the boundary element method (BEM) is frequently used because it reduces the computational efforts by applying discretization just on the boundaries of the model and provides a high level of accuracy for the problem (Tarinejad et al., 2019; Isari et al., 2019). In terms of the history of the use of the boundary element method, Friedman and Shaw (1962) were one of the first in formulations of time-domain boundary element analysis to solve multi-dimensional wave propagation problems. Niwa et al. (1980) used the time-domain boundary element method to solve elastodynamic problems. Then, Mansur (1983), Antes (1985), and Mansur and Brebbia (1985) proposed boundary element complete algorithms step by step to multidimensional elasto-dynamic problems. Karabalis and Beskos (1984) and Manolis et al. (1985) proposed threedimensional boundary element algorithms to solve multidimensional wave propagation problems. Zhao (1992) investigated the seismic body wave scattering incident to canyon topography. Later, Gaffet and Bouchon (1989) evaluated the dynamic behaviors of hills, indicating that resulting amplification depends on the height of the hill. Demirel and Wang (1987) used the Heaviside function to extract boundary element formulation that led to the lower precision of traction kernels. Yu et al. (2000) and Soares and Mansur (2009) has improved the accuracy of this formulation. Mossessian and Dravinski (1990a,

**Correspondence to:** Reza Tarinejad, Faculty of Civil Engineering, University of Tabriz, 29 Bahman Blvd, Tabriz 51666, Iran Tel: +98-4133392386

<sup>&</sup>lt;sup>†</sup>Associate Professor; <sup>‡</sup>Ph.D. Candidate; <sup>§</sup>Assistant Professor **Received** November 22, 2017; **Accepted** May 5, 2018

1990b) studied the scattering of elastic waves employing a three-dimensional canyon of arbitrary shape and used the indirect boundary integral equation method to investigate the steady state responses. Paolucci (2002) studied the dynamic response of several cases through the numerical wave propagation method based on the spectral element. In addition, he used analytical investigation for determining the fundamental frequency of vibration of topographic features due to earthquake ground shaking.

Kamalian et al. (2006) used a time-domain hybrid FEM/BEM method to analyze the site dynamic responses of various topographic features subjected to incident P and SV waves and proposed a comprehensive numerical parametric study on seismic behavior of canyons subjected to in-plane waves (Kamalian et al., 2007). Tarinejad (2008) used the three-dimensional boundary element method to evaluate the site effects on the amplification of spatially varying ground motion. A comprehensive parametric study for considering effective parameters on the displacement amplification caused by non-uniform excitation subjected to various angles of incident wave and frequencies, materials with different properties (e.g., damping and Poisson's ratio) of arbitraryshaped valleys was performed by Tarinejad et al. (2007). Sohrabi-Bidar et al. (2010) developed a time-domain boundary element method and investigated the seismic response of 3-D Gaussian-shaped valleys subjected to vertically propagating incident waves. Sohrabi-Bidar and Kamalian (2013) used boundary elements for the evaluation of the effects of three-dimensionality on the seismic response of Gaussian-shaped hills for simple incident pulses. They indicated that wave type, site geometry and wavelength are the effective parameters on the dynamic behavior of topography. Wu et al. (2016) investigated the spatially varying ground motions in V-shaped symmetric canyons. They proposed two methods to simulate spatially varying ground motions. First, the spatially varying ground motions are simulated by using the power spectral density and an empirical coherency loss function. Second, the spatially varying ground motions are simulated by using given ground motions and an empirical coherency loss function. Chopra and Wang (2010) evaluated the seismic response of arch dams due to spatially varying ground motions. They indicated that spatial variation has a profound influence on the dynamic behavior and earthquakeinduced stresses in the dam. Alves (2005) proposed a new method to simulate spatially varying ground motions for seismic analysis of a concrete arch dam. He used a transfer function that includes amplification and time delay. A method to generate non-uniform time series excitation using the records of three points, one point on the base and two points on both sides of the abutments, was proposed. It was indicated that dynamic response caused by non-uniform ground motion is different from uniform excitation.

According to the literature review, most of the research

regarding non-uniform ground motion generation are generally focused on the site effects of flat sites and a few are of concern to the topographic amplification. In this study, non-uniform ground motion is generated on the topographic feature using three-dimensional boundary element method in the time domain and the Pacoima Dam site is as a case study. Time delay between the ground motion of the base and the higher elevated points along the abutments of the canyon is characterized by determining the value of the time shift for which the cross-correlation of the records is maximized. Finally, a comparatively good statistical relationship with regard to the values extracted from the numerical analysis on the real model was proposed using the symbolic regression method, taking into consideration the most effective parameters in the investigation and production of nonuniform accelerations on the site and determination of the time delays. The accuracy of the proposed function is evaluated by regeneration of the non-uniform ground motion of the Pacoima Dam site during the 21 January, 2001 earthquake, which distinguishes this study from previous studies that are mostly two-dimensional or include simplified three-dimensional geometries.

### **2** Numerical formulation

In this study, numerical modeling is carried out using the time-domain boundary element method where the governing equation for an elastic, isotropic, and homogeneous body with a small amplitude displacement field can be written as:

$$\left(c_{\rm L}^2 - c_{\rm T}^2\right) \frac{\partial^2 \boldsymbol{u}_j}{\partial x_i \partial x_j} + c_{\rm T}^2 \frac{\partial^2 \boldsymbol{u}_i}{\partial x_j \partial x_j} + \boldsymbol{b}_i = \frac{\partial^2 \boldsymbol{u}_i}{\partial t^2}$$
(1)

in which  $\boldsymbol{u}_i$  denotes the displacement vector;  $\boldsymbol{b}_i$  denotes the body force vector and  $c_{\rm L}$  and  $c_{\rm T}$  are the spreading velocities of the compressional and shear waves, respectively, where  $c_{\rm L}^2 = (\lambda + 2\mu) / \rho$ ;  $c_{\rm T}^2 = \mu / \rho$ ,  $\lambda$  and  $\mu$  are the Lame constants, and  $\rho$  is the mass density. The governing boundary integral equation for an elastic, isotropic, homogeneous body can be obtained using the well-known weighted residual method as:

$$c_{ij}(\xi)\boldsymbol{u}_{j}(\xi,t) = \int_{\Gamma} \left\{ u_{ij}^{*}(x,\xi,t) * \boldsymbol{p}_{j}(x,t) \right\} d\Gamma - \int_{\Gamma} \left\{ p_{ij}^{*}(x,\xi,t) * \boldsymbol{u}_{j}(x,t) \right\} d\Gamma$$

$$(2)$$

in which  $p_j$  is traction in boundary  $\Gamma$ ,  $u_{ij}^*$  and  $p_{ij}^*$  are the transient displacement and traction fundamental solutions of Eq. (1), respectively, and denote the *j* th components of the displacements and tractions at point *x* at time *t* due to a unit point force applied in direction *i* at point  $\xi$  at preceding time  $\tau \, . \, u_{ij}^* \otimes \boldsymbol{p}_j$  and  $p_{ij}^* \otimes \boldsymbol{u}_j$  denote the Riemann convolution integrals and  $c_{ij}$  is the discontinuity term resulting from the singularity of the traction fundamental solution. In this study for seismic analysis, total displacement can be split into incident  $(\boldsymbol{u}_i^{\rm inc})$  and scattered  $(\boldsymbol{u}_i^{\rm sc})$  components, and the governing boundary integral equation should be modified as follows:

$$c_{ij}(\xi)\boldsymbol{u}_{j}(\xi,t) = \int_{\Gamma} \left\{ u_{ij}^{*}(\boldsymbol{x},\xi,t) * \boldsymbol{p}_{j}(\boldsymbol{x},t) \right\} \mathrm{d}\Gamma - \int_{\Gamma} \left\{ p_{ij}^{*}(\boldsymbol{x},\xi,t) * \boldsymbol{u}_{j}(\boldsymbol{x},t) \right\} \mathrm{d}\Gamma + \boldsymbol{u}_{i}^{\mathrm{inc.}}(\xi,t)$$
(3)

In homogenous topography problems, the boundary is a free surface, traction is null, and the above-mentioned equation for any point  $\xi$  in boundary  $\Gamma$  can be written as:

$$c_{ij}(\xi) \times \boldsymbol{u}_{j}(\xi,t) = -\int_{\Gamma} P_{ij}^{*}(x,\xi,t) \otimes \boldsymbol{u}_{j}(x,t) \times \mathrm{d}\Gamma + \boldsymbol{u}_{i}^{\mathrm{inc}}(\xi,t)$$
(4)

In order to solve the problem by numerical methods, the boundary integral equation must be expressed as a set of linear equations that when solved, the solution of the boundary value problem will be obtained. To transform the governing integral equation into the ideal form, it is separated in the time and then in the spatial domain. Finally, the obtained equations will be expressed in matrix form. The Cauchy principal value of the traction kernel integral for the singular integral is obtained together with the  $c_{ij}$  jump term indirectly using the rigid body motion technique that requires the body to have a closed boundary (Sohrabi-Bidar *et al.*, 2010; Sohrabi-Bidar and Kamalian, 2013).

#### **3** Generation of abutment records

In this study, Alves's (2005) algorithm was implemented to generate spatially non-uniform ground motion. Alves (2005) used a transfer function that includes amplification and time delay. He proposed a new method for generation of non-uniform time series excitation using the records of three points, one point on the base of the dam and two points along the abutments. The amplification is the amplitude of the transfer function between a base record and an abutment record; and the negative of the time delay multiplied by the frequency gives the phase of the transfer function. The Fourier transform of an abutment acceleration  $A_n(\omega)$  generated from the Fourier transform of a base acceleration  $A_m(\omega)$ is given by (Alves, 2005):

$$A_{n}(\omega) = \operatorname{Amp}_{n,m}(\omega) e^{-i\omega\tau_{n,m}(\omega)} A_{m}(\omega) \qquad (5)$$

where  $\operatorname{Amp}_{n,m}(\omega)$  is the amplification function,  $\tau_{n,m}(\omega)$  is the time delay function and  $\omega$  is frequency. The transfer function between a base and an abutment record can be written as (Alves, 2005):

$$TF(\omega) = \operatorname{Amp}_{n,m}(\omega) e^{-i\omega\tau_{n,m}(\omega)}$$
(6)

Site amplification was shown as a function of frequency and ratio of Fourier amplitude displacement computed from the abutment and base canyon ground motions (Alves, 2005). One of the characteristics of nonuniformity in the input ground motion is the time delay. Time delay is the difference in the arrival times of waves between the base of the dam and points higher along the abutments. In order to characterize the time delay between two records, the value of time lag resulting from maximum cross-correlation of the records must be determined. The cross-correlation between an abutment acceleration  $a_n(t)$  and a base acceleration  $a_m(t)$  can be written as (Alves, 2005):

$$c_{n,m}(\tau) = \int_{0}^{d} a_{n}(t+\tau)a_{m}(t)dt$$
  

$$\tau_{n,m} = \left\{\tau : c_{n,m}(\tau) \to \max\right\} \qquad -d \le \tau \le d \qquad (7)$$

where  $c_{n,m}(\tau)$  is the cross-correlation between two records.

#### 4 Case study research

This study focuses on simulating the spatially variable seismic ground motion and determining the arrival times of the waves between the base of the canyon and points higher along the abutments. For this purpose, 3D modeling of the Pacoima Dam topography was developed. Therefore, access to topographic information of the Pacoima Dam is important and necessary for model simulation.

#### 4.1 Pacoima Dam topography characteristics

In the numerical analysis, due to the sensitivity of spatially varying earthquake ground motion problems arising from the complex geometry characterization using a uniform cross section to canyon site modeling, inconsistent results were found. The difficulty of ground motion prediction demonstrates the necessity for developing a realistic model of the site that is capable of including all surface and bedrock topographic features (Tarinejad *et al.*, 2013, 2019; Hesami *et al.*, 2016; Barari *et al.*, 2017). A three-dimensional digital elevation model was developed in order to perform 3D modeling of the Pacoima Dam topography. The Pacoima Dam is a concrete arch dam located in the San Gabriel Mountains in Los Angeles County at N34.335 W118.396 (Tarinejad *et al.*, 2017).

*al.*, 2013). The Pacoima Dam topography was chosen for this study because it has experienced many earthquakes, and satisfies the assumption of a homogenous foundation due to the properties of the abutment rock. Based on studies performed on this dam site, Young's modulus of 12.1 GPa, and Poisson's ratio of 0.25 were used in the modeling. The site investigations on the Pacoima Dam site indicate that shear wave velocity was between 1500 and 2500 m/s for the foundation rock (Tarinejad *et al.*, 2013 and Taghavi Ghalesari *et al.*, 2019). The formulation mentioned in Section 2 was implemented in a computer code named BEMSA (boundary element method for seismic analyses) (Sohrabi-Bidar *et al.*, 2010, 2013). This model has been subjected to vertical propagation of the Ricker wave:

$$f(t) = A_{\max} \times [1 - 2 \times (\pi \times f_n \times (t - t_0))^2] \times e^{-(\pi \times f_p \times (t - t_0))^2}$$
(8)

where  $f_p$ ,  $t_0$  and  $A_{max}$  denote the predominant frequency, time shift parameter, and maximum amplitude of the time history, respectively. The 3D model of the site topography of the Pacoima Dam was prepared large enough to include all irregularities around the dam site, which is extended up to 5000 m from the dam centerline and comprises 1218 eight-node quadrilateral iso-parametric elements (see Fig. 2). The size of the elements was considered to be 40 m for a predominant frequency of 4 Hz, and the time shift parameter and maximum amplitude for the analysis were set to 0.25 s and 0.001 m, respectively. The average element size is 25 m in the center of the model for a predominant frequency higher than 4 Hz, and the time shift was set to 0.125 s.

#### **5** Discussion of the results

This section presents the ground responses in terms of seismic wave amplification and travel time due to topographic effect of the canyon through threedimensional boundary element analyses. For the obtained topographic amplification, a set of nine points (see Fig. 1) are considered along the canyon in various elevations. These points and corresponding elevations are illustrated in Table 1.

<b>Fable 1</b> Elevation of	f considered	points
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Considered points	Elevation relative to the canyon base (m)
BC	0
RCA, LCA	332
RCB, LCB	266
RCC, LCC	175
RCD, LCD	91



Fig. 1 (a) Pacoima Dam (b) location of considered points



(a)



Fig. 2 3D model of the Pacoima Dam site topography

#### 5.1 Topographic amplification

Figures 3 and 4 show the topographic amplification of ground motion for the Pacoima Dam canyon subjected to SH and P waves with the predominant frequencies of 1.5, 4, 6.5 and 9.0 Hz, respectively. The topographic amplification is computed from the ratio of Fourier amplitude of respective components of the motion at various points of the abutments to the corresponding component of the base motions of the canyon (point BC in Fig. 1). The results indicate that the pattern of topographic amplification depends on the type of the incident seismic wave and the predominant frequency. In addition, increasing the predominant frequency of incident waves results in increasing the effective frequency range and amplification. In terms of the wave type, the SH wave shows the highest amplification with a peak value at the frequency of 3.0 Hz at all of



Fig. 3 Topographic amplification of Pacoima Dam site subjected to incident SH wave in *x*-direction, with predominant frequency of 1.5, 4, 6.5, 9 Hz

the predominant frequencies. The P wave shows the amplification with a peak value at the frequency of 7.0 Hz at all of the predominant frequencies. A different pattern of topographic amplification is obtained for two points with the same elevation but located in the left and right abutments as illustrated in Figs. 3 and 4. It indicates that the topographic amplification is not only dependent on the elevation of the points but also on the surface geometry.

#### 5.2 Seismic wave travel time

In this section, to evaluate the time delay between two records, the time domain displacement responses of the Pacoima Dam site subjected to SH and P seismic waves is carried out through three-dimensional boundary element analysis. A comprehensive study was conducted by this model and it was subjected to various parameters such as predominant frequency (range of 1.5–9 Hz)



Fig. 4 Topographic amplification of Pacoima Dam site subjected to incident P wave in z-direction, with predominant frequency of 1.5, 4, 6.5, 9 Hz

and velocity (range of 1500–2500 m/s) of the incident wave. Time delay between the records of different points relative to the base point is determined by the cross-correlation function and are presented in Tables 2 and 3 corresponding to the different wave components (SH and P waves) and velocity of 1500 m/s. The results indicate that when the predominant frequency is increased, the time delay in most cases is decreased. As expected, the time delay is increased by increasing the elevations of the points.

Figures 5 and 6 illustrate the time-domain responses of two points, A and D, along the Pacoima Dam site subjected to incident SH waves in the x-direction. The results are compared for different predominant frequencies of incident seismic wave. Dynamic responses for frequencies of  $f_p = 1.5$ , 4.0, 6.5 and 9.0 Hz are presented in this section. Two basic quantities were obtained from the comparison between the abutment records to the base records from the boundary element method: amplification and time delay. These quantities can be used to create abutment records at the locations of various points on the abutment canyon from which records obtained at the base of the dam. This process was accomplished in the frequency domain. The time delay function is developed according to the results of seismic analysis of the Pacoima Dam site subjected to SH and P waves through the three-dimensional boundary element method.

According to the analyses performed on a set of models with different geometrical properties and wave characteristics, the following relationships (Eqs. (9), (10), (12), (13)) were proposed on the basis of symbolic regression technique. In this procedure, function identification searches for several expressions of functions that fit the given data set to identify the input variables that are related to the changes in the important influencing variables in order to express relationships in mathematical models, and to analyze their quality by optimization (genetic programming) procedures (Zelinka, 2004).

The proposed function contains the seismic wave characteristic and the canyon geometry. The time-delay between the displacement records of the abutment and the base of the canyon induced by the SH wave is defined by:

$$\tau_{n,m} (\text{left - abutment}) = (h_{n,m} / c) + (0.0574 / f_p) - 0.01807$$
(9)

$\Delta H$	$f_{\rm p} = 1.5$	$f_{\rm p} = 2$	$f_{\rm p} = 3$	$f_{\rm p} = 4$	$f_{\rm p} = 5.5$	$f_{\rm p} = 6.5$	$f_{\rm p} = 7.5$	$f_{\rm p} = 9$
0	0	0	0	0	0	0	0	0
61	0.075	0.075	0.05	0.025	0.015	0.03	0.03	0.03
91	0.1	0.075	0.1	0.05	0.045	0.06	0.06	0.06
120	0.1	0.1	0.1	0.075	0.075	0.075	0.075	0.075
157	0.125	0.0125	0.1	0.1	0.09	0.09	0.09	0.09
175	0.15	0.125	0.1	0.1	0.105	0.105	0.105	0.105
223	0.15	0.15	0.15	0.15	0.135	0.135	0.135	0.135
266	0.15	0.2	0.175	0.175	0.165	0.165	0.165	0.165
286	0.175	0.25	0.175	0.175	0.18	0.18	0.18	0.18
332	0.2	0.35	0.225	0.225	0.21	0.21	0.21	0.21

Table 2 Time delays between the records of the abutment and the base of the canyon for SH wave ( $c_s = 1500$  m/s)

Table 3 Time delays between the records of the abutment and the base of the canyon for P wave ( $c_p = 2550 \text{ m/s}$ )

$\Delta H$	$f_{\rm p} = 1.5$	$f_{\rm p} = 2$	$f_{\rm p} = 3$	$f_{\rm p} = 4$	$f_{\rm p} = 5.5$	$f_{\rm p} = 6.5$	$f_{\rm p} = 7.5$	$f_{\rm p} = 9$
0	0	0	0	0	0	0	0	0.00
61	0.075	0.075	0.05	0.025	0.03	0.03	0.03	0.03
91	0.1	0.1	0.075	0.05	0.045	0.045	0.05	0.04
120	0.1	0.1	0.1	0.05	0.075	0.06	0.06	0.05
157	0.125	0.1	0.1	0.075	0.07	0.07	0.07	0.07
175	0.125	0.125	0.1	0.1	0.08	0.08	0.08	0.07
223	0.13	0.12	0.11	0.11	0.1	0.09	0.09	0.09
266	0.135	0.135	0.13	0.13	0.11	0.11	0.11	0.11
286	0.135	0.135	0.135	0.135	0.12	0.12	0.12	0.12
332	0.16	0.16	0.16	0.16	0.14	0.14	0.14	0.14

$$\tau_{n,m}(\text{right - abutment}) = 0.9929 \times (h_{n,m} / c) + (0.0505 / f_{n}) - 0.0038$$
(10)

where  $\tau_{n,m}$  is the time-delay between the displacement records of two points (*n* and *m*),  $h_{n,m}$  is the height difference between two points of the canyon site, *c* is the shear wave velocity and  $f_p$  is the predominant frequency.

Figures 5–8 illustrate the time-domain response which is generated using three methods: time-domain response of boundary element method, Alves method using Eqs. (9) and (10) and weighted average amplification method instead of the amplification as described in the previous section. The proposed weighted average amplification factor for topographic features can be determined from the following equation:



Fig. 5 Comparison of the generated time-series at the points RCA and LCA subjected to the incident SH wave in the x-direction

$$\overline{\text{Amp}} = \frac{\sum_{i=1}^{n} h_i.\text{Amp}_i}{\sum_{i=1}^{n} h_i}$$
(11)

where  $\overline{\text{Amp}}$  is the proposed weighted average amplification factor, and  $h_i$  is the height of the point *i* from the base of the canyon.

The comparison of the time domain displacement of the points (A, D) for the left and right sides of the Pacoima

Dam due to the SH wave in Figs. 5 and 6 demonstrates that by increasing the predominant frequency, the match of the generated and computed records for the left and right points is improved when compared with the boundary element methods. Good agreement is obtained for the case of using the proposed weighted average amplification factor and time delay function in generation of the records and the results of the boundary element method. Figures 7 and 8 illustrate the time-domain response at



Fig. 6 Comparison of the generated time-series at the points RCD and LCD subjected to the incident SH wave in the x-direction

the two points (A, D) along the transverse direction of the Pacoima Dam site subjected to the incident P waves in the z-direction. The time-delay functions between two points for the P wave are proposed as follows:

$$\tau_{n,m}(\text{left - abutment}) = 0.922 \times (h_{n,m} / c) - (0.00044 / f_p) + 0.0099$$
(12)

$$\tau_{n,m}(\text{right - abutment}) = 0.9568 \times (h_{n,m} / c) + (0.0624 / f_p) - 0.0028$$
(13)

The comparison of the time domain displacement of the points (A, D) for the left and right sides of the canyon due to the P wave demonstrates that, by increasing the predominant frequency, the agreement between displacement records for the left and right points are



Fig. 7 Comparison of the generated time-series at the points RCA and LCA subjected to the incident P wave in the z-direction



Fig. 8 Comparison of the generated time-series at the point RCD and LCD subjected to the incident P wave in the z-direction

increasing when compared with the boundary element records. Similar to the SH wave, the results show that the proposed method indicate good agreement with the boundary element results. In order to indicate the efficiency and capability of the proposed approach, regeneration of the records of the January 13, 2001 earthquake on the Pacoima Dam site is performed and the results are compared to the recorded responses. A comparison between the time delays computed from the proposed function for each component of the January 13, 2001 earthquake records (Alves, 2005) and the recorded ones are shown in Table 4. Good agreement is obtained between the generated and recorded responses during the earthquake.

Table 4 Comparison between time delays computed from the base to the abutment stations for each component of the January 13, 2001 earthquake acceleration records and estimated from the presented time delay functions corresponding to  $f_p = 6.25$  (SH wave)

	E-W (stream)	Estimated		Vertical	Estimated
Base to right abutment	$\tau_{12,9} = 0.05$	0.0475	Base to right abutment	$\tau_{13,10} = 0.024$	0.028
Base to left abutment	$\tau_{15,9} = 0.04$	0.035	Base to left abutment	$\tau_{16,10} = -0.008$	0.01

# **6** Conclusions

In this study an approach was proposed for the generation of spatially variable seismic ground motions in topographic features and the Pacoima Dam site was selected as a case study. A computer code for simulating the seismic ground motions was developed and verified. For this purpose, the time domain 3D boundary element method was utilized. The time delay function was proposed and an approach to generate non-uniform ground motions on the Pacoima Dam site is developed. Efficiency and accuracy of the proposed functions and approaches were indicated by regeneration of the 21 January 2001 earthquake in the Pacoima Dam site. The results indicate good agreement between the recorded and generated responses.

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## Nomenclature

$C_{I}$	Compressional wave velocities	(m/s)
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- $c_{\rm T}$  Shear wave velocities (m/s)
- $\lambda$  Lame constant
- $\mu$  Lame constant
- $\rho$  Mass density (kg/m<sup>3</sup>)
- $\omega$  Circular frequency (rad/s)
- $u^*$  Transient displacement (m)
- $p^*$  Traction (N/m<sup>2</sup>)
- au Time delay function