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Application of wavelet transform in structural health monitoring

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Abstract: Structural health monitoring (SHM) is a process of implementing a damage detection strategy in existing structures to evaluate their condition to ensure safety. The changes in the material, geometric and/or structural properties affect structural responses, which can be captured and analyzed for condition assessment. Various vibration-based damage detection algorithms have been developed in the past few decades. Among them, wavelet transform (WT) gained popularity as an efficient method of signal processing to build a framework to identify modal properties and detect damage in structures. This article presents the state-of-the-art implementation of various WT tools in SHM with a focus on civil structures. The unique features and limitations of WT, and a comparison of WT and other signal processing methods, are further discussed. The comprehensive literature review in this study will help interested researchers to investigate the use of WT in SHM to meet their specific needs.

Keywords: wavelet transform; damage detection; modal properties; structural health monitoring; numerical simulations

1 Introduction

The changes of material and geometric properties of structures including boundary conditions, changes in loading conditions, deterioration with age, etc. may adversely affect the structure performance. Hence, the health condition of structures must be periodically inspected and monitored to ensure their safety. The process of using a measuring and sensing system to monitor performance and evaluate the health state is defined as structural health monitoring (SHM).

Vibration based SHM involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features, such as vibration characteristics, and statistical analysis to determine the current state of system health. It has drawn significant attention in recent years. Compared to other monitoring systems, this method has the advantage of monitoring the global nature of the vibration characteristics. The capability of identifying the modal properties paves the way to monitor not only a single individual structural component but the entire structure. Hence, large engineering structures such as bridges can be effectively monitored with a limited number of sensors and equipment.

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The basic idea behind vibration-based SHM is that damage usually modifies structural characteristics, which can be manifested through the observation and measuring of changes in vibration characteristics. Therefore, powerful signal processing tools capable of extracting subtle changes in the vibration signal are very important in order to detect damage in structures. Most vibration based SHM techniques require knowledge of pre-damaged status, and depend on measuring vibration characteristics and analyzing these data in the frequency domain. As vibration characteristics analysis is often performed in the frequency domain, most current vibration-based SHM methods implement fast Fourier transformation (FFT). FFT is one of the most widely used signal processing techniques in SHM. However, there are several restrictions for FFT. First, for multi-degree-of-freedom systems with strongly coupled modes, FFT is not usually able to give as accurate results as for single-degree-of-freedom systems (Staszewski, 1997). Second, FFT cannot obtain a good estimation of damping, especially for heavily damped systems. Finally, FFT does not pave the way to study the nature of the time series in the time frequency domain, thus the time information along the time series is lost. Therefore, FFT cannot depict the changes in natural frequencies over time which is fundamental in SHM (Tang *et al.*, 2011, Qiao *et al.*, 2012).

On the other hand, the WT has gained a great deal of attention recently due to its excellent capability in signal analysis. As a time frequency analysis tool, WT has the advantages of dealing with nonstationary, transient, and nonlinear signals. Even though FFT is a popular tool in analyzing the components of stationary signals (signals which have no change in the properties), it failed to analyze nonstationary signals (Sifuzzaman *et al.*, 2009). Further, WT allows constructing filters to stationary and nonstationary signals where FFT cannot. The main difference between FFT and WT is that wavelets are well localized in both the time and frequency domain whereas FFT is only localized in the frequency domain. The short-time Fourier transform (STFT) is also localized in both the time and frequency domain but has several issues with frequency and time resolution (Amezquita-Sanchez *et al.*, 2013). WT analysis allows studying the spectral characteristics of a time series as a function of time with better signal representation using multi resolution analysis. It clearly illuminates the changes of different periodic components along the time series. Moreover, wavelets have the ability to separate fine details in a signal. Very small wavelets are ideal to use to detect very fine features in a signal while large wavelets can be used to identify the coarse details. Also, WT has the ability of decomposing a signal into component wavelets (Sifuzzaman *et al.*, 2009). In wavelet theory, it is often possible to achieve good approximation of a given function using only a few coefficients, which is advantageous when compared to FFT. Wavelet theory has the capability of revealing some aspects in data signals such as breakdowns, discontinuities, trends, etc., which cannot be obtained by other signal processing tools. It can de-noise or compress a signal without appreciable degradation (Sifuzzaman *et al.*, 2009). WT has achieved the ability to overcome many of the limitations in Fourier analysis. Hence, WT is widely used not only in civil engineering field but in many other fields including mechanical systems and aerospace engineering as a signal processing tool in structural health monitoring, which shows that this can be generalized.

Reda Taha *et al.* (2006) provided an excellent review on WT applications for SHM. Since then, there have been new developments and extensive literature review on the topic. Amezquita-Sanchez and Adeli (2016) presented a review of signal processing techniques for vibration-based SHM with a focus on civil structures including buildings and bridges. The main objective of this article is to review recent developments on the application of WT in vibration based SHM with a focus on civil structures and present an in-depth discussion. The ultimate intent of this article is to provide the readers with a comprehensive review on the various aspects of WT that can meet their research needs in SHM.

2 Wavelet transform tools

WT are mathematical tools with which a signal can be reconstructed from wavelet representations. The major WT tools that are currently used in SHM are briefly discussed in this section.

A wavelet is a small wave which oscillates and decays quickly. It is a basis function (mother wavelet) used in an integral transform. A range of mother wavelet types have been used for engineering problems across multiple disciplines. Among them, the Haar (Garstecki *et al.*, 2005), Daubechies (Melhem and Kim, 2003), Symlet (Huang *et al.*, 2005), Morlet (Dien, 2008), Gabor (Slavic *et al.*, 2003), Gaussian (Perez-Ramirez *et al.*, 2016) and Mexican Hat (Lu and Hsu, 2003) types have been used successfully. Note that Morlet and Daubechies mother functions are the most commonly used types in SHM.

The underlying mathematical technique of the WT is similar to FT. Both transforms decompose signals into linear combinations of basis functions. However, the fundamental difference between them is that the WT has an infinite set of possible basis functions, while the FT has a single set of basis functions that includes only sine and cosine functions.

The continuous wavelet transform (CWT) used to decompose a function $x(t)$ into the frequency-time domain is defined in Eq. (1).

$$\mathrm{CWT}\{x(t)\}=W_x(\tau,s)=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty}x(t)\psi^*\left(\frac{t-\tau}{s}\right)\mathrm{d}t \qquad (1)$$

where τ and s is the dilation and translation parameters, respectively. ψ^* is the complex conjugate of $\psi(t)$, which is called the mother wavelet. The amplitude of the wavelet coefficient, $|W_x(\tau,s)|$ can be used to construct the wavelet amplitude map which illustrates the amplitude of the features in the original signal and variation with time. Once the amplitude map is obtained, local maxima indicate the natural frequencies of the system. WT is based on dilated scales and shifted windows, which have the ability to perform a reasonable time frequency resolution of a data signal which has contributed to widespread applications in engineering. A time domain signal is converted into WT in terms of the projection of the original signal on to a family of functions that are normalized dilations and translations of the wavelet transform. The mother wavelet function $\psi(t)$ dilates (scaled) and translates (shifted) as daughter wavelets. Scaling in WT means stretching or compressing it in the time domain. Smaller scales represent more compressed wavelets while larger scales produce more stretched wavelets. Further, the wavelet stretches into a long function to measure the low frequency movements, and it compresses into a short function in order to measure the high frequency movements.

In practical signal processing, discrete wavelet transform (DWT) is often employed by discretization of parameters in CWT. DWT of a discrete time sequence $x(n)$ is defined in equation (2).

$$C_{j,k}=2^{-\frac{j}{2}}\sum_n x(n)\psi(2^{-j}n-k) \qquad (2)$$

$2^{-\frac{j}{2}}\psi(2^{-j}n-k)$ are scaled and shifted versions of wavelet function $\psi(n)$ based on the values j (scaling coefficient) and k (shifting coefficient), and it is generally written as $\psi_{j,k}(n)$.

Both CWT and DWT are extendable from a one-dimensional to multidimensional case and implemented in a numerical way. The main advantage of CWT is the ability to analyze a signal at arbitrary scales and locations. CWT is highly redundant compared to DWT, thus it is relatively time consuming. DWT uses orthogonal functions for signal decomposition and its analysis is non-redundant and faster.

Wavelet multi-resolution analysis (WMRA) is a technique used to implement DWT with filters (Mallat, 1999). The WMRA of a continuous signal $x(t)$ can be expressed as

$$x(n) = \sum_{k=-\infty}^{\infty} c_{j_0,k}\phi_{j_0,k}(n) + \sum_{j=j_0}^{\infty}\sum_{k=-\infty}^{\infty} d_{j,k}\psi_{j,k}(n) \tag{3}$$

where $\phi_{j_0,m}(n) = 2^{-\frac{j_0}{2}}\phi(2^{-j_0}n-k)$ and $\psi_{j,m}(n) = 2^{-\frac{j}{2}}\psi(2^{-j}n-k)$ are scaled and translated versions of the scaling function $\phi(t)$ and the mother wavelet $\psi(t)$, respectively. In Eq. (4), the first term on the right side gives a low resolution of $x(n)$ at level j_0. For each j in the second term, a higher or fine resolution function including more detail of $x(n)$ is added. WMRA decomposes the signal into various resolution level; the data with coarse resolution (approximations) contain information about low frequency components and data with fine resolution (details) contain information about the high frequency components.

Wavelet packet transform (WPT), like WMRA, is a technique to decompose a signal repeatedly into successive low and high frequency components (Mallat, 1999). It differs from WMRA in that both approximations and details are decomposed further, which results in a more flexible and wider base for the analysis of signals. The wavelet packets are alternative basis functions formed by linear combinations of the usual wavelet functions. As a result, the WPT enables the extraction of features from signals that combine stationary and nonstationary characteristics with arbitrary time-frequency resolution.

A new time-frequency analysis method capable of analyzing nonstationary, nonlinear and noisy signals, synchrosqueezed wavelet transforms (SWT) was introduced by Daubechies *et al.* (2011). SWT relocated the CWT coefficients based on the frequency information, with the goal of obtaining a sharper representation in the time-frequency domain. It consists of two main steps. First, the main frequencies contained in the time series signal are estimated. Second, the signal is reconstructed by summing the components obtained. Hence, the SWT allows effective isolation of the main frequencies from noise.

Another more recent development in WT is the introduction of adaptive wavelet transform. Gilles (2013) introduced empirical wavelet transform (EWT) capable of decomposing a signal according to its contained information. The main idea is to extract the different modes of a signal by designing an appropriate wavelet filter bank. The wavelet filter bank is based on Fourier supports detected from the information contained in the processed signal spectrum. Fourier spectrum is used to estimate the modes or frequencies contained in the signal in order to build the wavelet filter bank. The new insight of this method lies in the fact that the corresponding dilation factors do not follow a prescribed scheme but are detected empirically.

3 Applications of WT in SHM

Doebling *et al.* (1996), Sohn *et al.* (2004), Carden and Fanning (2004), and Fan and Qiao (2011) conducted a detailed survey for structural health monitoring and damage detection studies on different structural systems. Among the various researched techniques, the WT tools have been proven to be among the successful methods for assessment of structural health and damage detection.

It is important to note that WT is being used across a variety of fields, such as mechanical engineering, image processing, biomedical engineering, robotics, etc. However, this study is focused on the application of WT in SHM with a focus on civil structures including buildings and bridges. In the literature review presented herein, the articles listed in Table 1 were selected for detailed review. The application of WT in SHM can be broadly categorized into two areas: modal and damping ratio identification, and damage detection. Table 1 summarizes the wavelet tools, chosen mother wavelet functions and the analyzed structures in the studies presented in the selected articles. The analyzed structures are classified as numerical, laboratory and real-life structures. For the studies in WT application in modal and damping ratio identification, the identified parameters in each study are listed in the table. A widely accepted definition of the levels of damage detection was provided by Rytter (1993). Rytter defined four levels of damage detection, which are detection (existence), determining location, quantification of severity, and prognosis of the remaining useful life. Table 1 lists the level of damage detection in each study presented in the selected articles in WT application in damage detection.

3.1 Modal and damping ratio identification

The WT is able to provide multiple resolutions in the time and frequency domains, which makes it an efficient signal processing algorithm that is capable of analyzing continuous and transient signals. It is capable of utilizing long time intervals (large window) for low frequency information, and short time intervals (small window) for high frequency information. Therefore, the WT provides an accurate location of the transient signals while simultaneously reporting the fundamental frequency and its low-order harmonics.

Table 1 Summary of reviewed literature

SHM Application: Modal and damping ratio identification

Authors	Wavelet tool	Mother wavelet	Analyzed structures (N - numerical, L - Laboratory, R - real-life)	Parameters identified
Kijewski and Kareem (2003)	CWT	Morlet	A tower in Japan (R)	Frequency and damping ratio
Slavic *et al.* (2003)	CWT	Gabor	A beam (L)	Damping ratio
Huang *et al.* (2005)	DWT	Daubechies	Three steel frames in shake table tests (L); a RC arch bridge (R)	Frequency, mode shape and damping ratio
Ji and Chang (2008)	CWT	Morlet	A steel cantilever beam (L); a bridge stay cable (R)	Mode shape and damping ratio
Chen *et al.* (2009)	CWT	Morlet	Mass-spring systems (N); a mass-spring vibration model (L)	Frequency and damping ratio
Shi and Chang (2012)	WMRA	dbX	A 10-story shear-beam building (N); a three-story building model (L)	Damping and stiffness
Wang *et al.* (2013)	CWT	Morlet	A mass-spring system (N); a cable under varying tension (L)	Frequency
Yan and Ren (2013)	CWT	Morlet, Mexhat, Meyer, Shannon	A four-story frame (N); an arch bridge (R)	Frequency and mode shape
Guo and Kareem (2015)	CWT	Morlet	Two tall buildings (R)	Frequency and damping ratio
Wang *et al.* (2016)	CWT	Morlet	A cable-stayed bridge (R)	Frequencies and damping ratio
Perez-Ramirez *et al.* (2016)	SWT	Different mother wavelets were tested and Gaussian was chosen	A four story building frame (L and R); a reinforced concrete bridge (R)	Frequency and damping ratio
Amezquita-Sanchez *et al.* (2017)	EWT	Gilles	A four-story frame (N); an eight-story steel frame (N); an 123 story tall building (R)	Frequency and damping ratio
Cantero *et al.* (2017, 2019)	CWT	Modified Littlewood-Paley	A three span reinforced concrete bridge (R); a steel girder bridge (R); a coupled vehicle-bridge model (L)	Frequency

SHM Application: Damage detection

Authors	Wavelet tool	Mother wavelet	Analyzed structures (N - numerical, L - Laboratory, R - real-life)	Level of damage detection
Hou *et al.* (2000)	CWT and DWT	Daubechies (db4)	A mass-spring system (N)	Existence
Okafor and Dutta (2000)	CWT	Daubechies (dbN)	A cantilevered beam (N, L)	Existence, location and severity
Yoon *et al.* (2000)	CWT	Morlet	A series of concrete beams (L)	Existence and severity (cracking stage)
Sun and Chang (2002)	WPT	Daubechies (db15)	A three-span continuous beam (N)	Existence, location and severity
Melhem and Kim (2003)	CWT	Daubechies	A concrete slab (L) and a concrete beam (L) under fatigue load	Existence
Hera and Hou (2004)	DWT	Daubechies (db4)	A four-story frame (N)	Existence and location
Ovanesova and Sua´rez (2004)	CWT and DWT	Biorthogonal, Haar	A fixed-end beam (N): a three-member frame (N)	Existence and location
Ren *et al.* (2008)	WPT	Daubechies (db5)	A scaled bridge model (L)	Existence and location
Park *et al.* (2008)	CWT	Morlet	A railroad track section (L)	Existence and damage classification
Noh *et al* (2011)	CWT	Morlet	A bridge column (L); a four-story steel frame in shake table test (N, L)	Severity (damage states under seismic excitations)
Feng *et al.* (2014)	WMRA	Reverse-biorthogonal	A spliced steel beam (L)	Crack location (quantitatively) and intensity (qualitatively)
Su *et al.* (2014)	CWT	Daubechies (db2 and db4)	An eight-story steel frame in shake table tests (L); a five-story shear building (N)	Existence and location

Table 1 Continued

SHM Application: Damage detection				
Authors	Wavelet tool	Mother wavelet	Analyzed structures (N - numerical, L - Laboratory, R - real-life)	Level of damage detection
Amezquita-Sanchez and Adeli (2015a)	SWT	Morlet	A scaled 38-story RC building (L)	Existence, location and severity
Gaviria and Montejo (2016)	CWT	Morlet	A five story shear building (N); A scaled two story shear building (L)	Existence, location and severity
Gholizad and Safari (2016)	CWT	Mexican hat	Double-layer space structures (N)	Existence and location
Shahsavari *et al.* (2017)	CWT	Symlet	A steel beam (L)	Existence and location
Abdulkareem *et al.* (2018)	CWT	Paul	Steel square plates (N, L)	Existence, location and severity
Pan *et al.* (2018)	WMRA	Morlet	A cable-stayed bridge (N)	Existence and damage states
Wang *et al.* (2019)	CWT	Mexican hat	A underground tunnel model (N, L)	Existence, location and severity

The scale in WT analysis is analogous to frequency in FFT. A graphical representation of wavelet coefficients plotted on a time-scale grid is called the scalogram. The wavelet coefficients take on maximum values at the instantaneous frequency, corresponding to dominant frequency components in the signal at each instant time. These define ridges in the time-frequency plane. By using the amplitude and phase of the signal extracted from the values of the wavelet coefficients along each ridge, time-domain-based feature extractions can be carried out.

Kijewski and Kareem (2003) discuss the considerations while applying WT in civil engineering, where structures are characterized by a longer period and more narrow banded responses. Frequency resolutions must be refined to insure modal separation, which results in an increase of end-effect errors that may influence the quality of wavelet amplitudes and modal properties such as damping. The study included guidelines for selection of wavelet central frequencies in the process of system identification in single- and multi-degree-of-freedom systems using the Morlet Wavelet. The wavelet central frequencies have significant impact on complete modal separation and end effect errors of the signals. Further, the simple padding scheme has been introduced in order to minimize the end effect error.

Slavic *et al.* (2003) suggested a damping ratio identification method using CWT with a Gabor wavelet. This also addressed the edge effect in damping identification. Numerical examples were built with known parameters. before they were applied to the real data. The damping ratio was identified using three methods according to the ridge detection; namely, the cross section method (CSM), which is based on the pre-known damped frequency, amplitude method (AM), which is based on the maxima of the CWT and Phase Method (PM) which relates to angular velocity. In this study, the effects of noise to the damping ratio were analyzed and it was concluded that AM and PM give better results among the three methods of damping identification due to feedback information about the noise. Further, damping identification at closed modes was analyzed in an experiment carried out according to the acquired acceleration for a uniform beam.

Huang *et al.* (2005) presented the use of DWT to determine modal parameters, i.e., natural frequencies, damping ratio, and mode shapes. Earthquake response data and free vibration data was acquired and WT with orthogonal wavelets was used to analyze the measured acceleration data. The accuracy of the procedure was verified using a numerical simulation of an earthquake response for a six story shear building. The effects of noise and mother wavelet functions on the analysis of dynamic responses were investigated. The authors used three different mother wavelets and concluded that they did not significantly influence the identified results. The mode shapes verification in the numerical simulation was carried out using the model assurance criterion values in between identified and theoretical mode shapes. The feasibility of the proposed methods was confirmed by applying to practical applications. The responses were recorded for a shaking table test performed in scaled steel frames and free vibration response for a five-span arch bridge. The presented results were in a good agreement with published results obtained from the subspace approach.

Ji and Chang (2008) proposed a CWT based approach to identify modal properties of line-like structures using digital images. Morlet wavelet was selected as the mother wavelet. Initially, a spatio-temporal displacement response was constructed from the acquired image sequences. The technique was illustrated using two applications, which were a cantilever beam in laboratory conditions and a bridge stay cable. Natural frequencies, damping ratios, and mode shapes were identified using the proposed method.

Chen *et al.* (2009) studied a CWT analysis to identify natural frequencies and damping ratios of single and multi DOF structures using the Morlet mother wavelet function. Both numerical and experimental structures were studied. Further, general guidelines for choosing the dilation parameter, translation parameter, and frequency parameter of the Morlet wavelet were discussed and provided.

Shi and Chang (2012) conducted laboratory and numerical experiments to identify time varying parameters, damping and stiffness coefficients of shear building models using a dbX wavelet. Displacements, velocities and accelerations were taken into analysis. The Kalman Filter was used in order to minimize the measurement noises. Assuming that the possible damage location of a building can be identified a priori, a substructural model containing both interface and internal restoring forces was formulated. The WMRA was then used to approximate the time-varying damping and stiffness parameters associated with the restoring forces.

Wang *et al.* (2013) proposed a method to identify instantaneous frequency using wavelet ridges of CWT extracted through a Morlet wavelet function. A penalty function was used in order to eliminate the noise effect of the data and wavelet ridges were extracted using a dynamic optimization technique. The proposed new method was verified using a numerical two degree of freedom spring mass dashpot system. The method was tested using a cable with varying tension force under laboratory conditions. The time varying frequencies were achieved for linear and sinusoidal varying tension forces applied on the cable. It was noticed that the error between the proposed and the theoretical method was below 1% and therefore the method proposed was highly effective in identifying instantaneous frequency in nonstationary signals.

Yan and Ren (2013) proposed a new method to conduct operational modal identification of a linear system using CWT. Several wavelet functions, including Morlet, Mexhat, Meyer, and Shannon wavelets were applied and a comparison of the results showed that the modal parameters can be well identified for different wavelet functions. Different scale discretion steps were also used and they seemed to have little effect on the results. The effect of noise is the most challenging factor in operational modal analysis. Therefore, a new theorem proved mathematically that CWT at different scales is independent of stationary excitations acting on structures. A real case study of a bridge in China was considered while comparing the results with a FE model of the same bridge.

Guo and Kareem (2015) introduced a nonstationary system identification method using CWT, transformed singular value decomposition (TSVD) and Laplace filtering. WT was initially utilized to uncover the time varying features of nonstationary data. TSVD was then used to identify the analysis region in the time frequency domain. Subsequently, Laplace wavelet filtering was adopted to extract impulse type signals in the analysis region using WT coefficients, thus enabling damping estimation and frequency from transient non stationary data extracted from impulse type signals. These properties can be identified using either wavelet modulus decay or Laplace wavelet parameters. Also, the mode shapes can be easily identified using singular value decomposition. The application of the method to two full-scale tall buildings demonstrated its advantages in revealing the time-varying nature of the frequency and damping from a transient signal and handling highly nonstationary data, which are appealing features when tracking the changes of structural conditions before, during and after extreme events.

Wang *et al.* (2016) presented a CWT analysis with a random decrement technique method for a cable stayed bridge during Typhoon Haihui cyclone. Wind characteristics and structural response were obtained to identify modal parameters, including natural frequencies and damping ratios. Morlet mother wavelet was selected. The instantaneous frequencies were identified from ridges of wavelet coefficients. The damping ratio was obtained from the wavelet phase and amplitude curves of the identified modes. The results were compared with FEM results and they appeared to be close. Four vertical, two torsional and one horizontal modes were taken into consideration. A further relationship between the modal parameters and wind speed was analyzed. The observed wind speed has less influence on natural frequencies and a remarkable effect on the damping ratio. The study concluded that, in general, WT provides accurate and reliable results with de-noising ability in modal parameters identification.

Perez-Ramirez *et al.* (2016) presented a new SWT-based methodology to identify natural frequencies and damping ratio. The RDT method was used to obtain free response from ambient data. Then the estimated free vibration acceleration response was decomposed by the SWT algorithm into individual mode components. Next the Hilbert transform was used to estimate both natural frequencies and damping ratios. In order to smoothen the results, a Kalman filter was used to obtain more reliable damping ratio values. The approach was initially validated using a three-DOF numerical model created using Matlab. Also a benchmark four-story steel frame structure was analyzed both numerically and experimentally. The results appeared to be in excellent agreement with those obtained by other researchers. The method was also applied to a real-life reinforced concrete bridge structure. It was shown that the proposed method can isolate the modes contained in the measured signal effectively and estimate the reliable natural frequencies and damping ratios even with a high level of embedded noise. Finally, it was concluded that the proposed method can account for noisy signals without any degradation while using a relatively low computational burden.

A new methodology was implemented by Amezquita-Sanchez *et al*. (2017) to identify natural frequencies and damping ratios of large civil structures by using a multiple signal classification algorithm (MUSIC), EWT and Hilbert transform. The MUSIC-EWT, presented by Amezquita-Sanchez and Adeli (2015b), is an adaptive algorithm capable of efficiently analyzing noisy nonstationary and nonlinear signals. In the method, the MUSIC algorithm is first used to estimate the contained frequencies in the signal corresponding to the natural frequencies of the structure, and build the appropriate boundaries to create the wavelet filter bank. Then, the time series signal is decomposed into different frequency bands through EWT. In order to validate the new methodology, a four-story steel frame structure, an eight-story steel frame structure with white noise, and 123-story high-rise building structure were analyzed as case studies. During the analysis, it was noticed that the proposed method can effectively account for noise without degrading the ability of extracting natural frequencies and damping ratios.

Cantero *et al*. (2017) conducted experimental and numerical studies to measure the evolution of modal properties of a bridge during a vehicle passage using the modified littlewood paley (MLP) basis function in CWT. The study also analyzed the changes in the mode shapes and frequency shifts of the separate systems (vehicle and bridge separately) with the location of the vehicle. Two case studies were conducted at a simply supported steel girder bridge and a three-span continuous reinforced concrete bridge. The further study presented in Cantero *et al*. (2019) indicated that the natural frequencies of several vehicle-bridge systems vary with vehicle position. It was noted that mechanical properties of the vehicle and the additional mass of the vehicle influence the vehicle-bridge system frequencies. Further, it was empirically shown that the frequency shifts also depend on the vehicle-to-bridge frequency ratio. A coupled vehicle-bridge model was tested in the lab as a verification example.

3.1.1 Case study: frequency and damping ratio identification of a steel girder bridge

A single span bridge in Holland, Michigan was selected as a case study for modal identification using CWT with Morlet mother function. The bridge has a 27.5 m span and a 13.7 m width, with a typical concrete deck supported by seven steel girders. The acceleration data was acquired using wireless sensors with two types of configurations as shown in Fig. 1. Table 2 presents parameters for each data set used in both configurations. Natural frequencies and damping ratios were identified using the CWT method and a comparison study was done between frequencies obtained using the CWT and FFT methods. The acquired acceleration data was initially filtered using the Butterworth filter to remove environmental noise effects. A sample time vs. acceleration signal is presented in Fig. 2. Matlab code was used to obtain FFT and CWT plots in Fig. 3. Figure 2 showed the time-domain representation of the recorded bridge dynamics. The FFT plot shown in Fig. 3(a) is the frequency-domain representation. It is obvious that the CWT scalogram shown in Fig. 3(b) is more capable of describing the changes in the system dynamics in both the time and frequency domains than Figs. 2 and 3(a) individually.

The summary of modal frequencies is presented in Table 3 and Table 4 for configuration 1 and 2, respectively.

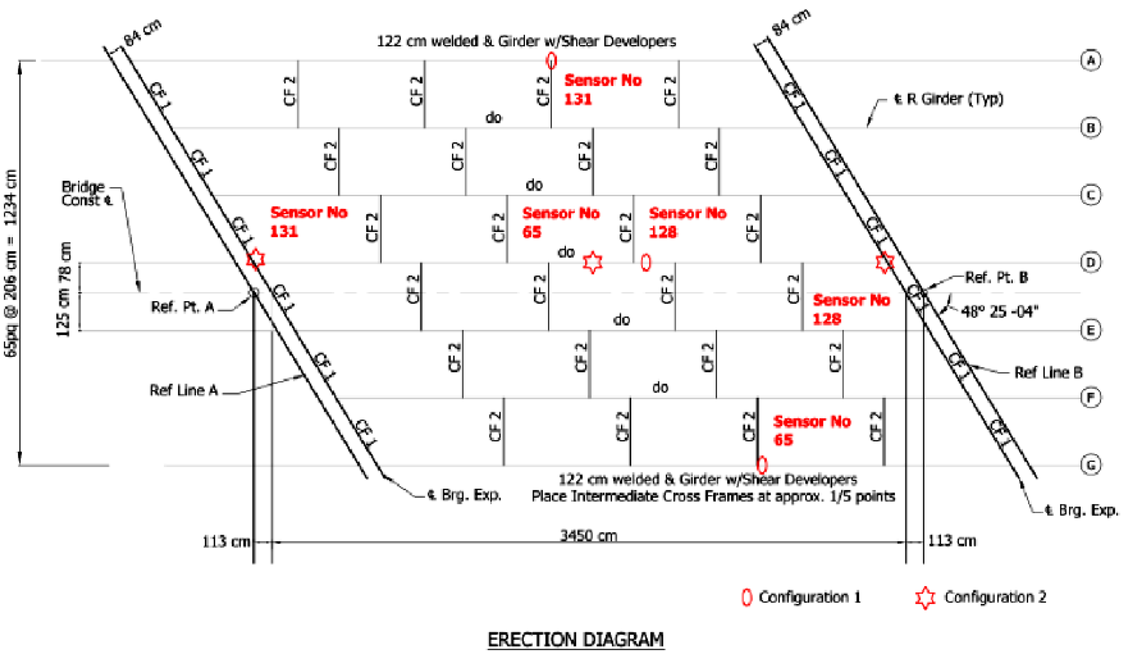

Fig. 1 Sensor configuration

In order to identify the compatibility of FFT and CWT methods, a percentage difference of modal frequencies were calculated and the majority of difference was less than 10% and show good agreement with each other. Further, CWT could find some of the frequencies which were not detected by FFT.

According to the amplitude map of the FFT and CWT plots, larger amplitudes were observed in sensor 65, which was placed on the middle span of the structure in the second configuration. This indicates both FFT and CWT power amplitude is sensitive to vibration response of the structure. In conclusion, all the modal frequencies were in good agreement and all the sampling frequencies indicated consistency between the results obtained using both FFT and CWT methods. Moreover, the mid-span of the bridge is more sensitive to both methods; therefore, sensor placement is highly recommended at the mid-span in order to identify clear peaks.

Table 2 Data set parameters

Data set	Configuration	Sampling frequency (Hz)	Sensor ID
Set 1	1	280	65, 128
Set 3		100	65, 128
Set 4		50	65, 128
Set 5	2	280	65, 128, 131
Set 6		100	65, 128, 131
Set 7		50	65, 128

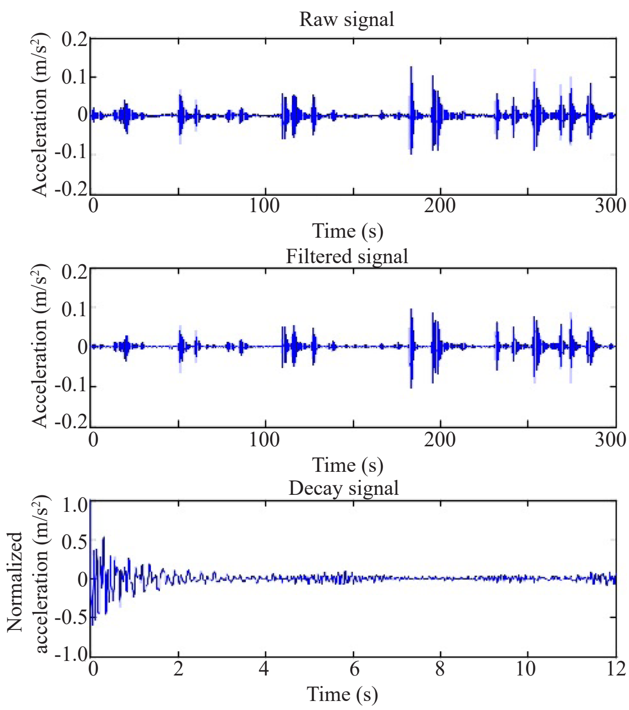

Fig. 2 Time vs. acceleration - configuration 1 - set 1 - sensor 65

The damping ratio was calculated using the CWT method for configuration 1 and 2. The amplitude spectrum obtained from a window parallel to the time axis at the first five frequencies in time, frequency and amplitude map was used to determine the damping ratio for the corresponding frequencies. The wavelet maps for first five modal frequencies and sample calculations for data set 1, sensor 65, in configuration one is shown in Fig. 4 and Table 5, respectively.

According to the analysis, all the figures showed the consistency of damping ratio values of the same sensor with different sampling rates in both the first and second configurations. Further, it indicated a good agreement between the values with different locations of the placed sensors in the structure. The results also showed that the majority of damping ratio values were between 2.8% - 1.5% in configuration 1 and 2.8%-1.2 % in configuration 2. Table 6 and Table 7 show a summary of damping ratio values corresponding to the first five frequencies in configuration 1 and 2, respectively.

3.2 Damage detection

In most studies in damage detection of structures, especially laboratory studies, damage was intentionally introduced into a structure. In some other studies when SHM was deployed on real-world structures, the investigators often define and quantify the damage that they seek. The increasing complexity of civil structures makes damage detection more challenging. As reported in the literature, WT has been used successfully for both cases.

Hou *et al.* (2000) presented a WT-based approach for damage detection. The Daubechies wavelet (db4) was selected as the mother wavelets in both CWT and DWT. The approach was applied to a simple structural SDOF model with three parallel breakage springs. It showed that structural damage may be detected by spikes in the details of the wavelet decomposition of the response data, and the locations of these spikes may accurately indicate the moments when the damage occurred. Further, the authors developed a detectability map based on the simple SDOF model which provides a quantitative relationship between the noise intensity and the damage level. It was generally concluded that the damage is more detectable for a weaker noise and more severe damage.

Structural damage detection in cantilevered beams was carried out by Okafor and Dutta (2000) using CWT. Damage was identified through the continuous wavelet coefficients obtained using Daubechies mother wavelet. A cantilever beam with reduced stiffness at different damage cases was analyzed using WT coefficients. The analysis was numerically performed using a finite element model in ANYSYS and an artificial excitation signal, respectively. It was observed that the peak point always occurred at the damaged location and its magnitude was proportional to the severity of the damage.

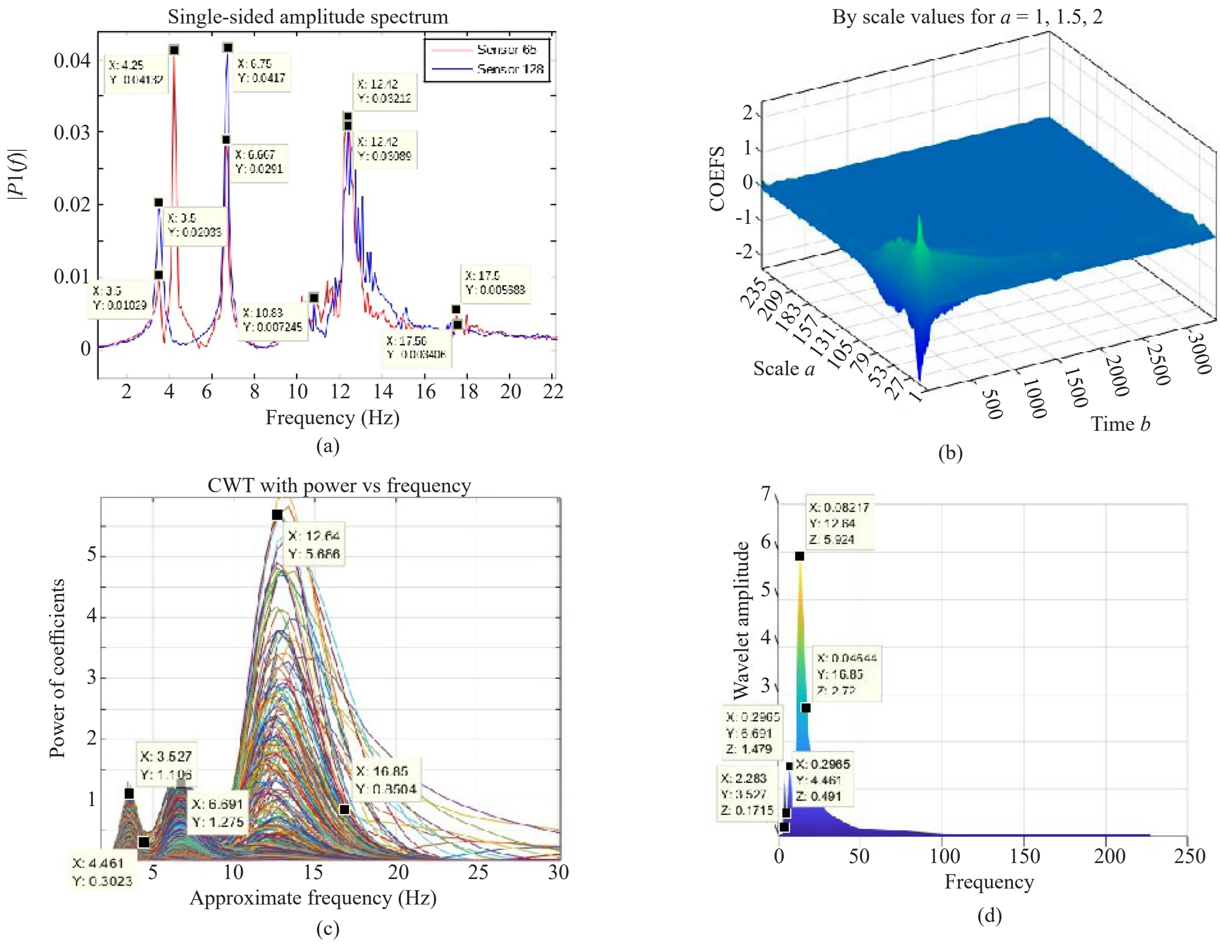

Fig. 3 FFT and CWT plots (a) FFT plot (b) CWT 3D scalogram (c) CWT power vs. frequency plot (d) CWT wavelet amplitude map

Table 3 Summary of FFT and CWT modal frequency values for configuration 1

Sensor number	Set 1 (280 Hz)			Set 3 (100 Hz)			Set 4 (50 Hz)		
	Frequency (Hz)		Diff (%)	Frequency (Hz)		Diff (%)	Frequency (Hz)		Diff (%)
	FFT	CWT		FFT	CWT		FFT	CWT	
65	3.5	3.53	0.9%	3.58	3.69	3.1%	3.4	3.87	13.8%
	4.25	4.46	4.9%	4.25	4.28	0.7%	4.4	4.28	2.7%
	6.67	6.69	0.3%	6.67	6.25	6.3%	6.8	6.25	8.1%
	12.42	12.64	1.8%	12.25	13.54	10.5%	12.4	13.54	9.2%
	17.5	16.85	3.7%	17.75	16.25	8.5%	15	16.25	8.3%
128	3.5	3.45	1.4%	3.58	3.61	0.8%	3.6	3.69	2.5%
	6.75	6.59	2.4%	6.67	6.77	1.5%	6.8	6.77	0.4%
	10.83	10.58	2.3%	11.83	10.83	8.5%	10.4	11.61	11.63%
	12.42	12.64	1.7%	12.33	12.5	1.4%	12.4	13.54	9.2%
	17.58	16.25	7.6%	17.67	16.25	8%	15	16.25	8.3%

Yoon *et al.* (2000) presented a damage detection approach in concrete beams based on Acoustic Emission (AE) signals and CWT using Morlet mother wavelet. Plain concrete beams without and with a notch, and reinforced concrete beams with different levels of corrosion in the reinforcement bars were tested in the lab. Different failure mechanisms such as micro-cracking, localized cracking, flexural cracking, and shear/bond cracking were characterized through acoustic emission (AE) responses. The WT analysis of AE signals was able to indicate damage mechanisms. The frequency shifts were noticed with respect to the changes in the crack

Table 4 Summary of FFT and CWT modal frequency values for configuration 2

Sensor Number	Set 5 (280 Hz)			Set 6 (100 Hz)			Set 7 (50 Hz)		
	Frequency (Hz)		Diff (%)	Frequency (Hz)		Diff (%)	Frequency (Hz)		Diff (%)
	FFT	CWT		FFT	CWT		FFT	CWT	
65	3.58	3.67	2.5%	3.58	3.61	0.8%	3.58	3.69	3.1%
	6.67	6.59	1.2%	6.67	6.77	1.5%	6.67	6.77	1.5%
	10.67	10.58	0.8%	10.5	10.83	3.1%	10.25	11.61	13.3%
	12.33	12.64	2.5%	12.33	12.5	1.4%	12.42	13.54	9 %
	15.08	15.17	0.6%	15	14.77	1.5%	15	16.25	8.3%
128	3.58	3.45	3.6%	3.67	3.61	1.6%	3.58	3.69	3.1%
	-	10.83	-	10.5	10.83	3.1%	6.67	6.25	6.3%
	12.33	12.64	2.5%	12.25	12.5	2%	10.17	10.16	0.1%
	-	-	-	-	14.77	-	12.42	13.54	9%
	17.75	16.85	5.1%	17.75	16.25	8.5%	15	16.25	8.3%
131	3.25	3.7	13.8%	3.67	3.61	1.6%	N/A	N/A	
	6.67	6.89	3.3%	6.58	6.25	5%			
	12.42	12.3	1%	12.25	12.5	2%			
	15.75	15.17	3.7%	15	14.77	1.5%			
	17.75	16.85	5.1%	17.83	16.25	8.9%			

Table 5 Set 1-sensor 65-configuration 1

Parameter	Value				
Natural Frequency f (Hz)	3.53	4.46	6.69	12.64	16.85
$T_i = 1/f$ (s)	0.27	0.24	0.16	0.08	0.06
m	14	17	23	13	25
t_1 (s)	1.4	1.41	1.65	0.5	0.1
t_2 (s)	5.38	3.06	5.38	1.5	1.5
$\left\|W_x(t,\omega_{di})\right\|$	7.08	1.38	0.6	3	1.98
$\left\|W_x(t+mT_i,\omega_{di})\right\|$	0.75	0.14	0.06	0.5	0.2
δ_i	0.17	0.14	0.1	0.14	0.09
ζ_i	0.028	0.022	0.016	0.023	0.015

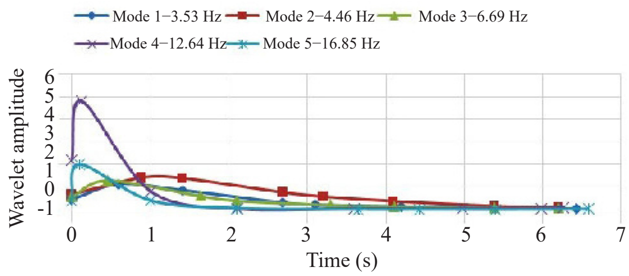

Fig. 4 Wavelet map - set 1-sensor 65-configuration 1

type. The scalograms also revealed that the high-energy signals in the later stage of damage were composed of several short closely spaced ridges.

Sun and Chang (2002) investigated the use of the WPT and the neural network model for damage assessment of civil engineering structures. Measured dynamic signals were initially decomposed into wavelet packet components. Selected component energies were then used as inputs for the neural network for various levels of damage assessment. A numerical study was performed using a three-span continuous beam under impact loads. Damage occurrence, location, and severity were determined using the proposed method. One limitation of the study was that the excitation needed to be repeatable, thus the ambient excitation could not be used.

Melhem and Kim (2003) performed damage detection in a Porland cement concrete slab on grade and a simply supported prestressed concrete beam under fatigue loads. Damage was detected by observing the changes of the ridge patterns in CWT scalograms before and after the damage occurrence. The study observed an increase in the number of clear ridges was caused by an

Table 6 Summary of frequency and damping ratio for configuration 1 using CWT

Sensor number	Set 1 (280 Hz)		Set 3 (100 Hz)		Set 4 (50 Hz)	
	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
65	3.53	2.8%	3.69	2.8%	3.87	2.7%
	4.46	2.2%	4.28	2.1%	4.28	2.3%
	6.69	1.6%	6.25	1.5%	6.25	1.9%
	12.64	2.3%	13.54	2.7%	13.54	2.6%
	16.85	1.5%	16.25	1.8%	16.25	1.8%
128	3.45	2.4%	3.61	3.2%	3.69	2.8%
	6.59	1.7%	6.77	1.6%	6.77	2.3%
	10.58	1.9%	10.83	1.9%	11.61	1.7%
	12.64	1.7%	12.5	1.9%	13.54	1.5%
	16.25	1.5%	16.25	1.5%	16.25	1.5%

Table 7 Summary of frequency and damping ratio for configuration 2 using CWT

Sensor number	Set 5 (280 Hz)		Set 6 (100 Hz)		Set 7 (50 Hz)	
	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio	Frequency (Hz)	Damping ratio
65	3.67	2.2%	3.61	2.3%	3.69	2.2%
	6.59	1.9%	6.77	1.6%	6.77	2.3%
	10.58	1.9%	10.83	1.8%	11.61	1.5%
	12.64	1.6%	12.5	1.8%	13.54	1.4%
	15.17	1.4%	14.77	1.7%	16.25	1.3%
128	3.45	2.5%	3.61	2.4%	3.69	2.1%
	-	-	10.83	1.4%	6.25	1.7%
	10.83	1.9%	12.5	1.4%	10.16	1.5%
	12.64	1.7%	14.77	1.4%	13.54	1.2%
	16.85	1.4%	16.25	1.3%	16.25	1.2%
131	3.7	2.8%	3.61	2.8%	N/A	N/A
	6.89	1.9%	6.25	1.9%		
	12.3	1.5%	12.5	1.6%		
	15.17	1.3%	14.77	1.2%		
	16.85	1.3%	16.25	1.3%		

increase in the number of cracks and when cracks grow, the magnitude of different ridges decreases. In addition, the authors concluded that low scale components which correspond to high frequency components of the signal do not change much but the low frequency components decrease significantly as the crack grows. Further, the authors compared FFT and CWT and concluded that CWT is more appropriate for nonstationary signals.

Hera and Hou (2004) presented an application of DWT analysis for damage detection for an ASCE four-story steel frame benchmark model. The response simulation data were generated by a FEM model of the structure subjected to stochastic wind load. Damage was induced by removing inter-story braces. It was found that damage due to sudden loss of structural elements and the time when it occurred can be clearly detected by spikes in the wavelet details. The damage location can be determined by the spatial distribution pattern of the observed spikes. The effects of noise and the severity of damage were also discussed in the study.

Ovanesova and Sua´rez (2004) performed a frequency analysis for a fixed concrete beam in order to estimate the effect of crack depths and it was found that crack depth had a small effect on frequencies although it was useful to provide a noticeable effect in localizing the crack. It was observed that wavelet coefficients indicate a good variation in discontinuity of the structure at cracked locations while showing clear irregularities in scalograms. The low scales indicated sharp and clear differences compared to the high scales. Further, damage detection of a simple plane frame using WT was also discussed and it showed WT was capable of revealing the discontinuities in the response signal. In addition, different mother wavelet functions were used and compared, and it was concluded that the biorthogonal wavelet was the most appropriate wavelet for crack detection in beams and frames.

Ren *et al.* (2008) performed damage detection of shear connectors of a scaled bridge structure model in the laboratory. The damage feature is characterized by the wavelet packet energy changes. Using measurements of hammer impact responses, the locations of the loosening of the shear connectors were effectively detected. The authors concluded that comparison between the vibration signals in the slab and the girder is more effective than comparing the vibration data in the slab before and after the damage state. The method presented in the study could only detect damage when a sensor is placed at a damaged location.

Park *et al.* (2008) proposed a wavelet based approach to detect damage in railroad tracks. A built-in sensing system associated with a damage detection algorithm was developed in the study. Both impedance and guided wavelet based damage detection methods were utilized simultaneously from the same active sensing system. The Morlet mother function was used for the CWT analysis. A two-step support vector machine (SVM) classifier was used as a robust pattern recognition tool for damage detection and classification. The applicability of the proposed method was verified by an experimental study of a railroad track section test specimen mounted with two piezoelectric patches.

Noh *et al.* (2011) developed three damage sensitive features (DSFs) using the CWT of earthquake responses of structures. The DSFs were functions of wavelet energies using the Morlet mother wavelet. The first DSF introduces how the wavelet energy at the natural frequency changes with respect to the damage increment. The second and third DSFs indicated how much the wavelet energy is spread out in time and how slowly the wavelet energy decays with time, respectively. The three DFS were applied to two sets of experimental data obtained from shake table tests of a bridge column and a four-story steel frame, and a set of simulated data. The results showed that the DSFs are directly correlated to damage states defined through story-drift ratio limits.

Feng *et al.* (2014) investigated a low-spatial-resolution fiber-optic sensor system for damage detection in structural members. A signal-processing approach based on decomposition of the distributed strain data using the stationary wavelet-transform method was introduced in the study. The process involved decomposition of the strain data and extraction of the approximate and detailed coefficients of the signal. The multiresolution analysis of the data involved further transformation of the approximate coefficients to extract crack features. In the verification experiment, the strain data of three 15-m long beams with two joints was obtained using the fiber-optic sensors for three different load cases. It was noticed that stationary wavelet transform has the ability to overcome the effects of system noise and frequency-peak shift distortions. For the three load cases, the WT approach was capable of detecting the locations of simulated defects with opening displacements larger than 50 μm. The proposed method did not require reference measurements of the healthy structure, but it did not make real-time measurements and required post-processing of the data.

Su *et al.* (2014) presented an efficient time varying autoregressive with exogenous input (TVARX) model to identify instantaneous modal parameters of a linear time varying structure and its substructures. CWT with Daubechies mother wavelets was employed with the model to correctly identify the instantaneous modal parameters in the frequency ranges of interest. By incorporating a substructural technique, the proposed approach was able to accurately locate potentially damaged floors of a shear building subjected to an earthquake. The effectiveness of the model for noisy responses was also considered and it was noted that it could be effectively used even for the noisy responses. The advantage of this method is that no reference data (before and after the damage occurred) is needed. The method was validated using numerically simulated earthquake response of a five-story shear building and shake table tests of an eight-story steel frame.

Amezquita-Sanchez and Adeli (2016) introduced a new synchrosqueezed wavelet transform-fractality dimension (SWT-FD) methodology to detect, locate and quantify the damage in structures. The methodology followed three steps. In step one, the SWT was used to de-noise the signal. In step two, a nonlinear dynamic measure based on chaos theory and FD was employed to detect features to be used for damage detection. In step three, a new structural damage index, based on the estimated FD values, was proposed as a measure of the condition of the structure and the damage location was obtained using the changes of the estimated FD values. Three different FD algorithms (Katz′s FD, Higuchi′s FD, and Box Dimension) were used in step two. The method was validated using the responses of a scaled model of a 38-story concrete high-rise structure to different levels of seismic excitations. It was demonstrated that the SWT integrated with either Katz′s FD or Box Dimension provides an effective tool for detecting and locating damage. SWT with Box Dimension was shown to be more effective than SWT with Katz′s FD for quantifying damage severity.

Gaviria and Montejo (2016) studied a modal analysis using free vibration response only (MAVFRO) and mass modification method with CWT and HT for signal processing and modal reconstruction. The Complex Morlet function was used as the mother wavelet in CWT analysis. The proposed method allowed dynamic properties such as natural frequencies and damping ratios to be identified, as well as the physical properties including mass and stiffness matrices. Further, the proposed method was applied to identify nonlinear damage on a numerical model of a reinforced concrete structure undergoing varying levels of seismic excitation. By identifying the condensed story stiffness, it allowed not only identification of the damage occurrence but was also able to locate and size the severity of the damage.

The damage detection method of space structures using two-dimensional (2D) continuous wavelet transform was proposed by Gholizad and Safari (2016). The isosurface of 2D wavelet coefficients, through applying the Mexican hat mother wavelet, was used to identify changes in mode shapes induced by the damage. The isosurface of intact and damaged mode shapes of a space structure are compared to indicate the location of the damage. Further, a two-step method was applied to alleviate the distortion of values caused by boundary conditions and noise effects. The method was numerically applied three types to double-layer space structures with nine scenarios of damage. The results showed that the method was able to detect all types of damage scenarios on both joints and tubular parts.

A statistical analysis was introduced by Shahsavari *et al.* (2017) for the detection and localization of damage along a beam. Laboratory tests were performed on a steel beam assembled with three bolted sections to simulate various levels of damage at two possible locations along the beam. A combination of various statistical methods and wavelet-based damage detection techniques using a Symlet wavelet were used along with the likelihood-based approach for the localization of damage. A CWT analysis was applied to the first mode of vibration of the beam. In order to extract the main patterns and eliminate the noise effects, a principal component analysis was performed on the wavelet coefficients. The scores of the first principal component were shown to be highly correlated with damage levels. Given that statistically significant damage was detected, a likelihood ratio test was proposed to determine the most likely location of damage along the beam.

A non-probabilistic WT method to evaluate the existence of both modelling error and measurement noise in vibration-based damage detection was presented by Abdulkareem *et al.* (2018). Since it is difficult to obtain unbiased distribution of uncertainties with probabilistic analysis methods, this method has an advantage to resolve the issues of uncertainties. The mode shapes of a damaged and undamaged structure were taken into consideration to extract wavelet coefficients to detect damage. The interval analysis method was used to obtain the upper and lower bounds of the wavelet coefficients. Based on CWT coefficient values, the coefficient increment factor of each segment of the structure was calculated, and the elemental possibility of damage existence was defined. Then a damage measurement index was established as an indicator of damage severity. The proposed method was applied numerically and experimentally to a four side fixed steel plate. The efficiency of the method was checked for different damage severity and noise levels.

Pan *et al.* (2018) developed a data driven framework for structural damage detection/condition assessment using a kernel function based support vector machine (SVM) integrated with enhanced feature extraction techniques using CWT (Morlet mother wavelet), Hilbert-Huang Transform (HHT) and Teager-Huang Transform (THT). Numerical analysis was carried out to verify the accuracy of the proposed method applied to a cable-stayed bridge. The impact of damage level, damage location, moving loading and sensor location was further analyzed through a parametric study. One conclusion of the authors was the WT has a significantly higher accuracy in noise interference than that of the HHT and THT. The study was based on simulated training data, thus more sources of variability must be well characterized by the training data for the algorithms to distinguish the damage from the operational and environmental variabilities.

Wang *et al.* (2019) proposed a method to identify damage in underground structures using a wavelet based residual force vector. In this study, a numerical finite element model of a metro tunnel was built and different damage locations with multiple levels of stiffness reductions were simulated. CWT using a Mexican hat mother wavelet was applied to the acceleration signals collected from the measurement points. The damage index was defined based on differences of the residual force vectors after damage and before damage. The damage index was calculated and used to locate the damage. The numerical results indicated that the method was able to successfully locate the damage in the simulated tunnel. Laboratory experiments were also conducted. A scaled aluminum pipe with different simulated damage was placed in a soil box and excited by a moving train load. The damage location was detected and the severity of the damage was also qualitatively identified.

3.2.1 Case study: damage detection of a cable-stayed bridge

A full-scale bridge benchmark problem was selected as a case study. The bridge is a three span cable-stayed bridge in China. The bridge was instrumented by a comprehensive structural health monitoring system. The acceleration response was collected by the Center of Structural Monitoring and Control at the Harbin Institute of Technology (HIT), China (Li *et al.*, 2014). For the detailed information about the bridge and the acceleration sensor configuration, refer to Li *et al.* (2014). Accelerations were monitored from January to July of 2008 and the data was made public by the Center of Structural Monitoring and Control at HIT. According to Li *et al.* (2014), on August 2008, several damage patterns were detected during an inspection of the bridge. The data collected on January 1, January 17, February 3, March 19, March 30, April 9, May 31, June 7, June 16, and July 31, 2008 was selected to represent the benchmark time history of the bridge from healthy status to damaged status. Data acquisition was performed for 24 hours with a sampling rate of 100 Hz. In this study, a damage detection algorithm based on Morlet WT was performed in order to detect the damage using the acquired acceleration data. The data was pre-processed using Butterworth filtering to reduce the effect of environmental factors. The analysis of ridges obtained in CWT scalograms was used to distinguish the healthy state and the damaged state of the bridge from January to July. In the meantime, the change of natural frequencies in the first five modes was analyzed.

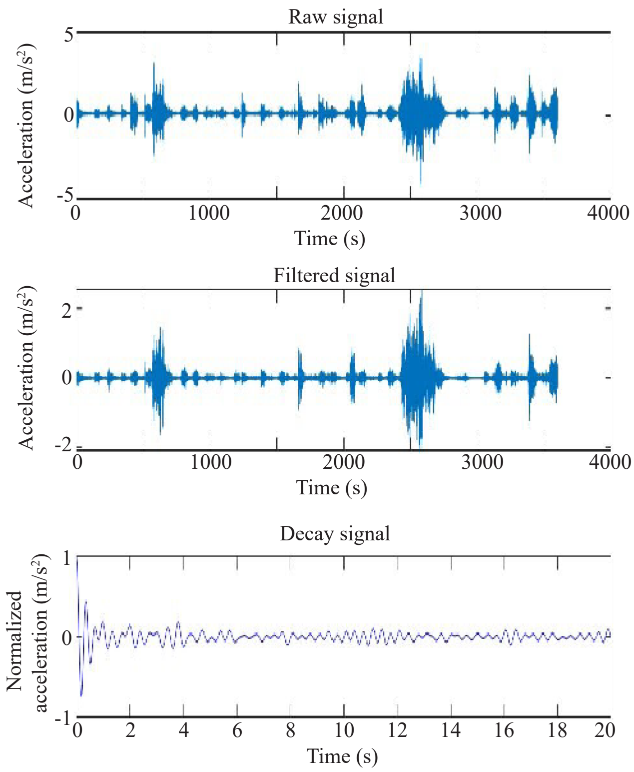

Fig. 5 Vibration-time history sample signal - January 17, 11th hour, sensor 2

The acceleration responses collected from January to July were processed using CWT, and frequency response of healthy status and damaged status were initially compared. Thereafter, wavelet based ridges were analyzed in scalograms obtained from the data acquired from the second accelerometer, in order to detect and confirm the damaged state. The eleventh hour's data was selected from January to July, which was the the same as used by the researchers at HIT. Butterworth filtering and the random decrement method were used as signal pre-processing tools.

In Fig. 5, the raw acceleration signal, filtered acceleration signal and corresponding decayed signal of January 17, when the bridge was in a health state, is presented. 200 sub-segments were used in order to decay the nonstationary signal using the random decrement method. Figure 6 represents the corresponding scalogram and frequency spectrum obtained using CWT. Frequency values for first seven modes were between 0.41 Hz – 1.45 Hz and it had a good agreement with the values obtained by HIT researchers using FFT, which was around 0.42 Hz – 1.44 Hz. The 2D scalogram presented in Fig. 6 indicates four clear ridges of scale ranges between 14 and 53. According to Melhem and Kim (2003), when the damage occurs, a decrease in natural frequencies and an increase of clear ridges are expected.

Figure 7 shows the scalograms at different stages from January to July. It indicates an abrupt increase of the number of clear ridges in July. Initially at the healthy status, there were around four clear ridges which continued until June 16, indicating that the bridge was still in a healthy state. Then, according to the vibration response on July 31, around 15 clear ridges were observed in addition to the initial four ridges. An instant increase in the number of ridges may indicate that the bridge has changed status from healthy to damaged by July 31. The damage might have occurred before July 31 but acceleration data between June 16 and July 31 was not available. In the 23rd hour of July 31, more ridges were observed and is believed to be due to the increment

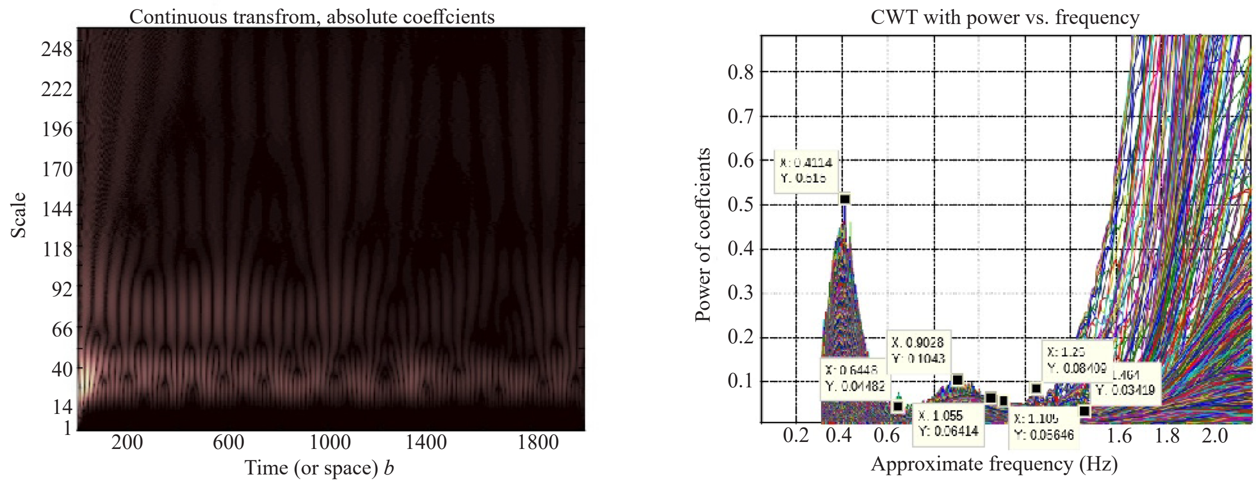

Fig. 6 Scalogram and frequency values at healthy status - January 17, 11th hour, sensor 2

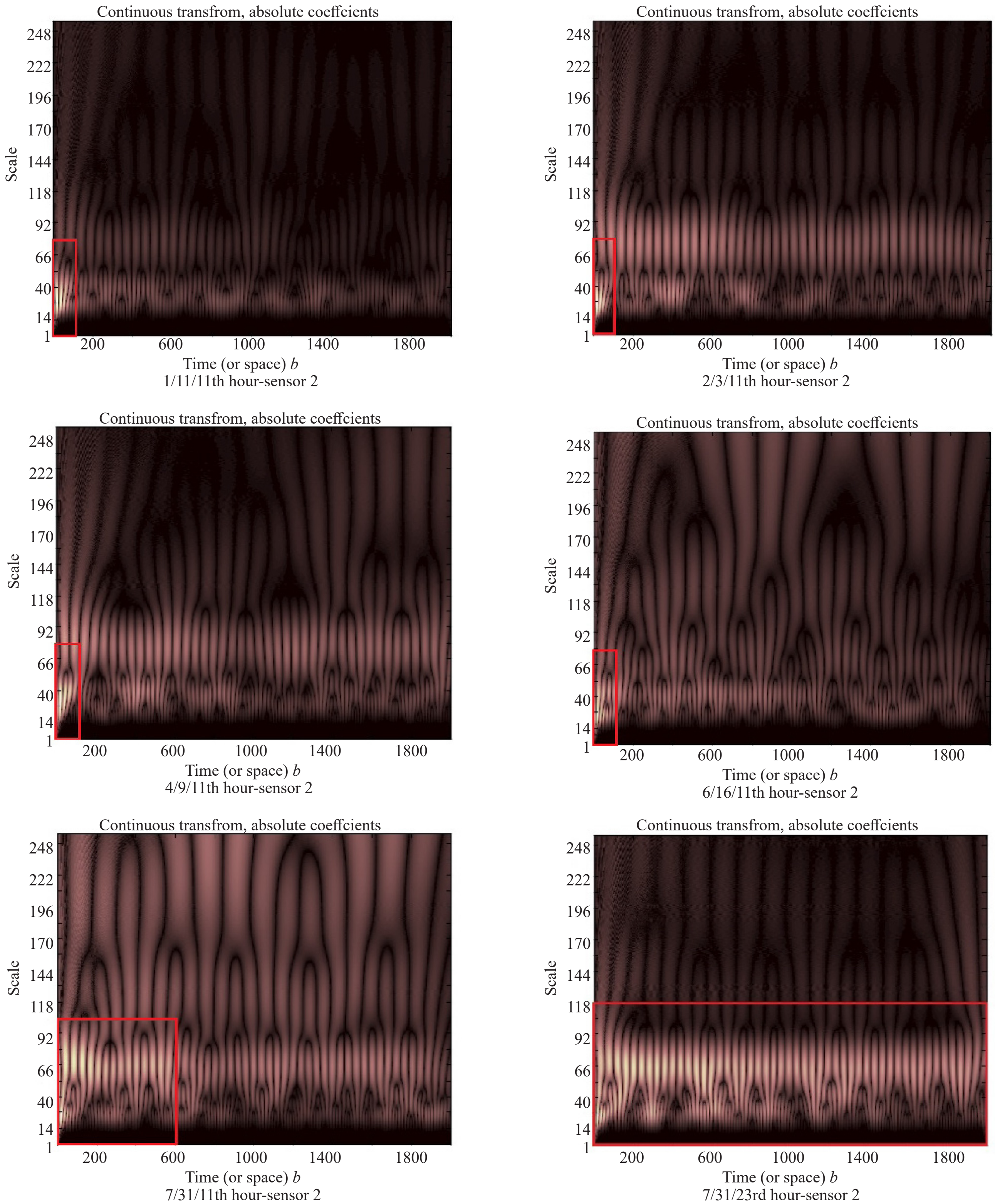

Fig. 7 2D Scalograms from January to July

of the damage. It appears that the use of the wavelet scalograms was useful for continuously monitoring progressively developing damage and its effect on the structural response.

The raw acceleration signal, filtered signal, and decayed signal is shown in Fig. 8 as per July 31. More fluctuations are visualized in the signal when compared to Fig. 5 at a healthy state. Moreover, the decrease in frequencies at the first seven modes is indicated in Fig. 9 with corresponding scalogram. The scalogram also indicates more fluctuations than in January 17. According to Li *et al.* (2014), HIT researchers also identified a decrease in frequency values at the first seven modes using FFT in the damaged state when compared to the healthy state. Further, they observed many fluctuations in the data signal at damaged status when compared to the healthy stage. Therefore, the results show good agreement with each other.

4 Conclusions

A significant amount of research has been done in the last few decades on implementation of WT in the field of SHM. This article presents an overview of the application of WT in SHM. Various WT tools have been introduced within the context of their implementation in SHM. Different features of the WT were discussed together with how researchers used these features in identifying vibration characteristics and detecting damage. Generally, as a signal-processing tool, WT possesses the advantages of excellent resolution in both time and frequency domains, good signal-to-noise ratio, and relative computational efficiency. The limitations of WT may be due to the requirement of multiple levels of decomposition, the impact of the mother wavelet selection, and possible spectral leakage. In the two case studies presented in this study, the application of WT in identification of modal properties such as natural frequencies, damping ratios, and mode shapes along with damage detection in structures, was demonstrated. Both experimental and numerical tests have been performed to evaluate the effectiveness of WT in different types of applications. The results were compared with FFT and they matched well. Both case studies concluded that WT performs very effectively and provides a reliable outcome.

Although much research has been carried out on the application of WT in SHM, most have only dealt with small and academic problems. As the advancement of the sensing technology has made a large amount of data on real-world structures available, there is still a relatively small amount of research that addresses SHM in large real-life structures. Challenges lie in developing automated online damage detection from the huge amount of collected data from large civil structures, as well as the severe noisiness of the data. WT provides a potentially effective and efficient tool for online SHM of large-scale structures, however, there is still a significant need for further research to enable WT-based methodologies to effectively handle noisy data and be accurate, scalable, and computationally efficient.

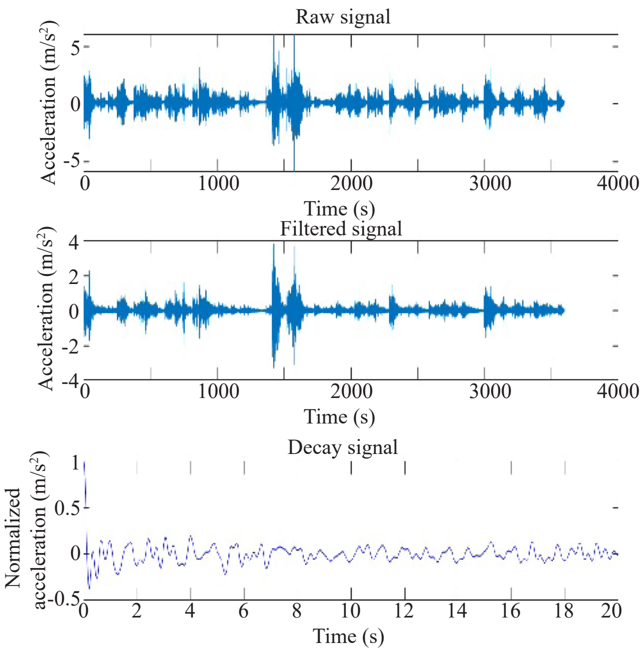

Fig. 8 Vibration-time history sample signal - July 31, 11th hour, sensor 2

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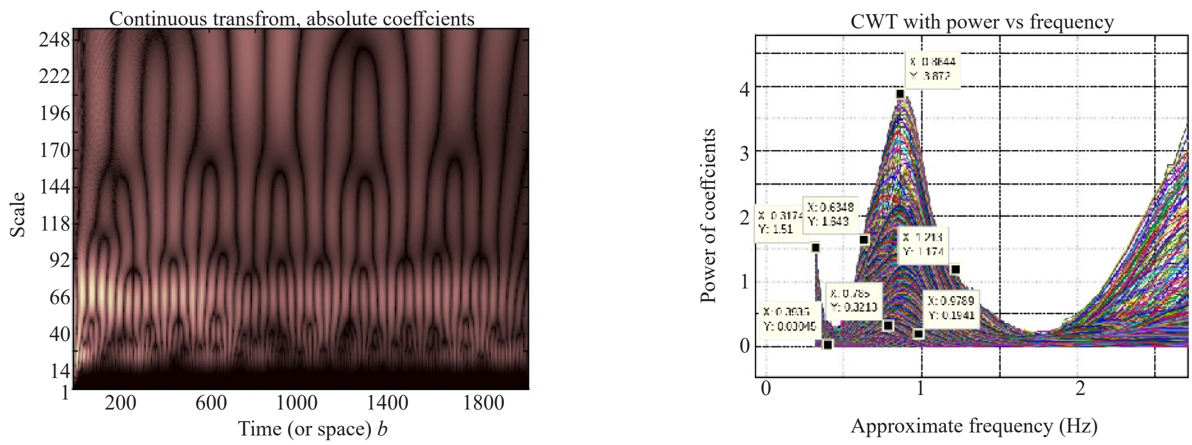

Fig. 9 Scalogram and frequency values at damaged status - July 31, 11th hour, sensor 2

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