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Equivalent damping of SDOF structure with Maxwell damper

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Abstract: To predict the maximum earthquake response of an SDOF structure with a Maxwell fluid damper or supplemental brace-viscous damper system using the seismic design response spectrum technique, a new approach is presented to determine the first- and second-order equivalent viscous damping and stiffness, the peak responses, and the damper force of the above structure. Based on the fact that the dynamic characteristics of a general linear viscoelastically damped structure are fully determined by its free vibration properties and the relaxation time constants of a Maxwell fluid damper and supplemental brace-viscous damper system in engineering practice are all small, the method of improved multiple time scales and the equivalent criterion in which all free vibration properties are the same are used to obtain the first- and second-order equivalent viscous damping and stiffness of the above structure in closed form. The accuracy of the proposed method is higher and significantly better than that of the modal strain energy method. Furthermore, in the parametric range of the requirements of the Chinese "Code for Seismic Design of Buildings", the error of the proposed second-order equivalent system for the above-mentioned engineering structure is not more than 0.5%.

Keywords: Maxwell damper; supplemental brace-viscous damper system; equivalent viscous damping; response spectrum method; maximum response of damper force

1 Introduction

Viscoelastic dampers (VED) are highly effective in mitigating the dynamic response of building structures induced by earthquake actions (Soong and Spencer, 2002; Spencer and Nagarajaiah, 2003; Takewaki, 2009; Tubaldi, 2015) and the response spectrum method is widely employed for the seismic analysis and design of structures with VED (Zambrano *et al*., 1996; Soong and Dargush, 1997; Christopoulos and Filiatrault, 2006). When this technique is applied to estimate the maximum earthquake response of a structure with VED, it is necessary and important to determine the approximately equivalent viscous damping of the viscoelastically damped structure (Chang *et al*., 1995; Zambrano *et al*., 1996; Palmeri, 2006).

The linear fluid orifice damper can be represented by the Maxwell model (Singh *et al*., 2003), the linear viscous damper in serial arrangement with its supporting brace can also be represented by the Maxwell model (Huang, 2009; Chen and Chai, 2011; Londono *et al*., 2013). The other dampers, such as fluid dampers with accumulators and even viscous shear wall dampers, especially when they are installed on deformable braces or dissipation assembly, may also be represented by the Maxwell model (Singh *et al*., 2003). Furthermore, the generalized Maxwell model can accurately describe the broad-band rheological behavior of common viscoelastic dampers (Park, 2001; Chang and Singh, 2009). Therefore, the dynamic characteristics and response of structures with Maxwell dampers have been widely studied (Soong and Dargush, 1997; Fu and Kasai, 1998; Singh *et al*., 2003; Palmeri and Ricciardelli, 2003; Yamada, 2008; Huang, 2009; Chen and Chai, 2011; Londono *et al*., 2013; Li *et al*., 2016).

The analytic methods for the dynamic response of structures with Maxwell dampers can be divided into exact and approximate methods. The exact method is a state-space approach (Soong and Dargush, 1997; Singh *et al*., 2003; Palmeri and Ricciardelli, 2003; Yamada, 2008; Chen and Chai, 2011). Although exact in nature, the state-space approach is computationally intensive due to its numerous internal variables. Furthermore, the physical insights offered by methods in the original

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space (e. g., the modal analysis) are lost in a state-space based approach. Consequently, it is difficult to use the exact method in combination with the design response spectrum to predict the maximum earthquake response of a structure. The approximate methods are dominated by the modal strain energy (MSE) method (Chang *et al*., 1995; Zambrano *et al*., 1996; Soong and Dargush, 1997; Fu and Kasai, 1998; Palmeri, 2006). In addition, the forcing decoupling method (Ou *et al*., 1998), complex damping method (Ou *et al*., 2007), and stochastic averaging method (Li *et al*., 2013) are also proposed to determine the equivalent damping and maximum response of viscoelastically damped structures.

At present, the MSE method and forcing decoupling method in combination with the design response spectrum have been applied to the seismic analysis and design of structures with viscoelastic dampers. Recently, the MSE method in combination with SAP2000 software is also used for numerical calculation of the seismic response of a twelve-story steel structure with Maxwell model-based linear Fluid Viscous Dampers (FVD) (Ras and Boumechra, 2016). The results show that the use of the passive control device FVD in steel structure generates a very significant reduction of the structural response when compared to the unbraced one.

However, there are two problems regarding the approximate methods. The first is that the complex modulus model originated from the sinusoidal testing of viscoelastic materials, and hence, strictly speaking, application of this model is limited to sinusoidal motion. The second is that all approximate methods are first-order approximate ones, which works well only for structures with small supplemental damping (Zambrano *et al*., 1996; Palmeri, 2006). Therefore, it is imperative to study the second-order approximate method so as to make it suitable for arbitrary exterior loadings.

Recently, the systematic design sensitivity analysis of structures with viscoelastic dampers has been presented (Lewandowski and Lasecka-Plura, 2016), and the methods enabling determination of the first- and second-order sensitivities of natural frequencies and non-dimensional damping ratios with respect to the parameters of dampers have been proposed. However, the above-mentioned methods for calculation of the sensitivities are the algebraically numerical methods, and it is difficult to use these methods to determine the approximately equivalent viscous damping with close form expression of the viscoelastically damped structure.

In this study, since the dynamic characteristics of a general linear viscoelastically damped structure are fully determined by its free vibration properties (Li *et al*., 2015), and the relaxation time constants of the Maxwell fluid damper (Soong and Dargush, 1997; Singh *et al*., 2003) and supplemental brace-viscous damper system (Chen and Chai, 2011; Londono *et al*., 2013) in engineering practice are all small, the multiple scale method (Nayfeh, 1973) is improved to construct the solution for free vibration of the structure. Based on the equivalent criterion in which all free vibration properties are the same, the analytic methods of first- and second-order equivalent viscous damping and stiffness in closed form of a single-degree-of freedom (SDOF) structure with a Maxwell fluid damper or supplemental brace-viscous damper system are proposed.

2 Equations of structure

For the SDOF structure with a Maxwell fluid damper or supplemental brace-viscous damper system, as show in Fig. 1, its equations of motion subjected to ground excitation can be written as

$$m\ddot{x} + c\dot{x} + kx + f_{\mathrm{d}} = -m\ddot{x}_{\mathrm{g}} \tag{1}$$

$$f_{\mathrm{d}} + \frac{c_{\mathrm{d}}}{k_{\mathrm{d}}}\dot{f}_{\mathrm{d}} = c_{\mathrm{d}}\dot{x} \tag{2}$$

where m, c, and k represent the mass, damping, and stiffness of the structure, respectively; x is the structural displacement, $\ddot{x}_{\mathrm{g}}$ is the ground acceleration, f_{d} is the force resisted by the Maxwell damper or brace-viscous damper system, and c_{d} and k_{d} are the damping and stiffness of Maxwell damper or brace-viscous damper system, respectively.

Equations (1) and (2) can be simplified as:

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \omega_0^2 x + f_{\mathrm{b}}(t) = -\ddot{x}_{\mathrm{g}} \tag{3}$$

$$f_{\mathrm{b}} + \lambda\dot{f}_{\mathrm{b}} = c_{\mathrm{b}}\dot{x} \tag{4}$$

where $\lambda = c_{\mathrm{d}}/k_{\mathrm{d}}$ is the relaxation time constant of Maxwell damper or brace-viscous damper system, and

$$\omega_0^2 = \frac{k}{m};\ 2\xi_0\omega_0 = \frac{c}{m};\ f_{\mathrm{b}} = \frac{f_{\mathrm{d}}}{m};\ c_{\mathrm{b}} = \frac{c_{\mathrm{d}}}{m} = 2\beta\omega_0 \tag{5}$$

where β is the damping ratio of the Maxwell damper or brace-viscous damper system (relative to the structural properties).

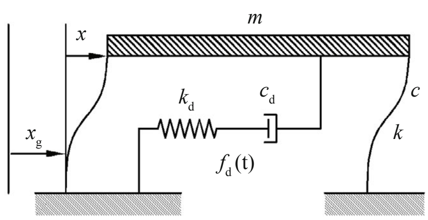

Fig. 1 Model of SDOF structure with Maxwell fluid damper or brace-viscous damper system

In engineering practice, for a linear Maxwell fluid orifice damper (Soong and Dargush, 1997; Singh *et al.*, 2003), its relaxation time constant λ is 0.006 s or 0.014 s, which is very small; for a supplemental brace-viscous damper system, in order to guarantee that this system is effective in mitigating the dynamic response, the relaxation time constant λ is also a small parameter (Chen and Chai, 2011; Londono *et al.*, 2013). For example, for a brace-viscous damper system, the Chinese "Code for Seismic Design of Buildings" (GB50011-2010, 2010) requires: $\omega_0\lambda \le 1/3$. The natural circular frequency ω_0 of a common SDOF structure is generally more than 10 rad/s; therefore, λ is comparatively small.

Because $f_b(t)$ is the disturbing term of Eq. (3), in order to reduce disturbance and raise analytical accuracy of the equivalent damping, the term $f_b(t)$ is represented in the following form to separate structural velocity and acceleration terms $f_b(t)$:

$$f_{\mathrm{b}} = c_{\mathrm{b}}\dot{x} - c_{\mathrm{b}}\lambda\ddot{x} + p_{\mathrm{b}} \tag{6}$$

where p_b is a new variable.

Thus, Eqs. (3) and (4) can be then changed into

$$\ddot{x} + \gamma(2\xi_0\omega_0 + c_{\mathrm{b}})\dot{x} + \omega_{\mathrm{n}}^2 x + \gamma p_{\mathrm{b}} = -\gamma\ddot{x}_{\mathrm{g}} \tag{7}$$

$$\gamma p_{\mathrm{b}} + \lambda(\gamma\dot{p}_{\mathrm{b}}) = \gamma c_{\mathrm{b}}\lambda^2\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} \tag{8}$$

where γp_b is the disturbing term of Eq. (7), ω_n is the natural frequency of the structural Eq. (7), and:

$$\gamma = \frac{1}{1 - c_{\mathrm{b}}\lambda} \quad ; \quad \omega_{\mathrm{n}}^2 = \gamma\omega_0^2 \tag{9}$$

From the following analysis, it can be shown that the analytic Eqs. (7) and (8) are superior to Eqs. (3) and (4) due to $\omega_0\lambda \le 1/3$; $\beta \le 0.3$; $1 < \gamma = (1\text{-}2\beta\omega_0\lambda)^{-1} \le 1.25$ in engineering practice. Considering Eqs. (3), (4), (7) and (8), and using the MSE method (Soong and Dargush, 1997),

$$f_{\mathrm{b}}(t) \approx K_{f_{\mathrm{b}}}x(t) + C_{f_{\mathrm{b}}}\dot{x}(t) \tag{10}$$

$$\gamma p_{\mathrm{b}}(t) \approx -K_{p_{\mathrm{b}}}x(t) - C_{p_{\mathrm{b}}}\dot{x}(t) \tag{11}$$

where

$$K_{f_{\mathrm{b}}} = \frac{c_{\mathrm{b}}\lambda\omega_0^2}{1+\lambda^2\omega_0^2} \quad ; \quad C_{f_{\mathrm{b}}} = \frac{c_{\mathrm{b}}}{1+\lambda^2\omega_0^2} \tag{12}$$

$$K_{p_{\mathrm{b}}} = \frac{\gamma c_{\mathrm{b}}\lambda^3\omega_{\mathrm{n}}^4}{1+\lambda^2\omega_{\mathrm{n}}^2} \quad ; \quad C_{p_{\mathrm{b}}} = \frac{\gamma c_{\mathrm{b}}\lambda^2\omega_{\mathrm{n}}^2}{1+\lambda^2\omega_{\mathrm{n}}^2} \tag{13}$$

Let

$$\eta_1 = \frac{K_{f_{\mathrm{b}}}}{\omega_0^2} \quad ; \quad \eta_2 = \frac{C_{f_{\mathrm{b}}}}{2\xi_0\omega_0} \tag{14}$$

$$\eta_3 = \frac{K_{p_{\mathrm{b}}}}{\omega_n^2} \quad ; \quad \eta_4 = \frac{C_{p_{\mathrm{b}}}}{\gamma(2\xi_0\omega_0 + c_{\mathrm{b}})} \tag{15}$$

where the parameters η_1 and η_2 represent the disturbing quantities produced by $f_b(t)$ with respect to Eq. (3), and the parameters η_3 and η_4 represent the disturbing quantities produced by $\gamma f_b(t)$ with respect to Eq. (7).

As discussed earlier, $\omega_0\lambda \le 1/3$; $\beta \le 0.3$; $1 < \gamma = (1\text{-}2\beta\omega_0\lambda)^{-1} \le 1.25$; and considering Eqs. (5), (9) and (12)-(15),

$$\frac{\eta_3}{\eta_1} = \frac{\gamma^2\lambda^2\omega_0^2(1+\lambda^2\omega_0^2)}{(1+\gamma\lambda^2\omega_0^2)} < \gamma^2\lambda^2\omega_0^2 \le 0.1736 \tag{16}$$

$$\frac{\eta_4}{\eta_2} = \frac{\gamma\lambda^2\omega_0^2(1+\lambda^2\omega_0^2)}{(1+\gamma\lambda^2\omega_0^2)}\cdot\frac{\xi_0}{(\xi_0+\beta)} < \gamma\lambda^2\omega_0^2 \le 0.1389 \tag{17}$$

Therefore, it can be concluded from Eqs. (16) and (17) that the structural analytic Eqs. (7) and (8) are superior to structural Eqs. (3) and (4) due to $\omega_0\lambda \le 1/3$and $\beta \le 0.3$ in engineering practice.

3 First-order equivalent system of structure

3.1 Analysis of first-order free vibration

Considering $\gamma c_b\lambda^2$ as a small perturbation parameter, let

$$\varepsilon c_0 = \gamma c_{\mathrm{b}}\lambda^2 \quad ; \quad \varepsilon\mu = \gamma(2\xi_0\omega_0 + c_{\mathrm{b}}) \tag{18}$$

where ε is a small parameter with an order of magnitude $O(\varepsilon) = O(\gamma c_b\lambda^2)$.

The free vibration equations of the systems of Eqs. (7) and (8) can be then given as

$$\ddot{x} + \varepsilon\mu\dot{x} + \omega_{\mathrm{n}}^2 x + \varepsilon p = 0 \tag{19}$$

$$p + \lambda\dot{p} = c_0\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} \tag{20}$$

Let

$$\varepsilon p = \varepsilon c_{p1}\dot{x} + \varepsilon k_{p1}x + O(\varepsilon^2) \tag{21}$$

where c_{p1} and k_{p1} are unknown damping and stiffness coefficients to be determined, respectively.

The free vibration Eqs. (19) and (20) can be changed into

$$\ddot{x}+\varepsilon(\mu+c_{p1})\dot{x}+\omega_1^2 x+\varepsilon(p-c_{p1}\dot{x}-k_{p1}x)=0 \tag{22}$$

$$p+\lambda\dot{p}=c_0\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} \tag{23}$$

where ω_1 respresents the first-order equivalent frequency to be determined, and

$$\omega_1^2=\omega_\mathrm{n}^2+\varepsilon k_{p1} \tag{24}$$

Because of $\varepsilon(p-c_{p1}\dot{x}-k_{p1}x)=O(\varepsilon^2)$, the first-order equivalent system of the free vibration Eqs. (22) and (23) (e.g., Eqs. (19) and (20)) can be expressed as

$$\ddot{x}+\varepsilon(\mu+c_{p1})\dot{x}+\omega_1^2 x=0 \tag{25}$$

The free vibration Eqs. (22) and (23) can be solved using the method of multiple scales (Nayfeh, 1973).

The desired functions $x(t)$ and $p(t)$ can be represented in terms of a series

$$x=x_0+\varepsilon x_1+O(\varepsilon^2) \tag{26}$$

$$p=p_0+\varepsilon p_1+O(\varepsilon^2) \tag{27}$$

where x_0 and p_0 are basically approximate solutions of x and p, respectively; x_1 and p_1 are first-order revised solutions of x and p, respectively.

The first, second and third derivatives of time are defined as follows,

$$\frac{\mathrm{d}}{\mathrm{d}t}=D_0+\varepsilon D_1+O(\varepsilon^2) \tag{28a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=D_0^2+2\varepsilon D_0 D_1+O(\varepsilon^2) \tag{28b}$$

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}=D_0^3+3\varepsilon D_0^2 D_1+O(\varepsilon^2) \tag{28c}$$

where $D_n=\partial/\partial T_n$, $T_n=\varepsilon^n t$ ($n=0, 1, 2, \ldots$).

Substituting the relationships of Eqs. (26), (27) and (28) into Eqs. (22) and (23), equating the coefficients at equal powers of ε, and limiting ourselves to the terms of the order of ε, a set of recurrent equations can be obtained.

$$D_0^2 x_0+\omega_1^2 x_0=0 \tag{29}$$

$$p_0+\lambda D_0 p_0=c_0 D_0^3 x_0 \tag{30}$$

$$D_0^2 x_1+\omega_1^2 x_1=-\mu D_0 x_0-2D_0 D_1 x_0+k_{p1}x_0-p_0 \tag{31}$$

It is found from Eqs. (29)–(31)

$$x_0(t)=\exp\left\{\mathrm{i}\,\omega_1 T_0-\frac{1}{2}\left[\mu-\frac{c_0\omega_1^2}{1+\lambda^2\omega_1^2}\right]T_1+\frac{\mathrm{i}}{2}\left[\frac{k_{p1}}{\omega_1}+\frac{c_0\lambda\omega_1^3}{1+\lambda^2\omega_1^2}\right]T_1\right\}+\mathrm{cc}+O(\varepsilon^2) \tag{32}$$

where $T_0=t$, $T_1=\varepsilon t$, the symbol cc denotes the expression to be complex conjugated to the preceding one.

Similarly, the free vibration equation of Eq. (25) can also be solved using the method of multiple scales. Thus, another solution is obtained

$$x_0(t)=\exp\left\{\mathrm{i}\,\omega_1 T_0-\frac{1}{2}\left[\mu+c_{p1}\right]T_1\right\}+\mathrm{cc}+O(\varepsilon^2) \tag{33}$$

Comparing Eq. (32) with Eq. (33) and considering the relationships of Eq. (18) and (24),

$$\varepsilon c_{p1}=-\frac{\gamma c_\mathrm{b}\lambda^2\omega_1^2}{1+\lambda^2\omega_1^2} \tag{34}$$

$$\varepsilon k_{p1}=-\frac{\gamma c_\mathrm{b}\lambda^3\omega_1^4}{1+\lambda^2\omega_1^2} \tag{35}$$

$$\omega_1^2=\omega_\mathrm{n}^2-\frac{\gamma c_\mathrm{b}\lambda^3\omega_1^4}{1+\lambda^2\omega_1^2} \tag{36}$$

Solving Eq. (36), we obtain

$$\omega_1^2=\frac{\lambda^2\omega_\mathrm{n}^2-1+\sqrt{(\lambda^2\omega_\mathrm{n}^2+1)^2+4(\gamma c_\mathrm{b}\lambda)\lambda^2\omega_\mathrm{n}^2}}{2\lambda^2(1+\gamma c_\mathrm{b}\lambda)} \tag{37}$$

As for a common SDOF engineering structure with a Maxwell damper or brace-viscous damper system, because of $\gamma c_\mathrm{b}\lambda<1$ and $\lambda^4\omega_\mathrm{n}^4 \ll 1$, the approximate solution of the first-order equivalent frequency can be obtained from the expression of Eq. (37)

$$\begin{aligned}\omega_1^2&=\frac{\lambda^2\omega_\mathrm{n}^2-1}{2\lambda^2(1+\gamma c_\mathrm{b}\lambda)}+\frac{1+\lambda^2\omega_\mathrm{n}^2(1+2\gamma c_\mathrm{b}\lambda)}{2\lambda^2(1+\gamma c_\mathrm{b}\lambda)}\cdot\sqrt{1-\frac{4(\gamma c_\mathrm{b}\lambda)\lambda^4\omega_\mathrm{n}^4(1+\gamma c_\mathrm{b}\lambda)}{[1+\lambda^2\omega_\mathrm{n}^2(1+2\gamma c_\mathrm{b}\lambda)]^2}}\\&=\omega_\mathrm{n}^2\left[1-\frac{(\gamma c_\mathrm{b}\lambda)\lambda^2\omega_\mathrm{n}^2}{1+\lambda^2\omega_\mathrm{n}^2(1+2\gamma c_\mathrm{b}\lambda)}+O(\lambda^6\omega_\mathrm{n}^6)\right]\end{aligned} \tag{38}$$

3.2 First-order equivalent system of structure

The first-order equivalent system of Eq. (25) of the free vibration Eqs. (19) and (20) of forced vibration systems of Eqs. (7) and (8) can be expressed as

$$\ddot{x} + c_1\dot{x} + \omega_1^2 x = 0 \tag{39}$$

where ω_1 is the first-order equivalent frequency given in Eq. (37), c_1 is first-order equivalent damping

$$c_1 = \varepsilon(\mu + c_{p1}) = \gamma\left(2\xi_0\omega_0 + \frac{c_\mathrm{b}}{1+\lambda^2\omega_1^2}\right) \tag{40}$$

Because the dynamic characteristics of a general linear viscoelastically damped structure is independent of its exterior loadings while fully determined by its free vibration properties (Li *et al*., 2015), the first-order equivalent system of the structural forced vibration systems of Eqs. (7) and (8) can be expressed as

$$\ddot{x} + c_1\dot{x} + \omega_1^2 x = -\gamma\ddot{x}_\mathrm{g} \tag{41}$$

3.3 First-order equivalent system of Maxwell damper

In order to analyze the maximum earthquake response of a Maxwell damper or brace-viscous damper system using the response spectrum technique, it is necessary to find the first-order equivalent system of the Maxwell damper or brace-viscous damper system.

Considering the relationships of Eqs. (7), (19) and (21),

$$\gamma p_\mathrm{b} = \varepsilon p = \varepsilon c_{p1}\dot{x} + \varepsilon k_{p1}x + O(\varepsilon^2) \tag{42}$$

From Eqs. (6), (34), and (35), the first-order equivalent system of the Maxwell damper or brace-viscous damper system can be expressed as

$$f_\mathrm{b} = \frac{f_\mathrm{d}}{m} = c_{f1}\dot{x} + k_{f1}x + O(\varepsilon^2) \tag{43}$$

where displacement x and velocity $\dot{x}$ can be calculated from Eq. (41), c_{f1} and k_{f1} are the first-order equivalent damping and stiffness of the damper system, respectively, which can be given as follows:

$$c_{f1} = \frac{c_\mathrm{b}}{1+\lambda^2\omega_1^2}\ ;\ k_{f1} = \frac{c_\mathrm{b}\lambda\omega_1^2}{1+\lambda^2\omega_1^2} \tag{44}$$

4 Second-order equivalent system of structure

4.1 Analysis of second-order free vibration

Representing the term γp_b of Eqs. (7) and (8) in the following form

$$\gamma p_\mathrm{b} = \gamma c_\mathrm{b}\lambda^2\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + \gamma c_\mathrm{b}\lambda^3 q \tag{45}$$

where q is a new variable.

Substituting Eq. (45) into Eqs. (7) and (8),

$$\ddot{x} + \gamma(2\xi_0\omega_0 + c_\mathrm{b})\dot{x} + \omega_\mathrm{n}^2 x + \gamma c_\mathrm{b}\lambda^2\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + \gamma c_\mathrm{b}\lambda^3 q = -\gamma\ddot{x}_\mathrm{g} \tag{46}$$

$$q + \lambda\dot{q} = -\frac{\mathrm{d}^4 x}{\mathrm{d}t^4} \tag{47}$$

Considering the relaxation time constant λ as a small perturbation parameter, let

$$\varepsilon = \lambda;\quad \varepsilon\mu = \gamma(2\xi_0\omega_0 + c_\mathrm{b});\quad \eta = \gamma c_\mathrm{b}\lambda \tag{48}$$

The free vibration equations of structural forced vibration systems of Eqs. (46) and (47) can then be written as

$$\ddot{x} + \varepsilon\mu\dot{x} + \omega_2^2 x + \varepsilon\eta\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + \varepsilon^2\eta q - \varepsilon^2 k_2 x = 0 \tag{49}$$

$$q + \lambda\dot{q} = -\frac{\mathrm{d}^4 x}{\mathrm{d}t^4} \tag{50}$$

where k_2 is the stiffness coefficient and ω_2 is the second-order equivalent frequency, which are to be determined. The interrelated relation is given as follows

$$\omega_2^2 = \omega_\mathrm{n}^2 + \varepsilon^2 k_2 \tag{51}$$

Let the second-order equivalent system of free vibration Eqs. (49) and (50) be in the following form

$$\ddot{x} + \varepsilon c_{q1}\dot{x} + \varepsilon^2 c_{q2}\dot{x} + \omega_2^2 x = 0 \tag{52}$$

where c_{q1} and c_{q2} are first- and second-order damping coefficients to be determined, respectively.

The method of multiple scales is used to construct the solutions of Eqs. (49) and (50). The desired functions $x(t)$ and $q(t)$ can be represented in terms of a series

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + O(\varepsilon^3) \tag{53}$$

$$q = q_0 + \varepsilon q_1 + \varepsilon^2 q_2 + O(\varepsilon^3) \tag{54}$$

where x_2 and q_2 are second-order revised solutions of x and q, respectively.

The first, second, third and fourth derivatives of time can then be defined as follows, respectively.

$$\frac{\mathrm{d}}{\mathrm{d}t} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + O(\varepsilon^3) \tag{55a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 (D_1^2 + 2D_0 D_2) + O(\varepsilon^3) \tag{55b}$$

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3} = D_0^3 + 3\varepsilon D_0^2 D_1 + 3\varepsilon^2 (D_0^2 D_2 + D_0 D_1^2) + O(\varepsilon^3) \tag{55c}$$

$$\frac{\mathrm{d}^4}{\mathrm{d}t^4} = D_0^4 + 4\varepsilon D_0^3 D_1 + 2\varepsilon^2 (2D_0^3 D_2 + 3D_0^2 D_1^2) + O(\varepsilon^3) \tag{55d}$$

When substituting the relationships of Eqs. (53), (54) and (55) into Eqs. (49) and (50) and carrying out the procedure described in detail in Section 3.1,

$$D_0^2 x_0 + \omega_2^2 x_0 = 0 \tag{56}$$

$$D_0^2 x_1 + \omega_2^2 x_1 = -2D_0 D_1 x_0 - \mu D_0 x_0 - \eta D_0^3 x_0 \tag{57}$$

$$q_0 + \lambda D_0 q_0 = -D_0^4 x_0 \tag{58}$$

$$\begin{aligned} D_0^2 x_2 + \omega_2^2 x_2 = & -(2D_0 D_1 x_1 + 2D_0 D_2 x_0 + D_1^2 x_0) - \\ & \mu(D_0 x_1 + D_1 x_0) - \eta(D_0^3 x_1 + 3D_0^2 D_1 x_0) - \\ & \eta q_0 + k_2 x_0 \end{aligned} \tag{59}$$

From Eqs. (56)–(59),

$$\begin{aligned} x_0(t) = \exp\Bigg\{ & \mathrm{i}\,\omega_2 T_0 - \frac{\mu - \eta\omega_2^2}{2} T_1 - \left[\frac{\eta\lambda\omega_2^4}{2(1+\lambda^2\omega_2^2)} + \frac{(\mu-\eta\omega_2^2)^2(\mathrm{i}\,\omega_2)}{8\omega_2^2} \right] T_2 + \\ & (\mathrm{i}\,\omega_2)\left[\frac{k_2}{2\omega_2^2} - \frac{\eta(\mu-\eta\omega_2^2)}{2} + \frac{\eta\omega_2^2}{2(1+\lambda^2\omega_2^2)} \right] T_2 \Bigg\} + \mathrm{cc} + \mathrm{O}(\varepsilon^3) \end{aligned} \tag{60}$$

where $T_2 = \varepsilon^2 t$.

Similarly, it can be obtained from the second-order equivalent Eq. (52) using the method of multiple scales

$$x_0(t) = \exp\left\{ \mathrm{i}\,\omega_2 T_0 - \frac{c_{q1}}{2} T_1 - \left[\frac{c_{q2}}{2} + \frac{c_{q1}^2 (\mathrm{i}\,\omega_2)}{8\omega_2^2} \right] T_2 \right\} + \mathrm{cc} + O(\varepsilon^3) \tag{61}$$

Comparing Eq. (60) and Eq. (61) and considering relationships of Eqs. (48) and (51),

$$\varepsilon c_{q1} = \varepsilon(\mu - \eta\omega_2^2) = \gamma[2\xi_0\omega_0 + c_{\mathrm{b}}(1-\lambda^2\omega_2^2)] \tag{62}$$

$$\varepsilon^2 c_{q2} = \frac{\varepsilon^2 \eta\lambda\omega_2^4}{1+\lambda^2\omega_2^2} = \frac{\gamma c_{\mathrm{b}}\lambda^4\omega_2^4}{1+\lambda^2\omega_2^2} \tag{63}$$

$$\begin{aligned} \varepsilon^2 k_2 &= \varepsilon^2 \eta\omega_2^2 \left[(\mu - \eta\omega_2^2) - \frac{\omega_2^2}{1+\lambda^2\omega_2^2} \right] \\ &= -\frac{\gamma c_{\mathrm{b}}\lambda^3\omega_2^4}{1+\lambda^2\omega_2^2} + \gamma^2 c_{\mathrm{b}}\lambda^2\omega_2^2 [2\xi_0\omega_0 + c_{\mathrm{b}}(1-\lambda^2\omega_2^2)] \end{aligned} \tag{64}$$

$$\omega_2^2 = \omega_{\mathrm{n}}^2 + \varepsilon^2 k_2 \tag{65}$$

Substitution of Eq. (64) into Eq. (65) yields

$$\begin{aligned} & (\gamma^2 c_{\mathrm{b}}^2 \lambda^6)\omega_2^6 + \lambda^2[1+\gamma c_{\mathrm{b}}\lambda(1-2\xi_0\omega_0\gamma\lambda)]\omega_2^4 + \\ & [1-\lambda^2(\omega_{\mathrm{n}}^2 + 2\xi_0\omega_0\gamma^2 c_{\mathrm{b}} + \gamma^2 c_{\mathrm{b}}^2)]\omega_2^2 - \omega_{\mathrm{n}}^2 = 0 \end{aligned} \tag{66}$$

The cubic Eq. (66) concerning ω_2^2 can be precisely solved in closed form (Abramowita and Stegun, 1965). As discussed earlier, $\gamma^2 c_{\mathrm{b}}^2 \lambda^6 << 1$, thus the approximate solution of the second-order equivalent frequency ω_2 can be obtained from Eq. (66) using perturbation approach (Nayfeh, 1973)

$$\omega_2^2 = y_0 + y_1 + y_2 + O(\gamma^6 c_{\mathrm{b}}^6 \lambda^{18}) \tag{67}$$

where

$$y_0 = \frac{A_1 + \sqrt{A_1^2 + 2\omega_{\mathrm{n}}^2 A_0}}{A_0} \tag{68a}$$

$$y_1 = -\frac{(\gamma^2 c_{\mathrm{b}}^2 \lambda^6) y_0^3}{y_0 A_0 - A_1} \tag{68b}$$

$$y_2 = -\frac{6(\gamma^2 c_{\mathrm{b}}^2 \lambda^6) y_0^2 y_1 + y_1^2 A_0}{2(y_0 A_0 - A_1)} \tag{68c}$$

$$A_0 = 2\lambda^2[1+\gamma c_{\mathrm{b}}\lambda(1-2\xi_0\omega_0\gamma\lambda)] \tag{68d}$$

$$A_1 = -1 + \lambda^2[\omega_{\mathrm{n}}^2 + \gamma^2 c_{\mathrm{b}}(2\xi_0\omega_0 + c_{\mathrm{b}})] \tag{68e}$$

4.2 Second-order equivalent system of structure

The second-order equivalent system of Eq. (52) of the free vibration Eqs. (49) and (50) of the forced vibration systems of Eqs. (7) and (8) can be expressed as

$$\ddot{x} + c_2\dot{x} + \omega_2^2 x = 0 \tag{69}$$

where ω_2 is second-order equivalent frequency calculated from Eq. (66) or Eq. (67), and c_2 is second-order equivalent damping given as

$$c_2 = \varepsilon c_{q1} + \varepsilon^2 c_{q2} = \gamma \left[2\xi_0\omega_0 + \frac{c_\mathrm{b}}{1+\lambda^2\omega_2^2} \right] \tag{70}$$

Similarly, carrying out the procedure described in detail in Section 3.2, the second-order equivalent system of the structural forced vibration systems of Eqs. (7) and (8) can be expressed as

$$\ddot{x} + c_2\dot{x} + \omega_2^2 x = -\gamma \ddot{x}_\mathrm{g} \tag{71}$$

4.3 Second-order equivalent system of Maxwell damper

Considering Eqs. (45), (46), (51) and (71),

$$\begin{aligned} \gamma p_\mathrm{b} &= (\omega_2^2 - \omega_\mathrm{n}^2)x + \gamma c_\mathrm{b}\left(\frac{1}{1+\lambda^2\omega_2^2} - 1 \right)\dot{x} + O(\lambda^3) \\ &= \varepsilon^2 k_2 x - \frac{\gamma c_\mathrm{b}\lambda^2\omega_2^2}{1+\lambda^2\omega_2^2}\dot{x} + O(\lambda^3) \end{aligned} \tag{72}$$

From Eqs. (6), (64) and (72), the second-order equivalent system of Maxwell system can be expressed as

$$f_\mathrm{b} = \frac{f_\mathrm{d}}{m} = c_{f2}\dot{x} + k_{f2}x + O(\lambda^3) \tag{73}$$

where displacement x and velocity $\dot{x}$ can be calculated from Eq. (71), and c_{f2} and k_{f2} are the second-order equivalent damping and stiffness of damper system, respectively, defined as follows

$$c_{f2} = \frac{c_\mathrm{b}}{1+\lambda^2\omega_2^2} \tag{74}$$

$$k_{f2} = \frac{c_\mathrm{b}\lambda\omega_2^2}{1+\lambda^2\omega_2^2} + (\gamma c_\mathrm{b}\lambda)\omega_2^2[2\xi_0\omega_0\lambda + c_\mathrm{b}\lambda(1-\lambda^2\omega_2^2)] \tag{75}$$

5 Analysis of maximum response of structure

5.1 Maximum response of structure

Structural first- and second-order equivalent system equations of Eq. (41) and Eq. (71) can be expressed in a uniform form

$$\ddot{x} + 2\xi_j\omega_j\dot{x} + \omega_j^2 x = -\gamma\ddot{x}_\mathrm{g} \qquad (j=1,2) \tag{76}$$

where ξ_1 and ξ_2 are first- and second-order equivalent damping ratios, respectively.

$$\xi_1 = \frac{c_1}{2\omega_1} \quad ; \quad \xi_2 = \frac{c_2}{2\omega_2} \tag{77}$$

Let $S_\mathrm{d}(\omega_j, \xi_j)$ and $S_\mathrm{v}(\omega_j, \xi_j)$ be displacement and velocity response spectrum of the oscillators $\delta_j(t)$, respectively, thus

$$\ddot{\delta}_j + 2\xi_j\omega_j\dot{\delta}_j + \omega_j^2\delta_j = -\ddot{x}_\mathrm{g} \qquad (j=1,2) \tag{78}$$

Using the response spectrum technique (Lin and Chang, 2003; Cacciola *et al*., 2004), the maximum response of structure can be expressed as:

$$|x(t)|_{\max} = \gamma S_\mathrm{d}(\omega_j, \xi_j) \qquad (j=1,2) \tag{79}$$

$$|\dot{x}(t)|_{\max} = \gamma S_\mathrm{v}(\omega_j, \xi_j) \qquad (j=1,2) \tag{80}$$

Since the methods of transforming from the acceleration response spectrum of the Chinese "Code for Seismic Design of Buildings" into displacement and velocity response spectrum have been found in Li *et al*., 2014a, 2014b, S_d and S_v can be calculated directly from the Chinese "Code for Seismic Design of Building", i.e.,

$$S_\mathrm{d}(\omega_j, \xi_j) = B_\mathrm{d}\alpha(\omega_j, \xi = 5\%)\,\mathrm{g}\,\omega_j^{-2} \tag{81}$$

$$S_\mathrm{v}(\omega_j, \xi_j) = B_\mathrm{v}\alpha(\omega_j, \xi = 5\%)\,\mathrm{g}\,\omega_j^{-1} \tag{82}$$

where $j = 1, 2$, g is acceleration of gravity, $\alpha(\omega_j, \xi = 5\%)$ is the earthquake-influenced coefficient of the oscillator with a period of $T_j = 2\pi/\omega_j$ and damping ratio $\xi = 5\%$ corresponding to the Chinese acceleration response spectrum, and B_d and B_v are displacement and velocity damping reduction factors, respectively, given by Lin and Chang (2003), i.e.,

$$B_\mathrm{d} = \frac{S_\mathrm{d}(\omega_j, \xi_j)}{S_\mathrm{d}(\omega_j, \xi = 5\%)} \quad ; \quad B_\mathrm{v} = \frac{S_\mathrm{v}(\omega_j, \xi_j)}{S_\mathrm{v}(\omega_j, \xi = 5\%)} \qquad (j=1,2) \tag{83}$$

5.2 Maximum response of Maxwell damper

From Eqs. (43) and (73), the variances of first- and second-order equivalent system of the Maxwell damper or brace-viscous damper system can be expressed in the following forms

$$\begin{aligned} \mathrm{E}[f_\mathrm{b}^2(t)] = {} & c_{fj}^2\,\mathrm{E}[\dot{x}^2(t)] + k_{fj}^2\,\mathrm{E}[x^2(t)] + \\ & 2c_{fj}k_{fj}\,\mathrm{E}[x(t)\dot{x}(t)] \qquad (j=1,2) \end{aligned} \tag{84}$$

where the operator E[•] represents the expectation operation of the quantity inside the bracket.

Making the assumption that the input excitation $\ddot{x}_{\rm g}(t)$ is a Gaussian stationary process with a zero mean value, the response $x(t)$ of a linear equivalent system of Eq. (76) subjected to a Gaussian process is also Gaussian. The derivative of a Gaussian, $\dot{x}(t)$, is Gaussian as well and the two processes, $x(t)$ and $\dot{x}(t)$, are mutually independent. As a result, the term E[$x(t)$ $\dot{x}(t)$] disappears due to the independent property (Soong and Grigoriu, 1993). Because the maximum value of a response is proportional to the root mean square of its process (Cacciola *et al.*, 2004), Eq. (84) can be represented by the peak response with their respective proportional factors as follows

$$\frac{1}{\eta_{f_{\rm b}}^2}\left|f_{\rm b}(t)\right|_{\max}^2=\frac{c_{fj}^2}{\eta_{\dot{x}}^2}\left|\dot{x}^2(t)\right|_{\max}^2+\frac{k_{fj}^2}{\eta_x^2}\left|x^2(t)\right|_{\max}^2 \qquad (j=1,2) \tag{85}$$

where $\eta_{f_{\rm b}}$, $\eta_{\dot{x}}$ and η_x are proportional factors corresponding to $f_{\rm b}(t)$, $\dot{x}(t)$ and $x(t)$, respectively, and are often assumed to have the same value in practice (Cacciola *et al.*, 2004). Thus, the above equation can be further simplified as

$$\left|f_{\rm b}(t)\right|_{\max}^2=c_{fj}^2\left|\dot{x}^2(t)\right|_{\max}^2+k_{fj}^2\left|x^2(t)\right|_{\max}^2 \qquad (j=1,2) \tag{86}$$

Substituting the relationships of Eqs. (79) and (80) into Eq. (86),

$$\left|f_{\rm b}(t)\right|_{\max}=\gamma\sqrt{c_{fj}^2S_{\rm v}^2(\omega_j,\xi_j)+k_{fj}^2S_{\rm d}^2(\omega_j,\xi_j)} \qquad (j=1,2) \tag{87}$$

6 Accuracy of equivalent system

The first, from the original structure in Eqs. (3) and (4), is the exact complex frequency response function $H_x(\mathrm{i}\omega)$ of the structural displacement x and can be expressed as

$$H_x(\mathrm{i}\omega)=-\frac{\lambda(\mathrm{i}\omega)+1}{\lambda(\mathrm{i}\omega)^3+(1+2\xi_0\omega_0\lambda)(\mathrm{i}\omega)^2+(2\xi_0\omega_0+\omega_0^2\lambda+c_{\rm b})(\mathrm{i}\omega)+\omega_0^2} \tag{88}$$

The exact stationary variance σ_x^2 of the structural displacement x, subjected to Gaussian stationary white noise $\ddot{x}_{\rm g}(t)$ with spectral density function $S_{\ddot{x}_{\rm g}}(\omega)=S_0$, can be expressed as

$$\sigma_x^2=\int_{-\infty}^{\infty}\left|H_x(\mathrm{i}\omega)\right|^2S_0\,\mathrm{d}\omega=\frac{\pi S_0(1+2\xi_0\omega_0\lambda+\omega_0^2\lambda^2)}{\omega_0^2[4\xi_0^2\omega_0^2\lambda+2\xi_0\omega_0(1+\omega_0^2\lambda^2+c_{\rm b}\lambda)+c_{\rm b}]} \tag{89}$$

The exact nonstationary variance $\sigma_x^2(t)$ of structural displacement x, subjected to several kinds of classical amplitude uniformly modulated nonstationary Gaussian white noise $\ddot{x}_{\rm g}(t)$ with correlation function $\mathrm{E}[\ddot{x}_{\rm g}(t)\cdot\ddot{x}_{\rm g}(t+\tau)]=2\pi S_0A(t)A(t+\tau)\delta(\tau)$, has been obtained in Li *et al.*, 2016,where S_0 is the constant spectral density, $A(t)$ is the deterministic amplitude function, and $\delta(\tau)$ is the dirac-delta function.

The second, from Eq. (76), is the approximate complex frequency response function $H_{xj}(\mathrm{i}\omega)$ given by the first- and second-order equivalent system for the original structure in Eqs. (3) and (4) can be expressed as

$$H_{xj}(\mathrm{i}\omega)=-\frac{\gamma}{(\mathrm{i}\omega)^2+2\xi_j\omega_j(\mathrm{i}\omega)+\omega_j^2} \qquad (j=1,2) \tag{90}$$

The corresponding approximate stationary variance σ_{xj}^2 subjected to the same stationary white noise can be expressed as

$$\sigma_{xj}^2=\frac{\pi\gamma^2S_0}{2\xi_j\omega_j^3} \qquad (j=1,2) \tag{91}$$

The corresponding approximate nonstationary variance $\sigma_{xj}^2(t)$ subjected to the same nonstationary white noise has also been obtained in the above-mentioned reference.

The third, the original structure in Eqs. (3) and (4), can be approximated by using the modal strain energy (MSE) method

$$\ddot{x}+2\xi_3\omega_3\dot{x}+\omega_3^2x=-\ddot{x}_{\rm g} \tag{92}$$

where

$$\omega_3^2=\frac{\omega_0^2\lambda^2+c_{\rm b}\lambda-1}{2\lambda^2}+\frac{\sqrt{(\omega_0^2\lambda^2+c_{\rm b}\lambda-1)^2+4\omega_0^2\lambda^2}}{2\lambda^2} \tag{93}$$

$$2\xi_3\omega_3=2\xi_0\omega_0+\frac{c_{\rm b}}{1+\lambda^2\omega_3^2} \tag{94}$$

The corresponding approximate complex frequency response function $H_{x3}(\mathrm{i}\omega)$ and approximate stationary variance σ_{x3}^2 given by the MSE method can be expressed, respectively, as follows

$$H_{x3}(\mathrm{i}\omega)=-\frac{1}{(\mathrm{i}\omega)^2+2\xi_3\omega_3(\mathrm{i}\omega)+\omega_3^2} \tag{95}$$

$$\sigma_{x3}^2=\frac{\pi S_0}{2\xi_3\omega_3^3} \tag{96}$$

The corresponding approximate nonstationary variance $\sigma_{x3}^2(t)$ can also be found in the reference mentioned above.

As a result, the accuracy of the proposed method or the MSE method can be investigated through a comparison of exact solutions of Eqs. (88) and (89) with approximate solutions of Eqs. (90) and (91) or approximate solutions of Eqs. (95) and (96). The accuracy can also be reviewed through the comparison of exact nonstationary response with approximate nonstationary response.

As discussed earlier, as for Maxwell fluid damper, λ = 0.014 s; as for brace-viscous damper system, $\omega_0\lambda \le 1/3$; moreover, the supplemental damping ratio provided by the damper system is generally required to be less than 25%, e. g., corresponding to $\beta \le 0.3$. Thus the classical parameters of calculation are selected as follows: ξ_0 = 0.05; ω_0 = 10, 20, 30 rad/s; $\omega_0\lambda$ = 1/5, 1/4, 1/3, 2/5; c_b = $2\beta\omega_0$, β = 0.1, 0.15, 0.20, 0.25, 0.30.

Figures 2-6 show the exact and approximate complex frequency response functions (FRF) (88), (90) and (92), respectively.

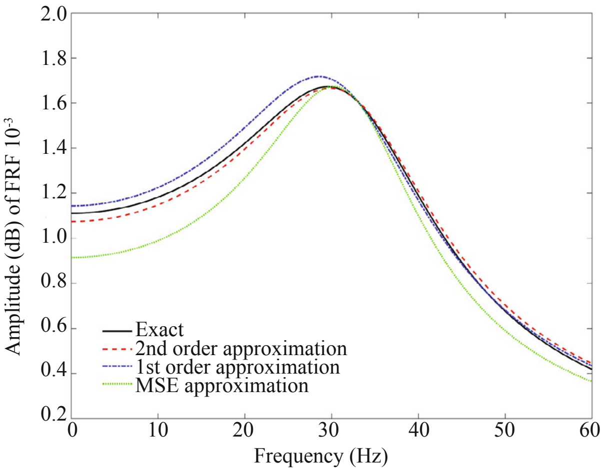

Fig. 2 FRF for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.30; ω_0 = 30 rad/s)

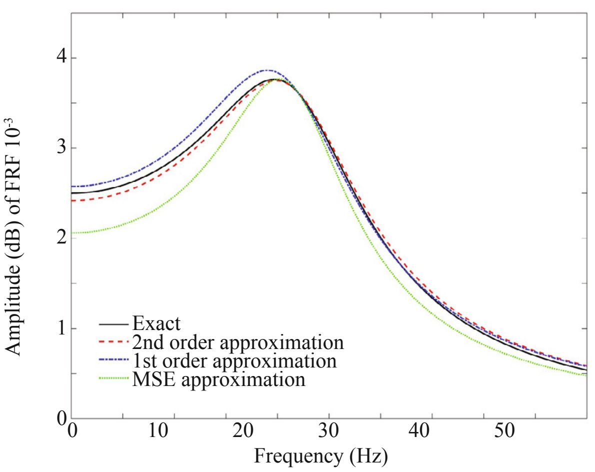

Fig. 3 FRF for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.30; ω_0 = 20 rad/s)

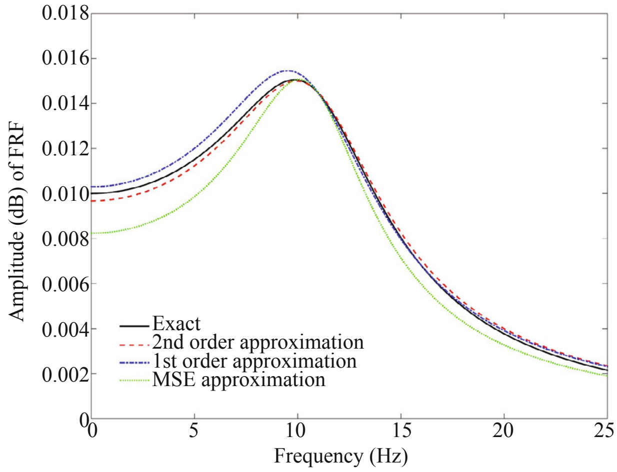

Fig. 4 FRF for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.30; ω_0 = 10 rad/s)

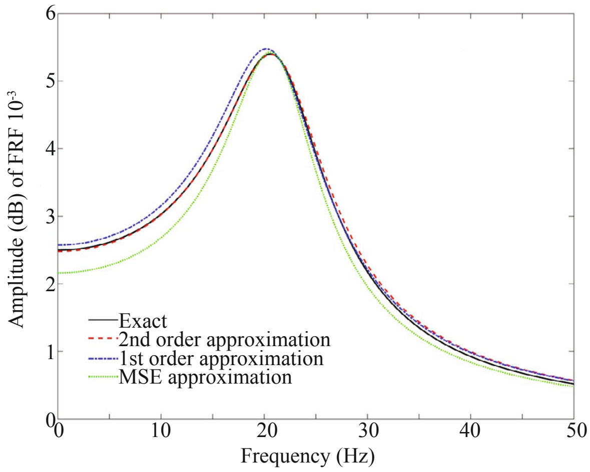

Fig. 5 FRF for case (ξ_0 = 0.05; $\omega_0\lambda$ = 2/5; β = 0.20; ω_0 = 20 rad/s)

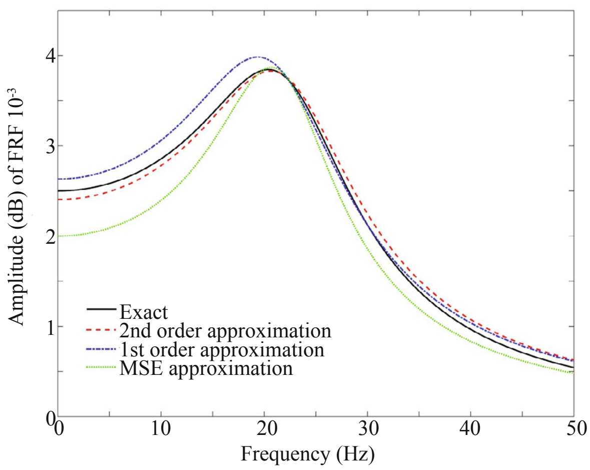

Fig. 6 FRF for case (ξ_0 = 0.05; $\omega_0\lambda$ = 2/5; β = 0.30; ω_0 = 20 rad/s)

As expected, the second-order approximation is more accurate than the first-order approximation. Moreover, the first- and second-order approximations are significantly better than the MSE approximation. It can also be observed from Fig. 2 to Fig. 6 that all approximations are more accurate for the smaller parameters $\omega_0\lambda$ or β.

Percentage errors in the root mean square of the structural displacement obtained using the first- and second-order approximate expression of Eq. (91) and MSE approximate expression of Eq. (96) are shown in Figs. 7-11.

Figures 12-14 show the exact and approximate nonstationary variances of the structural displacement under amplitude uniformly modulated nonstationary Gaussian white noise $\ddot{x}_g(t)$ with $A(t)$ = unit step function and the constant spectral density $S_0 = 0.0306\ \mathrm{m}^2/\mathrm{s}^3$.

Figures 15-17 also show the corresponding nonstationary results with respect to nonstationary white noise $\ddot{x}_g(t)$ with $A(t) = \mathrm{e}^{-0.6t}\text{-}\mathrm{e}^{-t}$ and $S_0 = 0.0306\ \mathrm{m}^2/\mathrm{s}^3$.

Apparently, the second-order approximation is more accurate than the first-order approximation, and the errors in the first- and second-order approximation are significantly smaller than the error in the MSE approximation, although the errors increase as the

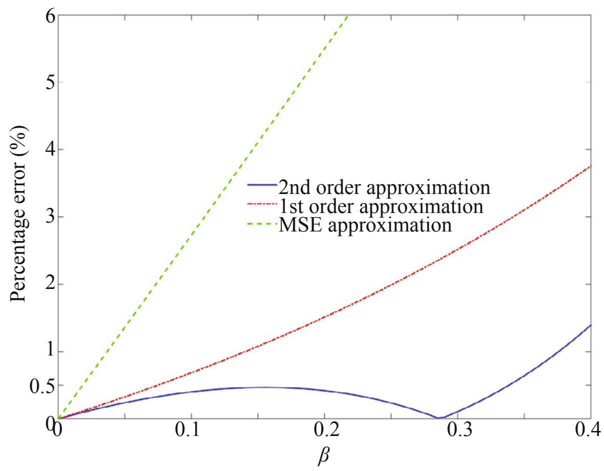

Fig. 7 Percentage error in the root mean square of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; ω_0 = 30 rad/s)

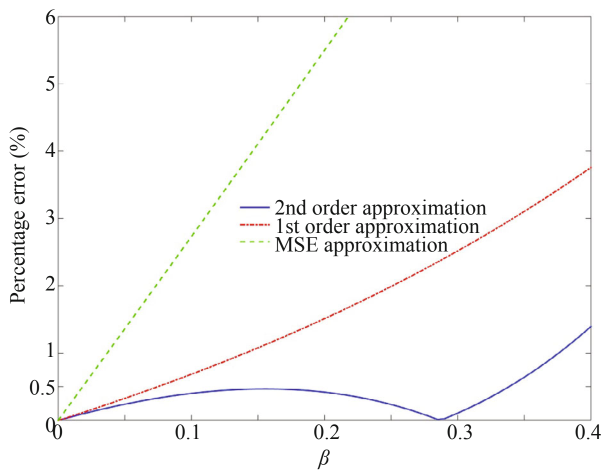

Fig. 8 Percentage error in the root mean square of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; ω_0 = 20 rad/s)

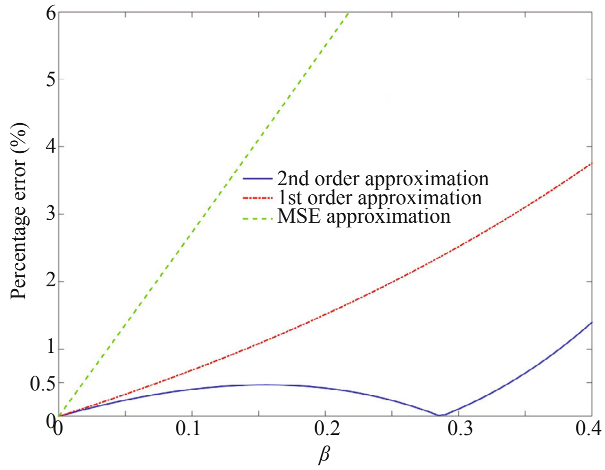

Fig. 9 Percentage error in the root mean square of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; ω_0 = 10 rad/s)

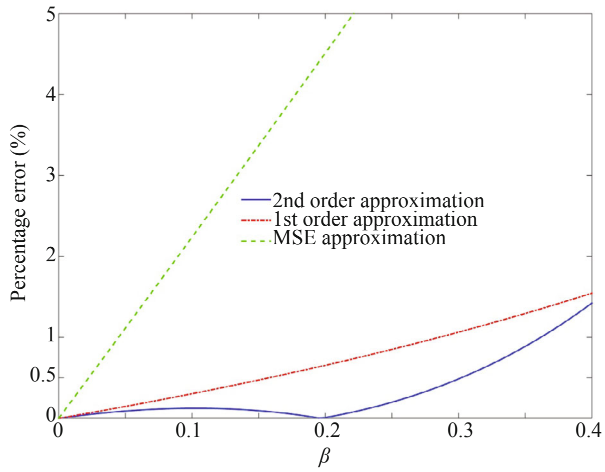

Fig. 10 Percentage error in the root mean square of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/4; ω_0 = 20 rad/s)

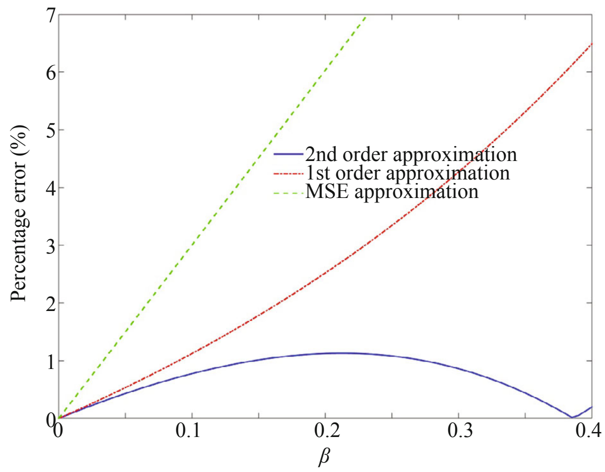

Fig. 11 Percentage error in the root mean square of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 2/5; ω_0 = 20 rad/s)

parameter $\omega_0\lambda$ *increases*. The error in the second-order approximation is less than 0.5%, corresponding to the range of parameter: ξ_0 = 0.05; $\omega_0\lambda \le 1/3$; $\beta \le 0.3$;

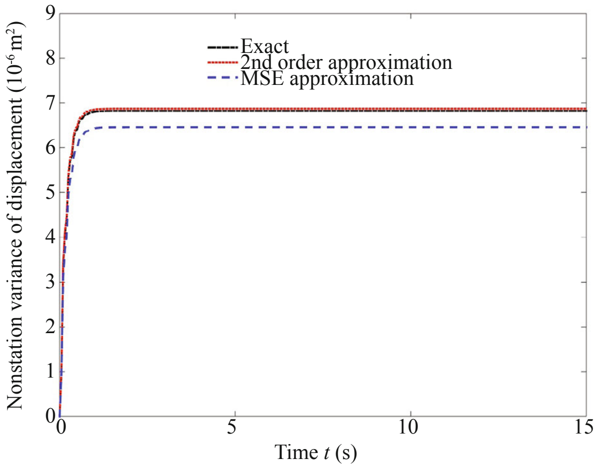

Fig. 12 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.1, ω_0 = 20 rad/s; $A(t)$ = unit step function)

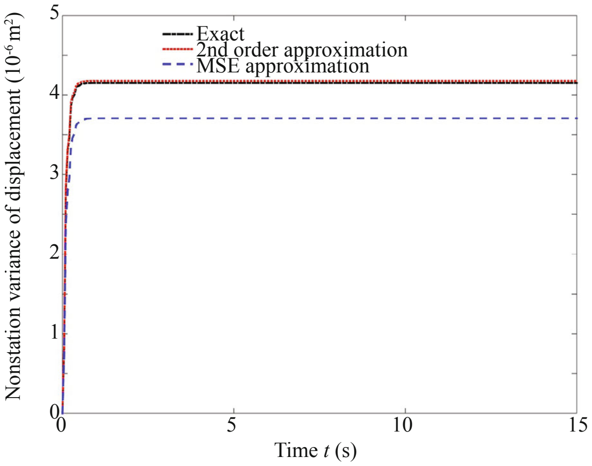

Fig. 13 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.2, ω_0 = 20 rad/s; $A(t)$ = unit step function)

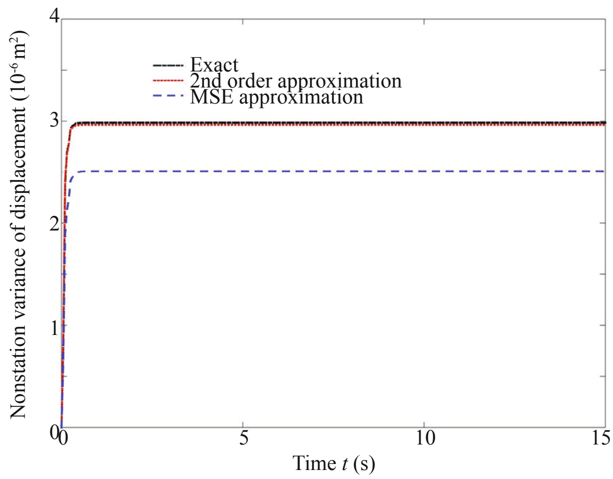

Fig. 14 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.3, ω_0 = 20 rad/s; $A(t)$ = unit step function)

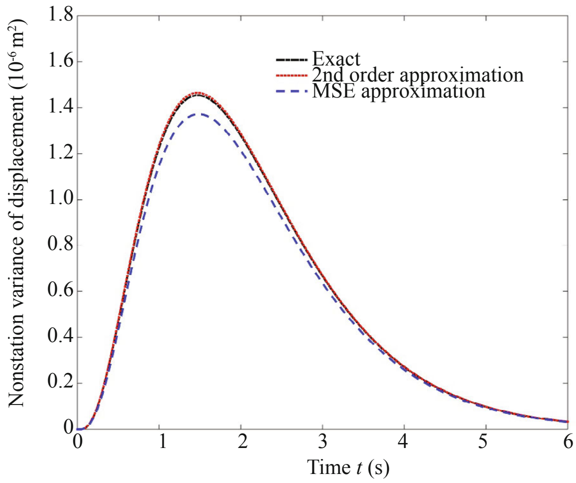

Fig. 15 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.1; ω_0 = 20 rad/s; $A(t) = e^{-0.6t} - e^{-t}$)

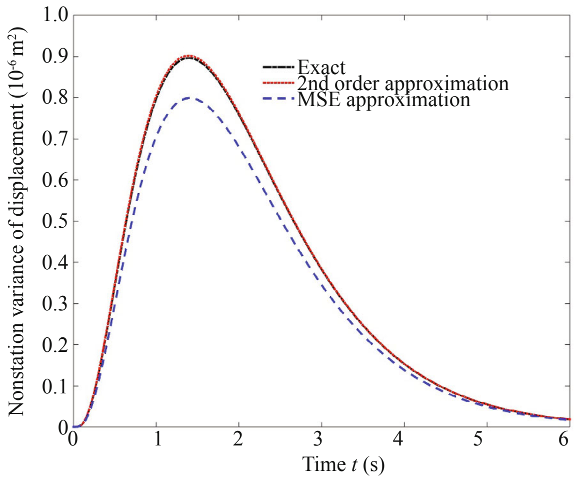

Fig. 16 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.2; ω_0 = 20 rad/s; $A(t) = e^{-0.6t} - e^{-t}$)

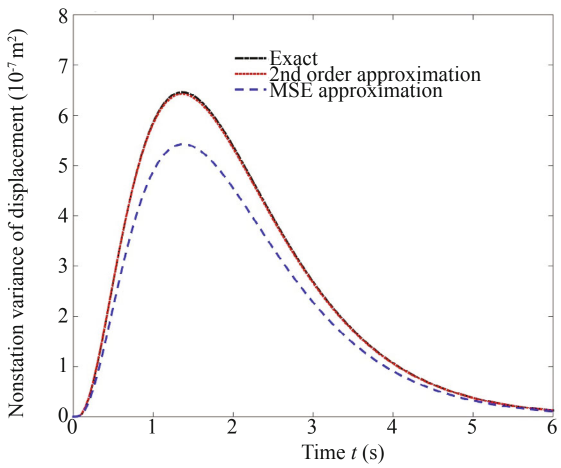

Fig. 17 Nonstationary variance of displacement for case (ξ_0 = 0.05; $\omega_0\lambda$ = 1/3; β = 0.3; ω_0 = 20 rad/s; $A(t) = e^{-0.6t} - e^{-t}$)

$10 \le \omega_0 \le 30$ rad/s. These results demonstrate that the proposed methods are of higher accuracy.

7 Conclusions

To predict the maximum earthquake response of an SDOF structure with a Maxwell fluid damper or supplemental brace-viscous damper system using the seismic design response spectrum technique, a new approach with higher accuracy is presented to (1) determine the first- and second-order equivalent damping and stiffness of the above structure in closed form using the multiple scales method; and (2) predict the peak responses of the above structure as well as the damper force. The following are a summary and conclusions.

(1) Based on the fact that the dynamic characteristics of a general linear viscoelastically damped structure are fully determined by its free vibration properties, using the equivalent criterion where free vibration properties of the structure are same, the equivalent problems of forced vibration of a general linear viscoelastically damped structure subjected to arbitrary exterior loadings can be strictly simplified as the equivalent problems of free vibration of a corresponding structure, which can provide the passable analytic approach to high-order equivalent damping and stiffness of a linear viscoelastically damped structure.

(2) The second-order, and even higher-order equivalent damping and stiffness of an SDOF structure with a Maxwell fluid damper or brace-viscous damper system can be obtained using the method of improved multiple time scales.

(3) The structural analytic Eqs. (7) and (8) are superior to structural Eqs. (3) and (4) due to the relaxation time constant of the damper system $\omega_0\lambda \le 1/3$ in engineering practice.

(4) The accuracy of the proposed methods is higher and significantly better than that of the modal strain energy method. In the parametric range of the requirements of the Chinese "Code for Seismic Design of Buildings", i.e., $\omega_0\lambda \le 1/3$, and supplemental damping ratio being less than 25%, the error of the proposed second-order equivalent system for a common SDOF engineering structure with a Maxwell fluid damper or brace-viscous damper system is less than 0.5%.

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