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Torsional vibration of a pipe pile in transversely isotropic saturated soil

Zheng Changjie[1, 2†], Hua Jianmin[1, 2‡] and Ding Xuanming[1, 2‡]

1. *College of Civil Engineering, Chongqing University, Chongqing 400045, China*

2. *Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing 400045, China*

Abstract: This study considers the torsional vibration of a pipe pile in a transversely isotropic saturated soil layer. Based on Biot's poroelastic theory and the constitutive relations of the transversely isotropic medium, the dynamic governing equations of the outer and inner transversely isotropic saturated soil layers are derived. The Laplace transform is used to solve the governing equations of the outer and inner soil layers. The dynamic torsional response of the pipe pile in the frequency domain is derived utilizing 1D elastic theory and the continuous conditions at the interfaces between the pipe pile and the soils. The time domain solution is obtained by Fourier inverse transform. A parametric study is conducted to demonstrate the influence of the anisotropies of the outer and inner soil on the torsional dynamic response of the pipe pile.

Keywords: torsional vibration; saturated soil; pipe pile; transversely isotropic medium

1 Introduction

The study of torsional vibrations of piles has attracted the attention of many researchers in the past several decades. Novak and Howell (1977,1978) proposed a plane strain model to investigate the dynamic torsional response of piles. Based on the plane strain model, Militano and Rajapakse (1999) studied the transient torsional and axial responses of a pile in multi-layered soil. Note that the stress gradient of the soil in the vertical direction was neglected in the plane strain model. Hu and Zhang (2007) assumed the soil as a viscoelastic layer and deduced a closed-form solution for the torsional dynamic response of the pile by considering the stress gradient of the soil in the vertical direction. Wu *et al*. (2016) adopted the fictitious soil pile model to analyze the torsional dynamic response of a pile embedded in layered soil. Pak and Abedzadeh (1992) studied the torsional traction of a cylindrical cavity in a half-space. Then Pak and Abedzadeh (1996) extended this scheme to the case of a rigid disk at the bottom of the cavity. Wang *et al.* (2008) studied the steady state torsional response of a pile in a saturated soil layer within the framework of the poroelastic theory originally presented by Biot (1956).

A common assumption in all of the above-mentioned studies is that the soil is an isotropic medium. However, the soil is often anisotropic due to the deposition history so that the properties in the horizontal and vertical directions are different. Liu and Novak (1994) investigated the dynamic response of single piles in transversely isotropic layered soil using the finite element method. Chen *et al.* (2008) investigated the transient torsional dynamic response of a pile embedded in transversely isotropic saturated soil based on the plane strain assumption. Wang *et al.* (2009) studied the torsional time-harmonic vibration of an end-bearing pile in a transversely isotropic saturated soil layer. Eskandari-Ghadi *et al*. (2012) studied the torsion vibration of a finite cylindrical cavity in a transversely isotropic half-space. Ardeshir-Behrestaghi (2014) extended it to the case of a bottom rigid disk. Wu *et al*. (2014) studied the vertical response of pile embedded in layered transversely isotropic soil.

The majority of the past studies focused on solid piles. In engineering practice, pipe piles are also widely used (Xu *et al.*, 2006; Liu *et al.*, 2007; Liu *et al.*, 2009; Ding *et al*., 2015). The action of pipe piles under dynamic loading is different from that of the solid piles because of the existence of the inner soil. Zheng *et al.* (2014) derived the frequency-domain solution of the torsional dynamic response of a pipe pile in isotropic saturated soil, in which the anisotropy of the soil was not

Correspondence to: Hua Jianmin, College of Civil Engineering, Chongqing University, Chongqing 400045, China
Tel: +86-13813953213
E-mail: hjm191@163.com
†Lecturer; ‡Associate Professor

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considered. The objective of this study is to extend the work of Zheng *et al.* (2014) by developing closed-form frequency domain solutions for the torsional vibration of a pipe pile in transversely isotropic saturated soil and semi-analytical solutions in the time domain. It is found that the anisotropy of the soil has a significant influence on the torsional vibration of the pipe pile. The solutions derived in this study can provide a theoretical guideline for dynamic pipe pile-soil interaction in a transversely isotropic medium.

2 Basic assumptions and computational model

The basic assumptions adopted here are as follows:

(1) The pile is elastic, and the pile bottom is fixed to a rigid base.

(2) The outer and inner soil are transversely isotropic saturated layers.

(3) The top surfaces of the soil layers are free and the bottoms are fixed.

(4) Only the circumferential displacements of the pile and soils are taken into account.

(5) The pipe pile is perfectly bonded to the soils, and there is no slip at the interfaces between the pile and soils.

(6) The initial conditions of the pile-soil system are static.

Figure 1 shows the computational model of the pile-soil system. A dynamic torque $m(t)$ is applied on the top of the pile. H is the length of the pile. r_1 and r_2 are the outer and inner radii of the pipe pile, respectively. $f_1(z,t)$ and $f_2(z,t)$ are the circumferential resistances developed at the outer and inner interfaces, respectively.

3 Governing equations

3.1 Governing equation of the soil

The governing differential equations for saturated soil in an axisymmetric system, in the absence of the vertical and radial displacements of the soil, can be

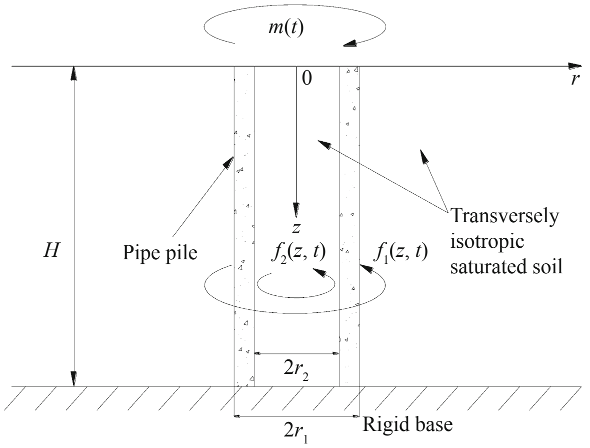

Fig. 1 Computational model

expressed as (Biot, 1956):

$$\frac{\partial \sigma_{r\theta}}{\partial r}+2\frac{\sigma_{r\theta}}{r}+\frac{\partial \sigma_{\theta z}}{\partial z}=\rho\frac{\partial^2 u_\theta}{\partial t^2}+\rho_{\mathrm{f}}\frac{\partial^2 w_\theta}{\partial t^2} \tag{1}$$

where $\sigma_{r\theta}$ and $\sigma_{\theta z}$ are the shear stress components of the soil; u_θ and w_θ are the circumferential displacements of the solid phase and fluid, respectively; and ρ and ρ_{f} are the densities of the bulk material and fluid, respectively.

The constitutive equations for a transversely isotropic medium can be written as (Lekhnitskii, 1963):

$$\sigma_{r\theta}=C_{66}\varepsilon_{r\theta} \tag{2}$$

$$\sigma_{\theta z}=C_{44}\varepsilon_{\theta z} \tag{3}$$

where C_{66} and C_{44} are the elastic constants; $\varepsilon_{r\theta}=\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}$, and $\varepsilon_{\theta z}=\frac{\partial u_\theta}{\partial z}$. In addition, $C_{44}=G_{\mathrm{v}}$ and $C_{66}=G_{\mathrm{h}}$, where G_{v} and G_{h} are the shear modulus of the soil in the vertical and horizontal planes, respectively.

Combining Eqs. (1)–(3), the governing equation of the transversely isotropic saturated soil can be expressed as:

$$G_{\mathrm{h}}(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2})u_\theta+G_{\mathrm{v}}\frac{\partial^2}{\partial z^2}u_\theta=\rho\frac{\partial^2}{\partial t^2}u_\theta+\rho_{\mathrm{f}}\frac{\partial^2}{\partial t^2}w_\theta \tag{4}$$

The governing equation of the fluid is (Biot, 1956):

$$\rho_{\mathrm{f}}\frac{\partial^2 u_\theta}{\partial t^2}+m\frac{\partial^2 w_\theta}{\partial t^2}+b\frac{\partial w_\theta}{\partial t}=0 \tag{5}$$

where b is a parameter accounting for the internal friction between the solid phase and fluid, and equal to the ratio between the fluid viscosity and the permeability of the soil. m is equal to the ratio between the fluid density and the porosity of the soil.

3.2 Governing equation of the pile

The pipe pile is assumed to be governed by the following one-dimensional wave equation:

$$G_{\mathrm{p}}J_{\mathrm{p}}\frac{\partial^2 \phi}{\partial z^2}-2\pi r_1^2 f_1-2\pi r_2^2 f_2=\rho_{\mathrm{p}}J_{\mathrm{p}}\frac{\partial^2 \phi}{\partial t^2} \tag{6}$$

where ϕ is the twist angle of the pipe pile; ρ_{p}, G_{p} and J_{p} denote the mass density, shear modulus and polar moment of inertia of the pipe pile, respectively; $J_{\mathrm{p}}=\frac{\pi}{2}(r_1^4-r_2^4)$.

4 Solitions for the governing equations

4.1 Solutions for the governing equations of the outer soil

The Laplace transform of a function $f(r,z,t)$ with respect to t is defined as:

$$F(z,r,s)=\int_0^{\infty} f(z,r,t)\mathrm{e}^{-st}\,\mathrm{d}t \tag{7}$$

Performing Laplace transform on Eqs. (4) and (5), the governing equations of the outer soil can be expressed as:

$$G_{\mathrm{h}1}(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2})U_{\theta 1}+G_{\mathrm{v}1}\frac{\partial^2}{\partial z^2}U_{\theta 1}=\rho_1 s^2 U_{\theta 1}+\rho_{\mathrm{f}} s^2 W_{\theta 1} \tag{8}$$

$$\rho_{\mathrm{f}} s^2 U_{\theta 1}+(m_1 s^2+b_1 s)W_{\theta 1}=0 \tag{9}$$

where the Subscript 1 means the variables and parameters corresponding to the outer soil; $U_{\theta 1}(z,r,s)$ and $W_{\theta 1}(z,r,s)$ are the Laplace transforms of $u_{\theta 1}(z,r,t)$ and $w_{\theta 1}(z,r,t)$.

Substituting Eq. (8) into Eq. (9) yields:

$$\delta_1(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2})U_{\theta 1}+\frac{\partial^2}{\partial z^2}U_{\theta 1}=\frac{s^2}{G_{\mathrm{v}1}}(\rho_1-\frac{\rho_{\mathrm{f}} s}{b_1+m_1 s})U_{\theta 1} \tag{10}$$

where $\delta_1=G_{\mathrm{h}1}/G_{\mathrm{v}1}$.

Using the method of separation of variables, $U_{\theta 1}$ can be easily obtained as:

$$U_{\theta 1}=\left[A_1 K_1(q_1 r)+B_1 I_1(q_1 r)\right]\left[C_1\cos(\alpha_1 z)+D_1\sin(\alpha_1 z)\right] \tag{11}$$

where $I_1(\ )$ and $K_1(\ )$ are the modified Bessel functions of the first and second kind of the first order, respectively; and α_1 and q_1 satisfy the following relationship:

$$q_1^2=\frac{s^2}{G_{\mathrm{v}1}\delta_1}(\rho_1-\frac{\rho_{\mathrm{f}} s}{b_1+m_1 s})+\frac{\alpha_1^2}{\delta_1} \tag{12}$$

The boundary conditions of the outer soil can be expressed as follows:

The surface of the outer soil layer is free, so the boundary conditions at $z=0$ can be expressed as:

$$\frac{\partial u_{\theta 1}}{\partial z}\Big|_{z=0}=\frac{\partial w_{\theta 1}}{\partial z}\Big|_{z=0}=0 \tag{13}$$

The bottom of the outer soil layer is fixed, so the boundary conditions at $z=H$ can be written as:

$$u_{\theta 1}\big|_{z=H}=w_{\theta 1}\big|_{z=H}=0 \tag{14}$$

The displacements of the outer soil at an infinite distance from the pipe pile are zero, so the boundary conditions at $r\to\infty$ can be expressed as:

$$u_{\theta 1}\big|_{r=\infty}=w_{\theta 1}\big|_{r=\infty}=0 \tag{15}$$

Substituting Eq. (11) into Eqs. (13)−(15) yields:

$$B_1=D_1=0 \tag{16}$$

$$\alpha_{1n}=\frac{(2n-1)\pi}{2H},\ n=1,2,3\cdots \tag{17}$$

Thus the circumferential displacement of the outer soil is obtained as:

$$U_{\theta 1}=\sum_{n=1}^{\infty}A_n K_1(q_{1n} r)\cos(\alpha_{1n} z) \tag{18}$$

The circumferential frictional resistance at the outer interface, which is equal to the circumferential shear stress of the outer soil at the outer interface, can be obtained as:

$$F_1=\delta_1 G_{\mathrm{v}1}\sum_{n=1}^{\infty}A_n q_{1n} K_2(q_{1n} r_1)\cos(\alpha_{1n} z) \tag{19}$$

where $F_1(z,s)$ is the Laplace transform of $f_1(z,t)$.

4.2 Solutions for the governing equations of the inner soil

A similar solving procedure of the outer soil can be followed for the inner soil. The general solution for the governing equations of the inner soil can be obtained as:

$$U_{\theta 2}=\left[A_2 K_1(q_2 r)+B_2 I_1(q_2 r)\right]\left[C_2\cos(\alpha_2 z)+D_2\sin(\alpha_2 z)\right] \tag{20}$$

where $q_2^2=\frac{s^2}{G_{\mathrm{v}2}\delta_2}(\rho_2-\frac{\rho_{\mathrm{f}}^2 s}{b_2+m_2 s})+\frac{\alpha_2^2}{\delta_2}$; $\delta_2=G_{\mathrm{h}2}/G_{\mathrm{v}2}$; $U_{\theta 2}(z,r,s)$ is the Laplace transform of $u_{\theta 2}(z,r,t)$; the Subscript 2 indicates that the variables and parameters correspond to the inner soil.

The boundary conditions for the inner soil can be expressed as:

The surface of the inner soil layer is free, so the boundary conditions at $z=0$ can be expressed as:

$$\frac{\partial u_{\theta 2}}{\partial z}\Big|_{z=0}=\frac{\partial w_{\theta 2}}{\partial z}\Big|_{z=0}=0 \tag{21}$$

The bottom of the inner soil layer is fixed, so the boundary conditions at $z=H$ can be written as:

$$u_{\theta 2}\big|_{z=H}=w_{\theta 2}\big|_{z=H}=0 \tag{22}$$

The displacements of the inner soil at the center are bounded, so the boundary conditions at $r=0$ can be expressed as:

$$u_{\theta 2}\big|_{r=0}<\infty,\ w_{\theta 2}\big|_{r=0}<\infty \tag{23}$$

Substituting Eq. (20) into Eqs. (21)−(23), one obtains:

$$A_2=D_2=0 \tag{24}$$

$$\alpha_{2n} = \frac{(2n-1)\pi}{2H} \tag{25}$$

The circumferential displacement of the inner soil is obtained as:

$$U_{\theta 2} = \sum_{n=1}^{\infty} B_n I_1(q_{2n} r)\cos(\alpha_{2n} z) \tag{26}$$

The circumferential shear frictional resistance at the inner interface can be obtained as:

$$F_2 = \delta_2 G_{v2} \sum_{n=1}^{\infty} B_n q_{2n} I_2(q_{2n} r_2)\cos(\alpha_{2n} z) \tag{27}$$

where $F_2(z,s)$ is the Laplace transform of $f_2(z,t)$.

4.3 Solution for the governing equation of the pile

Performing Laplace transform on Eq. (6) and substituting Eqs. (19) and (27) into it, the governing equation of the pipe pile is expressed as:

$$\frac{\partial^2 \Phi}{\partial z^2} - \frac{s^2}{C_p^2}\Phi = \frac{2\pi r_1^2}{G_p J_p} G_{h1} \sum_{n=1}^{\infty} A_n q_{1n} K_2(q_{1n} r_1)\cos(\alpha_{1n} z) + \frac{2\pi r_2^2}{G_p J_p} G_{h2} \sum_{n=1}^{\infty} B_n q_{2n} I_2(q_{2n} r_2)\cos(\alpha_{2n} z) \tag{28}$$

where $\Phi(z,s)$ is the Laplace transform of $\phi(z,t)$.

The solution for Eq. (28) can be easily obtained as:

$$\Phi = E_1 \cos(\frac{\omega}{C_p} z) + E_2 \sin(\frac{\omega}{C_p} z) + \sum_{n=1}^{\infty} \beta_{1n} A_n \cos(\alpha_{1n} z) + \sum_{n=1}^{\infty} \beta_{2n} B_n \cos(\alpha_{2n} z) \tag{29}$$

where $\omega = s/\mathrm{i}$; $C_p = G_p / \rho_p$;

$$\beta_{1n} = -\frac{2\pi r_1^2 \delta_1 G_{v1} q_{1n} K_2(q_{1n} r_1)}{\rho_p J_p (C_p^2 \alpha_{1n}^2 + s^2)};$$

$$\beta_{2n} = -\frac{2\pi r_2^2 \delta_2 G_{v2} q_{2n} I_2(q_{2n} r_2)}{\rho_p J_p (C_p^2 \alpha_{2n}^2 + s^2)}.$$

As mentioned earlier, the pile bottom is fixed and the pile top is subjected to a dynamic torque. Therefore the boundary conditions at the top and bottom of the pile are:

$$\frac{\partial \Phi}{\partial z}\Big|_{z=0} = -\frac{M(s)}{G_p J_p} \tag{30}$$

$$\Phi\big|_{z=H} = 0 \tag{31}$$

where $M(s)$ is the Laplace transform of $m(t)$.

The displacements at the interfaces of the pile and the soils are continuous:

$$U_{\theta 1}\big|_{r=r1} = r_1 \Phi \tag{32}$$

$$U_{\theta 2}\big|_{r=r2} = r_2 \Phi \tag{33}$$

Substituting Eqs. (18), (26) and (29) into Eqs. (32) and (33) yields:

$$\sum_{n=1}^{\infty} A_n K_1(q_{1n} r_1)\cos(\alpha_{1n} z) = r_1[E_1 \cos(\frac{\omega}{C_p} z) + E_2 \sin(\frac{\omega}{C_p} z) + \sum_{n=1}^{\infty} \beta_{1n} A_n \cos(\alpha_{1n} z) + \sum_{n=1}^{\infty} \beta_{2n} B_n \cos(\alpha_{2n} z)] \tag{34}$$

$$\sum_{n=1}^{\infty} B_n I_1(q_{2n} r_2)\cos(\alpha_{2n} z) = r_2[E_1 \cos(\frac{\omega}{C_p} z) + E_2 \sin(\frac{\omega}{C_p} z) + \sum_{n=1}^{\infty} \beta_{1n} A_n \cos(\alpha_{1n} z) + \sum_{n=1}^{\infty} \beta_{2n} B_n \cos(\alpha_{2n} z)] \tag{35}$$

Equations. (34) and (35) can also be expressed as:

$$r_2 \sum_{n=1}^{\infty} A_n K_1(q_{1n} r_1)\cos(\alpha_{1n} z) = r_1 \sum_{n=1}^{\infty} B_n I_1(q_{2n} r_2)\cos(\alpha_{2n} z) \tag{36}$$

It is found from Eqs. (17) and (25) that $\alpha_{1n} = \alpha_{2n}$. Given $\alpha_n = \alpha_{1n} = \alpha_{2n}$, it is obtained from Eq. (36) that:

$$B_n = \frac{r_2 K_1(q_{1n} r_1)}{r_1 I_1(q_{2n} r_2)} A_n \tag{37}$$

Multiplying $\cos(\alpha_n z)$ on both sides of Eq. (34) and integrating over the interval $z = [0, H]$, one obtains:

$$A_n = r_1 I_1(q_{2n} r_2)(\xi_{1n} E_1 + \xi_{2n} E_2) \tag{38}$$

where

$$\xi_{1n} = \frac{2\int_0^H \cos(\frac{\omega}{C_p} z)\cos(\alpha_n z)\mathrm{d}z}{H\left[K_1(q_{1n} r_1) I_1(q_{2n} r_2) - \beta_{1n} r_1 I_1(q_{2n} r_2) - \beta_{2n} r_2 K_1(q_{1n} r_1)\right]} \tag{39}$$

$$\xi_{2n} = \frac{2\int_0^H \sin(\frac{\omega}{C_p} z)\cos(\alpha_n z)\mathrm{d}z}{H\left[K_1(q_{1n} r_1) I_1(q_{2n} r_2) - \beta_{1n} r_1 I_1(q_{2n} r_2) - \beta_{2n} r_2 K_1(q_{1n} r_1)\right]} \tag{40}$$

Substituting Eq. (38) into Eq. (37), one can obtain:

$$B_n = r_2 K_1(q_{1n} r_1)(\xi_{1n} E_1 + \xi_{2n} E_2) \tag{41}$$

Substituting Eq. (29) into Eqs. (30) and (31) yields:

$$E_1 = \frac{C_p m(s)}{\omega G_p J_p}\tan(\frac{\omega}{C_p} H) \tag{42}$$

$$E_2 = -\frac{C_p m(s)}{\omega G_p J_p} \tag{43}$$

All of the coefficients have been determined. The twist angle of the pipe pile can be written as:

$$\varPhi = \frac{C_{\mathrm{p}} M(s)}{\omega G_{\mathrm{p}} J_{\mathrm{p}}} \left\{ \begin{aligned} & \tan\left(\frac{\omega}{C_{\mathrm{p}}} H\right) \cos\left(\frac{\omega}{C_{\mathrm{p}}} z\right) - \sin\left(\frac{\omega}{C_{\mathrm{p}}} z\right) \\ & + \sum_{n=1}^{\infty} \left[\beta_{1n} r_1 I_1(q_{2n} r_2) + \beta_{2n} r_2 K_1(q_{1n} r_1) \right] \left[\xi_{1n} \tan\left(\frac{\omega}{C_{\mathrm{p}}} H\right) - \xi_{2n} \right] \cos(\alpha_n z) \end{aligned} \right\} \tag{44}$$

The torsional complex impedance of the pile top is defined by:

$$K_{\mathrm{d}} = \frac{M(\mathrm{i}\omega)}{\varPhi(0, \mathrm{i}\omega)} \tag{45}$$

The complex impedance can be further expressed in terms of its non-dimensional real and imaginary parts as:

$$K_{\mathrm{d}} = \frac{G_{\mathrm{p}} J_{\mathrm{p}}}{r_1} \left(k + \mathrm{i} c \right) \tag{46}$$

where the parameters k and c represent the non-dimensional real and imaginary parts of the torsional impedance, respectively.

The time-domain twist angle of the pile can be obtained by numerical inverse Fourier transform:

$$\phi(z,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi(z, \mathrm{i}\omega) \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}\omega \tag{47}$$

5 Numerical results and analysis

The fundamental torsional vibration characteristics of a pipe pile in saturated soil has been studied in Zheng *et al.* (2014). Thus, numerical results presented herein are mainly focused on the influence of the anisotropies of soils on the impedance of the pile in the frequency domain and twist angle in the time domain. The software MATLAB is utilized for the numerical procedure. In the frequency domain, the upper limit of n=20 is sufficient for the convergence. In the time domain, the upper limit of 10000 is sufficient for the convergence of the inverse Fourier transform. The following parameter values in the analysis are adopted: H = 5 − 40 m; r_1= 0.5 m; r_2= 0.38 m; G_{p}=10 GPa; G_{v1}= G_{v2}=10 MPa; ρ_{p}= 2500 kg/m^3; ρ_1= ρ_2= 2000 kg/m^3; ρ_{f}= 1000 kg/m^3; m_1= m_2= 2500 kg/m^3; b_1= b_2=2500 N·s/m^2; δ_1= δ_2= 0.5 − 2.

5.1 Verification of the proposed solution

The present solution is verified by comparison with the solutions for a solid pile in transversely isotropic saturated soil proposed by Wang *et al.* (2009) and a pipe pile in isotropic saturated soil by Zheng *et al.* (2014). Given $r_2 = 0$, the solution of this study is reduced to that of a solid pile in transversely isotropic saturated soil. Figure 2 shows the comparison between the present solution and Wang *et al.* (2009), where $a_0 = \omega r_1 \sqrt{\rho_1 / G_{\mathrm{v1}}}$ is the non-dimensional frequency. Excellent agreement is obtained between the two solutions. Given δ_1= δ_2= 1, the solution of this study is reduced to that of a pipe pile in isotropic saturated soil. Figure 3 shows that the reduced solution of this study matches well with Zheng *et al.* (2014) for the isotropic saturated soil case. The validity of the present solution is therefore confirmed by the above two comparisons.

5.2 Frequency domain torsional impedance of pile

Figures 4 and 5 show the influence of the anisotropy of the outer soil on the torsional impedance of the pipe pile for H = 5 m, 40 m, respectively. The stiffness decreases with frequency, while the damping increases with frequency. It can be seen that the stiffness increases steeply with the increase of δ_1. This indicates that increasing the horizontal shear modulus of the outer soil can significantly increase the torsional impedance of the pipe pile. The increment caused by increasing δ_1 for long piles (H = 40 m) is more pronounced than that for short piles (H = 5 m). This is because the pile-soil coupled vibration causes larger soil resistance to longer piles, and the soil resistance mainly depends on the horizontal shear modulus. The damping of short piles increases with increasing δ_1. On the contrary, the damping of long piles decreases with increasing δ_1.

Figures 6 and 7 show the influence of the anisotropy of the inner soil on the torsional impedance of the pipe pile for H = 5 m, 40 m, respectively. In the low-frequency

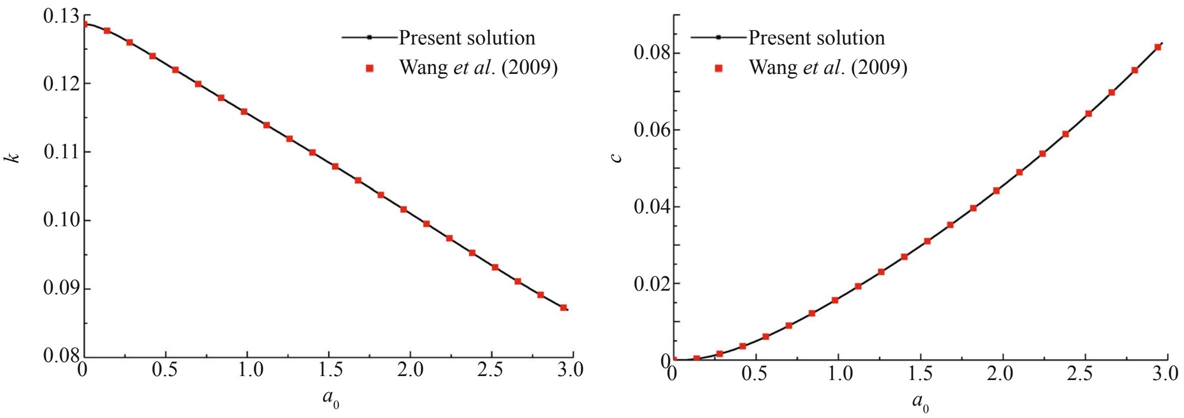

Fig. 2 Comparison with the solution for a solid pile in transversely isotropic saturated soil

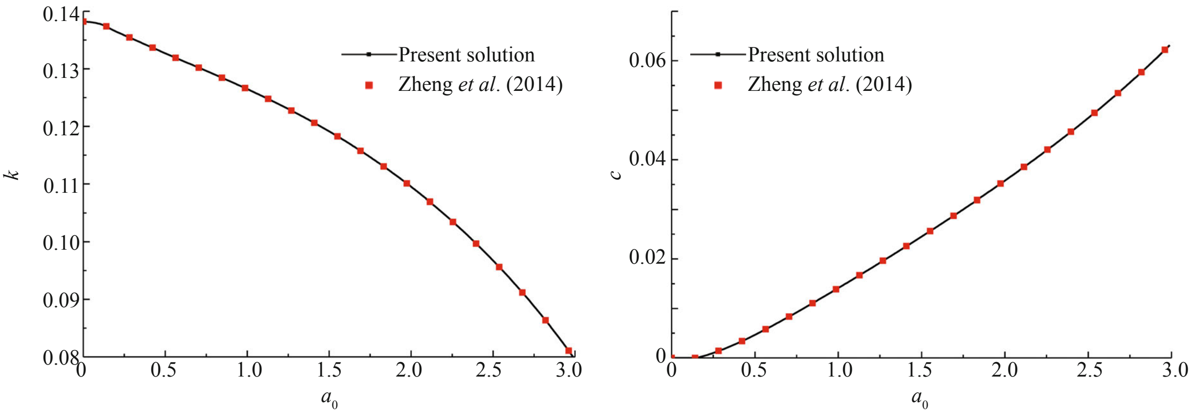

Fig. 3 Comparison with the solution for a pipe pile in isotropic saturated soil

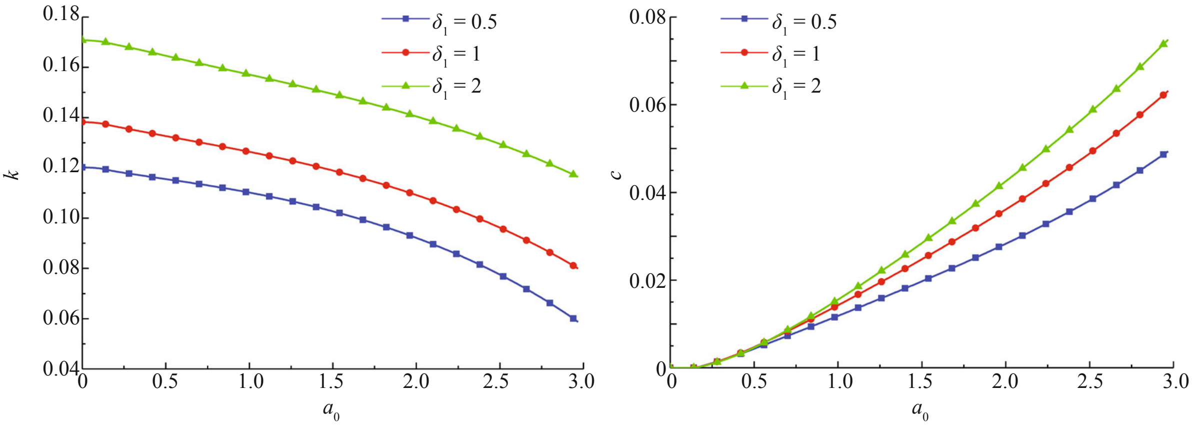

Fig. 4 Influence of the outer soil anisotropy on the torsional impedance of pile (H = 5 m)

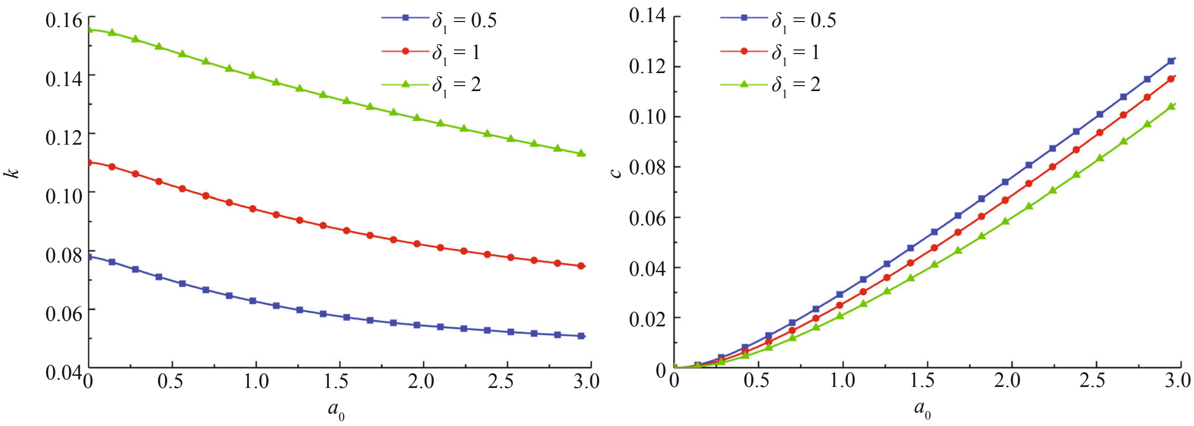

Fig. 5 Influence of the outer soil anisotropy on the torsional impedance of pile (H = 40 m)

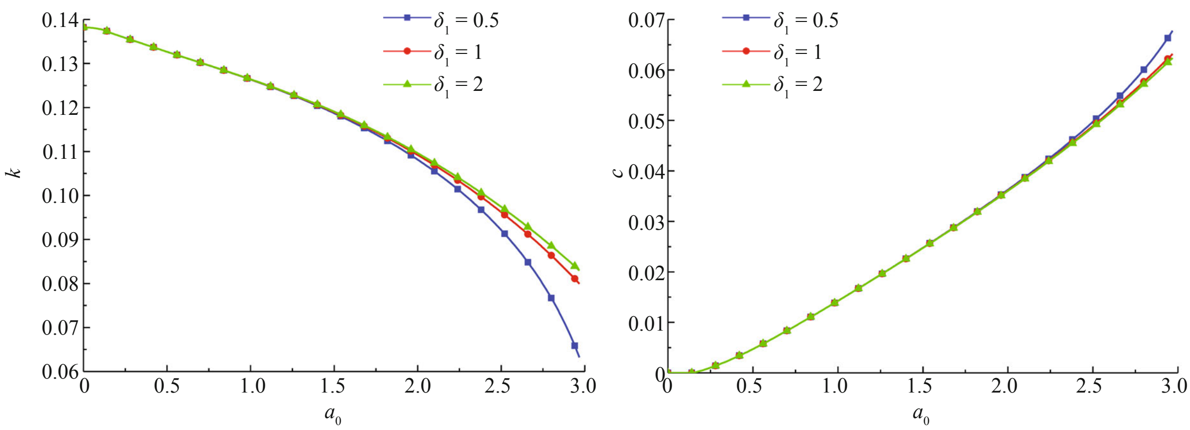

Fig. 6 Influence of the inner soil anisotropy on the torsional impedance of pile (H = 5 m)

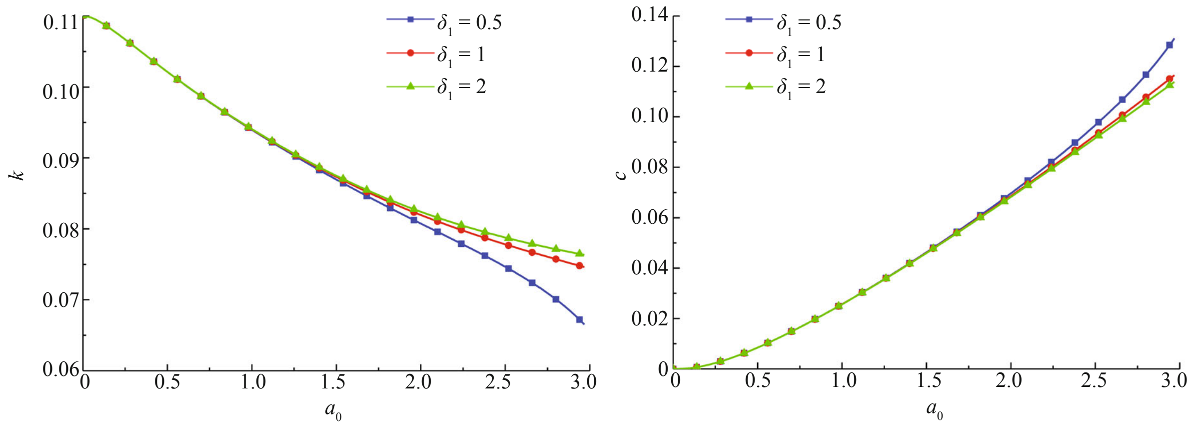

Fig. 7 Influence of the inner soil anisotropy on the torsional impedance of pile (H = 40 m)

range, the stiffness and damping show negligible change with the increase of δ_2. In higher frequency range, the stiffness increases and the damping decreases with the increase of δ_2. Furthermore, the stiffness and damping become less dependent on δ_2 with increasing δ_2. The influence of δ_2 doesn't show an obvious difference with the increase of pile length.

Figure 8 shows the influence of r_1 and r_2 on the complex impedance of the pile. With the increase of r_1 or decrease of r_2, the stiffness and damping increase. The area of the pile with r_1= 0.5 m and r_2= 0.2 m is approximately equal to that of the pile with r_1= 0.6 m and r_2= 0.38 m. It can be seen that the stiffness and damping of the pile with r_1= 0.6 m and r_2= 0.38 m is larger than that of the pile with r_1= 0.5 m and r_2= 0.2 m. With the same area, the stiffness and damping of the pile increase with the increase of the mean radius. This is due to the fact that the outerwall and inner wall of the pile increase and therefore the shear frictional resistances of soil increase with the increase of the mean radius. It is also noted that the curves of r_2= 0.2 m are close to that of the solid pile. It can be concluded that the action of the inner soil becomes negligible when $r_2/r_1 \leq 0.4$.

5.3 Time domain twist angle of pile

Figure 10 shows the influence of the anisotropy of the outer soil on the time histories of the non-dimensional twist angle of the pile head subjected to a triangular transient torque. The time history of the transient torsional load is shown in Fig. 9, where T_0 is the amplitude of the impulse load; $\tau = tC_p/H$ is the non-dimensional time. It can be seen from Fig. 10 that there is an obvious peak value in each curve which varies significantly with δ_1. With the increase of δ_1, the peak value of the twist angle of the pile head decreases, and the arrival time of the peak value also slightly decreases. It is also noted that δ_1 shows a more prominent effect on long piles (H = 40 m) than short piles (H = 5 m).

Figure 11 shows the influence of the anisotropy of the inner soil on the time histories of the non-dimensional twist angle of the pile head subjected to a triangular transient torque. For short piles (H = 5 m), the peak value of the twist angle of the pile head decreases slightly with the increase of δ_2, but the arrival time of the peak value shows negligible change. For long piles (H = 40 m), however, the anisotropy of the inner soil shows a negligible effect on the twist angle of the pile. This indicates that short piles are more dependent on δ_2 than long piles.

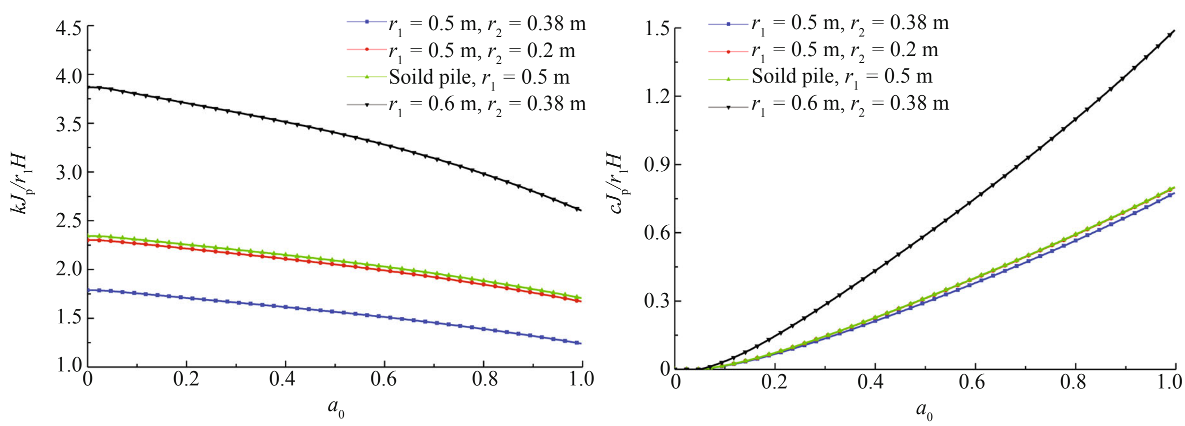

Fig. 8 Influence of the radii of pile on the torsional impedance of pile

6 Conclusions

The torsional vibration of a pipe pile in a transversely isotropic saturated soil layer has been investigated in this study. The frequency domain solution is derived by using the Laplace transform technique and the method of separation of variables. The time domain solution is obtained by the numerical inverse transform method. The solution proposed in this study is verified by comparison with some existing solutions. Some selected numerical results are presented to discuss the influence of the anisotropies of the outer and inner soil. The following conclusions are obtained:

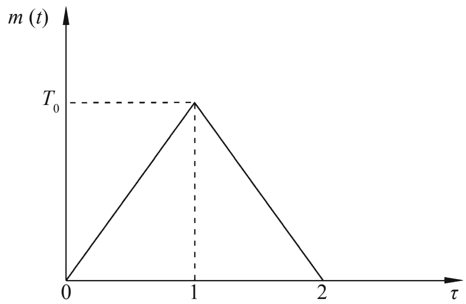

Fig. 9 Transient loading adopted in the analysis

(1) The stiffness greatly increases with increasing δ_1. The increment generated by increasing δ_1 for long piles is more pronounced than for short piles. The damping of short piles increases with increasing δ_1, but that of long piles decreases with increasing δ_1.

(2) In the low-frequency range, the stiffness and damping show negligible change with increasing δ_2. In the high-frequency range, the stiffness increases with increasing δ_2 and the damping decreases with increasing δ_2. The influence of δ_2 doesn't show an obvious change with the increase of the pile length.

(3) With the increase of r_1 or decrease of r_2, the impedance increases. With the same area, the stiffness and damping of the pile increase with the increase of the mean radius. The action of the inner soil becomes negligible when $r_2/r_1 \leq 0.4$.

(4) With the increase of δ_1, the twist angle of the pile in the time domain decreases, and the time of the peak value decreases. Long piles are more dependent on δ_1 than short piles.

(5) For short piles, with the increase of δ_2, the twist angle of the pile decreases, while the time of the

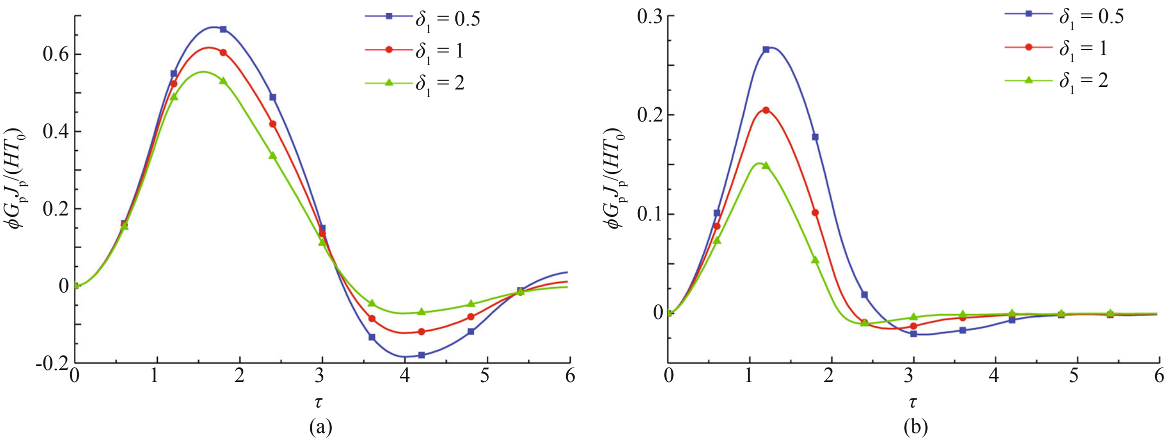

Fig. 10 Influence of the anisotropy of the outer soil on the non-dimensional twist angle of the pile: (a) *H* = 5 m; (b) *H* = 40 m

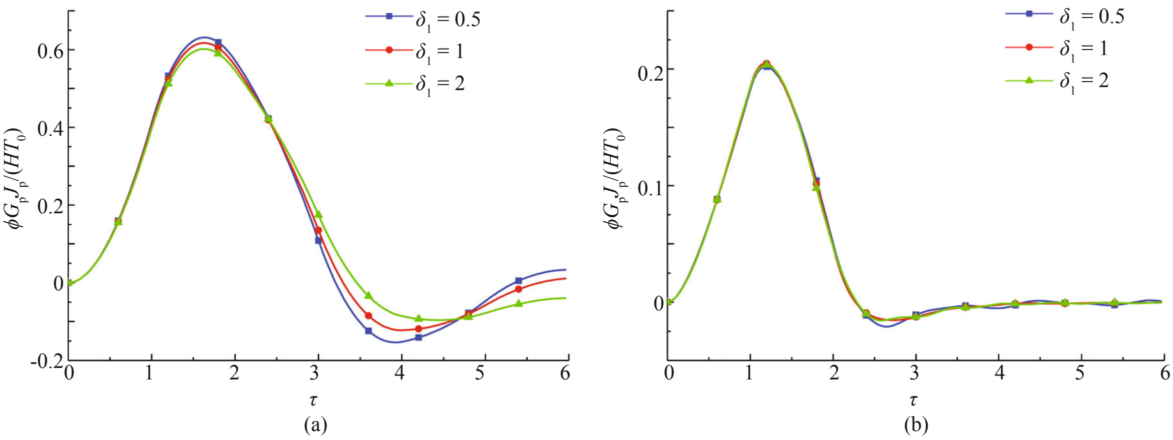

Fig. 11 Influence of the anisotropy of the inner soil on the non-dimensional twist angle of the pile: (a) *H* = 5 m; (b) *H* = 40 m

peak value shows little difference. For long piles, the anisotropy of the inner soil has a negligible effect on the twist angle.

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References

Ardeshir-Behrestaghi A, Eskandari-Ghadi M, Navayineya B and Vaseghi-Amiri J (2014), "Dynamic Reissner -sagoci Problem for a Transversely Isotropic Half-space Containing Finite Length Cylindrical Cavity," *Soil Dynamics and Earthquake Engineering*, **66**: 252–262.

Biot MA (1956) "Theory of Propagation of Elastic Waves in a Fluid-saturated Porous Solid. I: Low-frequency Range," *Journal of the Acoustical Society of America*, **28**(2): 168−178.

Chen G, Cai YQ, Liu FY and Sun HL (2008), "Dynamic Response of a Pile in a Transversely Isotropic Saturated Soil to Transient Torsional Loading," *Computers and Geotechnics*, **35**(2):165−172.

Ding XM, Liu HL, Chu J, *et al* (2015), "Time-domain Solution for Transient Dynamic Response of a Large-diameter Thin-walled Pipe Pile," *Earthquake Engineering & Engineering Vibration*, **14**(2): 239–251.

Eskandari-Ghadi M, Mahmoodian M, Pak RYS and Ardeshir-Behrestaghi A (2012), "Analytical Solution of Torsion Vibration of a Finite Cylindrical Cavity in a Transversely Isotropic Half-space," *ZAMM–Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, **92**(7): 583–595.

Hu CB and Zhang T (2007), "Soil-pile Interaction in Torsional Vibrations of a Pile in Viscious Damping Soil Layer," *Engeering Mechanics*, **24**(3):147−153. (in Chinese)

Lekhnitskii SG (1963), *Theory of Anisotropic Elastic Bodies*, Holden-Day, SanFrancisco, 1963.

Liu HL, Chu J and Deng A (2009), "Use of Large-diameter Cast–in Situ Concrete Pipe Piles for Embankment over Soft Clay," *Canadian Geotechnical Journal*, **46**(8): 915−927.

Liu HL, Ng CW, Fei K (2007), "Performance of a Geogrid-reinforced and Pile-supported Highway Embankment over Soft Clay: Case Study," *Journal of Geotechnical and Geoenvironmental Engineering*, **133**(12): 1483−1493.

Liu WM and Novak M (1994), "Dynamic Response of Single Piles Embedded in Transversely Isotropic Layered Media," *Earthquake Engineering and Structural Dynamics*, **23**(11): 1239−1257.

Militano G and Rajapakse RKND (1999), "Dynamic Response of a Pile in a Multi-layered Soil to Transient Torsional and Axial Loading," *Geotechnique*, **49**(1): 91−109.

Novak M and Howell F (1977), "Torsional Vibration of Pile Foundations," *Journal of Geotechnical Engineering Division*, ASCE, **103**: 271−285.

Novak M and Howell F (1978), "Dynamic Response of Pile Foundations in Torsion," *Journal of the Geotechnical Engineering Division*, ASCE, **104**(5): 535–552.

Pak RYS and Abedzadeh F (1992), "A Torsional Traction on an Open Finite Cylindrical Cavity, Proceedings of the Royal Society of London A: Mathematical," *Physical and Engineering Sciences*, **438**: 133–144.

Pak RYS and Abedzadeh F (1996), "A Torsional Contact Problem for an Indented Half-space," *Journal of Applied Mechanics*, **63**: 1–6.

Wang KH, Zhang ZQ, Chin JL and Xie KH (2008), "Dynamic Torsional Response of an End Bearing Pile in Saturated Poroelastic Medium," *Computers and Geotechnics*, **35**(3): 450–458.

Wang KH, Zhang ZQ, Chin JL and Xie KH (2009), "Dynamic Torsional Response of an End Bearing Pile in Transversely Isotropic Saturated Soil," *Journal of Sound and Vibration*, **327**(3): 440–453.

Wu WB, Liu H, M. El Naggar H, Mei GX and Jiang GS (2016), "Torsional Dynamic Response of a Pile Embedded in Layered Soil Based on the Fictitious Soil Pile Model," *Computers and Geotechnics*, **80**: 190–198.

Wu WB, Jiang GS, Huang SG and Leo CJ (2014), "Vertical Dynamic Response of Pile Embedded in Layered Transversely Isotropic Soil," *Mathematical Problems in Engineering*, Vol. 2014, Article ID 126916, 12 Pages, DOI: 10.1155/2014/126916.

Xu XT, Liu HL and Lehane BM (2006), "Pipe Pile Installation Effects in Soft Clay," *Geotechnical Engineering*, **159**(4): 285–296.

Zheng CJ, Liu HL, Ding XM and Lv YR (2014), "Torsional Dynamic Response of a Large-diameter Pipe Pile in Viscoelastic Saturated Soil," *International Journal for Numerical and Analytical Methods in Geomechanics*, **38**(16): 1724–1743.