

# Active control of highway bridges subject to a variety of earthquake loads

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**Abstract:** In this paper, a wavelet-filtered genetic-neuro-fuzzy (WGNF) control system design framework for response control of a highway bridge under various earthquake loads is discussed. The WGNF controller is developed by combining fuzzy logic, discrete wavelet transform, genetic algorithms, and neural networks for use as a control algorithm. To evaluate the performance of the WGNF algorithm, it is tested on a highway bridge equipped with hydraulic actuators. It controls the actuators installed on the abutments of the highway bridge structure. Various earthquakes used as input signals include an artificial earthquake, the El-Centro, Kobe, North Palm Springs, Turkey Bolu, Chi-Chi, and Northridge earthquakes. It is proved that the WGNF control system is effective in mitigating the vibration of the highway bridge under a variety of seismic excitation.

**Keywords:** structural control; wavelet transform; genetic algorithm; fuzzy logic; neural network; active control; nonlinear highway bridges

## 1 Introduction

With the continued deterioration of infrastructure in the United States, the need for healthy structures able to maintain its strength and serviceability throughout the length of their design life has become very important in structural engineering (FEMA, 2008; Kim *et al.*, 2014c). These events can vary greatly over time, creating a dynamic loading that cause large, time-varying displacements, velocities, and accelerations on the structure, which can impact the health of structures (Kim *et al.*, 2013a; Chong *et al.*, 2014). Due to large structural responses to the destructive forces, cracks can be created or increased and the overall and local strength also can be degraded, eventually leading to structural damage or collapse (Arsava *et al.*, 2014). Hence control systems can often be employed as a part of a structure in order to help the bridge or building act against lateral forces such as strong wind and earthquake events (Spencer and

Nagarajaiah, 2003).

Control systems are becoming increasingly researched and used on civil engineering structures to decrease and limit the responses of buildings or bridges during seismic events (Yao, 1972; Housner *et al.*, 1997). Control systems utilize devices that apply a force to a structure that offsets internal forces, displacements, and accelerations that are created during seismic events (Spencer *et al.*, 1999). Two common forms of control systems are passive and active control (Soong, 1990; Soong and Reinhorn, 1993; Cha *et al.*, 2012, 2013). Typical passive control systems are viscous liquid dampers and base isolators. These devices are designed and installed on a structure during construction that implement a non-adaptable control force during dynamic loading events (Soong and Dargush, 1997). Because they are installed during construction, it is difficult and sometimes impossible to modify the device during the lifetime of a structure (Soong and Dargush, 1997). The actuators used for active control systems consume a significant amount of power. The control signal is calculated using active control algorithms based on measured structural responses. The active control algorithms can determine control forces in near-real-time, depending on the required control force due to applied loadings on the structure (Spencer *et al.*, 1999; Kim *et al.*, 2010a, b, c; Cha and Agrawal, 2013). Therefore, it is important to develop an effective control model to implement the active controller for large civil structures. With this in mind, a new structural control strategy is proposed by integrating wavelet transform

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(WT), genetic algorithm (GA) based decentralized output feedback polynomial controls (DOFPC), and fuzzy logic theory and neural network (NN).

The first model used as a part of the integrated system is a fuzzy logic model. The fuzzy logic model has a main advantage of being used as a nonparametric method for system identification, and has been previously researched (Zadeh, 1965; Kim *et al.*, 2009a; Kim *et al.*, 2011; Mitchell *et al.*, 2012b; Kim *et al.*, 2015; Mohammadzadeh *et al.*, 2015), as well as general studies on the uncertainties and complexities of the dynamic system (Langari, 1999; Kim *et al.*, 2009b; Arsava *et al.*, 2013; Arsava *et al.*, 2015; Arsava and Kim, 2015). Using a Takagi-Sugeno (TS) model for fuzzy logic theory allows for a representation of complex nonlinear dynamic systems using fuzzy rules and linear system theory (Takagi and Sugeno, 1985; Yager and Filey, 1993; Johansen, 1994; Faravelli and Yao, 1996; Johansen and Babuška, 2003; Yan and Zhou, 2006; Chen *et al.*, 2007; Du and Zhang, 2008; Kim *et al.*, 2009a, 2009b; Kim *et al.*, 2013b; Kim *et al.*, 2014a, b). Therefore, fuzzy controllers, in particular, have been paid great attention from many investigators in the field of structural vibration control. To implement such effective fuzzy controllers into large civil structures, a variety of the fuzzy control design frameworks have been proposed for hazard mitigation of structures, including trial and error methods (Subramaniam *et al.*, 1996; Battaini *et al.*, 1998; Symans and Kelly, 1999; Loh *et al.*, 2003; Battaini *et al.*, 2004), linear quadratic Gaussian-based learning approach (Al-Dawod *et al.*, 2004), adaptation (Zhou *et al.*, 2003), sliding mode (Kim *et al.*, 2004; Alli and Yakut, 2005), among others. However, it would be difficult for all of the aforementioned methods to design a fuzzy controller using incomplete and incoherent measurements from large complex structures. Another disadvantage of using fuzzy logic as a model is that it needs a time consuming optimization process of the parameters; i.e., the optimization process can be complex and computationally intensive, leading itself to the inclusion of intelligent programming schemes such as GA and NN.

The use of a NN is to develop a learning mechanism that emulates that of the human brain, such that it creates a network of interlinked nodes. These nodes, being connected, compute an output from the input to the node, and create a series of links between all nodes. As previously mentioned, the use of a fuzzy inference system can be complex and difficult in computations. Using a NN in combination with a fuzzy inference system can create a model that is more efficient. The NN adjusts parameters throughout the entirety of computation. The regulated parameters improve performances and decreases errors of the system. It is able to learn patterns and make adjustments as needed to further create a more improved model because it emulates the human brain and its cognitive mechanism. Faravelli and Yao (1996) and Faravelli and Rossi (2002) used NN to train fuzzy

controllers for vibration control of a three-story building subject to earthquakes. Schurter and Roschke (2001) applied NN to optimal design of fuzzy logic-based controllers for seismic response control of a single story and four-story buildings. Tani *et al.* (1998) designed an active mass damper for vibration mitigation of a five-story building under earthquake inputs. The training effectiveness of NN has also been experimentally demonstrated using a five-story  $\frac{1}{2}$ -scaled steel frame structure (Hung *et al.*, 2003). Ning *et al.* (2009) designed a fuzzy sliding mode controller for a seismically excited highway bridge. Ozbulut and Hurlebaus (2011) designed superelastic-friction base isolators using NN for seismic response control of highway bridges under earthquake loads. Mitchell *et al.* (2012) proposed the use of NN for optimal operation of a high-rise building equipped with an active tuned mass damper using a fuzzy controller. They also used discrete wavelet transform to filter unwanted data from original measurements to be used for training and validating the fuzzy controller.

As another optimization scheme, GA has been adopted from structural engineers for hazard mitigation of large civil structures under earthquakes and/or strong wind loads. Ahlawat and Ramaswamy (2000; 2002; 2004) optimized the parameters of a fuzzy driven damping system using GA for buildings under earthquakes and wind loads. Wang and Lee (2002) designed a fuzzy sliding mode controller using GA for seismically excited buildings. Yan and Zhou (2006) proposed the use of GA for optimal design of fuzzy controllers for seismic response controls of buildings. Kim and Roschke (2006) used GA to design fuzzy logic controller for base isolated buildings subject to earthquake inputs. However, there is no study on the application of an advanced GA, an implicit redundant representation (IRR) GA (Cha and Agrawal, 2013) to the optimal adjustment of active fuzzy control forces for vibration control of highway bridge structures equipped with multiple active actuators.

However, due to the complexities of training the fuzzy controller using NNs and GAs, computation times can become excessive. Therefore, WT is used in conjunction with the combined fuzzy logic controller, GA-based DOFPC, and NNs to compress input data and decrease computation times. WTs, combined with the genetic neuro-fuzzy (GNF) model, leads to a wavelet-based GNF model (WGNF). The WT can be used to filter out high or low frequency components from a time series. The WT improves upon previous methods due to its ability to incorporate an adjustable window function. It allows a user to analyze particular data points in a time series, rather than the entire time window, which is the case in Fourier transforms. Fast Fourier transforms (FFT) have been previously used for structural health monitoring and vibration controls, but require a fixed time-window for the entire data set (Gurley and Kareem, 1999). This limitation of the FFT can induce difficulty when analyzing data for long periods of time, as in the case in structural health monitoring, and can lead to

missing key components, such as a particular damage point. The WT allows for an adjustable window, and therefore being able to look into any portion of a time series. WTs can also be used as a means of filtering, which is critical in the use of the WGNF model. As previously mentioned, the GNF system requires high computation times due to the stochastic learning mechanism of the GAs and NNs. Being able to decrease the amount of data points while still maintaining the important components allows for a reduced computational cost. The proposed model uses two levels of discrete WTs for compressing input data.

As previously mentioned, fuzzy logic controllers and neuro-fuzzy controllers have been widely researched. However, these controllers need extensive computation times to achieve adequate performances. Therefore, the creation of the new WGNF system provides for decreased computation times while maintaining the performance. However, no integrated approach for fuzzy logic, neural network, wavelet transform and genetic algorithm has been proposed for full-scale highway bridges under a variety of earthquake loads such that a fuzzy controller is optimally designed while the computational load is minimized. Thus, the creation of the WGNF system for means of structural control is innovative in its application to control systems for mitigation of structural responses of large civil engineering structures. This proposed control algorithm also requires less feedback information while decreasing the structural responses from other control systems from the structure in comparison to full state feedback controllers.

The organization of this paper is as follows: the WGNF algorithm is discussed in Section 2. Section 3 explains the highway bridge finite element model, followed by the simulation results. Concluding remark is given in Section 4.

## 2 Wavelet-filtered genetic neural fuzzy model (WGNF)

The WGNF is an integration model of the WT, GA-based DOFPC, NN, and fuzzy system. A least squares estimator and NN are used to train the membership function (MF) and the associated consequent parameters of a fuzzy model, respectively. The GA and wavelet are used to optimize the control forces and filter undesirable signals, respectively.

### 2.1 Takagi-Sugeno fuzzy model

The fuzzy modeling framework used in the WGNF model is as follows (Takagi and Sugeno, 1985; Kim *et al.*, 2009a).

$$R_j : \text{If } u_{FZ}^1 \text{ is } P_{1,j} \text{ and } u_{FZ}^2 \text{ is } P_{2,j} \dots \text{and } u_{FZ}^i \text{ is } P_{i,j}, \\ \text{then } z = f_j(u_{FZ}^1, \dots, u_{FZ}^i), \quad j = 1, 2, \dots, N_r. \quad (1)$$

where  $R_j$  is the  $j$ th rule,  $N_r$  is the rule number,  $P_{i,j}$  are  $j$ th fuzzy sets, and  $u_{FZ}^i$  are premise. The  $z = f_j(u_{FZ}^1, \dots, u_{FZ}^i)$  can be any linear equation. Using fuzzy interpolation methods, all the linear equations are blended as

$$y = \frac{\sum_{j=1}^{N_r} W_j(u_{FZ}^i) [f_j(u_{FZ}^1, \dots, u_{FZ}^i)]}{\sum_{j=1}^{N_r} W_j(u_{FZ}^i)} \quad (2)$$

where  $W_j(u_{FZ}^i) = \prod_{i=1}^n \mu_{P_{i,j}}(u_{FZ}^i)$ ,  $n$  is the total number of input variables and  $\mu_{P_{i,j}}(u_{FZ}^i)$  is the membership grade of  $u_{FZ}^i$  in  $P_{i,j}$ . However, it is difficult to optimize the consequent parameters of the fuzzy controller. Therefore, it is proposed to use NNs for effective optimization of the fuzzy control parameters.

### 2.2 Neural-fuzzy model

The configuration of a neuro-fuzzy model is shown in Fig. 1. The node in the 1st layer is determined as

$$F_{FZ}^{1,j} = \mu_{P_{i,j}}(u_{FZ}^i) \quad (3)$$

where,

$$\mu_{P_{i,j}}(u_{FZ}^i) = \exp\left[-(u - a_1)^2 / 2a_2^2\right] \quad (4)$$

where  $a_1$  and  $a_2$  are the user defined parameters. The node in the 2nd layer is calculated by

$$F_{FZ}^{2,j} = \mu_{P_{1,j}}(u_{FZ}^1) \times \mu_{P_{2,j}}(u_{FZ}^2) \times \dots \times \mu_{P_{i,j}}(u_{FZ}^i) \quad (5)$$

The 3rd layer normalizes the 2nd layer outputs

$$F_{FZ}^{3,j} = F_{FZ}^{2,j} / \sum_j \prod_{i=1}^n \mu_{P_{i,j}}(u_{FZ}^i) \quad (6)$$

The 4th layer integrates the consequent equation

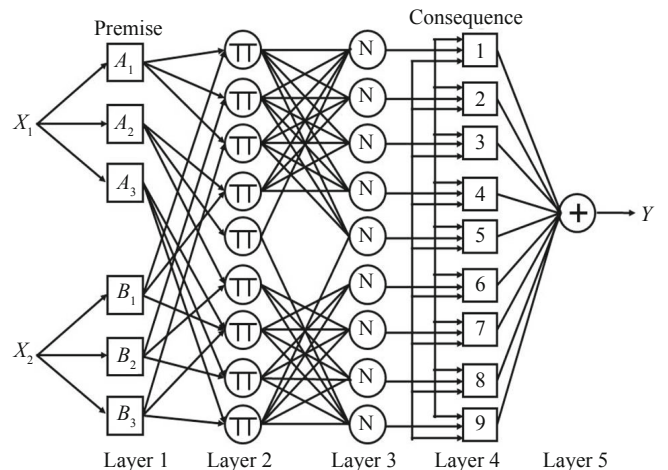


Fig. 1 Neuro-fuzzy model architecture

with the normalized 3rd layer.

$$F_{FZ}^{4,j} = F_{FZ}^{3,j} \cdot f_j = F_{FZ}^{3,j} \left[ f_j \left( u_{FZ}^1, \dots, u_{FZ}^i \right) \right] \quad (7)$$

Then the system output is calculated using the following last layer.

$$F_{FZ}^{5,j} = \frac{\sum_j \prod_{i=1}^n \mu_{P_{i,j}} \left( u_{FZ}^i \right) \left[ f_j \left( u_{FZ}^1, \dots, u_{FZ}^i \right) \right]}{\sum_j \prod_{i=1}^n \mu_{P_{i,j}} \left( u_{FZ}^i \right)} \quad (8)$$

In this algorithm, the number of iterations and the type and number of MF need to be optimized (Jang, 1993; Yang and Lin, 2005). However, the neuro-fuzzy model requires a set of optimal control force signals. Such optimal signals are generated using an advanced GA-based DOFPC system.

### 2.3 Advanced genetic algorithms

The optimal signals obtained from the DOFPC are used for training the WGNF control systems. The output of the DOFPC is expressed as (Cha and Agrawal, 2013).

$$v = \left| (e_0 + e_1 x + e_2 x^2 + e_3 x^3) + (g_0 + g_1 \dot{x} + g_2 \dot{x}^2 + g_3 \dot{x}^3) \right| \quad (9)$$

where  $x$  and  $\dot{x}$  are the drift and velocity, respectively.  $e_0, e_1, e_2, e_3, g_0, g_1, g_2,$  and  $g_3$  can be optimally determined. The optimal parameters of the DOFPC are determined using an IRR GA (Raich and Ghaboussi, 2000) with advanced search ability and a powerful dynamic encoding policy, and SGA (Goldberg, 1989). The gene locators (GL) and redundant segments in binary string allow the flexibility of the IRR, as shown in Fig. 2.

In order to indicate the starting binary of a gene instance, the GL is used. The gene instance has encoded information of design variables, and redundant segments have non-encoded information for current generation, but they can be used as encoded gene instances in later generations by genetic operators. The procedure of the optimization is as follows

Step (1): Random generation of the initial population to provide the two polynomial equations with initial coefficients.

Step (2): Response quantities of the structure are calculated by carrying out control analyses using the initial parameters.

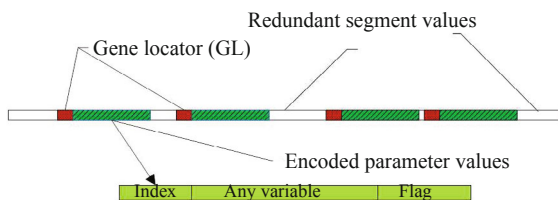


Fig. 2 IRR GA representation

Step (3): Selections of well-adapted individual strings using binary tournament selection scheme.

Step (4): New population as candidate solutions of the next generation is generated through crossover and mutation operator.

Step (5): The convergence of the objective function or the predefined maximum number of populations is checked.

If the evaluation criteria are satisfied, the algorithm stops. Otherwise, it goes back to Step (2). In order to search near optimal controller parameters, three different objective functions which were used at Step (3) in optimization procedure in previous paragraph are defined as peak base shear force ( $C_1$ ), peak overturning moments ( $C_2$ ), and the midspan peak displacement ( $C_3$ ) as shown in Table 1. The optimization objective function  $C$  is formulated as

$$C = \max(C_1) + \max(C_2) + \max(C_3) \quad (10)$$

where the maximum value to all the earthquake responses is used as each criteria. However, the optimization of the GNF controller is computationally expensive. Hence the number of input and output data points is compressed by using discrete WTs.

### 2.4 Wavelet transform

A discrete wavelet transform (DWT) can be described (Thuillard, 2001)

$$u_{FZ}^i = 2^{(s/2)} \sum_n \bar{x}(\bar{n}) \phi(2^s t - l) \quad (11)$$

where  $l, s,$  and  $\phi$  are the location, scale, and mother function, respectively. The original signal,  $\bar{x}(n)$ , is recalculated from

$$\bar{x}_i(\bar{n}) = \sum_l \sum_s u_{FZ}^i \Psi_{l,s}(\bar{n}) \quad (12)$$

To decompose the signals into both low and high frequency components, multi filter bank theory can be used (Taha *et al.*, 2004). It not only filters undesirable signals from the original data, but also reduces the total number of data points. To implement the multi filter bank system, the scaling and wavelet functions are used

$$\phi_{l,s} := 2^{s/2} \phi(2^s t - l) \quad (13)$$

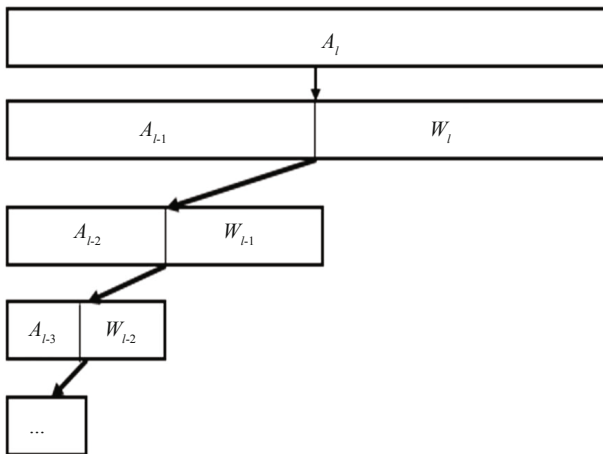
$$\Psi_{l,s} := 2^{s/2} \Psi(2^s t - l) \quad (14)$$

Figure 3 shows an example of such a decomposition process. The DWT-applied data sets are used for training the GNF control system.



**Table 1** Evaluation indices

Criteria	Formulation	Variables
Peak base shear force	$C_1 = \max \left\{ \frac{\max_{i,t}  \hat{F}_{bi}(t) }{\hat{F}_{0b}^{\max}} \right\}$	$i = 1, 2$ , for $x$ and $y$ directions, respectively. $\hat{F}_{bi}(t)$ is the $i$ th case base shear in the controlled structure, and $\hat{F}_{0b}^{\max}$ is the maximum of the base shears in $x$ and $y$ directions, respectively.
Peak overturning moments	$C_2 = \max \left\{ \frac{\max_{i,t}  M_{bi}(t) }{M_{0b}^{\max}} \right\}$	$i = 1, 2$ , for $x$ and $y$ directions, respectively. $M_{bi}(t)$ is the overturning moment in the controlled structure for the $i$ th case, and $M_{0b}^{\max}$ is the maximum of the overturning moments in both directions in the uncontrolled structure, respectively.
Peak displacement at the midspan	$C_3 = \max \left\{ \frac{\max_{i,t}  \hat{y}_{mi}(t) }{\hat{y}_{0b}^{\max}} \right\}$	$\hat{y}_{mi}(t)$ is the displacement at the midspan of the controlled structure for the $i$ th case, and $\hat{y}_{0b}^{\max}$ is the maximum of the midspan displacement in both directions in the uncontrolled structure, respectively.

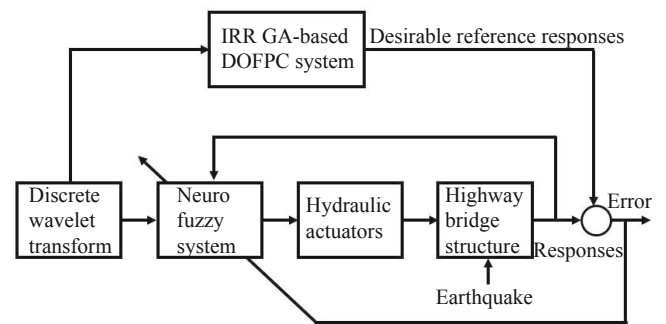
**Fig. 3** Wavelet transform-based multi-resolution analysis framework

## 2.5 Wavelet-based GNF controller

The DWT is effective in removing undesirable noises from the original signals. Daubechie wavelet-based filtered data are used as input signals to the GNF model. The configuration of the WGNF controller is shown in Fig. 4. The WGNF controller optimally determines the control force of an actuator within a highway bridge. Both acceleration and displacement responses are used as input signals of the GNF controller. To assess the control performance, extensive simulations were performed on a highway bridge under a variety of earthquake loads.

## 3 Example

The WGNF control system is tested on a benchmark highway bridge (Agrawal *et al.*, 2009) equipped with sixteen active control actuators. The performances of the proposed WGNF system are compared with a benchmark controller to show its effectiveness in improving structural performance.

**Fig. 4** WGNF architecture

## 3.1 Highway bridge

A benchmark bridge finite element model was developed based on an existing structure located at the crossing of the 91 and 5 highways in Orange County of California (Agrawal *et al.*, 2009). A prestressed concrete box-girder is used with continuous two spans of 58.5 m. The deck has a width of 12.95 m and 15 m for the east and west spans, respectively. The bridge carries four lanes of traffic on the top columns of 6.9 m in height. The location of the bridge is within 20 km of two faults, the Whittier-Ellsinore and Newport-Inglewood fault zones, showing a great need for structural control due to its susceptibility to seismic events. Figure 5 shows the bridge schematic. By assuming a bilinear stress-strain relationship, the bridge pier model was developed such that it behaves linearly until the first yielding point develops. It is also assumed that the bridge model has bi-directional seismic behaviors. In order to investigate the performance of the proposed controller, six earthquake ground motions are used, including North Palm Springs (1986), TCU084 component of Chi-Chi earthquake, Taiwan (1999), El Centro component of Imperial Valley earthquake (1940), Rinaldi component of Northridge earthquake (1994), Bolu component of Duzce, Turkey

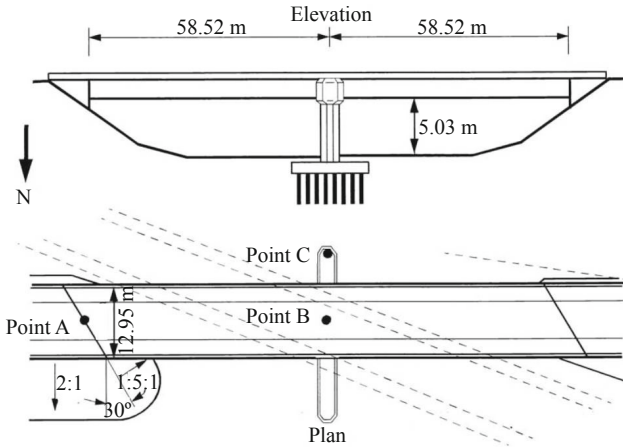


Fig. 5 Benchmark highway bridge

earthquake (1999), and Nishi-Akashi component of Kobe earthquake (1995). These earthquakes are all near-field earthquakes with soil-types from A to D based on NEHRP classification.

### 3.2 Control system design

Many simulations were performed to determine the best arrangement of control forces. It was determined that the computation of only two control forces are needed to calculate. This bridge is equipped with sixteen actuators in each  $x$ - and  $y$ -directions. The WGNF model was trained using the control signals of the DOFPC employing GA (Cha and Agrawal, 2013). It is assumed that the measurement noise is a zero mean Gaussian white noise with the signal to noise ratio of 10%. Four Gaussian membership functions are used for constructing the fuzzy sets. The total iteration of 30 was conducted. The training time for modeling the fuzzy controller results in 1368 s (i.e., 22.8 min). Figure 6 shows the initial (before training) and final MFs (after training). As a benchmark control strategy, linear quadratic Gaussian (LQG) controller was considered. LQG control is an integration of linear quadratic regular, which is a full state feedback controller and Kalman estimator, which is able to reduce states to obtain comparable performance without needing full state feedback.

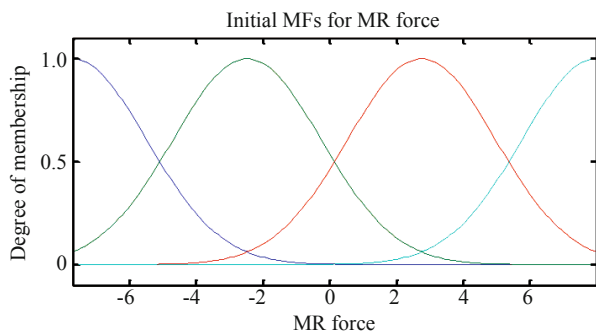


Fig. 6 Membership functions before and after training

As a means of validation and comparison, several evaluation indices are used. These indices compare structural responses and control outputs of the proposed WGNF system with those of the uncontrolled structure, showing how much each index is reduced. These indices are presented in Table 2 (Agrawal *et al.*, 2009)

The  $\hat{F}_{bi}(t)$  is the shear force at the  $i$ th degree of freedom of the controlled structure,  $\hat{F}_{0b,max}$  is the uncontrolled maximum shear force,  $\hat{M}_{bi}(t)$  is the overturning moment,  $\hat{M}_{0b,max}$  is the uncontrolled maximum overturning moment,  $\hat{y}_{mi}(t)$  is the midspan displacement,  $\hat{y}_{0m,max}$  is the uncontrolled maximum midspan displacement,  $|\hat{y}_{mi}(t)|$  is the midspan acceleration,  $\hat{y}_{0m,max}$  is the uncontrolled maximum acceleration,  $|\hat{y}_{bi}(t)|$  is the abutment displacement,  $\hat{y}_{0b,max}$  is the uncontrolled maximum abutment displacement,  $\hat{\Phi}_i(t)$  is the ductility,  $\hat{\Phi}_{max}$  is the uncontrolled maximum ductility,  $d\hat{E}_i$  is the dissipated energy of curvature at the column,  $\hat{E}_{max}$  is the uncontrolled maximum dissipated energy of the curvature at the column,  $\hat{N}_{c,d}$  is the number of plastic connections of the control system,  $\hat{N}_d$  is the number of plastic connections of the uncontrolled system,  $|\cdot|$  is the absolute value,  $\|\cdot\|$  is the normalized value,  $\hat{f}_i(t)$  is the control force,  $\hat{W}$  is the seismic weight of the system,  $\hat{d}_i(t)$  is the device stroke,  $\hat{x}_{0m,max}$  is the uncontrolled maximum bearing deformation,  $\hat{P}_i(t)$  is the instantaneous power required for the control device,  $\hat{x}_{0m,max}$  is the uncontrolled maximum velocity of bearing, and  $\hat{x}_{c,k}$  is the state.

Table 3 shows the simulation results of the proposed WGNF control systems compared to another active control system. In Table 3, the evaluation results that are best performed values are highlighted in bold. Using these 21 evaluation indices, it is shown that the proposed WGNF controller's performances are quite superior to those of the benchmark LQG control algorithm for all the El-Centro, Kobe, North Palm Springs, Turkey Bolu and Rinadi earthquakes.

Note that the same number of control devices ( $J_{19}$ ) is used for implementing both the WGNF and LQG controllers. It is clear from Table 3 that the maximum peak responses of the WGNF control system are quite competitive with those for the LQG. In particular, the peak evaluation criteria  $J_2, J_3, J_5-J_{11}, J_{13}, J_{14}, J_{16}$  and  $J_{18}$  are significantly reduced during most of the ground

**Table 2 Control performance evaluation criteria**

$J_1 = \max \left\{ \frac{\max  \hat{F}_{bi}(t) }{\hat{F}_{0b,max}} \right\}$	$J_2 = \max \left\{ \frac{\max  \hat{M}_{bi}(t) }{\hat{M}_{0b,max}} \right\}$	$J_3 = \max \left\{ \frac{\max  \hat{y}_{mi}(t) }{\hat{y}_{0m,max}} \right\}$
$J_4 = \max \left\{ \frac{\max  \ddot{\hat{y}}_{mi}(t) }{\ddot{\hat{y}}_{0m,max}} \right\}$	$J_5 = \max \left\{ \frac{\max  \hat{y}_{bi}(t) }{\hat{y}_{0b,max}} \right\}$	$J_6 = \max \left\{ \frac{\max  \hat{\phi}_i(t) }{\hat{\phi}_{max}} \right\}$
$J_7 = \max \left\{ \frac{\max \int d\hat{E}_i}{\hat{E}_{max}} \right\}$	$J_8 = \max \left\{ \frac{\hat{N}_{c,d}}{\hat{N}_d} \right\}$	$J_9 = \max \left\{ \frac{\max \ \hat{F}_{bi}(t)\ }{\ \hat{F}_{0b,max}\ } \right\}$
$J_{10} = \max \left\{ \frac{\max \ \hat{M}_{bi}(t)\ }{\ \hat{M}_{bi,max}\ } \right\}$	$J_{11} = \max \left\{ \frac{\max \ \hat{y}_{mi}(t)\ }{\ \hat{y}_{0m,max}\ } \right\}$	$J_{12} = \max \left\{ \frac{\max \ \ddot{\hat{y}}_{mi}(t)\ }{\ \ddot{\hat{y}}_{0m,max}\ } \right\}$
$J_{13} = \max \left\{ \frac{\max \ \hat{y}_{bi}(t)\ }{\ \hat{y}_{0b,max}\ } \right\}$	$J_{14} = \max \left\{ \frac{\max \ \hat{\phi}_i(t)\ }{\ \hat{\phi}_{max}\ } \right\}$	$J_{15} = \max \left\{ \max \left( \frac{\hat{f}_i(t)}{\hat{W}} \right) \right\}$
$J_{16} = \max \left\{ \max \left( \frac{\hat{d}_i(t)}{\hat{x}_{0m,max}} \right) \right\}$	$J_{17} = \max \left\{ \frac{\max \left[ \sum_i \hat{P}_i(t) \right]}{\hat{x}_{0m,max} \hat{W}} \right\}$	$J_{18} = \max \left\{ \frac{\max \left[ \sum_i \int_0^{t_r} \hat{P}_i(t) \right]}{\hat{x}_{0m,max} \hat{W}} \right\}$
$J_{19} = \# \text{ of control devices}$	$J_{20} = \# \text{ of required sensors}$	$J_{21} = \dim(\hat{x}_{c,k})$

**Table 3 Control performance evaluation**

	Npalm		ChiChi		ElCentro		Rina dir		Turk		Kobe		Ave		Max	
	L	W	L	W	L	W	L	W	L	W	L	W	L	W	L	W
$J_1$	0.95	0.96	0.88	0.74	0.79	0.80	0.90	0.78	0.91	0.90	0.79	0.89	0.85	0.85	0.95	0.96
$J_2$	0.77	0.80	0.97	0.96	0.74	0.73	0.98	0.96	0.98	0.98	0.70	0.78	0.86	0.87	0.98	0.98
$J_3$	0.82	0.82	0.80	0.68	0.78	0.80	0.87	0.71	0.75	0.72	0.70	0.79	0.79	0.75	0.87	0.82
$J_4$	0.79	0.94	0.88	0.98	0.88	1.0	0.84	0.98	0.80	0.91	0.90	0.94	0.85	0.96	0.90	1.0
$J_5$	0.94	0.82	0.80	0.64	0.64	0.46	0.88	0.68	0.71	0.69	0.59	0.45	0.76	0.62	0.94	0.82
$J_6$	0.77	0.80	0.74	0.56	0.74	0.73	0.85	0.61	0.46	0.36	0.70	0.78	0.71	0.64	0.85	0.80
$J_7$	0	0	0.51	0.35	0	0	0.62	0.39	0.33	0.11	0	0	0.24	0.14	0.62	0.39
$J_8$	0	0	0.67	0.67	0	0	1.0	1.0	0.33	0.33	0	0	0.33	0.33	1.0	1.0
$J_9$	0.74	0.80	0.89	0.84	0.68	0.72	0.87	0.80	0.89	0.84	0.71	0.79	0.80	0.80	0.89	0.84
$J_{10}$	0.70	0.74	0.83	0.79	0.64	0.69	0.88	0.82	0.53	0.48	0.61	0.75	0.70	0.71	0.88	0.82
$J_{11}$	0.70	0.75	0.78	0.70	0.66	0.71	0.80	0.70	0.61	0.56	0.73	0.77	0.71	0.70	0.80	0.77
$J_{12}$	0.72	0.86	0.79	0.85	0.69	0.80	0.80	0.84	0.79	0.88	0.80	0.88	0.77	0.85	0.80	0.88
$J_{13}$	0.48	0.37	0.78	0.67	0.48	0.46	0.82	0.68	0.52	0.42	0.47	0.41	0.59	0.50	0.82	0.68
$J_{14}$	0.70	0.74	0.65	0.38	0.64	0.69	0.83	0.58	0.24	0.12	0.71	0.75	0.63	0.54	0.83	0.75
$J_{15}$	0.01	0.01	0.02	0.03	0.01	0.01	0.02	0.03	0.01	0.02	0.01	0.01	0.01	0.02	0.02	0.03
$J_{16}$	0.90	0.79	0.77	0.62	0.59	0.42	0.80	0.62	0.71	0.69	0.51	0.44	0.71	0.60	0.90	0.79
$J_{17}$	0.05	0.05	0.11	0.16	0.02	0.03	0.11	0.12	0.07	0.08	0.07	0.03	0.07	0.08	0.11	0.16
$J_{18}$	0.01	0.01	0.02	0.02	0.00	0.00	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.02	0.02
$J_{19}$	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
$J_{20}$	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
$J_{21}$	28	20	28	20	28	20	28	20	28	20	28	20	28	20	28	20
Number of response quantities for which best performance													11	15	9	16

motions: the maximum of peak midspan displacement ( $J_3$ ), peak bearing deformation ( $J_5$ ), peak ductility ( $J_6$ ), peak displacement energy ( $J_7$ ) are decreased by approximately 6%, 15%, 6%, and 59%, respectively, over those by the LQG controller, although maximum midspan acceleration performance ( $J_4$ ) of the LQG controller is better than the proposed WGNF controller. It should be noted that structural displacement responses are directly related to safety of the bridge structures. Moreover, the maximum of the normed base shear ( $J_9$ ), normed overturning moment ( $J_{10}$ ), normed midspan displacement ( $J_{11}$ ), normed bearing deformation ( $J_{13}$ ), normed ductility ( $J_{14}$ ), and peak stroke ( $J_{16}$ ) are reduced by approximately 5%, 7%, 4%, 21%, 15%, and 14%, respectively, compared to the LQG controller.

In summary, the WGNF is equal to or better, than the LQG in 15 of 21 evaluation indices for average values for all the six earthquake simulation cases. The WGNF is also equal to or better, than the LQG in 16 of 21 evaluation indices for maximum responses. In particular, the WGNF is more effective than the LQG in reducing displacement responses, which is directly related to structural safety. Furthermore, it needs to be noted that the WGNF controller uses only 4 sensors while the LQG needs 12 sensors to implement the controllers (see  $J_{20}$ ). Plus, the WGNF needs 20 states to implement the fuzzy controller into the bridge model while 28 states are required for the LQG, which means that the WGNF uses less computational resources than the LQG.

#### 4 Conclusions

This paper develops a wavelet-filtered genetic neuro-fuzzy system (WGNF) as a means for vibration controls of highway bridges excited by various earthquake loads. The proposed WGNF system combines aspects of discrete wavelet transforms, genetic algorithm, neural networks, and fuzzy logic theory. The WGNF system is trained using an artificial earthquake, which combines the characteristics of various earthquake accelerations. To evaluate the control performance of the WGNF controller, a full-scale highway bridge benchmark control structure is investigated. The highway bridge benchmark control models considers bi-directional ground motions and uses bi-linear bridge pier model. It is shown from the simulation results that the WGNF system effectively mitigates structural responses of the highway bridge equipped with sixteen hydraulic actuators. The WGNF system reduces the power consumption, control force magnitude, and the required number of sensors installed on the highway bridge. The WGNF system also greatly reduces computation time of control forces in comparison with other control algorithms. The comparison based on 21 evaluation criteria with six historical ground motions classified as near-field earthquakes shows that the proposed controller is more effective to reduce the key damage indices such as peak displacement, which

are directly related to the bridge collapse. Moreover, the WGNF control system uses less number of sensors compared to traditional LQG control system. It should be noted that the proposed control system is more reliable in real application based on using reduced number of sensors.

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### List of acronyms

Acronym	Interpretation
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WGNF	Wavelet-filtered genetic-neuro-fuzzy
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WT	Wavelet transform
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GA	Genetic algorithm
DOFPC	Output feedback polynomial controls
TS	Takagi-Sugeno
NN	Neural network
IRR	Implicit redundant representation
GNF	Genetic neuro-fuzzy
FFT	Fast Fourier transform
MF	Membership functions
CWT	Continuous wavelet transform
MRA	Multi-resolution analysis
LQG	Linear quadratic Gaussian