Earthq Eng & Eng Vib (2012) 11: 243-256 **DOI**: 10.1007/s11803-012-0114-4

A simplified multisupport response spectrum method

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Abstract: A simplified multisupport response spectrum method is presented. The structural response is a sum of two components of a structure with a first natural period less than 2 s. The first component is the pseudostatic response caused by the inconsistent motions of the structural supports, and the second is the structural dynamic response to ground motion accelerations. This method is formally consistent with the classical response spectrum method, and the effects of multisupport excitation are considered for any modal response spectrum or modal superposition. If the seismic inputs at each support are the same, the support displacements caused by the pseudostatic response become rigid body displacements. The response spectrum in the case of multisupport excitations then reduces to that for uniform excitations. In other words, this multisupport response spectrum method is a modification and extension of the existing response spectrum method under uniform excitation. Moreover, most of the coherency coefficients in this formulation are simplified by approximating the ground motion excitation as white noise. The results indicate that this simplification can reduce the calculation time while maintaining accuracy. Furthermore, the internal forces obtained by the multisupport response spectrum method are compared with those produced by the traditional response spectrum method in two case studies of existing long-span structures. Because the effects of inconsistent support displacements are not considered in the traditional response spectrum method, the values of internal forces near the supports are underestimated. These regions are important potential failure points and deserve special attention in the seismic design of reticulated structures.

Keywords: multisupport excitation; uniform excitation; response spectrum method; coherency coefficients; simplification; comparative study

1 Introduction

In 1941, the concept of a response spectrum was proposed by Biot. Without sufficient records of strong earthquakes, the response spectrum method had no practical application at that time. Nonetheless, the number of strong motion records has increased greatly over the past thirty years, so the response spectrum method can now be used to evaluate current designs. Because the classical response spectrum method has a clear physical interpretation, a concise form, and convenient applications, it has been more and more widely applied to structural seismic design.

The classical response spectrum method based on uniform excitation has been widely applied in seismic design codes in many countries and has sufficient precision for most structures. Nevertheless, current response spectrum theory results in large errors for long-span structures because of the time–space variation of seismic ground motion. Hence, it is necessary to propose a modified response spectrum method for use in cases of multiple support excitations.

Berrah and Kausel (1992, 1993) modified the traditional CQC (Complete Quadratic Combination) method by introducing two coefficients to simulate multisupport seismic excitation. Their method, however, cannot consider pseudostatic effects. One coefficient is used to adjust each spectrum value of the supports and the other is used to modify the mode correlation coefficients to consider the time–space variation of seismic ground motion. Yamumura and Tanaka (1990) divided supports into several groups based on their location and local soil layer. Supports in the same group were assumed to be completely coherent, while different groups were assumed to be noncoherent with each other. An approximate response spectrum method was then suggested. This method, however, cannot take into account the wave passage effect or loss of coherence. Trifunac and Todorovska (1997) treated the stiffness of columns as weights to determine a reference point for each column's movement. Hence, multiple support

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Supported by: Major Program of National Science Foundation of China Under Grant No. 90715005; Program for New Century Excellent Talents in University Under Grant No. NCET-07-0186; Doctoral Fund of Ministry of Education of China Under Grant No. 200802860007

Received February 2, 2010; **Accepted** March 2, 2012

inputs were translated into a uniform seismic input, and the traditional response spectrum method was extended to solve the problem for multisupport excitation. In line with random vibration theory, Kiureghian and Neuenhofer (1992) presented an exact derivation of a new response spectrum method that can consider the wave passage effect, coherence loss and the local soil layer effect. This method is the most rigorous, but it involves the cross-power spectral density function and numerical integration and thus requires long computational times. This method is too cumbersome to be accepted by designers.

Current research has focused on simplifying the multisupport response spectrum method. Based on theoretical results by Burdisso and Singh (1987a,b), Loh and Ku (1995) proposed an approximate response spectrum method that translates the integral of the coherence function into a sum of a series of elementary functions. Based on random vibration theory, Li and Li (2005) developed a response spectrum method for seismic response analysis of linear, multi-degree-of-freedom structures under multisupport excitations. Various response quantities are obtained from proposed combination rules in terms of the mean response spectrum. This method makes it possible to apply the response spectrum to seismic reliability analysis of structures. Furthermore, simplified procedures that can dramatically enhance the efficiency of computing the spectral parameters and correlation coefficients in the combination rules are also given in Li and Li (2005). Kahan *et al.* (1996) simplify the coherency function given the condition of small spatial variations in the seismic motion, and their results show that the simplified method can maintain sufficient accuracy when the coherence loss ratio is less than 2×10^{-3} s/m and the apparent velocity is more than 200 m/s. Yu and Zhou (2008) developed a complex multisupport response spectrum (CMSRS) method for seismic analysis of a nonclassically damped linear system. The CMSRS method can properly account for the effects of correlation between the support motions as well as between the modal displacement and velocity responses of the structure. It also provides reasonable and acceptable estimates of the peak response, the response spectra at the support points and the coherency function.

In the multisupport response spectrum method, several effects should be considered, including the wave passage effect, the dispersion effect and the local soil effect. The spans of buildings, however, are spatially confined, so it is rare for buildings to be located on several different soil layers. Thus, the effects of soil layers local to individual supports are not considered in this paper. A simplified multisupport response spectrum method with a clear physical interpretation is presented. In this method, the structural response consists of two components for a structure whose first natural period is less than 2 s. One component is the pseudostatic response caused by the inconsistent motions of the structural supports, and the other is the structural dynamic response due to ground motion accelerations. If the seismic inputs at each support are the same, the support displacements caused by the pseudostatic response become rigid body displacements and do not lead to any strains and stresses on the structure. The response spectrum for multisupport excitations then reduces to that for uniform excitations. Moreover, most of the coherency coefficients in the formulation are simplified in this paper by approximating the ground motion excitation as white noise. This approximation is justifiable in light of the frequency response function. Furthermore, the internal forces from the multisupport response spectrum method and the traditional response spectrum method are compared for two case studies of existing long-span structures. The numerical results show that the magnitudes of the internal forces near the supports are underestimated by the traditional response spectrum method. These regions are always important in the overall performance of the structure and are deserving of special attention in reticulated structure design.

2 Equations of motion under multiple support excitations (Clough and Penzien, 1993)

Under the absolute coordinate system relative to the center of the earth, the dynamic equation for a discrete, n-degree-of-freedom linear system subjected to m support motions can be described in matrix form

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{M}_{\mathrm{c}} \\ \boldsymbol{M}_{\mathrm{c}}^{\mathrm{T}} & \boldsymbol{M}_{\mathrm{g}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}} \\ \ddot{\boldsymbol{u}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C} & \boldsymbol{C}_{\mathrm{c}} \\ \boldsymbol{C}_{\mathrm{c}}^{\mathrm{T}} & \boldsymbol{C}_{\mathrm{g}} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{u}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{K}_{\mathrm{c}} \\ \boldsymbol{K}_{\mathrm{c}}^{\mathrm{T}} & \boldsymbol{K}_{\mathrm{g}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{F} \end{Bmatrix} \tag{1}$$

where $\boldsymbol{u}$ is the m-vector of prescribed support displacements; $\boldsymbol{x}$ is the n-vector of the displacements of the unconstrained degrees of freedom; $\boldsymbol{F}$ is the m-vector of the reaction forces in the supported degrees of freedom; $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ represent the $N\times N$ mass matrix, the damping matrix, and the stiffness matrix of the unconstrained degrees of freedom, respectively; $\boldsymbol{M}_{\mathrm{g}}$, $\boldsymbol{C}_{\mathrm{g}}$, and $\boldsymbol{K}_{\mathrm{g}}$ represent the $M\times M$ mass matrix, the damping matrix, and the stiffness matrix of the support degrees of freedom; and $\boldsymbol{M}_{\mathrm{C}}$, $\boldsymbol{C}_{\mathrm{C}}$, $\boldsymbol{K}_{\mathrm{C}}$ represent the $N\times M$ mass matrix, the damping matrix, and the stiffness matrix of the couplings between the unconstrained and the support degrees of freedom, respectively. In the general case, $\boldsymbol{M}_{\mathrm{C}}$ is assumed to be a zero matrix.

It is common to decompose $\boldsymbol{x}$ into the quasi-static displacement $\boldsymbol{x}^{\mathrm{s}}$ and the dynamic relative displacement $\boldsymbol{x}^{\mathrm{d}}$, namely,

$$\boldsymbol{x} = \boldsymbol{x}^{\mathrm{d}} + \boldsymbol{x}^{\mathrm{s}} \tag{2}$$

Setting the dynamic term of Eq. (1) equal to zero, the quasi-static displacement $\boldsymbol{x}^{\mathrm{s}}$ can be obtained as

$$\boldsymbol{x}^{\mathrm{s}} = -\boldsymbol{K}^{-1}\boldsymbol{K}_{\mathrm{c}}\boldsymbol{u} = \boldsymbol{R}\boldsymbol{u} \tag{3}$$

in which $\boldsymbol{R} = -\boldsymbol{K}^{-1}\boldsymbol{K}_c$ and is called the influence matrix.

Substituting Eq. (2) and Eq. (3) into Eq. (1) results in

$$\boldsymbol{M}\ddot{\boldsymbol{x}}^{\mathrm{d}} + \boldsymbol{C}\dot{\boldsymbol{x}}^{\mathrm{d}} + \boldsymbol{K}\boldsymbol{x}^{\mathrm{d}} = -(\boldsymbol{MR} + \boldsymbol{M}_{\mathrm{c}})\ddot{\boldsymbol{u}} - (\boldsymbol{CR} + \boldsymbol{C}_{\mathrm{c}})\dot{\boldsymbol{u}} \quad (4)$$

The damping term on the right-hand side of Eq. (4) is omitted because it is small compared to the inertial force term. Noting that $\boldsymbol{M}_c = \boldsymbol{0}$, Eq. (4) becomes

$$\boldsymbol{M}\ddot{\boldsymbol{x}}^{\mathrm{d}} + \boldsymbol{C}\dot{\boldsymbol{x}}^{\mathrm{d}} + \boldsymbol{K}\boldsymbol{x}^{\mathrm{d}} = -\boldsymbol{MR}\ddot{\boldsymbol{u}} \quad (5)$$

3 Analysis of the *i*-th mode

Equation (5) can be uncoupled by the mode-superposition method. Setting

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}\boldsymbol{y}(t) \quad (6)$$

and multiplying by $\boldsymbol{\Phi}$ on both sides of Eq. (5) results in

$$\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi}\ddot{\boldsymbol{y}} + \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{\Phi}\dot{\boldsymbol{y}} + \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\Phi}\boldsymbol{y} = -\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{MR}\ddot{\boldsymbol{u}}(t) \quad (7)$$

and a set of n-uncoupled equations

$$\ddot{y}_i(t) + 2\zeta_i\omega_i\dot{y}_i(t) + \omega_i^2 y_i(t) = -\beta_i\ddot{u}_i(t) \qquad (i=1,2\ldots\ldots n) \quad (8)$$

in which β_i is the participation coefficient for the ith vibration mode under simple support excitation.

$$\beta_i = \frac{\boldsymbol{\phi}_i^{\mathrm{T}}\boldsymbol{MH}}{\boldsymbol{\phi}_i^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\phi}_i} \quad (9a)$$

$$\ddot{u}_i(t) = \boldsymbol{A}_i\ddot{\boldsymbol{u}}(t) = \sum_{k=1}^{m} A_{ik}\ddot{u}_k \qquad (k=1, 2...\mathrm{m}) \quad (9b)$$

$$\boldsymbol{A}_i = \frac{\boldsymbol{\phi}_i^{\mathrm{T}}\boldsymbol{MR}}{\boldsymbol{\phi}_i^{\mathrm{T}}\boldsymbol{MH}} = A_{ik} \qquad (k=1, 2...\mathrm{m}) \quad (9c)$$

where $\boldsymbol{A}_i$ is a row vector with m elements, m is the number of supports, and $\boldsymbol{H}$ is a column vector with n elements indicating the direction of the earthquake excitation. The values of the elements in $\boldsymbol{H}$ are either 1 or 0 depending on whether or not the direction of excitation is consistent with the direction of the degree of freedom in $\boldsymbol{M}$.

Equation (8) is quite similar to the equation of motion under uniform excitation. Thus, the spectral density function can be used to relate the structural dynamic response to the ground motion, as shown in Eq. (10)

$$G_{\ddot{y}_i}^{\mathrm{p}}(\omega) = \beta_i^2\left|H_i(\omega)\right|^2 G_{\ddot{u}_i}(\omega) \quad (10)$$

where $G_{\ddot{y}_i}^{\mathrm{p}}(\omega)$ is the spectral density function of the structure's response $\ddot{y}_i$ under multisupport excitation; the superscript p indicates multiple support excitation; $G_{\ddot{u}_i}(\omega)$ is the spectral density function of the ground exciting $\ddot{u}_i$ (Eq. (9b)); $H_i(\omega)$ is a transfer function with the expression $H_i(\omega) = \dfrac{-\omega^2}{-\omega^2 + i\cdot 2\xi_i\omega_i\omega + \omega_i^2}$.

Based on the definition of the coherency function, one obtains

$$\gamma_{kl}(\omega) = \frac{G_{\ddot{u}_k\ddot{u}_l}(\omega)}{(G_{\ddot{u}_k\ddot{u}_k}(\omega)G_{\ddot{u}_l\ddot{u}_l}(\omega))^{1/2}} \quad (11)$$

in which $G_{\ddot{u}_k\ddot{u}_l}(\omega)$ is the cross-power spectral density function between points k and l.

In general, the spans of long-span spatial structures are less than 300 m. Therefore, all supports can be assumed to stand on the same soil layers. Also, the distance of the seismic wave from the bedrock to the surface is generally much longer than the separation between the structural supports. Hence, the auto-power spectral density $G_{\ddot{u}_g}(\omega)$ can be supposed to be the the same at each support, and Eq. (11) can be translated into

$$G_{\ddot{u}_k\ddot{u}_l}(\omega) = \gamma_{kl}(\omega)G_{\ddot{u}_{\mathrm{g}}}(\omega) \quad (12)$$

Based on Eq. (12) and Eq. (9b), one gets (Berrah and Kausel, 1992)

$$G_{\ddot{u}_i}(\omega) = \sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik}A_{il}G_{\ddot{u}_k\ddot{u}_l}(\omega) = (\sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik}A_{il})\gamma_{kl}(\omega)G_{\ddot{u}_{\mathrm{g}}}(\omega) \quad (13)$$

Substituting Eq. (13) into Eq. (10), Eq. (10) becomes

$$G_{\ddot{y}_i}^{\mathrm{p}}(\omega) = (\sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik}A_{il})\beta_i^2\left|H_i(\omega)\right|^2\gamma_{kl}(\omega)G_{\ddot{u}_{\mathrm{g}}}(\omega) \quad (14)$$

The relationship of the spectral density function to the structural response under uniform excitation and the ground motion can be expressed as

$$G_{\ddot{y}_i}(\omega) = \beta_i^2\left|H_i(\omega)\right|^2 G_{\ddot{u}_{\mathrm{g}}}(\omega) \quad (15)$$

According to random process theory, the variances of the structural response under uniform excitation and multisupport excitation are

$$\sigma_{\ddot{y}_i}^{\mathrm{p}\,2} = \int_0^{+\infty} G_{\ddot{y}_i}^{\mathrm{p}}(\omega)\mathrm{d}\omega \quad (16a)$$

$$\sigma_{\ddot{y}_i}^{\;2} = \int_0^{+\infty} G_{\ddot{y}_i}(\omega)\mathrm{d}\omega \quad (16b)$$

Using Eq. (14), Eq. (15), Eq. (16a) and Eq. (16b), one gets

$$\sigma_{\ddot{y}_i}^{\mathrm{p}\,2} = \sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik}A_{il}\rho_{ikl}^{\;2}\sigma_{\ddot{y}_i}^{\;2} \quad (17)$$

in which

$$\rho_{ikl}{}^2 = \frac{\int_0^{+\infty} \mathrm{Re}(\gamma_{kl}(\omega)\left|H_i(\omega)\right|^2 G_{\ddot{u}_g}(\omega))\mathrm{d}\omega}{\int_0^{+\infty} \left|H_i(\omega)\right|^2 G_{\ddot{u}_g}(\omega)\mathrm{d}\omega} \tag{18}$$

$D(\omega_i,\zeta_i)$ denotes the response spectrum in the case of uniform excitation, while $D^{\mathrm{p}}(\omega_i,\zeta_i)$ is the response spectrum in the case of multisupport excitation. The maximum responses and the response variances of stationary random processes are related as follows (Berrah and Kausel, 1993)

$$D^{\mathrm{p}}(\omega_i,\xi_i) = p_{\ddot{y}_i}^{\mathrm{p}}\sigma_{\ddot{y}_i}^{\mathrm{p}} \tag{19a}$$

$$D\ (\omega_i,\xi_i) = p_{\ddot{y}_i}\sigma_{\ddot{y}_i} \tag{19b}$$

For practical considerations, the peak factors are simplified by assuming that the peak factors of the nodal responses are the same (Berrah and Kausel, 1993; Kiureghian, 1981; Smeby and Kiureghian, 1985). Substituting Eq. 19(a) and Eq. 19(b) into Eq. (17) results in

$$D^{\mathrm{p}}(\omega_i,\zeta_i) = [\sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik}A_{il}\rho_{ikl}{}^2]^{1/2} D\ (\omega_i,\zeta_i) \tag{20}$$

Equation (20) relates the response spectrum of mode i under uniform excitation to that of mode i under multisupport excitation.

4 Modal combination rules

Using the derivation method in Berrah (1992) and defining $\ddot{v}_i(t) = -\beta_i\ddot{u}_i(t)$, Eq. (8) can be expressed in another form

$$\ddot{y}_i(t) + 2\zeta_i\omega_i\dot{y}_i(t) + \omega_i^2 y_i(t) = \ddot{v}_i(t) \tag{21}$$

From random vibration theory, the power spectral density function of the acceleration response $\ddot{z}(t)$ associated with the zth structural degree is

$$G_{\ddot{z}}^{\mathrm{p}}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n}\phi_{zi}\phi_{zj}\,\mathrm{Re}(H_i^*(\omega)H_j(\omega)G_{\ddot{v}_i\ddot{v}_j}(\omega)) \tag{22}$$

Then, the variance of $\ddot{z}(t)$ can be expressed as

$$\sigma_{\ddot{z}}^{\mathrm{p}2} = \sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{Re}[\int_0^{+\infty} (H_i^*(\omega)H_j(\omega))\phi_{zi}\phi_{zj}G_{\ddot{v}_i\ddot{v}_j}(\omega)\mathrm{d}\omega] \tag{23}$$

Define

$$\sigma_{ij}^{\mathrm{p}2} = \mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)G_{\ddot{v}_i\ddot{v}_j}(\omega)\mathrm{d}\omega \tag{24a}$$

$$\sigma_i^{\mathrm{p}2} = \int_0^{+\infty}\left|H_i(\omega)\right|^2 G_{\ddot{v}_i}(\omega)\mathrm{d}\omega \tag{24b}$$

Thus

$$\rho_{ij}^{\mathrm{p}} = \frac{\sigma_{ij}^{\mathrm{p}}}{(\sigma_i^{\mathrm{p}}\sigma_j^{\mathrm{p}})^{1/2}} \tag{25}$$

$$\sigma_{\ddot{z}}^{\mathrm{p}} = [\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_i\beta_j\phi_{zi}\phi_{zj}\rho_{ij}^{\mathrm{p}2}\sigma_i^{\mathrm{p}}\sigma_j^{\mathrm{p}}]^{1/2} \tag{26}$$

Random vibration theory allows the maximum responses and the response variances of a stationary random process to be related by the peak factors (Berrah and Kausel, 1993). For practical considerations, the peak factors are simplified by assuming they are coincident for the various modes (Berrah and Kausel, 1993; Kiureghian, 1981; Smeby and Kiureghian, 1985), so one gets

$$D_{\ddot{z}}^{\mathrm{p}} = [\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_i\beta_j\phi_{zi}\phi_{zj}\rho_{ij}^{\mathrm{p}2}D_i^{\mathrm{p}}D_j^{\mathrm{p}}]^{1/2} \tag{27}$$

Combining Eq. (9b) and Eq. (21) results in the following equation:

$$G_{\ddot{v}_i\ddot{v}_j}(\omega) = \beta_i\beta_j\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{jl}G_{\ddot{u}_k\ddot{u}_l}(\omega) \tag{28}$$

In the case of multiple support excitation, substituting Eq. (28) into Eq. (24a) gives

$$\sigma_{ij}^{\mathrm{p}2} = \mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)\beta_i\beta_j\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{jl}G_{\ddot{u}_k\ddot{u}_l}(\omega)\mathrm{d}\omega$$

$$= \beta_i\beta_j\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{jl}\,\mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)G_{\ddot{u}_k\ddot{u}_l}(\omega)\mathrm{d}\omega \tag{29}$$

while in the case of uniform excitation, we have

$$\sigma_{ij}{}^2 = \beta_i\beta_j\,\mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)G_{\ddot{u}_g}(\omega)\mathrm{d}\omega \tag{30}$$

Defining

$$\rho_{ijkl}{}^2 = \frac{\mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)G_{\ddot{u}_k\ddot{u}_l}(\omega)\mathrm{d}\omega}{\mathrm{Re}\int_0^{+\infty} H_i^*(\omega)H_j(\omega)G_{\ddot{u}_g}(\omega)\mathrm{d}\omega} \tag{31}$$

and noting Eq. (30), Eq. (29) can be rewritten as

$$\sigma_{ij}^{\mathrm{p}2} = \beta_i\beta_j\sigma_{ij}{}^2\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{jl}\rho_{ijkl}{}^2 \tag{32}$$

Substituting Eq. (16) and Eq. (32) into Eq. (25) results in

$$\rho_{ij}^{\mathrm{p}} = \frac{\sigma_{ij}^{\mathrm{p}}}{(\sigma_i^{\mathrm{p}}\sigma_j^{\mathrm{p}})^{1/2}} =$$

$$\frac{\sigma_{ij}}{(\sigma_i\,\sigma_j)^{1/2}}\frac{\left[\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{jl}\rho_{ijkl}{}^2\right]^{1/2}}{(\sum_{k=1}^{m}\sum_{l=1}^{m}A_{ik}A_{il}\rho_{ikl}{}^2)^{1/4}(\sum_{k=1}^{m}\sum_{l=1}^{m}A_{jk}A_{jl}\rho_{jkl}{}^2)^{1/4}} \tag{33}$$

Defining $\rho_{ij} = \dfrac{\sigma_{ij}}{(\sigma_i \sigma_j)^{1/2}}$ in the case of uniform excitation, Eq. (33) becomes

$$\rho_{ij}^{\mathrm{p}} = \rho_{ij} \frac{[\sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik} A_{jl} \rho_{ijkl}{}^2]^{1/2}}{(\sum_{k=1}^{m}\sum_{l=1}^{m} A_{ik} A_{il} \rho_{ikl}{}^2)^{1/4} (\sum_{k=1}^{m}\sum_{l=1}^{m} A_{jk} A_{jl} \rho_{jkl}{}^2)^{1/4}} \quad (34)$$

Equation (34) shows the relationship between the combination rules in the cases of multiple support excitation and uniform excitation.

5 Simplified response spectrum of multiple support excitations

Kiureghian and Neuenhofer (1992) show that the coupling between the pseudostatic and dynamic responses almost vanishes for structures with first vibration periods less than 2 s. In fact, the first vibration periods of most spatially latticed structures are less than 2 s, and the coupling term can consequently be neglected in practical considerations. Hence, a simplified acceleration response spectrum for multiple support excitations can be expressed as follows

$$E[\max|\ddot{z}(t)|] = [\sum_{k=1}^{m}\sum_{l=1}^{m} r_{zk} r_{zl} \rho_{\ddot{u}_k \ddot{u}_l}{}^2 \ddot{u}_{k,\max} \ddot{u}_{l,\max} + \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}^{\mathrm{p}2} \beta_i \beta_j \phi_{zi} \phi_{zj} D_i^{\mathrm{p}} D_j^{\mathrm{p}}]^{1/2} \quad (35)$$

where r_{zk} and r_{zl} are the components of matrix $\boldsymbol{R}$ (see Eq. (3)) associated with row z and column k and with row z and column l, respectively, and $\ddot{u}_{k,\max}$ and $\ddot{u}_{l,\max}$ denote the peak acceleration inputs for supports k and l, respectively. The pseudostatic displacement coupling coefficient is $\rho_{\ddot{u}_k \ddot{u}_l} = \mathrm{Re}\int_0^{+\infty} \gamma_{kl}(\omega) G_{\ddot{u}_{\mathrm{g}}}(\omega)\mathrm{d}\omega / \int_0^{+\infty} G_{\ddot{u}_{\mathrm{g}}}(\omega)\mathrm{d}\omega$. The derivation of the first term in Eq. (35) is not the focus of this paper, but it can be seen in detail in Kiureghian and Neuenhofer (1992). The expressions for D_i^{p} and D_j^{p} are shown in Eq. (20).

There is a clear physical interpretation of Eq. (35): the maximum structural response includes two parts—the pseudostatic response and the dynamic response. The expression for the pseudostatic response given in Kiureghian and Neuenhofer (1992) is adopted in this paper. The pseudostatic response is caused by different displacements of the support points and differs from uniform excitation. It is easy to predict that if the seismic input were a uniform excitation, the pseudostatic displacement associated with the first term in Eq. (35) would become a rigid body displacement and would not cause any internal forces in the structure. The maximum structural response would then be caused only by the action of ground acceleration on the structure, and the response spectrum method for multisupport excitation would reduce to that for uniform excitation. The multiple support excitation method can thus be regarded as a modification of the traditional uniform excitation method. Furthermore, the response spectrum method under multisupport excitation can be taken as an extension of the modal combination rules of the traditional response spectrum method.

6 Discussions on the simplification of coherency coefficients

Equation (35) shows that both the pseudostatic response and the dynamic response are accounted for by the coherency coefficients, such as $\rho_{\ddot{u}_k \ddot{u}_l}$, ρ_{ikl} and ρ_{ij}. All of the values of these coefficients were obtained by integration. As more vibration modes and supports are included in the combination, it takes longer to compute $\rho_{\ddot{u}_k \ddot{u}_l}$, ρ_{ikl} and ρ_{ij}. To enable the use of the multisupport response spectrum method in practical engineering applications, simplifications of the coherency coefficients are investigated.

The Kanai-Tajimi spectrum is used as the auto-power spectral density function $G_{\ddot{u}_{\mathrm{g}}}(\omega)$

$$G_{\ddot{u}_{\mathrm{g}}}(\omega) = \frac{\omega_{\mathrm{g}}^4 + 4\xi_{\mathrm{g}}^2 \omega_{\mathrm{g}}^2 \omega^2}{\left(\omega_{\mathrm{g}}^2 - \omega^2\right) + 4\xi_{\mathrm{g}}^2 \omega_{\mathrm{g}}^2 \omega} G_0 \quad (36)$$

where G_0 stands for a scale factor. The values of ω_{g} and ξ_{g} are listed in Table 1 (Sun and Jiang, 1991).

The following coherency function model is adopted in this paper (Liu *et al*., 2004):

$$\gamma_{kl}(\mathrm{d}x, \mathrm{d}y, \omega) = \exp(-\beta_1 \mathrm{d}x - \beta_2 \mathrm{d}y) \bullet \exp((-\alpha_1 \mathrm{d}x^{\frac{1}{2}} - \alpha_2 \mathrm{d}y^{\frac{1}{2}})(\frac{\omega}{2\pi})^2) \exp(\mathrm{i}\frac{\omega \mathrm{d}y}{V_{\mathrm{app}}}) \quad (37)$$

where $\alpha_1 = a/f + bf + c$, $\alpha_2 = d/f + ef + g$, β_1 = 0.0014, β_2 = 0.001951, a = 0.0686, b = 0.0026, c = 0.0316, d = 0.0015, e = 0.0001, g = 0.0018, and dx and dy are the projection distances between points k and l perpendicular to and parallel to the wave direction, respectively.

Table 1 Values of ω_{g} and ξ_{g} corresponding to different site conditions

	Stiff soils	Middle stiff soils	Soft soils
ω_{g}	17.9	13.3	8.8
ξ_{g}	0.45	0.49	0.54

6.1 Simplification of coefficient $\rho_{\ddot{u}_k\ddot{u}_l}$

In traditional simplified calculations, the Kanai-Tajimi spectrum model, which is used to describe seismic ground motion, is usually replaced with white noise. Figure 1 shows a comparison between white noise and the Kanai-Tajimi spectrum model for three kinds of soils: stiff soils, moderately stiff soils and soft soils.

Figure 1 indicates that as the distance between two points increases, the value of $\rho_{\ddot{u}_k\ddot{u}_l}$ decreases gradually, which is similar to the trend of the coherency function of seismic ground motion. As shown in Fig. 1, the values of $\rho_{\ddot{u}_k\ddot{u}_l}$ are higher for soft soils and moderately stiff soils than for stiff soils, because the coherency function of the ground motion decreases with increasing frequency, and so the low frequency components have greater influences on $\rho_{\ddot{u}_k\ddot{u}_l}$. Consequently, it is invalid to use white noise for the ground acceleration model. Otherwise, the values of $\rho_{\ddot{u}_k\ddot{u}_l}$ will be overestimated, especially for stiff soils, for which the maximum error can reach 100%.

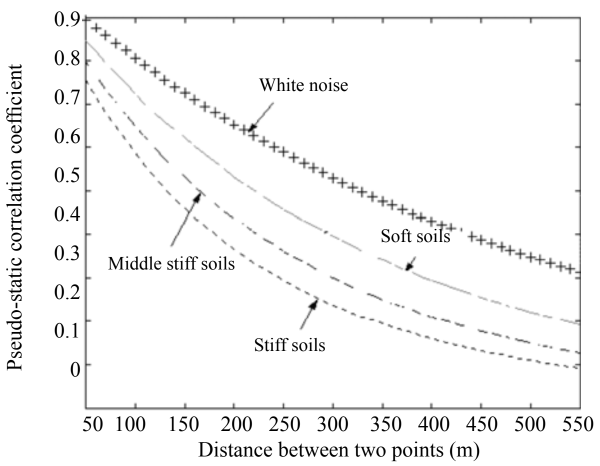


Fig. 1 Simplification of $\rho_{\ddot{u}_k\ddot{u}_l}$

6.2 Simplification of coefficient ρ_{ikl}

The coefficient ρ_{ikl} is simplified by approximating the ground motion excitation as white noise. This approximation is justifiable in light of the frequency response function. The procedure is as follows:

The Kanai-Tajimi spectrum model (i.e., the ground motion model) is replaced by white noise, and comparisons are made for stiff soils, moderately stiff soils, and soft soils (see Fig. 2). As shown in Fig. 2, the errors introduced by replacing the Kanai-Tajimi spectrum model with white noise are very small for stiff soils and moderately stiff soils. Nevertheless, there are large errors in soft soils when the frequency is above 1.5 Hz. These errors are acceptable, however, in practical applications because the structural response caused by the high frequency components is small.

It follows from Eq. (18) that the frequency response function $\left|H_i(\omega)\right|^2$ (see Eq. (38)) can be regarded as a filter on each modal response. Concretely speaking, $\left|H_i(\omega)\right|^2$ indicates what fraction of the excitation energy in a given frequency is transferred to mode i. In other words, $(1-\left|H_i(\omega)\right|^2)$ stands for the filtered energy, as shown in Fig. 3.

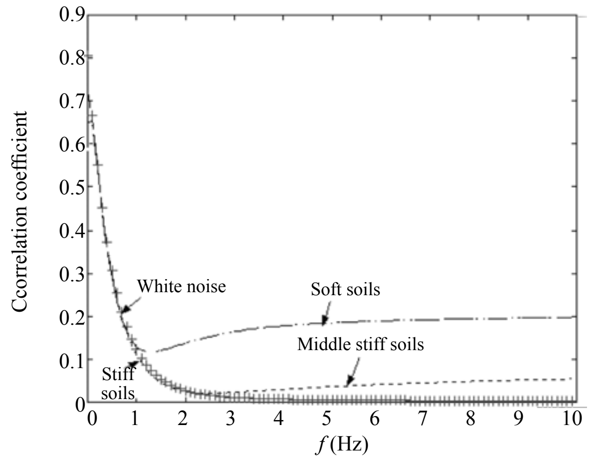


Fig. 2 Simplification of the ground motion model for ρ_{ikl}

$$\left|H_i(\omega)\right|^2 = H_i^*(\omega) * H_i(\omega) = [(\omega_i^2-\omega^2)^2+(2\zeta_i\omega_i\omega)^2]^{-1} \tag{38}$$

where ω_i and ζ_i denote the circular frequency and the damping ratio of mode i, respectively.

Since the value of ζ_i is far less than 1.0, the function $\left|H_i(\omega)\right|^2$ has a sharp peak value (i.e., $(2\zeta_i\omega_i\omega)^{-2}$) at the frequency ω_i, and $(2\zeta_i\omega_i\omega)^{-2}$ decreases to $\frac{1}{2}(2\zeta_i\omega_i\omega)^{-2}$ within a small change in frequency $\zeta_i\omega_i$ (see Fig. 3). Thus, the main contributions from $\int_0^{+\infty}\mathrm{Re}(\left|H_i(\omega)\right|^2\gamma_{kl}(\omega)G_{\ddot{u}_g}(\omega))\mathrm{d}\omega$ and $\int_0^{+\infty}\left|H_i(\omega)\right|^2G_{\ddot{u}_g}(\omega)\mathrm{d}\omega$ arise at frequencies near ω_i, and the contributions from other frequencies are small.

Because of the small value of the structural damping ratio (for example, the damping ratio of spatially latticed structures is always within 0.05, far less than 1.0), structures have strong frequency selectivity for seismic waves. It is a fair approximation that only the frequency

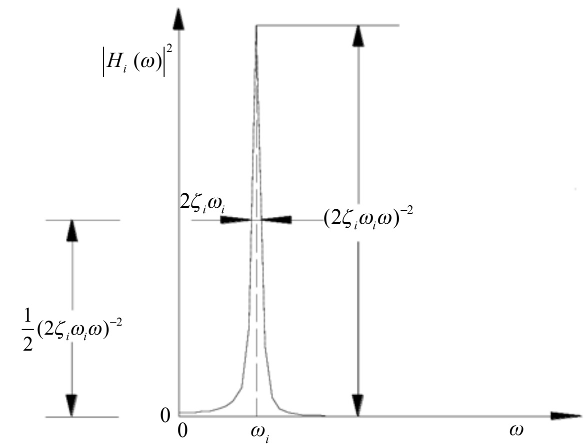


Fig. 3 Frequency response function $\left|H_i(\omega)\right|^2$

component of a seismic wave that is coincident with the structural frequency will contribute to the response of a particular vibration mode and will make no contribution to the responses of other vibration modes. In other words, the other frequency components of the seismic wave will not contribute to the response corresponding to this vibration mode. Therefore, Eq. (18) can be transformed into

$$\rho_{ikl} = \frac{\mathrm{Re}(\left|H_i\,(\omega_i)\right|^2 \gamma_{kl}(\omega_i) G_{\ddot{u}_g}(\omega_i))}{\left|H_i\,(\omega_i)\right|^2 G_{\ddot{u}_g}(\omega_i)} \tag{39}$$

Comparisons are made with four kinds of soil to verify the precision of Eq. (39), and the results are shown in Fig. 4.

Figures 4(a) to 4(d) show that the errors caused by simplifying the frequency response function are small enough to be ignored in stiff soils, moderately stiff soils and white noise. In soft soils, the errors introduced by this simplification are large at high frequencies. Nonetheless, the response caused by the high frequencies contributes only a small amount of the total structural response, and the values of ρ_{ikl} are low at high frequencies, so the global errors can be neglected. As a result, this simplification of ρ_{ikl} is acceptable.

6.3 Simplification of coefficient ρ_{ij}

ρ_{ij} is the combination coefficient for mode i and mode j under uniform excitation. In the simplified calculation, the ground motion spectrum is assumed to be white noise rather than the Kanai-Tajimi spectrum model. Comparisons are made based on four kinds of soils (see Fig. 5).

It can be seen from Fig. 5 that it is feasible to simplify the seismic excitations as white noise for stiff and moderately stiff soils. In soft soils, the errors introduced by this simplification are small when the frequencies of two modes are close to each other, but become large when the two frequencies are far from each other. In present design codes, this simplified method is widely used for practical analysis with simple support excitations. Figure 5 shows that this simplification is generally acceptable.

7 Application of the multisupport response spectrum method in practical engineering

7.1 AMECO (aircraft maintenance engineering company) hangar

The Capital International Airport hangar is 306 m

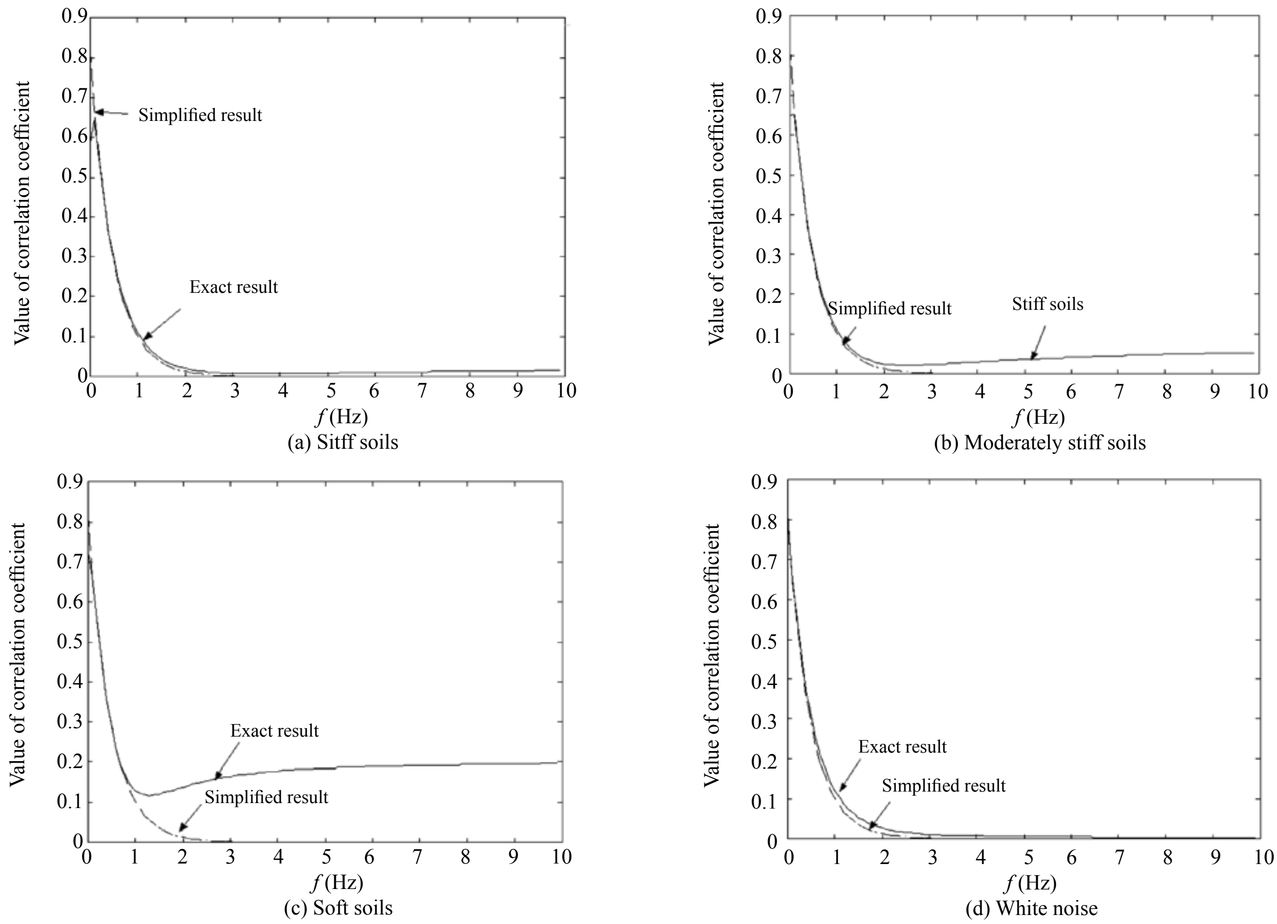


Fig. 4 Simplification of the frequency response function for ρ_{ikl}

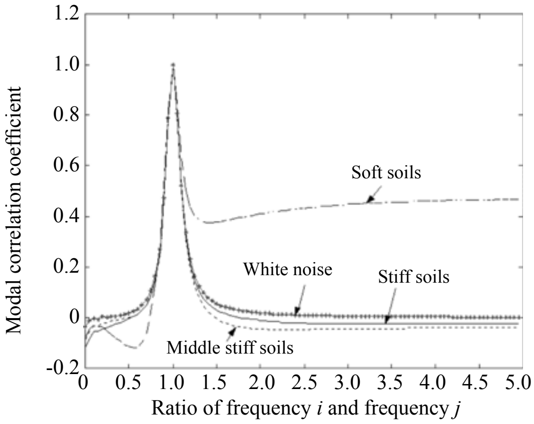


Fig. 5 Simplification of the combination coefficient ρ_{ij}

long (153 m+153 m), 90 m wide and 40 m tall. The total area is 35,993 m^2, and the total weight is 5,500 t. This structure is the largest hangar in China and one of the largest hangars in the world. The steel roof of the hangar is composed of three substructures: the gate bridge, the middle-beam truss, and the diagonal square pyramid grid structure. The whole structure is supported by reinforced concrete columns.

Photos of the hangar structure and the structural plane arrangements are shown in Figs. 6 through 8. Figure 8 shows that only one column is located on the axis of the door, and there are no inner columns so that four 747-400-type airplanes can be repaired inside the hangar structure.

The structural model (see Fig. 7) possesses 24 supports, 4,318 joints, and 14,902 members. The representative gravity load value is 21.0 kN on each joint. The first 90 natural frequencies of the system

Fig. 6 AMECO hangar

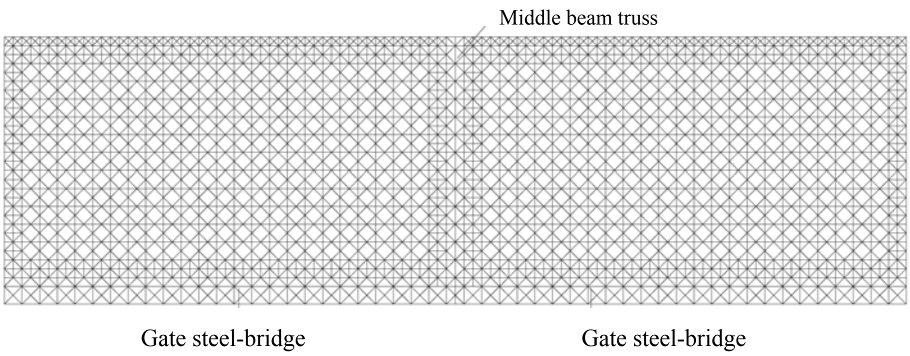


Fig. 7 Structural plan arrangement

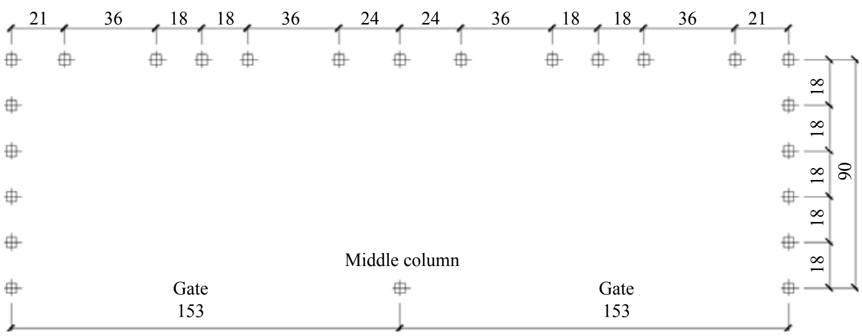


Fig. 8 Column net plan arrangement (unit: m)

are included. The damping ratios are assumed to be a constant 0.02 in this paper. The Kanai-Tajimi spectrum model (i.e., Eq. (36)) with $\omega_g = 23.3\,\mathrm{rad/s}$, $\xi_g = 0.42$ and $G_0 = 36.8\,\mathrm{cm^2/s^2}$ is adopted for the ground acceleration. Vertical earthquake excitation is considered, and Eq. (37) is used as the vertical coherency function. The maximum vertical seismic coefficient $\alpha_{v\max}$ is 0.137, in accordance with the Chinese seismic design code for buildings. The apparent velocity V_{app} is set to 500 m/s. Table 2 shows the calculation results.

The magnitudes and distributions of internal forces obtained from the response spectrum method and the time history analysis are compared in Figs. 9 and 10.

The ratios of the axial forces from the two methods are illustrated in Fig. 9. There are 1,192 members with the ratio less than 1, and 2,184 members with a ratio the ratio at around 1. Members with the ratio at about 1.2 and 1.4 are 3,281 and 814, respectively. For ratios around 1.6 and 1.8, the corresponding members are 563 and 310, respectively. In addition, five members possess a ratio larger than 2. Obviously, the number of members with a ratio around 1.2 is the largest, which accounted for 38.9 percent of the total members. The members with a ratio smaller than 1 are fewer, accounting for 14.1 percent. Nearly 85.9 percent of the total members have ratios that fall between 1 and 2.

The distributions for ratios of axial forces are shown in Fig. 10. For the top and bottom layers, members with ratios around 1 are greater than those with ratios larger than 1. Members with ratios greater than 1 are more common in the middle layer. However, their axial forces are smaller and have little influence on the integral safety degree of the structure. For most web members of the upper and lower layers, ratios are around 1 or less. It is concluded that for the space trusses, the results obtained from the multi-support response spectrum method are approximate to and slightly larger than those from the multi-support time history method.

The numerical results using multiple support excitation and simple support excitation are compared (see Fig. 11). This paper also attempts to explain the differences between them to provide some guidance for future designs.

These results indicate that the seismic responses under multiple support excitations are larger than those under simple support excitation in the upper and lower layers near the supports. In the middle layer, the values of the internal forces of most members under multiple support excitations are higher than those under simple support excitation. The internal forces on some members are more than doubled. The responses of the web members located in the peripheral areas of the roof under multisupport excitations are stronger than those under uniform excitation. The main reason for this phenomenon is the inconsistent movements of the supports, which lead to a large pseudostatic effect. Thus, it can be concluded that the internal forces of the space truss structure members near the supports are underestimated under simple support excitation. These regions are always weak locations in seismic design, so they merit special attention.

Table 2 Number of members subjected to different ranges of internal forces

Range (kN)	Top layer	Middle layer	Bottom layer	Upper web members	Lower web members
0–10	2	105	1	1228	1179
10–30	483	439	607	552	589
30–50	749	127	671	23	32
50–70	194	4	137	3	4
≥70	0	0	12	5	6
Total	1428	675	1428	1811	1810

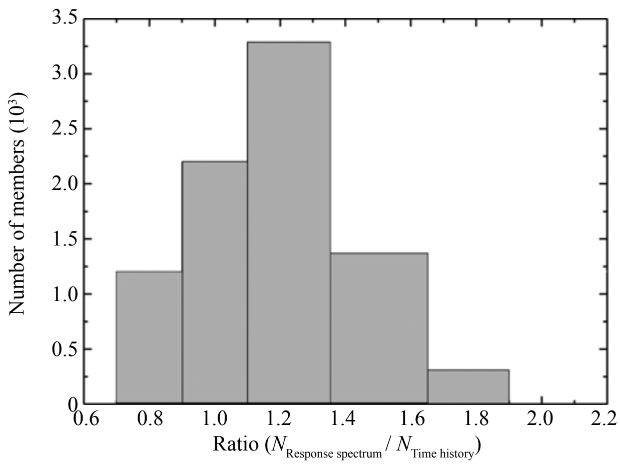
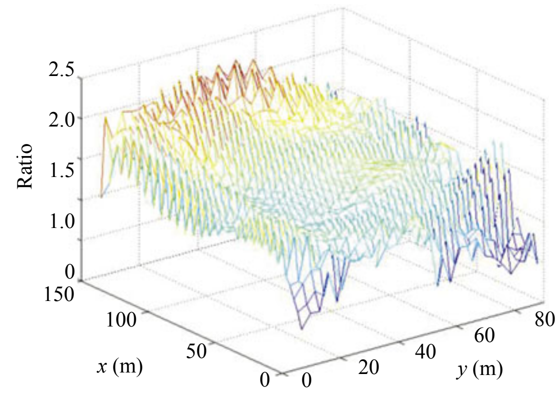


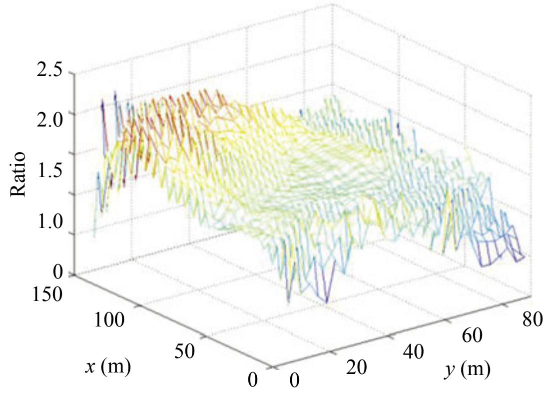
Fig. 9 Comparisons of axial force amplitude between response spectrum and time history methods

7.2 Stadium at the Harbin Institute of Technology

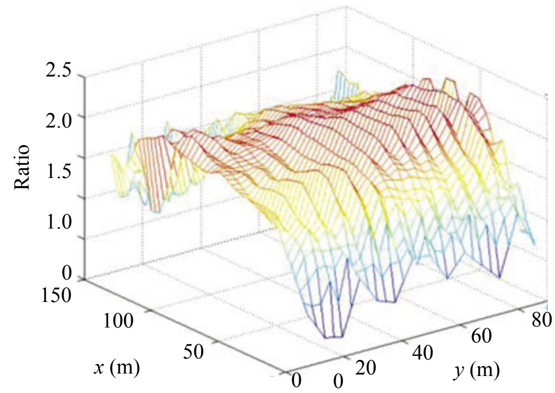
Figures 12 and 13 show the stadium at the Harbin Institute of Technology. Its main body is a double-layer shell 168.3 m long, 35.12 m wide, and 4,300 m^2 in area, and the span in the direction of wave propagation is 151.2 m. There are 86 supports (see Fig. 14), 847 joints, and 3,134 members. The representative gravity load



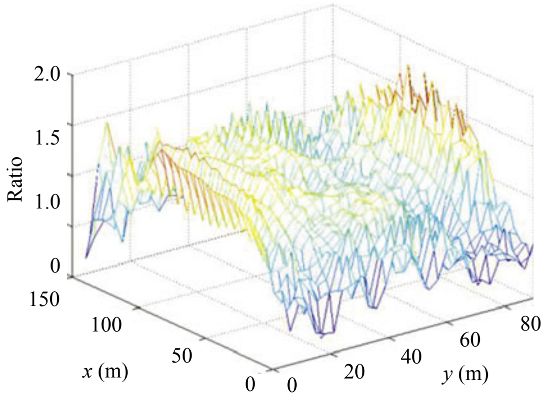
(a) Top layer



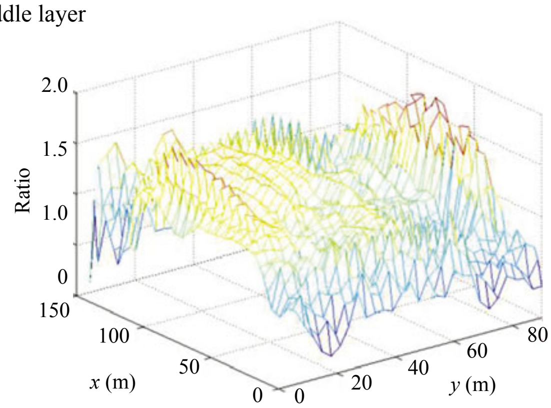
(b) Bottom layer



(c) Middle layer

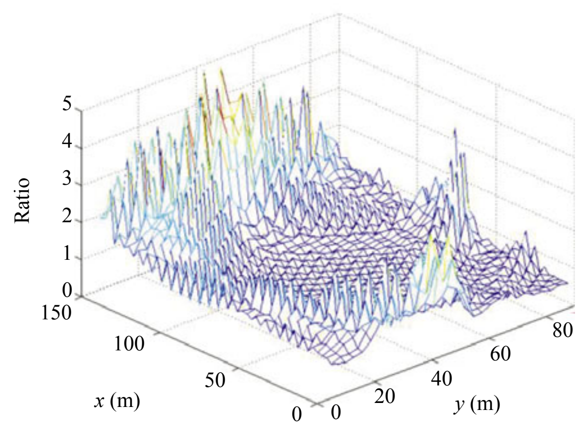


(d) Upper web members

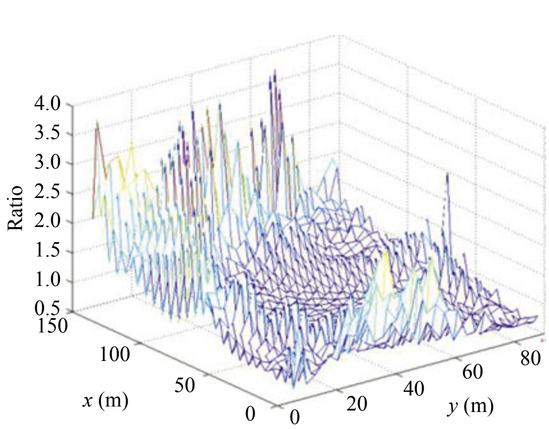


(e) Lower web members

Fig. 10 Distribution of axial force ratios (ratio = response spectrum / time history)

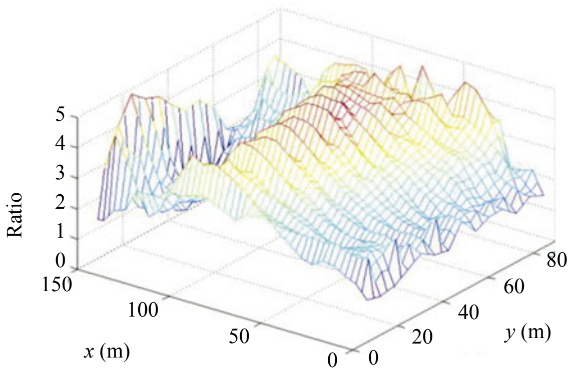


(a) Top layer

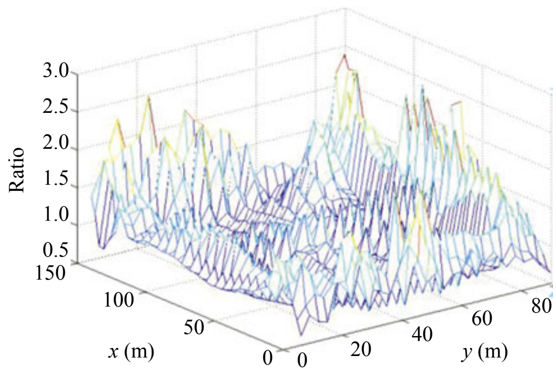


(b) Bottom layer

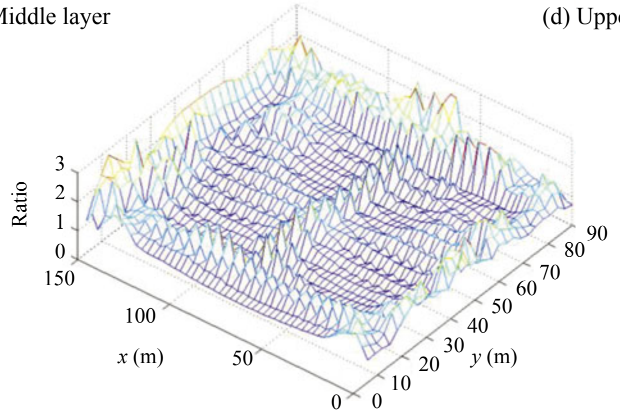
Fig. 11 Distribution of axial force ratios (ratio = multisupport / simple support)



(c) Middle layer



(d) Upper web members



(e) Lower web members

Fig. 11 Continued

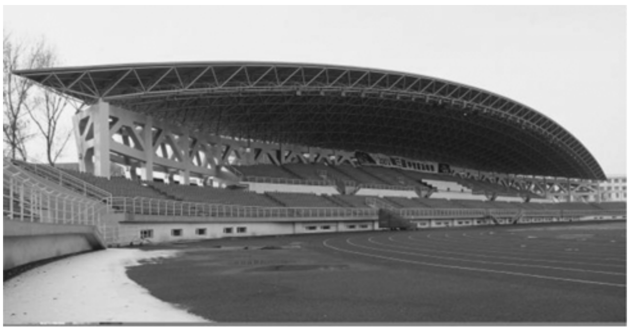


Fig. 12 Stadium at the Harbin Institute of Technology

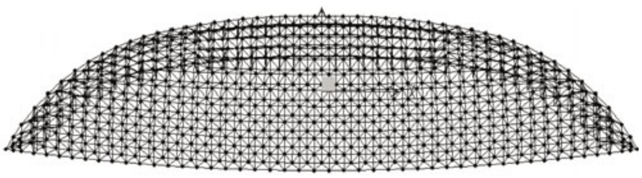


Fig. 13 Structural model

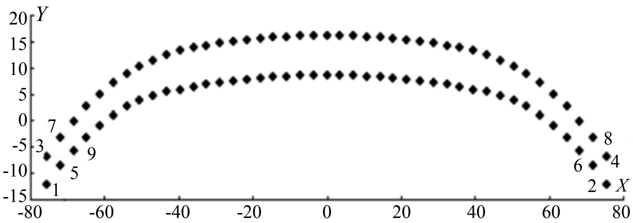


Fig. 14 Layout of the stadium supports

value is 0.42 kN/m^2. The first 60 natural frequencies of the system are selected according to an iterative trial calculation. The mode damping ratios are 0.02. The power spectral density of the ground acceleration and the coherency function model are the same as those in Section 7.1. The results are shown in Table 3.

A comparison of the internal forces and their distributions from the response spectrum method and the time history analysis method was still performed, as shown in Figs. 15 and 16.

Figure 15 shows the ratios of axial forces from the

Table 3 Number of members subjected to different ranges of internal forces

Range (kN)	Top layer	Web members	Bottom layer
0–10	595	1277	625
10–20	259	211	82
20–30	12	55	23
30–40	0	28	16
≥40	0	17	4
Total	866	1588	750

two methods. Note that members with ratios of less than 1 are 219, and those around 1 are 492. Members with ratios close to 1.2 and 1.4 are 1,024 and 785, respectively. Members corresponding to ratios around 1.6 and 1.8 are 308 and 365, respectively. In addition, there are 11 members which have a ratio larger than 2. It can be seen that members with a ratio around 1.2 account for the greatest number, which is about 31.96% of the total. The proportion of members with a ratio of less than 1 is 6.84%. Members with a ratio between 1 and 2 are nearly 93% of the total amount.

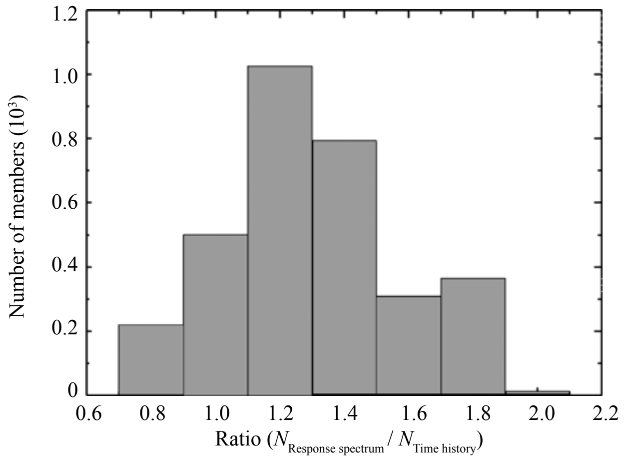


Fig. 15 Comparisons of axial force amplitude between response spectrum and time history methods

Figure 16(a) indicates that the ratios of axial forces for the most top chords are close to 1. For some members at the two longitudinal ends of the reticulated shell, the ratios are less than 1. However, the internal forces of these members are so small that the selected sections always have enough safety margins and the whole structure is confirmed to be safe. The same case is also observed in Fig. 16(b). In Fig. 16(c), the ratios of axial forces for most web members are close to or slightly larger than 1. Therefore, the multi-support response spectrum method can provide results that are close to and a little larger than those of the multi-support time history method for the reticulated shells.

The numerical results using multiple support excitation and simple support excitation are compared in Fig. 17.

It is clear from Fig. 17 that the structural responses under multiple support excitations are larger than those under uniform excitation in the upper layer, lower layer and web members near the supports because multisupport excitations account for the effects of the inconsistent motions of each support. In the central part of the structure, the responses under multiple support excitations are similar to those under simple support excitation. The internal forces in the reticulated shell members near the supports are underestimated under simple support excitation. The areas near the supports are important but weak regions that deserve special emphasis in the design of reticulated structures.

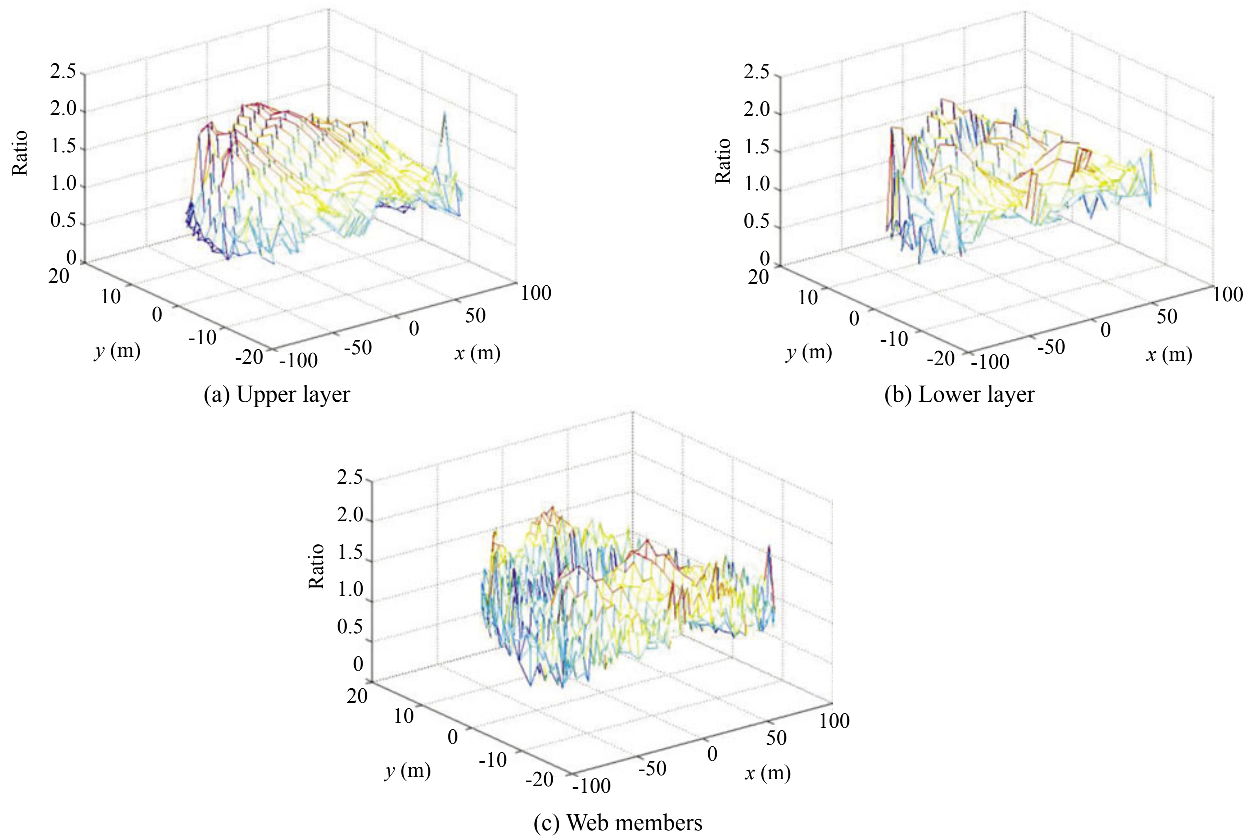


Fig. 16 Distribution of axial force ratios (ratio = response spectrum / time history)

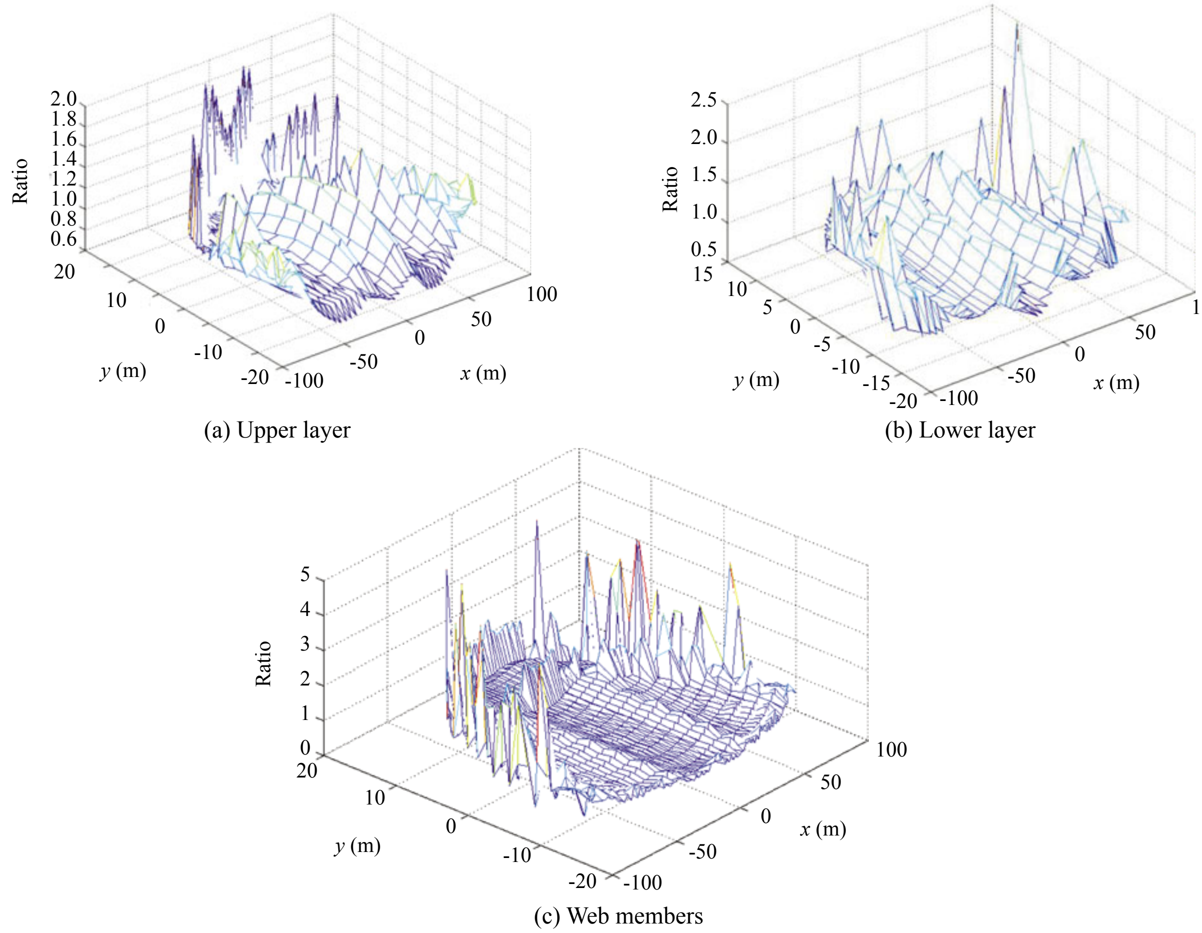


Fig. 17 Distribution of axial force ratios (ratio = multisupport / simple support)

8 Conclusions

(1) A simplified response spectrum method for multisupport excitations is proposed. This method provides a clear physical interpretation, i.e., the structural response is the sum of two components. The first component is the pseudostatic response caused by inconsistent support movements, and the second is the dynamic response caused by ground motion acceleration. This method is formally consistent with simple support excitation; however, the effects of multisupport excitations are considered for any modal response spectrum or modal combination. Under uniform excitation, the support motions are the same, and the displacements due to the pseudostatic response become rigid body displacements. These displacements do not result in any structural internal forces, and dynamic coherency coefficients under multisupport excitations become 1.0. Hence, the multisupport response spectrum method reduces to the traditional response spectrum method. From this perspective, multisupport excitation is a modification of uniform excitation, and the corresponding response spectrum method can be considered to be an extension of the combination rules of vibration modes under simple support excitation.

(2) Most of the coherency coefficients in the formulation of the response spectrum are simplified in this paper. In Eq. (35), it is found that the pseudostatic response and the dynamic response are obtained from a combination of these coherency coefficients. Nonetheless, calculating these coefficients requires a long computation time. This is especially true when a large number of supports and modes are included, because the coefficients' values must be obtained by integration. The simplification is accomplished by approximating the ground motion excitation as white noise. This approximation is justifiable in light of the frequency response function. Several numerical results indicate that the simplified method is valid under appropriate conditions.

(3) Based on comparisons of the structural internal forces predicted by the response spectrum method under multisupport and simple support excitation, it is concluded that the effects of inconsistent support movements cannot be ignored. The internal forces of the layer and web members near the supports are underestimated under simple support excitation. The areas near the support are always important regions, so they deserve special attention in the seismic design of grid structures and reticulated shells.

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