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A modified method for simulating non-stationary multi-point earthquake ground motion

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Abstract: A spectral-representation-based algorithm is proposed to simulate non-stationary and stochastic processes with evolutionary power, according to a prescribed non-stationary cross-spectral density matrix. Non-stationary multi-point seismic ground motions at different locations on the ground surface are generated for use in engineering applications. First, a modified iterative procedure is used to generate uniformly modulated non-stationary ground motion time histories which are compatible with the prescribed power spectrum. Then, ground motion time histories are modeled as a non-stationary stochastic process with amplitude and frequency modulation. The characteristic frequency and damping ratio of the Clough-Penzien acceleration spectrum are considered as a function of time in order to study the frequency time variation. Finally, two numerical examples are presented to validate the efficiency of the proposed method, and the results show that this method can be effectively applied to the dynamic seismic analysis of long and large scale structures.

Keywords: multi-support excitation; spatial correlation; power spectrum-compatibility; non-stationary; ground motion simulation

1 Introduction

The spatial variation of seismic ground motions has a significant effect on the dynamic response of long and large scale structures (Saxena *et al*., 2000). Although many actual earthquake records have been collected, they are not sufficient to carry out a deterministic seismic response analysis for structures considering multi-point seismic inputs. Thus, it is important to establish a reasonable synthesized earthquake method and input ground motion models for improved seismic response analysis of long and large scale structures.

Spatial variation of seismic ground motions has received much attention following the installation of dense earthquake monitor arrays. In particular, the SMART-1 (Strong Motion Array in Taiwan) has provided an abundance of data for various magnitude records that have been extensively studied by scientists. In recent years, a wide variety of techniques to simulate spatially variable seismic ground motions have been proposed (Hao *et al*., 1989; Conte *et al*., 1992; Ramadan and Novak, 1993; Zerva, 1992 and 1994). Among them, one commonly used method is the spectral representation method (Zerva, 1992 and 1994). ARMA (auto-regressive-moving-average) techniques (Conte *et al*., 1992) and the coherency function approximation method based on Fourier series (Ramadan and Novak, 1993) have also been extensively employed. Applying the theory of Hao *et al*. (1989), Dong *et al*. (2007) presented an improved local convergent method to simulate multi-point earthquake ground motions. Zerva and Zervas (2002) suggested that an appropriate simulation technique should be successful in matching the characteristics of the simulated motions with those of the target field, with a substantial savings in the cost of computation.

However, the power spectrum computed from synthesized seismic ground motions generated by using the above methods cannot adequately match the prescribed power spectrum. Therefore, the fitted method for seismic ground motions with a target spectrum has been adopted by researchers. Using the transition relationship between the response spectrum and the power spectrum, Chen *et al*. (1983) synthesized artificial earthquake ground motions fitted with the standard response spectrum. Considering phase spectrum through the phase correction technique, Hu and He (1986) suggested a simulated ground motion method from the response spectrum. Based on the phase difference spectrum, generating a non-stationary ground motion in both the time and frequency domains was proposed by Yang and Jiang (2002). However, most of the ground

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motion synthesizing methods were available only for one location. Hence, it is necessary to present a method to simulate multi-point seismic ground motions that are compatible with the target spectrum.

Using a prescribed non-stationary cross-spectral density matrix, an improved spectral representation, based on the simulation algorithm, is presented to simulate non-stationary and multi-variant stochastic processes with an evolutionary power spectrum. For an important application of multi-point earthquake ground motion simulation, an iterative scheme is introduced to generate seismic ground motion time histories at different locations on the ground surface that are compatible with a prescribed power spectrum, and correlated to a given coherence function.

2 Theoretical foundation

2.1 Simulation formula

The mean value of one-dimensional, multi-variant non-stationary stochastic vector processes with components $f_1^0(t), f_2^0(t), \ldots, f_n^0(t)$, equals to zero,

$$\varepsilon\left[f_j^0(t)\right] = 0, \quad j = 1, 2, \cdots, n \tag{1}$$

Then, the cross-correlation matrix can be given by

$$\boldsymbol{R}^0(t, t+\tau) = \begin{bmatrix} R^0{}_{11}(t,t+\tau) & R^0{}_{12}(t,t+\tau) & \cdots & R^0{}_{1n}(t,t+\tau) \\ R^0{}_{21}(t,t+\tau) & R^0{}_{22}(t,t+\tau) & \cdots & R^0{}_{2n}(t,t+\tau) \\ \vdots & \vdots & \ddots & \vdots \\ R^0{}_{n1}(t,t+\tau) & R^0{}_{n2}(t,t+\tau) & \cdots & R^0{}_{nn}(t,t+\tau) \end{bmatrix} \tag{2}$$

and cross spectral density matrix can be given by

$$\boldsymbol{S}^0(\omega, t) = \begin{bmatrix} S^0{}_{11}(\omega,t) & S^0{}_{12}(\omega,t) & \cdots & S^0{}_{1n}(\omega,t) \\ S^0{}_{21}(\omega,t) & S^0{}_{22}(\omega,t) & \cdots & S^0{}_{2n}(\omega,t) \\ \vdots & \vdots & \ddots & \vdots \\ S^0{}_{n1}(\omega,t) & S^0{}_{n2}(\omega,t) & \cdots & S^0{}_{nn}(\omega,t) \end{bmatrix} \tag{3}$$

Note that the vector process is non-stationary, therefore, the cross-correlation matrix is a function of two time variables t and $t + \tau$ in which t is the time and τ is the time lag, while the cross-spectral density matrix is a function of frequency ω and time t.

Based on the theory of evolutionary power spectrum for non-stationary stochastic processes (Priestley, 1988), the elements of the cross-spectral density matrix are defined as:

$$S^0{}_{jj}(\omega, t) = \left|A_j(\omega, t)\right|^2 S_j(\omega), \quad j = 1, 2, \cdots, n \tag{4}$$

$$S_{jk}^0(\omega, t) = A_j(\omega, t) A_k(\omega, t) \sqrt{S_j(\omega) S_k(\omega)}\, \Gamma_{jk}(\omega), \quad j, k = 1, 2, \cdots, n;\ j \neq k \tag{5}$$

where $A_j(\omega, t)$ ($j = 1, 2, \ldots, n$) are the modulating functions of $f_1^0(t), f_2^0(t), \ldots, f_n^0(t)$, respectively; and $S_j(\omega)$ ($j = 1, 2, \ldots, n$) are the (stationary) power spectral density functions of $f_1^0(t), f_2^0(t), \ldots, f_n^0(t)$, respectively; $\Gamma_{jk}(\omega)$ ($j, k = 1, 2, \ldots, n, j \neq k$) are the complex coherence functions between $f_j^0(t)$ and $f_k^0(t)$. Equations (4) and (5) imply that the modulating function $A_j(\omega, t)$ represents the change of the evolutionary power spectrum, relative to the (stationary) power spectral density function $S_j(\omega)$.

Consequently, for any instant time t, the diagonal elements of the cross-spectral density matrix are real and non-negative functions of ω, satisfying:

$$S^0{}_{jj}(\omega, t) = S^0{}_{jj}(-\omega, t), \quad j = 1, 2, \cdots, n; \tag{6}$$

while the off-diagonal elements are generally complex functions of ω, satisfying:

$$S_{jk}^0(\omega, t) = S_{jk}^{0*}(-\omega, t), \quad j, k = 1, 2, \cdots, n, j \neq k; \tag{7}$$

$$S_{jk}^0(\omega, t) = S_{kj}^{0*}(\omega, t), \quad j, k = 1, 2, \cdots, n, j \neq k; \tag{8}$$

where the asterisk denotes a complex conjugate. Equation (8) indicates that the cross-spectral density matrix $\boldsymbol{S}^0(\omega, t)$ is a Hermitian matrix for any value of t.

To simulate a one-dimensional, multi-variant non-stationary stochastic process with time and frequency modulation at every time instant t, the cross-spectral density matrix $\boldsymbol{S}^0(\omega, t)$ can be decomposed into the following product by Cholesky's method,

$$\boldsymbol{S}^0(\omega, t) = \boldsymbol{H}(\omega, t) \boldsymbol{H}^{\mathrm{T}*}(\omega, t) \tag{9}$$

where $\boldsymbol{H}(\omega, t)$ is a lower triangular matrix:

$$\boldsymbol{H}(\omega, t) = \begin{bmatrix} H_{11}(\omega,t) & & & \\ H_{21}(\omega,t) & H_{22}(\omega,t) & & \\ \vdots & \vdots & \ddots & \\ H_{n1}(\omega,t) & H_{n2}(\omega,t) & \cdots & H_{nn}(\omega,t) \end{bmatrix} \tag{10}$$

The diagonal elements of $\boldsymbol{H}(\omega, t)$ satisfy

$$H_{jj}(\omega, t) = H_{jj}(-\omega, t), \quad j = 1, 2, \cdots, n \tag{11}$$

If the off-diagonal elements $H_{jk}(\omega, t)$ are written in a polar coordinate system

$$H_{jk}(\omega, t) = \left|H_{jk}(\omega, t)\right| \mathrm{e}^{\mathrm{i}\theta_{jk}(\omega, t)}, \quad j = k+1, k+2, \cdots, n;\ k = 1, 2, \cdots, n-1; \tag{12}$$

where

$$\theta_{jk}(\omega,t)=\arctan\left(\frac{\mathrm{Im}\left[H_{jk}(\omega,t)\right]}{\mathrm{Re}\left[H_{jk}(\omega,t)\right]}\right) \tag{13}$$

where Im and Re are the imaginary and the real part of a complex number, respectively, and $H_{jk}(\omega, t)$ satisfies

$$\left|H_{jk}(\omega,t)\right|=\left|H_{jk}(-\omega,t)\right|,\quad j=k+1,k+2,\cdots,n;\quad k=1,2,\cdots,n-1; \tag{14}$$

$$\theta_{jk}(\omega,t)=-\theta_{jk}(-\omega,t),\quad j=k+1,k+2,\cdots,n;\quad k=1,2,\cdots,n-1; \tag{15}$$

When matrix $\boldsymbol{S}^0(\omega, t)$ is decomposed by Eqs. (9)-(10), the non-stationary stochastic vector process $f_j^0(t)$ (j = 1, 2,…, n) can be simulated by the following series (Deodatis, 1996):

$$f_j(t)=2\sum_{m=1}^{j}\sum_{l=1}^{N}\left|H_{jm}(\omega_l,t)\right|\sqrt{\Delta\omega}\times\cos\left[\omega_l t-\theta_{jm}(\omega_l,t)+\varphi_{ml}\right],\quad j=1,2,\cdots,n \tag{16}$$

For Eq. (16), only the correlation between the current location j and those preceding locations j -1 is considered. Thus, all the locations should be correlative with each other due to reflections and refractions of the seismic waves in the heterogeneous medium. Considering the above reason, the improved equation is given as

$$f_j(t)=2\sum_{m=1}^{n}\sum_{l=1}^{N}\left|H_{jm}(\omega_l,t)\right|\sqrt{\Delta\omega}\times\cos\left[\omega_l t-\theta_{jm}(\omega_l,t)+\varphi_{ml}\right],\quad j=1,2,\cdots,n \tag{17}$$

where

$$\omega_l=l\Delta\omega,\quad l=1,2,\cdots,N \tag{18}$$

$$\Delta\omega=\frac{\omega_u}{N} \tag{19}$$

$$\theta_{jm}(\omega_l,t)=\arctan\left(\frac{\mathrm{Im}\left[H_{jm}(\omega_l,t)\right]}{\mathrm{Re}\left[H_{jm}(\omega_l,t)\right]}\right) \tag{20}$$

in which ω_u represents a cut-off frequency; then, the elements of the cross-spectral density matrix in Eq. (3) are assumed to be zero for any time instant t if the frequency, ω is greater than ω_u. The φ_{ml} (m = 1, 2,…, n, l = 1, 2,…, N) in Eq. (17) is a sequence of independent random phase angles distributed uniformly over the interval [0, 2π].Therefore, based on Eqs. (17)-(20), a sample function $f_j^{(p)}(t)$ (j = 1, 2,…, n) of the simulated non-stationary stochastic vector process $f_j(t)$ (j = 1, 2, …, n) can be obtained by replacing the sequences of random phase angles φ_{1l}, φ_{2l},…, φ_{nl} (l = 1, 2,…, N) with their respective p-th realizations $\varphi^{(p)}_{1l}$, $\varphi^{(p)}_{2l}$,…, $\varphi^{(p)}_{nl}$ (l = 1, 2,…, N):

$$f_j^{(p)}(t)=2\sum_{m=1}^{n}\sum_{l=1}^{N}\left|H_{jm}(\omega_l,t)\right|\sqrt{\Delta\omega}\times\cos\left[\omega_l t-\theta_{jm}(\omega_l,t)+\varphi_{ml}^{(p)}\right],\quad j=1,2,\cdots,n \tag{21}$$

2.2 Uniformly modulated non-stationary seismic ground motion

For uniformly modulated non-stationary seismic ground motions, the modulating functions $A_j(\omega, t)$ (j = 1, 2,…, n) are independent of the frequency ω:

$$A_j(\omega,t)=A_j(t),\quad j=1,2,\cdots,n \tag{22}$$

Then, the components of the non-stationary stochastic vector process can be expressed as

$$f_j^0(t)=A_j(t)u_j^0(t),\quad j=1,2,\cdots,n \tag{23}$$

where $u_j^0(t)$ (j = 1, 2,…, n) are the components of the stationary stochastic vector process, where the mean values are equal to zero.

$$\varepsilon\left[u_j^0(t)\right]=0,\quad j=1,2,\cdots,n \tag{24}$$

The stationary cross-spectral density matrix is given by

$$\boldsymbol{S}^0(\omega)=\begin{bmatrix} S_1(\omega) & \sqrt{S_1(\omega)S_2(\omega)}\Gamma_{12}(\omega) & \cdots & \sqrt{S_1(\omega)S_n(\omega)}\Gamma_{1n}(\omega) \\ \sqrt{S_2(\omega)S_1(\omega)}\Gamma_{21}(\omega) & S_2(\omega) & \cdots & \sqrt{S_2(\omega)S_n(\omega)}\Gamma_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{S_n(\omega)S_1(\omega)}\Gamma_{n1}(\omega) & \sqrt{S_n(\omega)S_2(\omega)}\Gamma_{n2}(\omega) & \cdots & S_n(\omega) \end{bmatrix} \tag{25}$$

The elements of matrix $\boldsymbol{S}^0(\omega)$ have been defined in Eqs. (4) and (5).

To simulate the one-dimensional, multi-variant stationary stochastic process $u_j^0(t)$ (j = 1, 2,…, n), its cross-spectral density matrix $\boldsymbol{S}^0(\omega)$ must be decomposed into the following product by Cholesky's method.

$$\boldsymbol{S}^0(\omega)=\boldsymbol{L}(\omega)\boldsymbol{L}^{\mathrm{T}*}(\omega) \tag{26}$$

where superscript T denotes the transpose of a matrix; $\boldsymbol{L}(\omega)$ is a lower triangular matrix:

$$\boldsymbol{L}(\omega)=\begin{bmatrix} l_{11}(\omega) & & & \\ l_{21}(\mathrm{i}\omega) & l_{22}(\omega) & & \\ \vdots & \vdots & \ddots & \\ l_{n1}(\mathrm{i}\omega) & l_{n2}(\mathrm{i}\omega) & \cdots & l_{nn}(\omega) \end{bmatrix} \tag{27}$$

Generally, diagonal elements are real and non-negative functions of ω; while off-diagonal elements are complex functions of ω.

Once the matrix $\boldsymbol{S}^0(\omega)$ is decomposed by Eqs. (26)-(27), the synthesized expression $u_j(t)$ from the stationary stochastic vector process $u_j^0(t)$ (j = 1, 2,…, n), can be simulated by the following series (Hao *et al*., 1989), instead of Eq. (17):

$$u_j(t) = \sum_{m=1}^{n}\sum_{l=1}^{N-1} A_{jm}(\omega_l)\cos(\omega_l + \theta_{jm} + \varphi_{ml}), \quad j = 1,2,\cdots,n \tag{28}$$

where amplitudes $A_{jm}(\omega_l)$ and phase angles $\theta_{jm}(\omega_l)$ are to be determined by considering correlation relationships between location j and m; φ_{ml} is the random phase angle uniformly distributed in the range of zero and 2π. Note that φ_{ml} and φ_{rs} are mutually independent as $m \neq r$, and $l \neq s$. The amplitudes and phase angles in Eq. (28) can be expressed as

$$\begin{aligned} A_{jm}(\omega_l) &= \sqrt{4\Delta\omega}\left|l_{jm}(\mathrm{i}\omega_l)\right| \\ \theta_{jm}(\omega_l) &= \arctan\frac{\mathrm{Im}\left[l_{jm}(\mathrm{i}\omega_l)\right]}{\mathrm{Re}\left[l_{jm}(\mathrm{i}\omega_l)\right]} \end{aligned} \tag{29}$$

Consequently, the stationary stochastic processes $u_j(t)$ (j = 1, 2,…, n) are obtained based on Eqs. (28) and (29). Using the FFT technique, simulation of the stationary stochastic vector process can be performed with great computational efficiency (Qu and Wang, 1998). Thus, Eq. (28) can be expressed in the frequency domain as follows:

$$U_j(l) = \sum_{j=1}^{n}\frac{1}{2}A_{jm}(l)\mathrm{e}^{\mathrm{i}[\theta_{jm}(l)+\varphi_{ml}]}, \quad j = 1,2,\cdots,n \tag{30}$$

To obtain the simulated $u_j(t)$, the Fourier spectrum for stationary stochastic vector process is first synthesized, and then the stationary stochastic vector process can be obtained by using inverse Fourier transformation technique.

2.3 Simulation of multi-point seismic ground motion compatible with prescribed power spectrum

Based on a prescribed non-stationary cross-spectral density matrix, the simulation algorithm presented in this paper can be used to generate sample functions of a general non-stationary stochastic vector process with an evolutionary power spectrum. The problem with this approach is that the power spectrum for the generated sample functions does not adequately coincide with the prescribed power spectrum.

An improved algorithm that generates uniformly modulated non-stationary seismic ground motion time histories compatible with the prescribed power spectrum is proposed herein. The target acceleration power spectrum and modulating functions $A_j(t)$ associated with each point located on the ground surface are assigned. In addition, complex coherence functions $\Gamma_{jk}(\omega)$ (j, k = 1, 2,…, n, $j \neq k$) between random pairs of points are prescribed. The simulation procedures of the acceleration time histories performed in the frequency domain are shown in Fig. 1.

3 Numerical examples

To validate the proposed algorithm on a simulation of non-stationary stochastic vector processes, two examples of synthesizing non-stationary earthquake ground motions are presented. For the first example, seismic ground motions are modeled as a uniformly modulated non-stationary stochastic vector process, and using an iterative scheme in the frequency domain, sample functions compatible with the prescribed power spectrum are generated. In the second example, seismic ground motions are modeled as a non-stationary stochastic vector process with amplitude and frequency modulation, and based on a target cross-spectral density matrix, sample functions that consider coherence between an arbitrary pair of points on the ground surface are synthesized.

3.1 Example 1

The ground motion time histories are modeled as a one-dimensional, uniformly modulated non-stationary stochastic vector process. Selecting four points on the ground surface along a straight line parallel to the direction of the seismic wave propagation, time histories corresponding to the four points are generated. The configuration of the four points on the ground surface is shown in Fig. 2. Their coordinate values are 0 m, 1 m, 101 m and 1000 m, respectively. Referring to the iterative procedures in Fig. 1, the sample functions of the acceleration time histories at the four points are generated. Each non-stationary acceleration time history is independently iterated to fit a prescribed acceleration power spectrum. This iteration process is carried out in the frequency domain and sufficient convergence precision is satisfied after several iterations. The coherence between pairs of acceleration time histories is also shown to have good agreement through the analysis.

In order to simulate the multiple-support seismic ground motions, three basic components are required: (a) target power spectral density function; (b) coherency function; and (c) modulating function.

The target spectral density function is modeled in Clough and Penzien form (Clough and Penzien, 1975):

$$S_j(\omega) = S_{0j}\left(\frac{1+4\xi_{gj}^2(\omega/\omega_{gj})^2}{[1-(\omega/\omega_{gj})^2]^2+4\xi_{gj}^2(\omega/\omega_{gj})^2}\right)\bullet \left(\frac{(\omega/\omega_{fj})^4}{[1-(\omega/\omega_{fj})^2]^2+4\xi_{fj}^2(\omega/\omega_{fj})^2}\right), \quad j = 1,2,3,4 \tag{31}$$

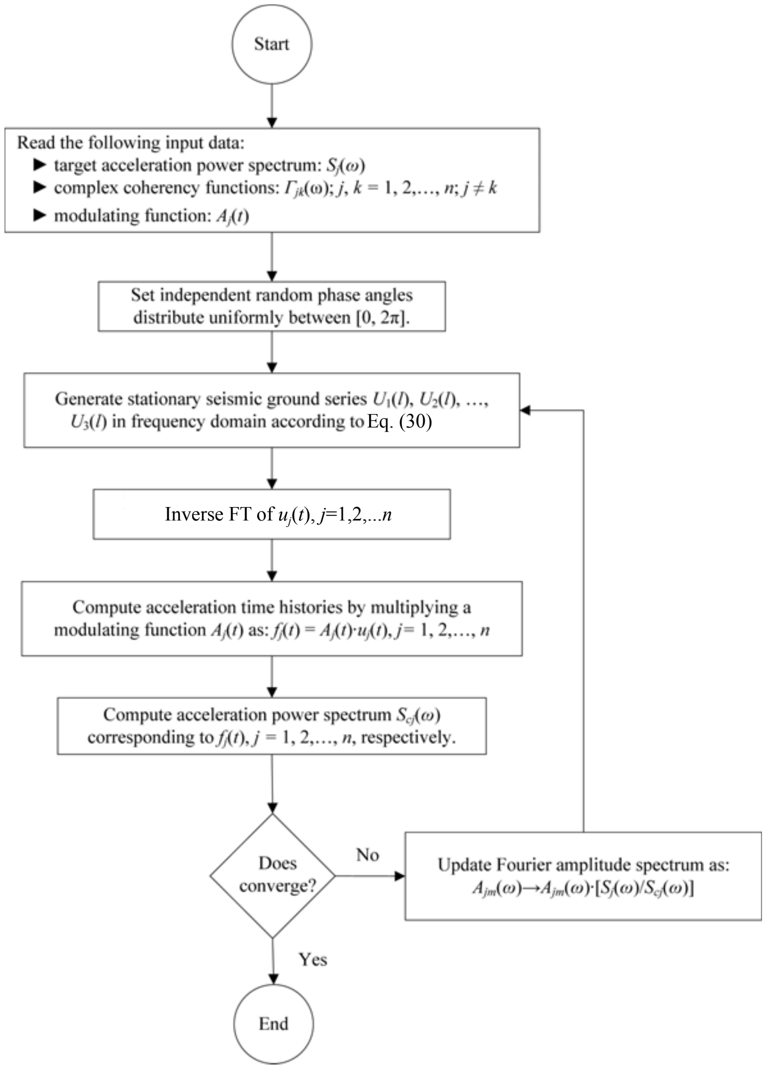


Fig. 1 Flowchart of iteration in simulating power spectrum compatible acceleration time histories

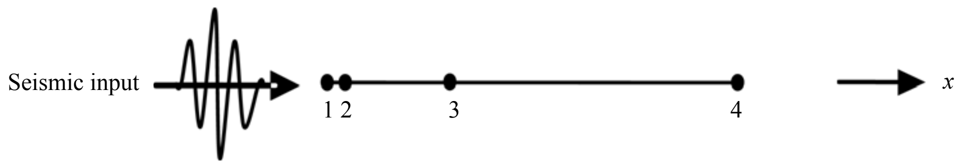


Fig. 2 Configuration of points 1, 2, 3 and 4 on the ground surface

where S_{0j} is a scale factor depending on the ground motion intensity; ω_{gj} and ξ_{gj} are filter parameters of the Kanai-Tajimi model representing the natural frequency and damping of the soil layer at point j, respectively; and ω_{fj} and ξ_{fj} are the parameters of a second filter which is introduced to assure a finite power for the ground displacement. For medium stiffness soil, ω_{gj} = 10.0 rad/s, ξ_{gj} = 0.4, ω_{fj} = 1.0 rad/s, ξ_{fj} = 0.6 (Der Kiureghian and Neuenhofer, 1992).

The coherence function adopted is one of the empirical models suggested by Qu *et al*. (1996), as

$$\gamma_{jm}(\mathrm{i}\omega, d_{jm}) = \exp(-a(\omega)d_{jm}^{b(\omega)})\exp\left(-\mathrm{i}\omega\frac{d_{jm}}{v_{\mathrm{app}}}\right),$$
$$j,m = 1,2,3,4 \quad j \neq m \tag{32}$$

where d_{jm} is the horizontal projected distance along the wave propagation direction between location j and m; v_{app} is the apparent wave velocity; and $a(\omega)$ and $b(\omega)$ are the functions of frequency that can be expressed as

$$a(\omega) = a_1\omega^2 + a_2$$
$$b(\omega) = b_1\omega + b_2 \tag{33}$$

where a_1, a_2, b_1 and b_2 are empirical parameters. For medium stiff soil, a_1 = 1.678e-5, a_2 = 1.219e-3, b_1 = -5.5e-3, b_2 = 0.7674 (Qu *et al*., 1996).

The modulating function chosen in this study is presented by Monti *et al*. (1996) as

$$\zeta(t) = \begin{cases} \left(\dfrac{t}{t_1}\right)^2, & 0 \le t \le t_1 \\ 1, & t_1 \le t \le t_2 \\ \exp\left(\dfrac{t-t_2}{t_{\max}-t_2}\ln\beta\right) & t_2 \le t \le t_{\max} \end{cases} \tag{34}$$

where t_1 and t_2 are the ramp duration and decay starting time, respectively; $t_{\max}$ is the time-history duration; and β is the ratio of the envelope amplitude at $t_{\max}$ to that during the stationary phase ($t_1 \le t \le t_2$). The parameters used in the modulating function are t_1 = 3 s, t_2 = 13 s, $t_{\max}$ = 24 s, β = 0.02.

The generated acceleration time histories are shown in Fig. 3. Due to the specified modulating function, the acceleration time histories at points 1, 2, 3 and 4 reflect the non-stationary characteristics of ground motion.

A comparison of the prescribed power spectrum and the power spectrum generated from acceleration time histories for points 1, 3 and 4 is shown in Fig. 4, respectively. Since the spatial distance between points 1 and 2 is short, the comparison for point 2 is neglected.

From Figs. 4(a) and 4(c), the simulated power spectrums from the non-fitted time histories do not show good agreement with the target one in the lower frequency region. From Figs. 4(b) and 4(d), the simulated power spectrum from the fitted time histories is in good agreement with target one; therefore, the accuracy of the simulated seismic ground motions is improved.

Comparisons of coherency from the simulated ground motion time histories and the target coherency function for points 1, 2, 3 and 4 are shown in Fig 5. The curves labeled “simulated 1” and “simulated 2”denote the synthesized seismic ground motions with and without adopting an iterative scheme, respectively. The simulated coherency values can match the target coherency function well. The coherency between two

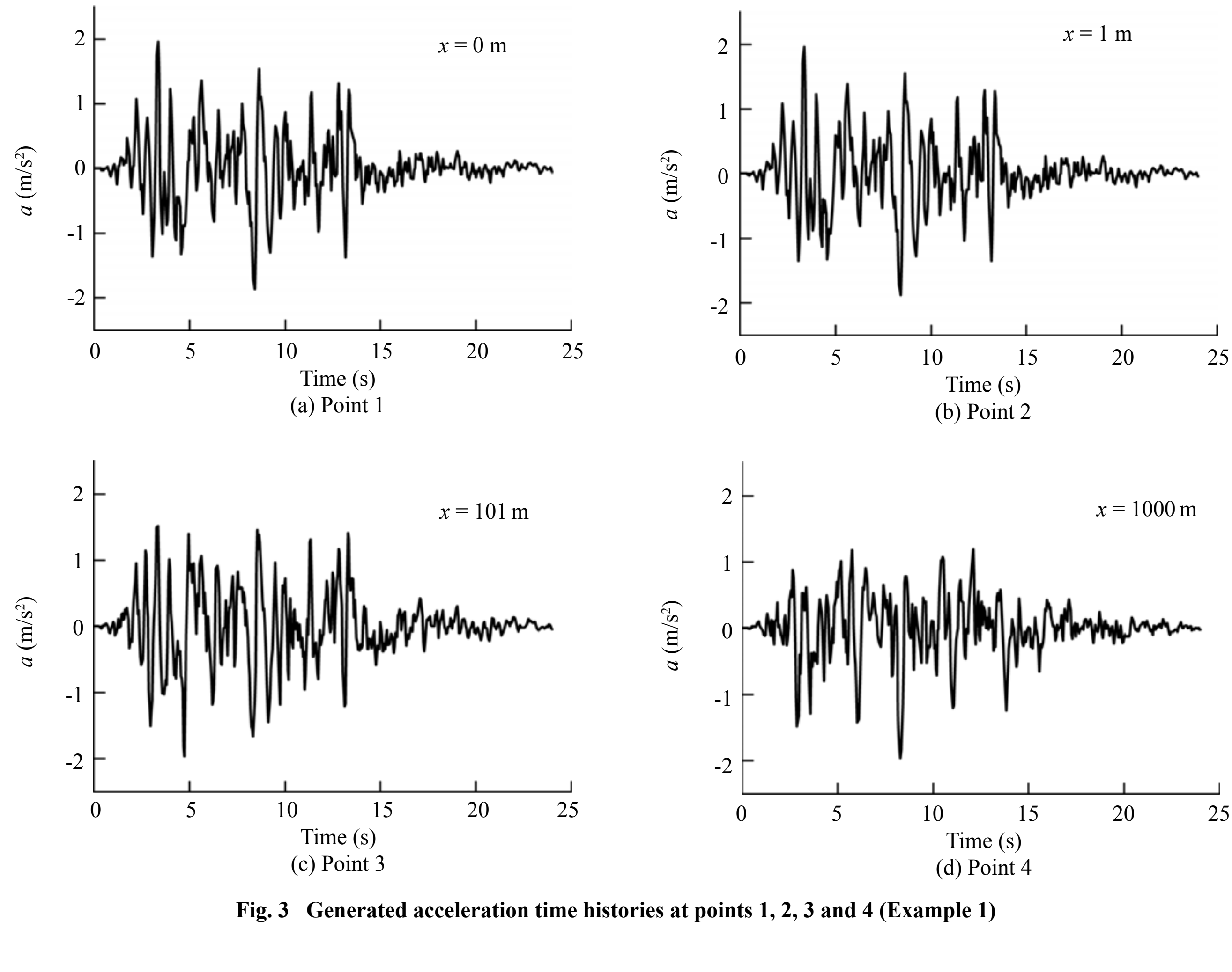
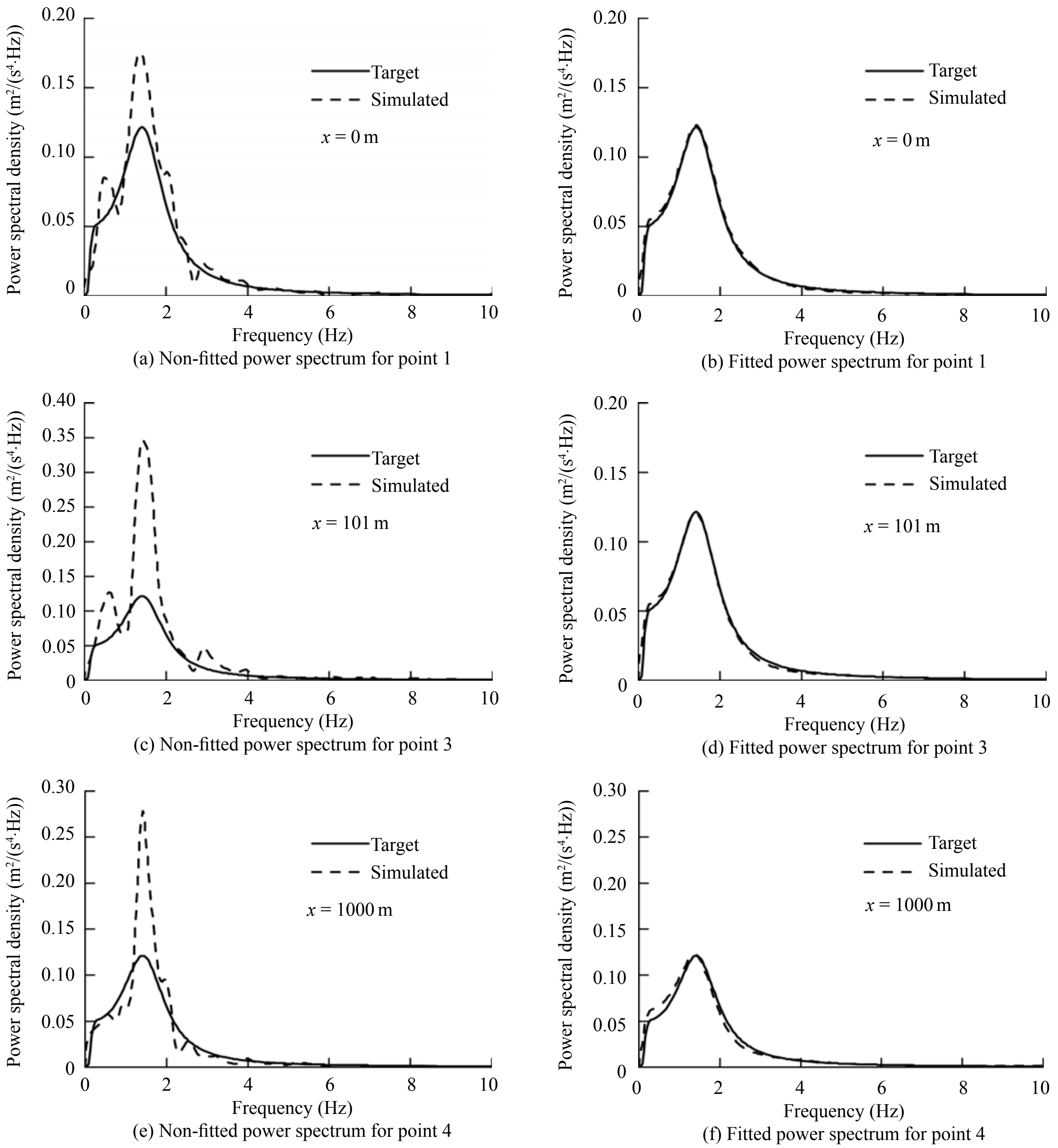


Fig. 3 Generated acceleration time histories at points 1, 2, 3 and 4 (Example 1)



(a) Non-fitted power spectrum for point 1
(b) Fitted power spectrum for point 1
(c) Non-fitted power spectrum for point 3
(d) Fitted power spectrum for point 3
(e) Non-fitted power spectrum for point 4
(f) Fitted power spectrum for point 4

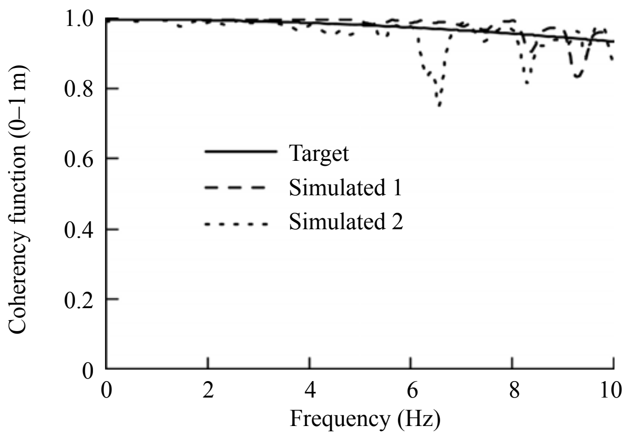
Fig.4 Comparison of the power spectrum of generated ground motions with the targets

points becomes weaker as the frequency increases. Since the spatial distance between point 1 and point 2 is much shorter than between point 1 and point 4, the coherency between point 1 and 2 is stronger than the one between point 1 and 4. It is also indicated that the coherency between pairs of points is hardly affected by using an iterative scheme. As a result, the fitted seismic ground motion is accurate and reliable.

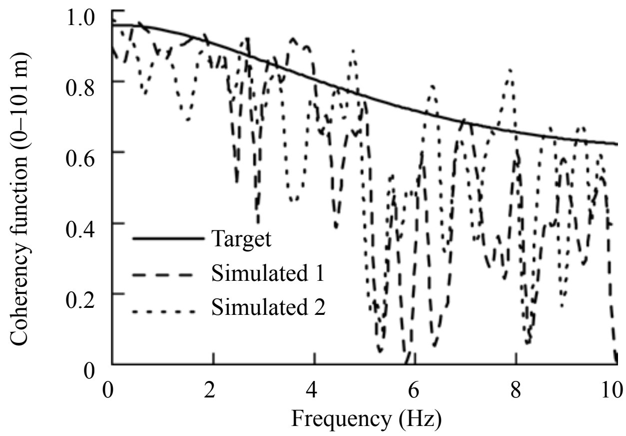
Note that the simulated seismic ground motion time histories using theiterative procedures shown in Fig. 1 have the following characteristics: (I) compatible with prescribed power spectrum; (2) correlated with the given coherence function; and (3) have amplitude varying as a function of time on the basis of a prescribed modulating function.

3.2 Example 2

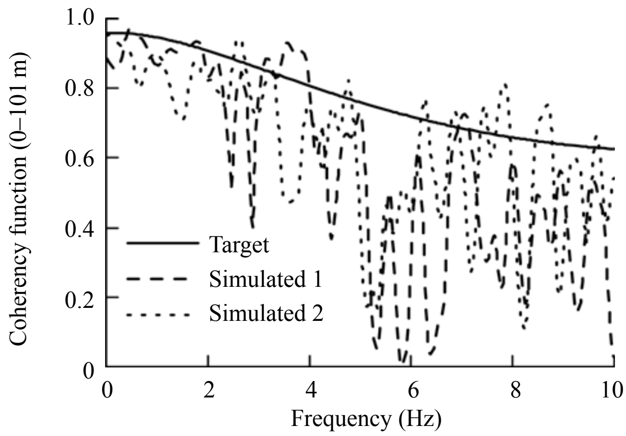
Figure 6 displays an acceleration record of the Niigata earthquake in 1964. The record shows both



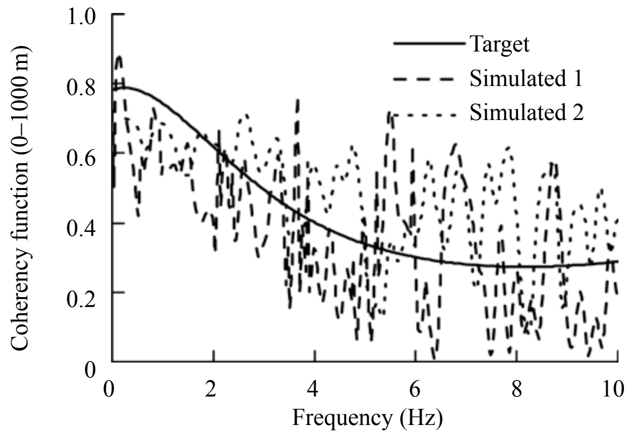
(a) Coherency function between points 1 and 2



(b) Coherency function between points 1 and 3



(c) Coherency function between points 2 and 3



(d) Coherency function between points 1 and 4

Fig. 5 Comparison of coherency function for simulated ground motions with the targets (Example 1)

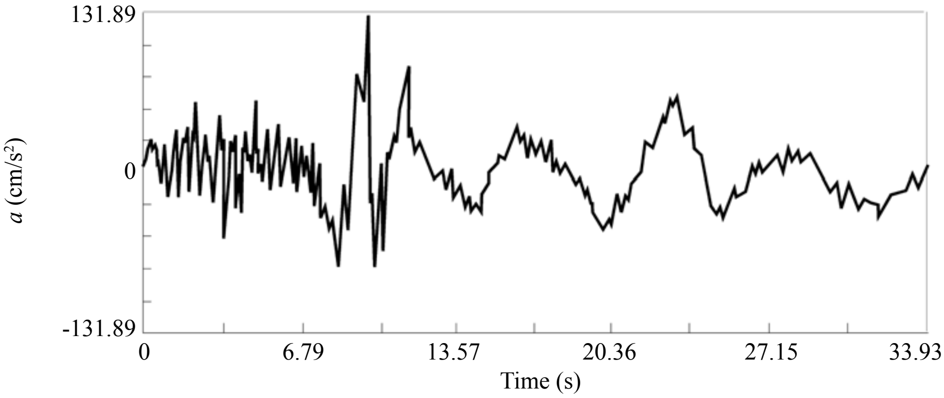


Fig. 6 Acceleration record from Niigata earthquake in 1964

amplitude and frequency variation with respect to time. In particular, an abrupt change of its frequency content between approximately 8 and 10 s can be found. During this period, the frequency content changes from a relatively broad band of frequencies to a single low frequency. The reason is due to soil liquefaction. The objective of this example is to synthesize multi-point artificial ground motions with changeable frequency and amplitude.

Ground motion time histories were modeled as a non-stationary stochastic vector process with amplitude and frequency modulation, which means that both the amplitude and the frequency of the ground motion are varied with time. Sample functions for the seismic ground motion time histories are generated through Eq. (17).

As in example 1, four points were also selected with the same configuration as above, and the Clough-Penzien spectrum (Clough and Penzien, 1975) is also chosen to model the power spectral density functions $S_j(\omega)$ (j = 1, 2, 3, 4) of the acceleration time histories $f_j^0(t)$ (j = 1, 2, 3, 4), respectively. To consider the variation of the frequency content with time, the characteristic frequency ω_{gj} and damping ratio ξ_{gj} of the ground motions are defined as a function of time and expressed as (Deodatis and Shinozuka, 1988):

$$\omega_{gj}(t)=\begin{cases}15.56\ \text{rad/s} & 0\ \text{s}\le t\le 4.5\ \text{s}\\ 27.12t_p^3-40.68t_p^2+15.56 & 4.5\ \text{s}\le t\le 5.5\ \text{s}\\ 2.0\ \text{rad/s} & t\ge 5.5\ \text{s}\end{cases},\quad j=1,2,3,4 \tag{35}$$

$$\xi_{gj}(t)=\begin{cases}0.64 & 0\ \text{s}\le t\le 4.5\ \text{s}\\ 1.25t_p^3-1.875t_p^2+0.64 & 4.5\ \text{s}\le t\le 5.5\ \text{s}\\ 0.015 & t\ge 5.5\ \text{s}\end{cases},\quad j=1,2,3,4 \tag{36}$$

where $t_p = t - 4.5$ s and

$$\omega_{fj}(t)=0.1\omega_{gj}(t),\ \xi_{fj}(t)=\xi_{gj}(t),\quad j=1,2,3,4 \tag{37}$$

The expressions of $\omega_{gj}(t)$ and $\xi_{gj}(t)$ describe a sudden drop of the characteristic frequency and damping ratio during the period from t = 4.5 to 5.5 s. The parameters which determine the intensity of the acceleration varied with respect to time are

$$S_{0j}(t)=\frac{\sigma^2}{\pi\omega_{gj}(t)\left[2\xi_{gj}(t)+\dfrac{1}{2\xi_{gj}(t)}\right]},\quad j=1,2,3,4 \tag{38}$$

where the standard deviation σ is equal to 100 cm/s^2 at every time instant. The coherency function developed by Qu *et al*. (1996) is used again.

Here, it should be pointed out that the definitions for $S_j(\omega)$ (j = 1, 2, 3, 4) no longer imply that the modulating function $A_j(\omega, t)$ represents a change in the evolutionary power spectrum, relative to the (stationary) power spectral density function $S_j(\omega)$. Note that $S_j(\omega)$ is now a function of both frequency and time since the characteristics of a frequency and damping ratio of the ground are functions of time, and $A_j(\omega, t)$ is a function

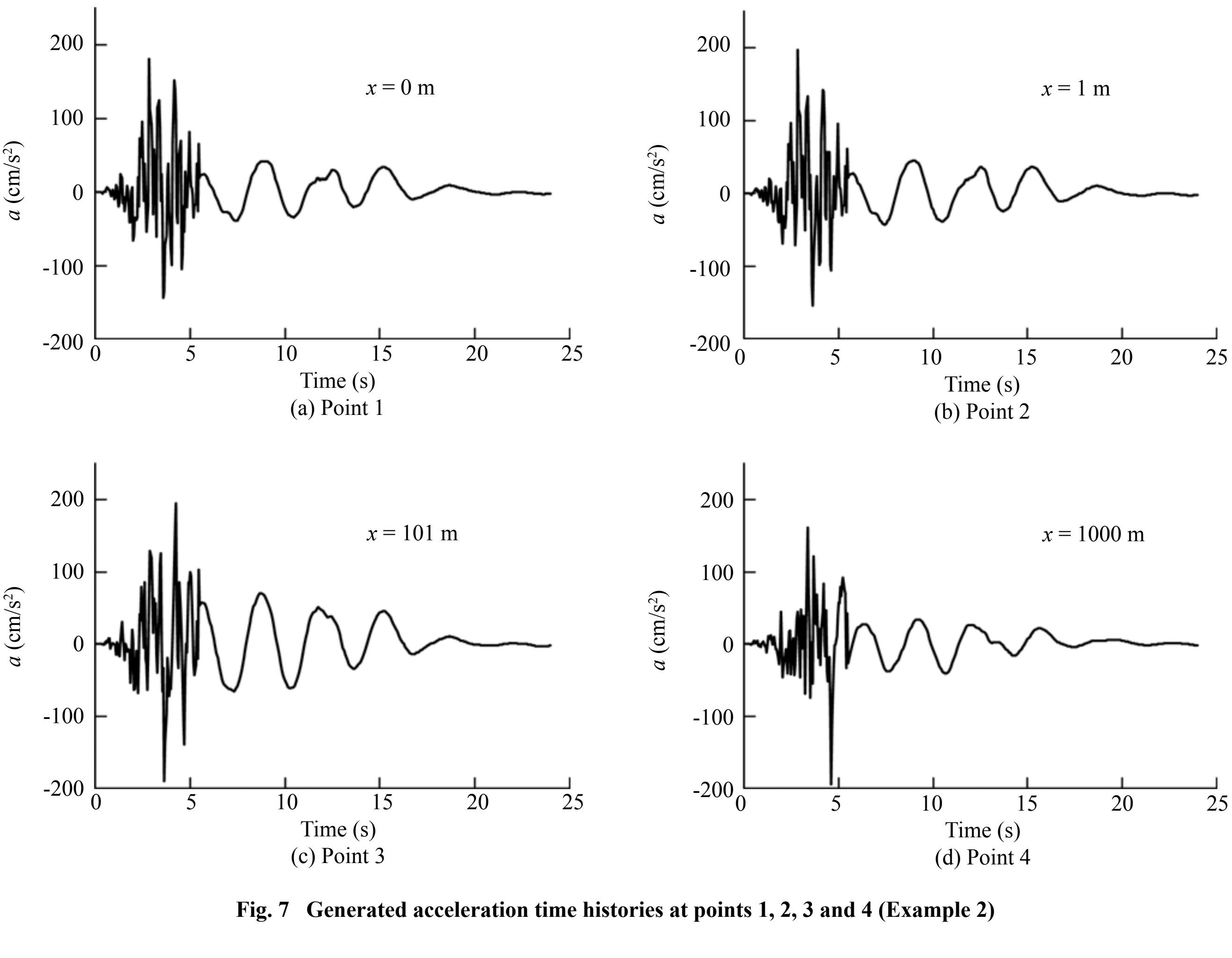


Fig. 7 Generated acceleration time histories at points 1, 2, 3 and 4 (Example 2)

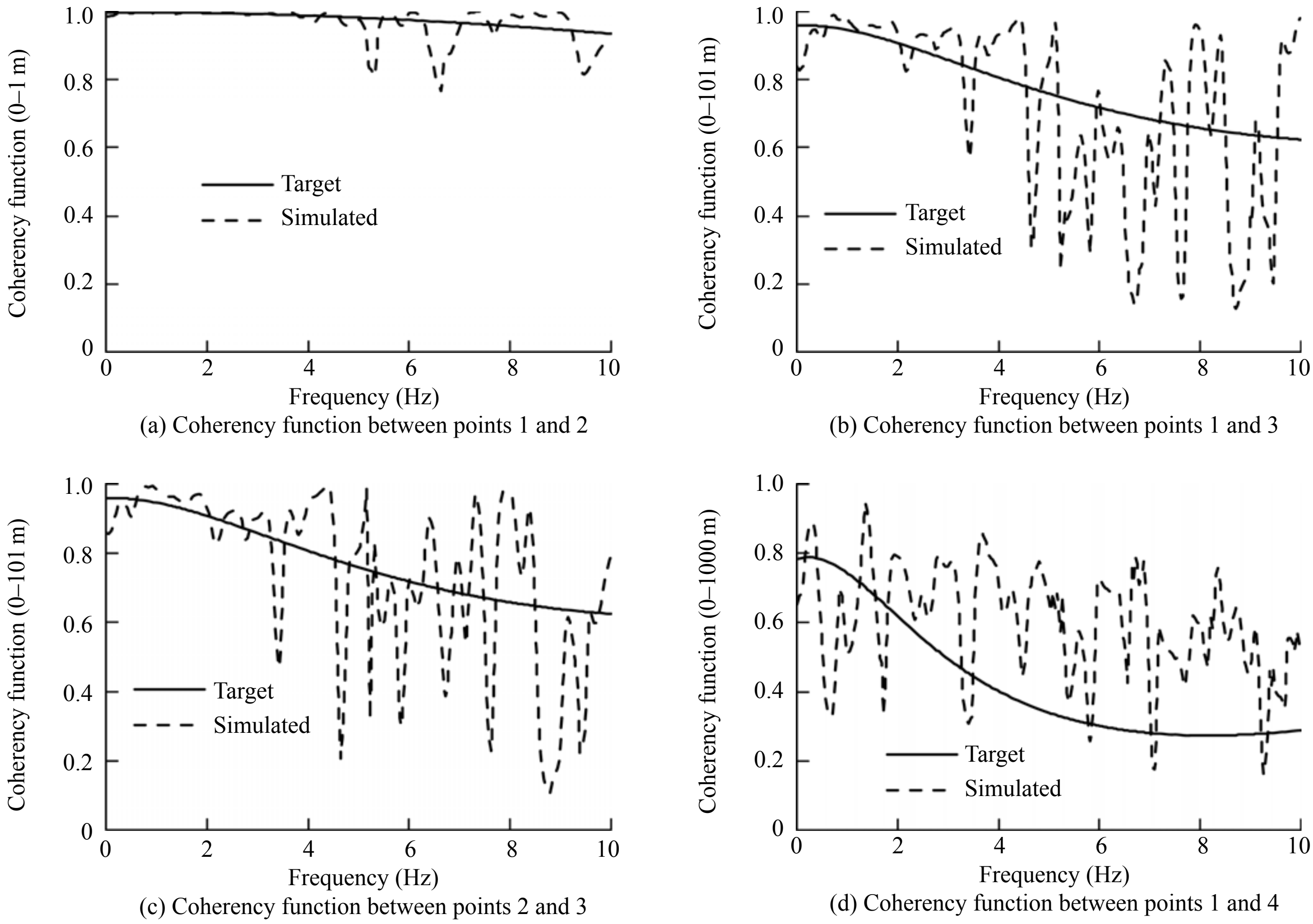


Fig. 8 Comparison of coherency function for simulated ground motions and the targets (Example 2)

of time only.

The generated acceleration time histories are shown in Fig. 7. It is obvious that the frequency and amplitude variation characteristics of the 1964 Niigata earthquake acceleration record are reproduced very well in the synthesized four-point ground motion time histories.

The comparisons between the coherency function from the simulated acceleration time histories and target coherency function are displayed in Fig. 8. The coherency functions from the simulated sample functions agree well with those from the target.

4 Summary

A spectral-representation-based simulation algorithm is presented to generate sample functions of a non-stationary, stochastic process with evolutionary power on the basis of its prescribed non-stationary cross-spectral density matrix. An important application of the algorithm is to synthesize non-stationary multi-point earthquake ground motions.

The proposed simulation algorithm is composed of two parts. First, the seismic ground motion time histories are modeled as a uniformly modulated non-stationary stochastic vector process by an iterative procedure shown in Fig. 1. Then, the seismic ground motion time histories are modeled as a non-stationary stochastic vector process with amplitude and frequency modulation.

Two examples are provided to demonstrate the capabilities of the proposed algorithm. In the first example, the proposed iterative procedure is applied to simulate seismic ground motion time histories at four points on the ground surface which are compatible with the prescribed power spectrum. The results show that the power spectrum calculated from the generated acceleration time histories is in good agreement with the target power spectrum. In the second example, due to the characteristic frequency and damping ratio of the ground motions changing with time, the simulated seismic ground motions can reproduce the frequency and amplitude variation characteristics of the actual earthquake record.

The power spectral density function reflects the statistical characteristic of seismic ground motions very well. Thus, it is of practical significance in simulating multi-point earthquake ground motions compatible with the target power spectrum. The ground motion time histories generated by the method presented in this paper can be directly used as an input for the dynamic seismic response analysis of large span structures such as bridges, lifelines, dams, oil/gas pipelines and so on.

References

Chen YQ, Liu XH and Gong SL (1983), "The Artificial Earthquake Ground Motions Compatible with Standard Response Spectra," *Journal of Building Structures*, **4**(1): 34–43. (in Chinese)

Clough RW and Penzien J (1975), *Dynamics of structures*, New York: McGrawHill.

Conte JP, Pister KS and Mahin SA (1992), "Non-Stationary ARMA Modeling of Seismic Ground Motion," *Soil Dynamics and Earthquake Engineering*, **11**: 411–426.

Deodatis G (1996), "Non-stationary Stochastic Vector Processes: Seismic Ground Motion Applications," *Probabilistic Engineering Mechanics*, **11**: 149–168.

Deodatis G and Shinozuka M (1988), "Auto-regressive Model for Non-stationary Stochastic Processes," *Journal of Engineering Mechanics,* ASCE, **114**(11): 1995–2012.

Der Kiureghian A and Neuenhofer A (1992), "Response Spectrum Method for Multi-support Seismic Excitations," *Earthquake Engineering and Structural Dynamics*, **21**: 713–740.

Dong RB, Zhou J and Feng X (2007), "A Local Convergent Method for Simulating Multi-point Earthquake Ground Motion," *Journal of Sound and Vibration*, **26**(4): 5–9. (in Chinese)

Hao H, Oliveira CS and Penzien J (1989), "Multiple-Station Ground Motion Processing and Simulation Based on SMART-1 Array Data," *Nuclear Engineering and Design*, **111**: 293–310.

Hu YX and He X (1986), "Phase Angle Consideration in Generating Response Spectrum-Compatible Ground Motion," *Earthquake Engineering and Engineering Vibration*, **6**(2): 37–51. (in Chinese)

Monti G, Nuti C and Pintio E (1996), "Nonlinear Response of Bridges Under Multi-support Excitation," *Journal of Structural Engineering*, **122**(5): 476–484.

Priestley MB (1988), *Non-linear and Non-stationary Time Series Analysis*, New York: Academic Press.

Qu TJ and Wang QX (1998), "Simulation of Spatial Correlative Time Histories of Multi-point Ground Motion, Part I: Fundamental Formulas," *Earthquake Engineering and Engineering Vibration*, **18**(1): 8–15. (in Chinese)

Qu TJ, Wang JJ and Wang QX (1996), "A Practical Model for the Power Spectrum of Spatially Variant Ground Motion," *Acta Seismologica Sinica*, **9**(1): 69–79.

Ramadan O and Novak M (1993), "Simulation of Spatially Incoherent Random Ground Motions," *Journal of Engineering Mechanics*, **119**(5): 997–1016.

Saxena V, Deodatis G and Shinozuka M (2000), "Effect of Spatial Variation of Earthquake Ground Motion on the Nonlinear Dynamic Response of Highway Bridges," *Proceedings of 12th World Conference on Earthquake Engineering*, Auckland, New Zealand.

Yang QS and Jiang HP (2002), "Generation of Response-spectrum-compatible Ground Motions Based on Phase-difference Spectrum," *Earthquake Engineering and Engineering Vibration*, **22**(1): 32–38. (in Chinese)

Zerva A (1992), "Seismic Ground Motion Simulations from a Class of Spatial Variability Model," *Earthquake Engineering and Structural Dynamics*, **21**(4): 351–361.

Zerva A (1994), "On the Spatial Variation of Seismic Ground Motions and Its Effect on Lifelines," *Engineering Structure*, **16**: 534–546.

Zerva A and Zervas V (2002), "Spatial Variation of Seismic Ground Motions: an Overview," *Applied Mechanics Reviews*, **55**(3): 271–297.