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# An improved modal pushover analysis procedure for estimating seismic demands of structures

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**Abstract:** The pushover analysis (POA) procedure is difficult to apply to high-rise buildings, as it cannot account for the contributions of higher modes. To overcome this limitation, a modal pushover analysis (MPA) procedure was proposed by Chopra *et al*. (2001). However, invariable lateral force distributions are still adopted in the MPA. In this paper, an improved MPA procedure is presented to estimate the seismic demands of structures, considering the redistribution of inertia forces after the structure yields. This improved procedure is verified with numerical examples of 5-, 9- and 22-story buildings. It is concluded that the improved MPA procedure is more accurate than either the POA procedure or MPA procedure. In addition, the proposed procedure avoids a large computational effort by adopting a two-phase lateral force distribution..

**Keywords:** seismic demand estimation; pushover analysis; improved modal pushover analysis; two-phase lateral force distribution; capacity curve

## 1 Introduction

Recently, there has been a growing interest in displacement-based seismic design (DBSD), where the displacement or interstory-drift, rather than the lateral force, is considered as the basic demand parameter in the design, evaluation and rehabilitation of structures. Estimating seismic demands at low performance levels, such as life safety and collapse prevention, requires explicit consideration of the inelastic behavior of the structure. While nonlinear response history analysis (RHA) is the most rigorous procedure to compute seismic demands, current civil engineering practice prefers to use nonlinear static procedures (NSP) or pushover analysis (POA) (Chopra *et al.*, 2004). The POA has been widely used for its conceptual simplicity, computational attractiveness and capability of providing satisfactory predictions of seismic demands for low- and medium-rise structures if the inelastic action is distributed over the height of the structures (Lawson *et al.*, 1994; Bracci *et al.*, 1997; Sasaki *et al.*, 1998; Krawinkler and Seneviratna, 1998; Fajfar, 1999; Gupta and Krawinkler, 1999; Kunnath and Gupta, 2000; Elnashai, 2001; Chopra *et al.*, 2001; Fajfar, 2002; Ye and Pan, 2000; Yang *et al.*, 2000; Yin *et al.*, 2003; Sun *et al.*, 2003; Zhou and LU, 2004). The POA is based on two basic assumptions: (1) the response is controlled by the fundamental mode of the structure; and (2) the mode shape remains unchanged after the structure yields. Obviously, the POA does not account for the contribution of higher modes to the structural response; therefore, it is difficult to apply to high-rise buildings (Gupta and Krawinkler, 1999; Kunnath and Gupta, 2000; Goel and Chopra, 2005; Zhai, 2005).

Based on structural dynamics theory, Chopra and Goel (2001) proposed a modal pushover analysis (MPA) to include higher mode contributions into the total seismic demand. In MPA, the seismic demand due to individual terms in the modal expansion of the effective earthquake forces is determined by a POA using the inertial force distribution for each mode. Combining these ‘modal’ demands due to the first two or three terms of the expansion provides an estimate of the total seismic demand on inelastic systems. Because the MPA procedure can include the contributions of higher modes, it provides a better estimation of seismic demands. However, invariable lateral force distributions are still adopted in the MPA; therefore, it cannot overcome the same limitations as the POA induced by the second assumption. In this paper, the MPA procedure

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is improved by considering the redistribution of inertia forces after the structure yields. This improved procedure is verified by three examples.

## 2 Brief description of MPA procedure

The details of the MPA procedure can be found in Chopra and Goel (2001), Chopra and Goel (2002), and Chopra and Chintanapakdee (2003). To better understand the method proposed in this paper, it is summarized below:

(1) Compute the structural natural frequencies $\omega_n$ and modes $\boldsymbol{\varphi}_n$. In practical applications, only the first two or three modes are needed.

(2) For the $n$th mode, develop the pushover curve (base shear-top displacement curve) using force distribution $\boldsymbol{s}_n^*$ defined as

$$\boldsymbol{s}_n^*=\boldsymbol{M}\boldsymbol{\varphi}_n \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix of the structure.

(3) Idealize the pushover curve as a bilinear curve as shown in Fig. 1(a).

(4) Convert the idealized pushover curve to the force-deformation relationship ($F_{sn}/L_n - D_n$) of $n$th-mode inelastic SDF system as shown in Fig. 1(b), here,

$$\frac{F_{sn}}{L_n}=\frac{V_{bn}}{M_n^*}, \qquad D_n=\frac{u_{rn}}{\Gamma_n\varphi_{rn}}, \qquad M_n^*=L_n\Gamma_n \tag{2}$$

where $\Gamma_n$ is the $n$th modal participation factor, and $M_n^*$ is the effective modal mass and determine the initial elastic vibration period $T_n$ and yielding deformation $D_{ny}$.

(5) Compute the peak deformation $D_n$ of the $n$th-mode inelastic SDF system by nonlinear history analysis, or using inelastic design spectrum.

(6) Calculate the peak roof displacement $u_{rn}$ associated with the $n$th-mode inelastic SDF system from

$$u_{rn}=\Gamma_n\varphi_{rn}D_n \tag{3}$$

(7) From the pushover database, extract values of any desired responses $r_n$ at the peak roof displacement $u_{rn}$.

(8) Repeat steps 3-7 for the first few “modes”.

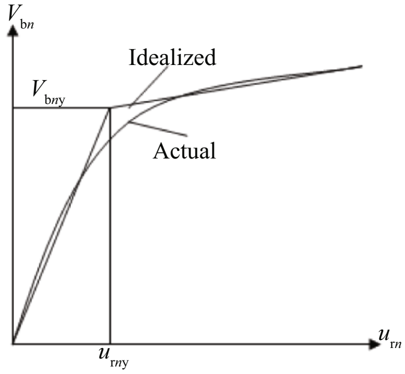
(9) Determine the total seismic demand $r_{\text{total}}$:

$$r_{\text{total}}=\sqrt{\sum r_n^2} \tag{4}$$

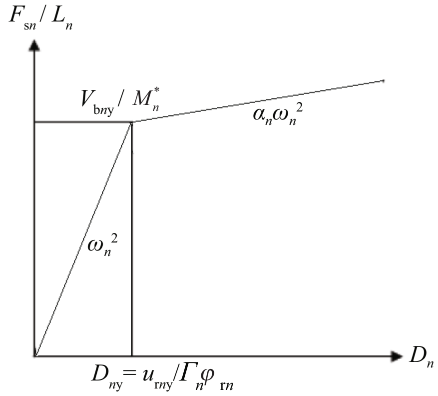
## 3 Improved MPA procedure

Though the MPA procedure can consider the contributions of multi-modes in estimating structural seismic demands, this procedure cannot avoid the weakness of applying invariable lateral force distribution. In the present study, attempts are made to consider the redistribution of inertia forces after the structure yields.

Obviously, the structural stiffness changes after it yields, so the displacement shape vector also changes. The most idealized procedure may be the one that uses the time-variant floor displacement vector as the displacement shape vector and the product of the floor displacement vector and the structural mass matrix as the force distribution at each applied-load step. However, such a procedure requires much computational work. Note that though the stiffness of structures after yielding changes continuously as shown in Fig. 1, with regard to the idealized bilinear curve, the major concern is the stiffnesses $k_1$ and $k_2$, if the analysis is conducted based on the tangential stiffness after the structure yields. As a result, it is suggested that after establishing the idealized bilinear curve, a POA is once again conducted in two




(a) Pushover curve and the idealized bilinear curve




(b) Converted force-displacement relationship for equivalent SDF system

**Fig. 1 Idealized pushover capacity curve and converted curve for equivalent SDF system**

phases: one before and one after the structure yields. In the first phase, the POA is performed by using the first three elastic natural modes of the structure considered, i.e., the same as in the MPA. In the second phase, the POA is performed only for the first mode and the lateral force distribution is based on assuming that the floor displacement vector (distribution) at the initial yielding point is the displacement shape vector and neglecting the contribution of the higher-modes. This is due to the following consideration. It is recognized that the higher-mode equivalent SDOF systems do not contribute much to the inelastic response when the structure's displacement reaches the target displacement, and that the errors arising from elastic computation in calculating the response of higher-mode equivalent SDOF systems can be neglected (Gupta and Kunnath, 2000).

The improved MPA procedure is summarized by implementing the following steps.

(1) Determine the pushover curves and the desired response values $r$ of each-mode system with the MPA procedure.

(2) Obtain the idealized bilinear capacity curve for the first mode system from Step 3 of the MPA procedure described in Section 2.

(3) Determine the displacements vector $\boldsymbol{\varphi}_{1y}$ for all floors at the yielding point with the POA of the first - mode system. The displacements vector is then used as the force distribution of the second phase of the POA for the first mode system.

(4) Repeat the POA for the force distribution: $\boldsymbol{s}_1^* = \boldsymbol{M}\,\boldsymbol{\varphi}_{1y}$ from the structure yielding point, and the values of response quantities, such as base shear, top displacement, etc, are newly obtained. Then, a new pushover curve can be determined by combining these new results and those obtained before the structure yielded.

(5) Determine the response value $r_1$ according to the new pushover capacity curve of the first-mode system obtained from Step 4.

(6) Determine the total seismic demand $r_{\rm total}$ with the SRSS combination rule by combining the result obtained from Step 5 and the results of other higher-mode systems obtained from Step 1.

## 4 Examples

Analyses of low-, moderate- and high-rise buildings are presented to verify the improved MPA procedure. In conducting RHA and POA, the computer program IDARC-2D (Version4.0) developed by the State University of New York at Buffalo (Reinhorn, *et al.*, 2001) is used in this study. IDARC-2D is a program for the inelastic damage analysis of buildings.

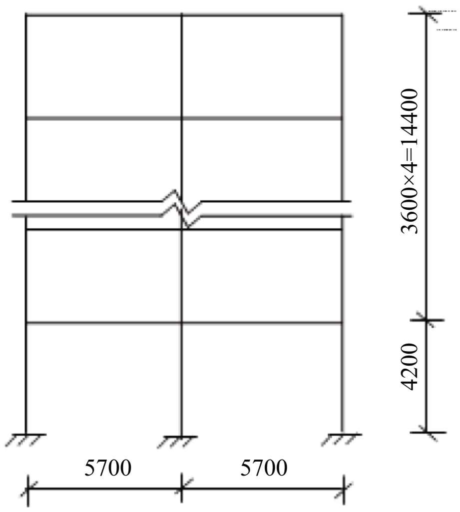



**Fig. 2 Analysis model of the example building 1 (unit: mm)**

**Table 1(a) Dimensions and reinforcement details of beams for example building 1**

| Story number | Concrete properties (MPa) | Dimensions (mm×mm) | | Main reinforcement areas (mm$^2$)/Hoop bar(Diameter & Spacing) (mm) | | | |
|---|---|---|---|---|---|---|---|
| | | | | External span | | Internal span | |
| | | External span | Internal span | Support section | Midspan section | Support section | Midspan section |
| 1st~5th story | C40 | 650×250 | - | 1964/ Φ10@100 | 1964/ Φ10@120 | - | - |

**Table 1(b) Dimensions and reinforcement details of columns for example building 1**

| Story number | Concrete properties (MPa) | Dimensions (mm×mm) | | Main reinforcement areas (mm$^2$)/Hoop bar(Diameter & Spacing) (mm) | |
|---|---|---|---|---|---|
| | | External columns | Internal columns | External columns | Internal columns |
| 1st~5th story | C40 | 700×700 | 700×700 | 2281/ Φ8@100 | 2281/ Φ8@100 |

### 4.1 Example 1

The building is a 5-story, 2-span RC frame structure as shown in Fig. 2. The dimensions and reinforcement details of the beams and columns are given in Tables 1(a) and (b), respectively.

This structure is a typical low-rise structure. Its seismic response is mainly contributed from the first mode, and the contributions of higher modes can be neglected. Thus, if the inverted triangle distribution (the first modal force distribution) is adopted, the error of the POA induced by its first assumption can be neglected. The main error is caused by the second assumption of POA, because the lateral force distribution cannot reflect the change of mode shape after the structure yields. The capacity curves obtained from POA with different lateral force distributions are given in Fig. 3. Here, the lateral force distributions include inverted triangle distribution, uniform distribution, and the improved two-phase load distributions described in the previous section. In Fig. 3, the capacity curves obtained by using incremental dynamic analysis (IDA) under four ground motions recorded at different site soil conditions and their mean curve are also given, which can be considered as the accurate capacity curve of the building considered. The ground motions used are Tianjin (1976, soft soil site), Ninghe (1976, soft soil site), Northridge (1994, Medium soil site) and El Centro (1940, Medium soil site). Note that the capacity curve for the two-phase force distribution proposed in this study is closer to the accurate capacity curve than that obtained from the uniform distribution and inverted triangle distribution. It indicates that the two-phase is more reasonable than the two force distributions mentioned above and can provide a better estimate of structural seismic demands for low-rise structures.

### 4.2 Example 2

The characteristics of the mid-rise structure are taken from Chopra and Goel (2002) with slight modifications. It is a 9-story steel frame structure with a total height of 37.17m as shown in Fig. 4. In the analysis, two types of steel were used for the structure: Q345 for columns and Q235 for beams. The details of the cross sections of the beams and columns and story masses are listed in Table 2.

The structure is analyzed in four ways: (1) POA with inverted triangle distribution; (2) MPA; (3) improved MPA; and (4) IDA under three ground motions (EI Centro, Taft, and Shandon). The structural capacity curves for the first-, second- and third-mode systems obtained from MPA analysis are shown in Fig. 5,

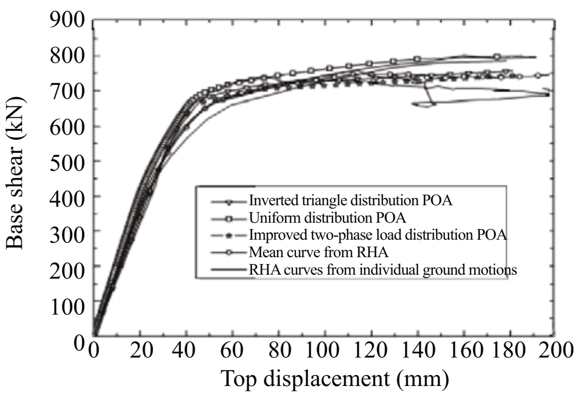



**Fig. 3 Effect of lateral force distribution on capacity curves for the example building 1**

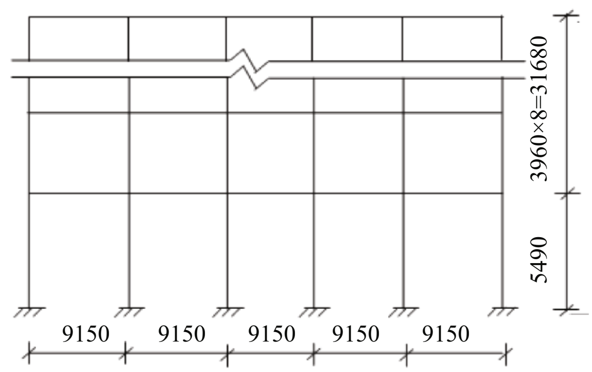



**Fig. 4 Analysis model of the example building 2 (unit: mm)**

**Table 2 Details of cross sections for beams and columns and story masses for example building 2**

| Floor No. | Cross section of beams | Cross section of columns | Story masses ($10^6$kg) |
|---|---|---|---|
| 9 | W24×68 | W14×257 | 1.07 |
| 8 | W27×84 | W14×283 | 0.989 |
| 7 | W30×99 | W14×283 | 0.989 |
| 6 | W36×135 | W14×370 | 0.989 |
| 5 | W36×135 | W14×370 | 0.989 |
| 4 | W36×135 | W14×455 | 0.989 |
| 3 | W36×135 | W14×455 | 0.989 |
| 2 | W36×160 | W14×455 | 0.989 |
| 1 | W36×160 | W14×500 | 1.01 |

together with the first-mode system capacity curve from the improved MPA. It shows that the capacity curve from the improved MPA is the same as that from MPA analysis for the first mode system before the structure yielded. However, the difference becomes much greater after the structure yields.

A comparison of story drift ratios of different analysis procedures is shown in Fig. 6(a), together with the mean values from the RHA under the ground motions stated above. Fig. 6(b) shows the errors of story drift ratios from POA, MPA and improved MPA, compared with those from RHA. From Fig. 6, it can be concluded that:

(1) Compared to POA with inverted triangle lateral force distribution, the improved MPA has a higher accuracy in estimating story drifts. It is also seen that the peak story drift occurs at the lower floors for POA, while it occurs at the higher floors in the improved MPA.

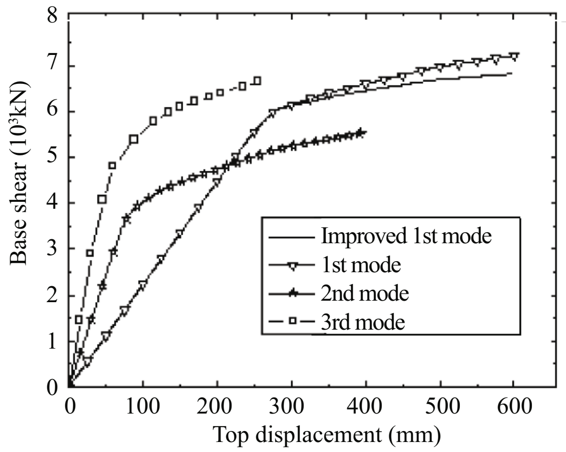
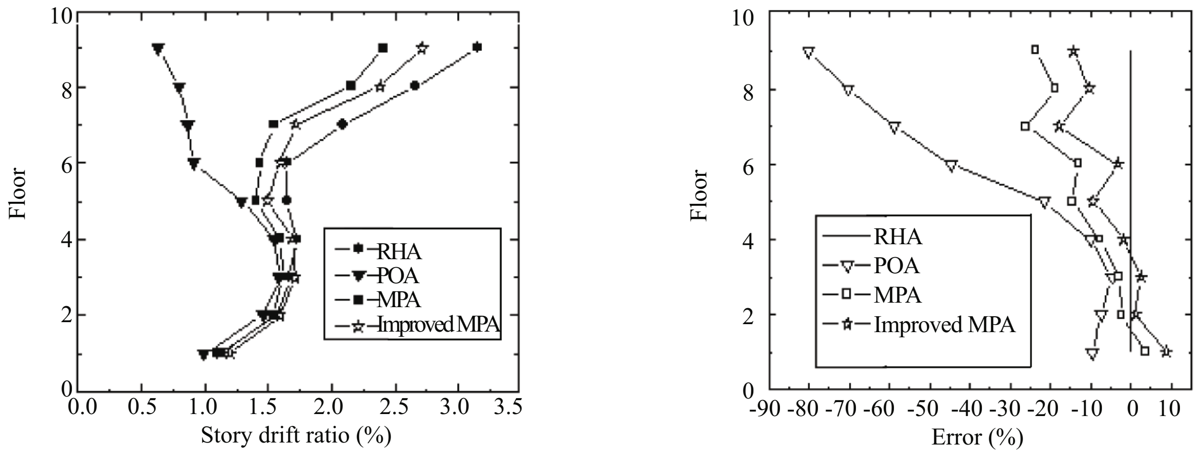



**Fig. 5 Capacity curves of first three-modes SDF systems of the example building 2**




(a) Story drift ratio

(b) Error of story drift ratio

**Fig. 6 Comparison of story drift ratios and their errors obtained by different procedures for the example building 2**

(2) The improved MPA also provides a better estimate of story drift than the MPA due to the consideration of the redistribution of inertia forces after the structure yields.

### 4.3 Example 3

The high-rise is a 22-story RC frame-shear wall structure, and its plan view is shown in Fig.7. The dimensions and reinforcement details of the structure are given in Table 3.

The RC frame-shear wall structure is analyzed in the same way as described in Example 2 for the steel frame structure. The pushover lateral load patterns include inverted triangle distribution, MPA distribution, and the improved MPA distribution. The ground motions of EI Centro (1940), Taft (1952), and Shandon (1966) are selected for the RHA.

As was done in example building 2, the calculated results using four different analysis procedures for example building 3 are plotted in Fig. 8. It is concluded that the improved MPA procedure also provides increased accuracy in estimating seismic demands compared to both POA and MPA for high-rise RC frame-shear wall structures.

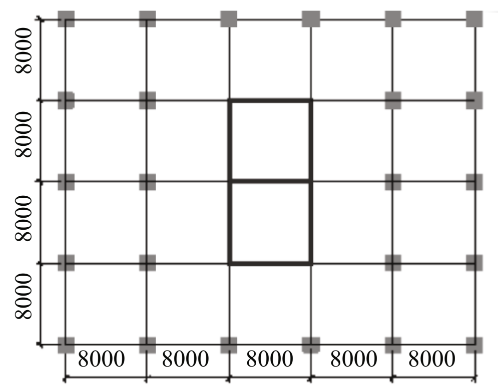



**Fig. 7 Plane view of the example building 3 (unit: mm)**

**Table 3 Dimensions and reinforcement details of the example building 3**

| | |
|---|---|
| Story heights (mm) | 4200 (1st story), 5000 (2nd-8th story), 4300 (9th-22th story) |
| Concrete properties (MPa) | C60 (1st-3rd story), C50 (4th-10th story), C40 (11th-17th story), C30 (18th-22th story) |
| Dimensions of cross sections for columns (mm×mm) | 1300×1300 (1st-3rd story), 1200×1200 (4th-8th story), 1100×1100 (9th-16th story), 1000×1000 (17th-22th story) |
| Main reinforcement areas for columns ($mm^2$) | 4926 (1st-3rd story), 4426 (4th-7th story ), 3706 (8th-16th story ), 3574 (17th-22th story ) |
| Dimensions of cross sections for beams (mm×mm) | 500×700 (1st-22th story ) |
| Main reinforcement areas for beams ($mm^2$) | 6872 (1st-3rd story), 5890 (4th-22th story ) |
| Depth of shear walls (mm) | 600 (1st-5th story), 500 (6th-8th story), 400 (9th-17th story), 300 (18th-22th story) |
| Main reinforcement ratios for shear walls (%) | 0.51 (1st-3rd story), 0.45 (4th-22th story ) |

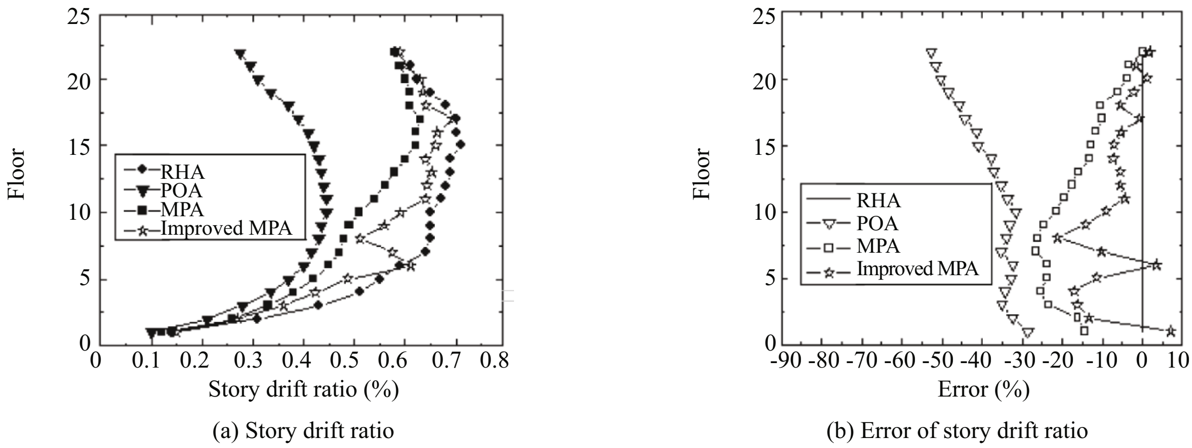



**Fig. 8 Comparison of story drift ratios and their errors obtained by different procedures for the example building 3**

## 5 Conclusions

Recognizing that the modal pushover analysis procedure (MPA) procedure proposed by Chopra and Goel (2001) cannot avoid the shortcoming of adopting invariable lateral force distribution, an improved MPA procedure is presented to estimate the seismic demands of structures. In the proposed procedure, two types of lateral force distributions are used before and after the structure yields to consider the redistribution of inertia forces after the structure yields. To demonstrate the accuracy of the proposed procedure, three numerical examples of 5-, 9- and 22-story buildings under different seismic excitations are provided. From the comparisons, the following conclusions can be drawn:

(1) For low-rise structures, the improved MPA procedure, in which only two-phase first mode force distributions are needed, can provide a better estimate of seismic demands than with traditional lateral force distribution, such as lateral inverted triangle and uniform distribution. Thus, the two-phase first mode force distribution is suggested in practical seismic design for low-rise buildings.

(2) For high-rise structures, the improved MPA procedure is more accurate in estimating seismic demands than both the POA and MPA procedures, because this improved procedure can not only include the contributions of higher modes but also involve the effect of the redistribution of inertia forces after the structure yields. Additionally, the improved MPA procedure also can avoid the large computational resources required by adopting a time-variable displacement shape vector at each applied-load step.

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