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Seismic spatial effects on long-span bridge response in nonstationary inhomogeneous random fields

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Abstract: The long-span bridge response to nonstationary multiple seismic random excitations is investigated using the PEM (pseudo excitation method). This method transforms the nonstationary random response analysis into ordinary direct dynamic analysis, and therefore, the analysis can be solved conveniently using the Newmark, Wilson-θ schemes or the precise integration method. Numerical results of the seismic response for an actual long-span bridge using the proposed PEM are given and compared with the results based on the conventional stationary analysis. From the numerical comparisons, it was found that both the seismic spatial effect and the nonstationary effect are quite important, and that both stationary and nonstationary seismic analysis should pay special attention to the wave passage effect.

Keywords: earthquake; nonstationary; random vibration; multiple excitation; long-span bridge

1 Introduction

Earthquakes are in essence random, and so the random vibration based aseismic design of structures has been gradually accepted by the earthquake engineering community. So far, however, only the comparatively simple stationary random vibration analysis of structures has been put to practical use. Nonstationary random vibration analysis is still thought to be too complicated to use, particularly if the structure considered has many degrees of freedom and/or multiple supports.

In general, a typical strong motion earthquake record can be divided into three stages. In the first stage, the intensity of the ground motion increases, which mainly reflects the motion of P waves. The intensity of the ground motion remains the strongest in the second stage, which is mainly contributed from the S waves. The ground motion will decrease in the last stage. This complete seismic motion is usually regarded as a nonstationary random process. If the nonstationary characteristics are assumed to occur only for the intensity of motion, this random process is usually regarded as a uniformly modulated evolutionary random process (Priestley, 1967; Lin, 1967). However, if the ground motion power spectral density (PSD) curve also varies with time, in other words, both the intensity and the energy distribution with ground motion frequency depend on time, then the ground motion can be regarded as a non-uniformly modulated evolutionary random process (Priestly, 1967), which is nonstationary. Usually, when the intensity of the seismic motion in its second stage appears to be stationary, and at the same time, if the time interval of this stage is much longer (i.e., three times or more) than the fundamental vibration period of the structure under consideration, a simplified, stationary-based random analysis is considered to be acceptable as a replacement for the nonstationary analysis. However, the fundamental vibration period of many long-span bridges range from 10 to 20 seconds, while the stationary portion of a typical strong earthquake is usually only 20-30 seconds. Therefore, nonstationary analyses are necessary for these bridges. The conventional nonstationary random analyses are very inefficient. However, by using the pseudo excitation method (Lin *et al.*, 1997; Lin and Zhang, 2004) combined with the precise integration method (Zhong and Williams, 1995; Lin *et al.*, 1995), such analyses, whether for uniformly or non-uniformly modulated evolutionary random excitations, have become easier and much more efficient. The numerical analysis results for an actual long-span suspension bridge obtained by using this scheme are provided and compared with the results obtained from the more traditional stationary analysis. It is shown that for these long-span bridges, both the seismic spatial effects and the nonstationary effect may

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be very important and should be accounted for in the seismic analysis.

2 PEM for single evolutionary random excitation

Consider a non-uniformly modulated evolutionary random excitation $f(t)$ defined as (Priestly, 1967)

$$f(t)=\int_{-\infty}^{\infty} A(\omega,t)e^{i\omega t}d\alpha(\omega) \tag{1}$$

in which $A(\omega,\ t)$ is a slowly varying non-uniform modulation function and α satisfies the equation

$$E\left[d\alpha^*(\omega_1)d\alpha(\omega_2)\right]=S_{xx}(\omega_1)\delta(\omega_2-\omega_1)d\omega_1 d\omega_2 \tag{2}$$

here $S_{xx}(\omega_1)$ is the auto-PSD of the stationary random process $x(t)$.

It is difficult to perform the Riemann-Stieltjes integration of Eq.(1) in engineering applications. However, these difficulties can be avoided by using PEM as follows. First, the following pseudo excitation (Lin *et al.*, 1997) is formulated:

$$\tilde{f}(\omega,t)=A(\omega,t)\sqrt{S_{xx}(\omega)}e^{i\omega t} \tag{3}$$

Then, a structure subjected to a single seismic random excitation is considered. The equation of motion is

$$\boldsymbol{M}\ddot{\boldsymbol{y}}+\boldsymbol{C}\dot{\boldsymbol{y}}+\boldsymbol{K}\boldsymbol{y}=-\boldsymbol{M}\boldsymbol{e}\ddot{x}_g(t) \tag{4}$$

in which the ground acceleration $\ddot{x}_g(t)$ is an evolutionary random process taking the form of $f(t)$ in Eq. (1), and $\boldsymbol{e}$ is a constant vector characterizing the distribution of inertia forces. Assuming the pseudo acceleration of $\ddot{x}_g(t)$ has the form of Eq. (3), then Eq. (4) becomes

$$\boldsymbol{M}\ddot{\tilde{\boldsymbol{y}}}+\boldsymbol{C}\dot{\tilde{\boldsymbol{y}}}+\boldsymbol{K}\tilde{\boldsymbol{y}}=-\boldsymbol{M}\boldsymbol{e}A(\omega,t)\sqrt{S_{xx}(\omega)}e^{i\omega t} \tag{5}$$

where $S_{xx}(\omega)$ multiplied by $|A(\omega,t)|^2$ is the auto-PSD of the ground acceleration $\ddot{x}_g(t)$. When the structure is initially at rest, the time history $\tilde{\boldsymbol{y}}(\omega,t)$ and, if necessary, the time histories of its arbitrary linear responses $\tilde{\boldsymbol{u}}(\omega,t)$ and $\tilde{\boldsymbol{v}}(\omega,t)$ of interest can be calculated. Thus, the auto- or cross-PSD matrices of the corresponding structural random responses $\boldsymbol{u}(t)$ and $\boldsymbol{v}(t)$ can be obtained from

$$\boldsymbol{S}_{uu}(\omega,t)=\tilde{\boldsymbol{u}}^*(\omega,t)\tilde{\boldsymbol{u}}^T(\omega,t) \tag{6}$$

$$\boldsymbol{S}_{uv}(\omega,t)=\tilde{\boldsymbol{u}}^*(\omega,t)\tilde{\boldsymbol{v}}^T(\omega,t) \tag{7}$$

For uniformly modulated evolutionary random excitations, Eq. (1) reduces to

$$f(t)=g(t)x(t) \tag{8}$$

and $A(\omega,\ t)$ in Eqs. (3) and (5) should be replaced by $g(t)$.

3 PEM for multiple evolutionary random excitations

3.1 Fully coherent excitations

To consider the phase-lags between ground excitations, i.e., the wave passage effect, the zero-mean-valued evolutionary random excitation vector $\boldsymbol{f}(t)$ to which the structure is subjected should be as follows:

$$\boldsymbol{f}(t)=\begin{Bmatrix} F(t-t_1)\\ F(t-t_2)\\ \vdots \\ F(t-t_n)\end{Bmatrix}=\begin{Bmatrix} F_1(t)\\ F_2(t)\\ \vdots \\ F_n(t)\end{Bmatrix}=\begin{Bmatrix} a_1\int_{-\infty}^{\infty}A(\omega,t-t_1)e^{i\omega(t-t_1)}d\alpha_1(\omega)\\ a_2\int_{-\infty}^{\infty}A(\omega,t-t_2)e^{i\omega(t-t_2)}d\alpha_2(\omega)\\ \vdots \\ a_n\int_{-\infty}^{\infty}A(\omega,t-t_n)e^{i\omega(t-t_n)}d\alpha_n(\omega)\end{Bmatrix} \tag{9}$$

in which all $F_j(t)$ have the identical form, although they have different initial time instants t_j (j=1, 2,···,n); $A(\omega,t)$ is a slowly varying non-uniform modulation function; and a_j (j=1, 2,···,n) are given real numbers. $F(t)$ represents a stationary random process, and its auto-PSD $S_{FF}(\omega)$ is known. Using the Wiener-Khintchine relation and denoting $\tau=\tau_l-\tau_k$, one obtains

$$E\left[\boldsymbol{f}(\tau_k)\boldsymbol{f}^T(\tau_l)\right]$$
$$=E\left[\begin{Bmatrix} a_1\int_{-\infty}^{\infty}A(\omega_1,\tau_k-t_1)e^{-i\omega_1(\tau_k-t_1)}d\alpha_1^*(\omega_1)\\ a_2\int_{-\infty}^{\infty}A(\omega_1,\tau_k-t_2)e^{-i\omega_1(\tau_k-t_2)}d\alpha_2^*(\omega_1)\\ \vdots \\ a_n\int_{-\infty}^{\infty}A(\omega_1,\tau_k-t_n)e^{-i\omega_1(\tau_k-t_n)}d\alpha_n^*(\omega_1)\end{Bmatrix}\begin{Bmatrix} a_1\int_{-\infty}^{\infty}A(\omega_2,\tau_l-t_1)e^{i\omega_2(\tau_l-t_1)}d\alpha_1(\omega_2)\\ a_2\int_{-\infty}^{\infty}A(\omega_2,\tau_l-t_2)e^{i\omega_2(\tau_l-t_2)}d\alpha_2(\omega_2)\\ \vdots \\ a_n\int_{-\infty}^{\infty}A(\omega_2,\tau_l-t_n)e^{i\omega_2(\tau_l-t_n)}d\alpha_n(\omega_2)\end{Bmatrix}^T\right]$$

$$
=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\omega_1,t_1)\cdot
$$

$$
\begin{bmatrix}
E\left[\mathrm{d}\alpha_1^*(\omega_1)\mathrm{d}\alpha_1(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_1)-\mathrm{i}\omega_1(\tau_k-t_1)} & E\left[\mathrm{d}\alpha_1^*(\omega_1)\mathrm{d}\alpha_2(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_2)-\mathrm{i}\omega_1(\tau_k-t_1)} & \cdots & E\left[\mathrm{d}\alpha_1^*(\omega_1)\mathrm{d}\alpha_n(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_n)-\mathrm{i}\omega_1(\tau_k-t_1)} \\
E\left[\mathrm{d}\alpha_2^*(\omega_1)\mathrm{d}\alpha_1(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_1)-\mathrm{i}\omega_1(\tau_k-t_2)} & E\left[\mathrm{d}\alpha_2^*(\omega_1)\mathrm{d}\alpha_2(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_2)-\mathrm{i}\omega_1(\tau_k-t_2)} & \cdots & E\left[\mathrm{d}\alpha_2^*(\omega_1)\mathrm{d}\alpha_n(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_n)-\mathrm{i}\omega_1(\tau_k-t_2)} \\
\vdots & \vdots & & \vdots \\
E\left[\mathrm{d}\alpha_n^*(\omega_1)\mathrm{d}\alpha_1(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_1)-\mathrm{i}\omega_1(\tau_k-t_n)} & E\left[\mathrm{d}\alpha_n^*(\omega_1)\mathrm{d}\alpha_2(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_2)-\mathrm{i}\omega_1(\tau_k-t_n)} & \cdots & E\left[\mathrm{d}\alpha_n^*(\omega_1)\mathrm{d}\alpha_n(\omega_2)\right]\mathrm{e}^{\mathrm{i}\omega_2(\tau_l-t_n)-\mathrm{i}\omega_1(\tau_k-t_n)}
\end{bmatrix} \boldsymbol{A}(\omega_2,t_2)
$$

$$
=\int_{-\infty}^{\infty} \boldsymbol{A}(\omega,t_1)\begin{bmatrix} 1 & \mathrm{e}^{\mathrm{i}\omega(t_1-t_2)} & \cdots & \mathrm{e}^{\mathrm{i}\omega(t_1-t_n)} \\ \mathrm{e}^{\mathrm{i}\omega(t_2-t_1)} & 1 & \cdots & \mathrm{e}^{\mathrm{i}\omega(t_2-t_n)} \\ \vdots & \vdots & & \vdots \\ \mathrm{e}^{\mathrm{i}\omega(t_n-t_1)} & \mathrm{e}^{\mathrm{i}\omega(t_n-t_2)} & \cdots & 1 \end{bmatrix} \boldsymbol{A}(\omega,t_2)\mathrm{e}^{\mathrm{i}\omega\tau}S_{FF}(\omega)\mathrm{d}\omega
$$

$$
=\int_{-\infty}^{\infty} \boldsymbol{A}(\omega,t_1)\boldsymbol{V}^*\boldsymbol{R}_0\boldsymbol{V}^{\mathrm{T}}\boldsymbol{A}(\omega,t_2)\mathrm{e}^{\mathrm{i}\omega\tau}S_{FF}(\omega)\mathrm{d}\omega \tag{10}
$$

in which

$$
\boldsymbol{A}(\omega,t)=\mathrm{diag}\left[a_1A(\omega,t-t_1),\, a_2A(\omega,t-t_2),\cdots,\, a_nA(\omega,t-t_n)\right]
$$

$$
\boldsymbol{V}=\mathrm{diag}\left[\mathrm{e}^{-\mathrm{i}\omega t_1},\mathrm{e}^{-\mathrm{i}\omega t_2},\cdots,\mathrm{e}^{-\mathrm{i}\omega t_n}\right] \tag{11}
$$

$$
\boldsymbol{R}_0=\boldsymbol{q}_0\boldsymbol{q}_0^{\mathrm{T}}=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}
$$

$\boldsymbol{R}_0$ is a square matrix and $\boldsymbol{q}_0$ is a column vector, all elements in $\boldsymbol{R}_0$ and $\boldsymbol{q}_0$ are unity. Assume $\boldsymbol{y}(t)$ is an arbitrary response vector excited by $\boldsymbol{f}(t)$, which can be expressed by

$$
\boldsymbol{y}(t)=\int_0^t \boldsymbol{h}(t-\tau)\boldsymbol{f}(\tau)\mathrm{d}\tau \tag{12}
$$

Thus, if $\boldsymbol{y}_k(t_k)$ and $\boldsymbol{y}_l(t_l)$ are two arbitrary response vectors, then their covariance matrix is

$$
\boldsymbol{R}_{y_ky_l}(t_k,t_l)=E\left[\boldsymbol{y}_k(t_k)\boldsymbol{y}_l^{\mathrm{T}}(t_l)\right]
$$

$$
=\int_0^{t_k}\int_0^{t_l}\boldsymbol{h}_k(t_k-\tau_k)E\left[\boldsymbol{f}(\tau_k)\boldsymbol{f}^{\mathrm{T}}(\tau_l)\right]\boldsymbol{h}_l^{\mathrm{T}}(t_l-\tau_l)\mathrm{d}\tau_k\mathrm{d}\tau_l \tag{13}
$$

Substituting Eqs. (10) and (11) into Eq. (13) gives

$$
\boldsymbol{R}_{y_ky_l}(t_k,t_l)=\int_{-\infty}^{\infty}\boldsymbol{I}_k^*\boldsymbol{I}_l^{\mathrm{T}}S_{FF}(\omega)\mathrm{d}\omega \tag{14}
$$

in which

$$
\boldsymbol{I}_k(\omega,t_k)=\int_0^{t_k}\boldsymbol{h}_k(t_k-\tau_k)\boldsymbol{A}(\omega,\tau_k)\boldsymbol{V}\boldsymbol{q}_0\mathrm{e}^{\mathrm{i}\omega\tau_k}\mathrm{d}\tau_k \tag{15}
$$

When $t_k=t_l=t$, Eq. (14) gives the cross-PSD matrix between $\boldsymbol{y}_k$ and $\boldsymbol{y}_l$ as

$$
\boldsymbol{S}_{y_ky_l}(\omega,t)=\boldsymbol{I}_k^*(\omega,t)\boldsymbol{I}_l^{\mathrm{T}}(\omega,t)S_{FF}(\omega) \tag{16}
$$

If let $k = l$, Eq. (16) gives the auto-PSD matrix of $\boldsymbol{y}_k$. It is known from Eq. (15) that $\boldsymbol{I}_k(\omega, t)$ is the response to $\boldsymbol{A}(\omega,t)\boldsymbol{V}\boldsymbol{q}_0\mathrm{e}^{\mathrm{i}\omega t}$. Therefore, by constituting the following pseudo excitation

$$
\tilde{\boldsymbol{f}}(t)=\boldsymbol{A}(\omega,t)\boldsymbol{V}\boldsymbol{q}_0\sqrt{S_{FF}(\omega)}\mathrm{e}^{\mathrm{i}\omega t} \tag{17}
$$

the resulting response will be

$$
\tilde{\boldsymbol{y}}_k(\omega,t)=\sqrt{S_{FF}(\omega)}\boldsymbol{I}_k(\omega,t) \tag{18}
$$

Thus, from Eq. (16), the following equation is obtained.

$$
\boldsymbol{S}_{y_ky_l}(\omega,t)=\tilde{\boldsymbol{y}}_k^*(\omega,t)\tilde{\boldsymbol{y}}_l^{\mathrm{T}}(\omega,t) \tag{19}
$$

3.2 Partially coherent excitations

If the partial coherency between the excitations is taken into account, the matrix $\boldsymbol{R}_0$ in Eq. (11) will be replaced by the following matrix $\boldsymbol{R}$:

$$\boldsymbol{R}=\begin{bmatrix}1 & \rho_{12} & \cdots & \rho_{1n}\\ \rho_{21} & 1 & \cdots & \rho_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{n1} & \rho_{n2} & \cdots & 1\end{bmatrix} \tag{20}$$

$\boldsymbol{R}$ is usually a real symmetric matrix with very low order n, which can be easily decomposed into

$$\boldsymbol{R}=\sum_{j=1}^{r}\alpha_j\boldsymbol{\varphi}_j^*\boldsymbol{\varphi}_j^{\mathrm{T}} \tag{21}$$

α_j is the j-th non-zero eigenvalue of matrix $\boldsymbol{R}$, the corresponding normalized mode is $\boldsymbol{\varphi}_j$ (j=1, 2, r, $r\le n$), r is the rank of $\boldsymbol{R}$. Therefore, the corresponding excitation PSD matrix can be expressed by

$$\begin{aligned}\boldsymbol{S}_{ff}(\omega,t)&=S_{FF}(\omega)\boldsymbol{A}(\omega,t)\boldsymbol{V}^*\boldsymbol{R}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}(\omega,t)\\&=\sum_{j=1}^{r}\alpha_jS_{FF}(\omega)\boldsymbol{A}(\omega,t)\boldsymbol{V}^*\boldsymbol{\varphi}_j^*\boldsymbol{\varphi}_j^{\mathrm{T}}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}(\omega,t)\\&=\sum_{j=1}^{r}\tilde{\boldsymbol{f}}_j^*(t)\tilde{\boldsymbol{f}}_j^{\mathrm{T}}(t)\\&=\sum_{j=1}^{r}\boldsymbol{S}_{f_jf_j}(\omega,t)\end{aligned} \tag{22}$$

Thus the global excitation PSD-matrix is decomposed into the sum of r sub-PSD-matrices. The pseudo excitation corresponding to the j-th sub-PSD-matrix is

$$\tilde{\boldsymbol{f}}_j(t)=\boldsymbol{A}(\omega,t)\boldsymbol{V}\boldsymbol{\varphi}_j\sqrt{\alpha_jS_{FF}(\omega)}\mathrm{e}^{\mathrm{i}\omega t} \tag{23}$$

When the structure is initially at rest, the time histories $\tilde{\boldsymbol{y}}_{kj}(t)$ and $\tilde{\boldsymbol{y}}_{lj}(t)$ for j=1,2, r can be calculated at a specific frequency ω. Thus, the global cross-PSD matrix between these two responses at this ω is

$$S_{y_ky_l}(\omega,t)=\sum_{j=1}^{r}\tilde{\boldsymbol{y}}_{kj}^*(t)\tilde{\boldsymbol{y}}_{lj}^{\mathrm{T}}(t) \tag{24}$$

For uniformly modulated evolutionary random excitations, $A(\omega,t)$ in Eqs.(9), (10) and (11) should be replaced by $g(t)$, and all equations in this section remain valid.

PEM turns the nonstationary equations of random vibration into deterministic transient equations, which can be analyzed in terms of the well-known Newmark or Wilson-θ schemes. The efficiency will be further increased if the precise integration method is alternatively used (Lin *et al.*, 1995; Zhong, 2004), which is briefly described in section 5.

4 Expected extreme values of nonstationary random processes

The evaluation of the peak amplitude of the response of the structure to nonstationary seismic excitations has also received much attention (Shrikhande and Gupta, 1997; Zhao and Liu, 2001). Previously, it was implemented only for very simple structures. By using PEM, however, it can be extended for use on very complicated structures.

In order to evaluate the expected extreme value of response of the structure to nonstationary Gaussian excitations, the duration, during which the intensity of the excitation peaks exceeds 50% of the maximum peak intensity, denoted by $[t_0,\ t_0+\tau]$, is taken as the equivalent stationary duration to evaluate the desired expected extreme values. Once the time-dependant PSD of any arbitrary response $y(t)$, $S_{yy}(\omega,t)$, has been obtained over the equivalent duration using PEM, the equivalent stationary PSD over that duration is

$$S_{yy}^{'}(\omega)=\frac{1}{\tau}\int_{t_0}^{t_0+\tau}S_{yy}(\omega,t)\ \mathrm{d}t \tag{25}$$

To compute the extreme value of responses, the parameters t_0 and τ are

$$t_0=t_1/\sqrt{2},\quad \tau=t_2+\ln 2/c-t_1/\sqrt{2} \tag{26}$$

Thus, the equivalent stationary random responses can be calculated (Davenport, 1961; Lin and Zhang, 2004).

5 Precise integration method

Equation (4) can be written as

$$\boldsymbol{M}\ddot{\tilde{\boldsymbol{y}}}+\boldsymbol{C}\dot{\tilde{\boldsymbol{y}}}+\boldsymbol{K}\tilde{\boldsymbol{y}}=\boldsymbol{f}(t) \tag{27}$$

in which $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are assumed to be time-invariant matrices of $n\times n$ orders, and $\boldsymbol{f}(t)$ is the external force vector. The initial displacement $\tilde{\boldsymbol{y}}(0)$ and the initial velocity $\dot{\tilde{\boldsymbol{y}}}(0)$ of the system are both null. Combining equation of motion (27) with the identity $\dot{\tilde{\boldsymbol{y}}}=\dot{\tilde{\boldsymbol{y}}}$ lead to the first-order equation of motion in the state space as follows:

$$\dot{\boldsymbol{v}}=\boldsymbol{H}\boldsymbol{v}+\boldsymbol{r} \tag{28}$$

in which

$$\boldsymbol{H}=\begin{bmatrix}0 & \boldsymbol{I}\\ \boldsymbol{B} & \boldsymbol{D}\end{bmatrix},\quad \boldsymbol{v}=\begin{Bmatrix}\tilde{\boldsymbol{y}}\\ \dot{\tilde{\boldsymbol{y}}}\end{Bmatrix},\quad \boldsymbol{r}=\begin{Bmatrix}0\\ \boldsymbol{M}^{-1}\boldsymbol{f}(t)\end{Bmatrix},$$
$$\boldsymbol{B}=-\boldsymbol{M}^{-1}\boldsymbol{K},\quad \boldsymbol{D}=-\boldsymbol{M}^{-1}\boldsymbol{C} \tag{29}$$

The homogeneous solution of Eq. (28) is

$$\boldsymbol{v}_h(t)=\boldsymbol{T}(\tau)\boldsymbol{c} \tag{30}$$

in which

$$\boldsymbol{T}(\tau)=\exp(\boldsymbol{H}\tau) \tag{31}$$

Consider the current integration interval $t\in(t_k,\ t_{k+1})$, $\tau=t-t_k$. When τ=0 or $t=t_k$, $\boldsymbol{T}(\tau)=\boldsymbol{I}$ and therefore $\boldsymbol{c}$ is a constant vector. If the particular solution to Eq. (28), $v_p(t)$, is temporarily assumed to have been found, then the general solution of Eq. (28) is

$$v(t)=T(\tau)\left[v(t_k)-v_p(t_k)\right]+v_p(t) \tag{32}$$

Now, let $t=t_{k+1}$, or $\tau=t_{k+1}-t_k$ which represents a full step size, then $v(t_{k+1})$ can be obtained from Eq. (32). In order to compute $T(\tau)$ accurately, it is desirable to subdivide the step τ into m=2^N equal intervals, i.e.

$$\Delta t=\tau/m=2^{-N}\tau \tag{33}$$

For application purposes, the use of N=20 is sufficient, because it leads to $\Delta t\approx 10^{-6}\tau$. Such a small Δt is in general much less than the highest natural vibration period of any practical discretized system.

Using Taylor expansion

$$\exp(H\times\Delta t)\approx I+T_{a0} \tag{34}$$

in which

$$T_{a0}=(H\times\Delta t)+(H\times\Delta t)^2/2!+(H\times\Delta t)^3/3!+(H\times\Delta t)^4/4! \tag{35}$$

Substituting Eq. (34) into Eq. (31) gives

$$T(\tau)=\left(\exp(H\times\Delta t)\right)^m=\left(I+T_{a0}\right)^m \tag{36}$$

Note that

$$I+T_{ai}=(I+T_{a,i-1})^2=(I+2\times T_{a,i-1}+T_{a,i-1}\times T_{a,i-1}),\quad (i=1,\ 2,\ \cdots,\ N) \tag{37}$$

so that clearly

$$I+T_{ai}=(I+T_{a,N-1})^2=(I+T_{a,N-2})^4=\cdots=(I+T_{a0})^m=T(\tau) \tag{38}$$

Equations (37) and (38) suggest the following computing strategy. In order to avoid the loss of significant digits in the matrix $T(\tau)$, it is necessary to compute T_{a1} directly from T_{a0}, compute T_{a2} directly from T_{a1} , etc. by using

$$T_{ai}=2\times T_{a,i-1}+T_{a,i-1}\times T_{a,i-1},\qquad (i=1,\ 2,\ \cdots,\ N) \tag{39}$$

Then $T(\tau)$ should be computed from

$$T(\tau)\approx I+T_{aN} \tag{40}$$

In Eq. (40), the error is caused by the truncation of the Taylor expansion of Eq. (35). It is generally negligibly small because when N=20, the first term ignored by the truncation is near $\mathrm{O}(\Delta t^5)=10^{-30}\mathrm{O}(\tau^5)$, which is usually less than or close to the round-off errors of ordinary computers. In Eq. (32), $v_p(t)$ can be derived for different loading forms, as follows:

5.1 Linear loading (HPD-L) form

Assume that the loading varies linearly within t79time step (t_k,t_{k+1}) , i.e.

$$r=r_0+r_1\times(t-t_0) \tag{41}$$

in which r_0 and r_1 are time invariant vectors. The particular solution of Eq. (28) is then (Zhong and Williams, 1995; Lin *et al.*, 1995)

$$v_p(t)=(H^{-1}+It)(-H^{-1}r_1)-H^{-1}(r_0-r_1t_k) \tag{42}$$

Substituting Eq. (41) into Eq. (32) gives the HPD-L (High Precision Direct integration —Linear) formula

$$v(t_{k+1})=T(\tau)\left[v(t_k)+H^{-1}(r_0+H^{-1}r_1)\right]-H^{-1}(r_0+H^{-1}r_1+r_1\tau) \tag{43}$$

The time interval is $\tau=t_{k+1}-t_k$.

5.2 Sinusoidal loading (HPD-S) form

If the applied loading is sinusoidal within the time region $t\in(t_k,\ t_{k+1})$, then

$$r(t)=r_1\sin\omega t+r_2\cos\omega t \tag{44}$$

in which r_1 and r_1 are time invariant vectors. Substituting Eq. (44) into Eq. (28) enables the particular solution to be obtained (Lin *et al.*, 1995; Zhong, 2004)

$$v_p(t)=v_1\sin\omega t+v_2\cos\omega t \tag{45}$$

in which

$$v_1=(\omega I+H^2/\omega)^{-1}(r_2-Hr_1/\omega)$$
$$v_2=(\omega I+H^2/\omega)^{-1}(-r_1-Hr_2/\omega) \tag{46}$$

Substituting Eq. (45) into Eq. (32) gives the general solution of Eq. (28), i.e., the HPD-S direct integration formula

$$v(t_{k+1})=T(\tau)\left[v(t_k)-v_1\sin\omega t_k-v_2\cos\omega t_k\right]+v_1\sin\omega t_{k+1}+v_2\cos\omega t_{k+1} \tag{47}$$

The time interval $\tau=t_{k+1}-t_k$ can cover an arbitrary segment, or even many periods, of a sinusoidal wave because no matter how large the step size may be, exact responses will be obtained provided the matrix $T(\tau)$ has been generated accurately and instability has not occurred.

5.3 Polynomial-modulated sinusoidal loading (HPD-P) form

If the applied loading is polynomial-modulated sinusoidal within the time region $t\in(t_k,\ t_{k+1})$:

$$\boldsymbol{r}(t)=\left[\boldsymbol{r}_0+\boldsymbol{r}_1 t+\boldsymbol{r}_2 t^2\right](\alpha\sin\omega t+\beta\cos\omega t) \quad (48)$$

in which $\boldsymbol{r}_0$, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are time invariant vectors, α and β are given constants. The particular solution of Eq. (28) is then (Lin, Shen and Williams 1995a,b; Lin and Zhang 2004)

$$\boldsymbol{v}_p(t)=\left(\boldsymbol{a}_0+\boldsymbol{a}_1 t+\boldsymbol{a}_2 t^2\right)\sin\omega t+\left(\boldsymbol{b}_0+\boldsymbol{b}_1 t+\boldsymbol{b}_2 t^2\right)\cos\omega t \quad (49)$$

in which

$$\begin{aligned}\boldsymbol{a}_i&=\left(\boldsymbol{H}^2+\omega^2\boldsymbol{I}\right)^{-1}\left(-\boldsymbol{H}\boldsymbol{P}_{ia}+\omega\boldsymbol{P}_{ib}\right)\\ \boldsymbol{b}_i&=\left(\boldsymbol{H}^2+\omega^2\boldsymbol{I}\right)^{-1}\left(-\boldsymbol{H}\boldsymbol{P}_{ib}-\omega\boldsymbol{P}_{ia}\right)\end{aligned}\quad (i=2,1,0) \quad (50)$$

and

$$\begin{aligned}&\boldsymbol{P}_{2a}=\alpha\boldsymbol{r}_2 && \boldsymbol{P}_{2b}=\beta\boldsymbol{r}_2\\ &\boldsymbol{P}_{1a}=\alpha\boldsymbol{r}_1-2\boldsymbol{a}_2 && \boldsymbol{P}_{1b}=\beta\boldsymbol{r}_1-2\boldsymbol{b}_2 \quad (i=2,1,0)\\ &\boldsymbol{P}_{0a}=\alpha\boldsymbol{r}_0-\boldsymbol{a}_1 && \boldsymbol{P}_{0b}=\beta\boldsymbol{r}_0-\boldsymbol{b}_1\end{aligned} \quad (51)$$

5.4 Exponentially decaying sinusoidal loading (HPD-E) form

Suppose that the applied loading varies according to the following exponentially decaying sinusoidal law within the time region $t\in(t_k,\ t_{k+1})$

$$\boldsymbol{r}(t)=\exp(\alpha t)(\boldsymbol{r}_1\sin\omega t+\boldsymbol{r}_2\cos\omega t) \quad (52)$$

in which $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are time invariant vectors. Substituting Eq. (52) into Eq. (28) enables the particular solution to be obtained (Lin *et al.*, 1995; Zhong, 2004) as

$$\boldsymbol{v}_p(t)=\exp(\alpha t)\left(\boldsymbol{v}_1\sin\omega t+\boldsymbol{v}_2\cos\omega t\right) \quad (53)$$

in which

$$\begin{aligned}\boldsymbol{v}_1&=\left[(\alpha\boldsymbol{I}-\boldsymbol{H}^2)+\omega^2\boldsymbol{I}\right]^{-1}\left[(\alpha\boldsymbol{I}-\boldsymbol{H})\boldsymbol{r}_1+\omega\boldsymbol{r}_2\right]\\ \boldsymbol{v}_2&=\left[(\alpha\boldsymbol{I}-\boldsymbol{H}^2)+\omega^2\boldsymbol{I}\right]^{-1}\left[(\alpha\boldsymbol{I}-\boldsymbol{H})\boldsymbol{r}_2-\omega\boldsymbol{r}_1\right]\end{aligned} \quad (54)$$

Then, substituting Eq. (54) into Eq. (32) gives the general solution of Eq. (28), i.e., the HPD-E direct integration formula

$$\begin{aligned}\boldsymbol{v}(t_{k+1})=&\boldsymbol{T}(\tau)\left[\boldsymbol{v}(t_k)-\exp(\alpha t_k)\left(\boldsymbol{v}_1\sin\omega t_k+\boldsymbol{v}_2\cos\omega t_k\right)\right]\\ &+\exp(\alpha t_{k+1})\left(\boldsymbol{v}_1\sin\omega t_{k+1}+\boldsymbol{v}_2\cos\omega t_{k+1}\right)\end{aligned} \quad (55)$$

6 Numerical comparisons with the corresponding stationary analysis

Based on the stationary random seismic response analysis of the Wanxin Bridge performed by Lin *et al.* 2004, the nonstationary random seismic response analysis is extended herein. The FE model of the Wanxin Suspension Bridge is shown in Fig.1.

The ground acceleration response spectrum was generated based on the National Criteria of People's Republic of China (2001) with a regional fortification intensity 7, site-category 2, and seismic classification 1. In the mode-superposition, 100 modes were used.

Figures 2 through 4 show the expected extreme value distributions of axial forces, transverse shear forces and vertical shear forces along the deck under horizontally traveling P, SH and SV waves. The results are compared with those from the corresponding stationary random vibration analyses under ground motion conditions: (1) Uniformly (i.e., at an apparent wave speed $v_{app}=\infty$); (2) with a limited apparent wave speed, v_{app} which is 3 km/s for P waves and 2 km/s for S waves, meaning that the wave passage effect was taken into account; or 3) with the same apparent wave speeds as above and using the QWW model (Qu *et al.*, 1996) to account for the incoherence effect. Therefore, each of Figs. 2-4 has six curves, three for stationary random responses and three for nonstationary random responses.

The nonstationary random excitation model $z(t)=g(t)x(t)$ was used, where the auto-PSD of $x(t)$ was assumed to be the same as for the stationary excitation (Lin *et al.*, 2004). The frequency domain and the parameters used also remained the same. The modulation function had the form

$$g(t)=\begin{cases}I_0\left(t/t_1\right)^2 & 0\le t\le t_1\\ I_0 & t_1\le t\le t_2\\ I_0\exp\left[c(t-t_2)\right] & t\ge t_2\end{cases} \quad (56)$$

with t_1=8.0s, t_2=20.0s and c=0.20. The duration of the earthquake was $t\in[0,25\text{s}]$, and the time step-size was Δt=0.5s. The precise integration HPD-P, HPD-S and HPD-E forms were used for the three parts of $g(t)$.

Figures 2-4 show that for this long-span bridge, the wave passage effect is quite important in the seismic analysis. Comparatively, the incoherence effect is not as important. In addition, whether for uniform or differential ground motion, the nonstationary responses are always smaller than the corresponding stationary responses. In other words, the stationary-based analyses generally provide conservative results. Typically, the results from nonstationary-based analyses are reduced about 10%-20%.

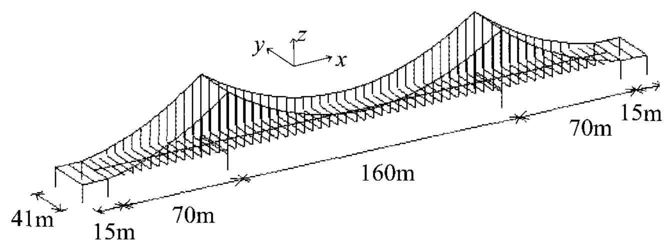

Fig. 1 FE model of Wanxin Suspension Bridge

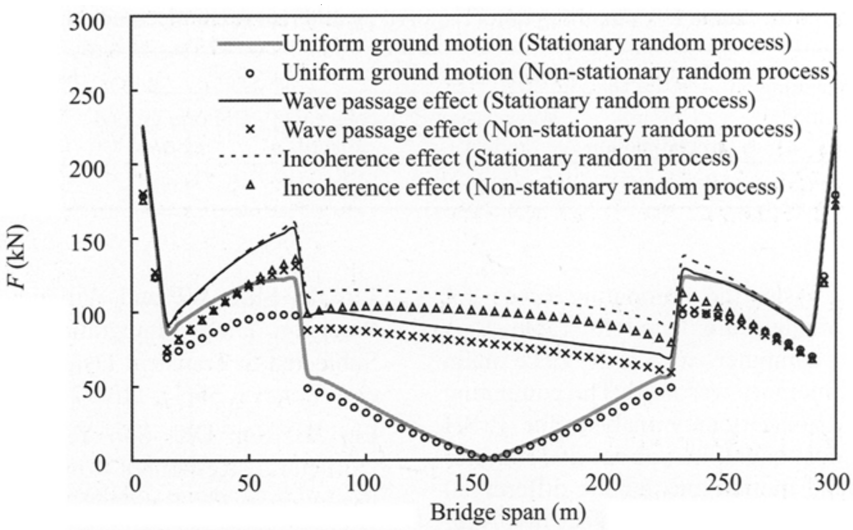

Fig. 2 Axial force distribution along the deck under P waves

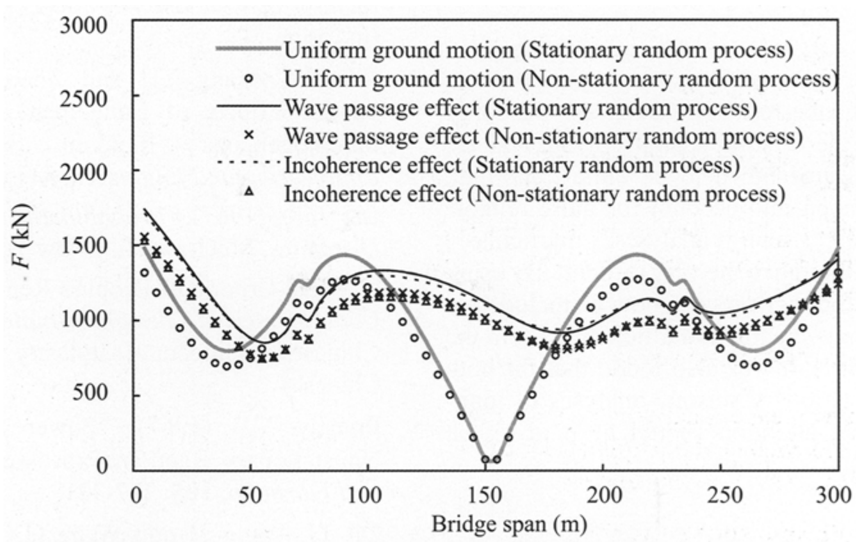

Fig. 3 Transverse shear force distribution along the deck under SH waves

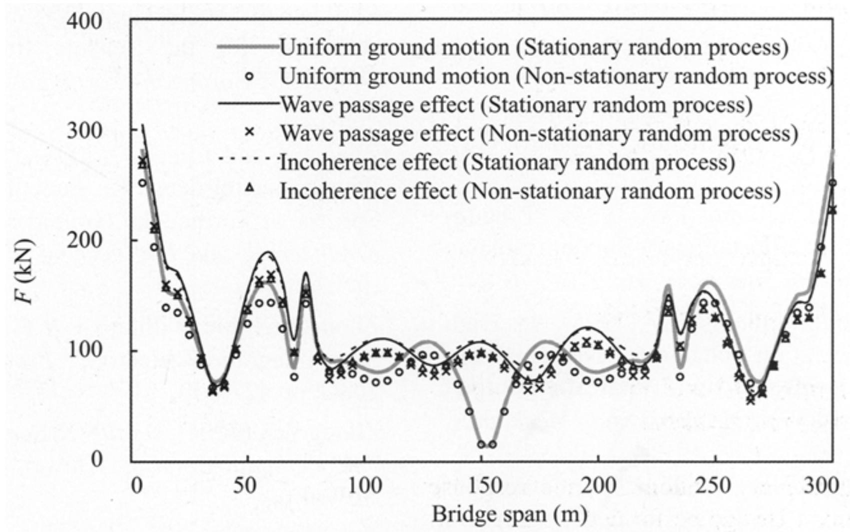

Fig. 4 Vertical shear force distribution along the deck under SV waves

Table 1 Computing times required by different excitation models

Multi- excitation models	Computing 100 modes	Stationary			Nonstationary		
		Uniform ground motion	Wave passage effect	Incoherence effect	Uniform ground motion	Wave passage effect	Incoherence effect
Computing times	23s	9s	10s	127s	297s	319s	4503s

In the above analysis, the computing times for different excitation models are listed in Table 1. A Pentium-4 personal computer with 3.0 GHz main frequency and 512M memory was used. The computing times required by the nonstationary analyses for P, SH or SV waves are almost equal, i.e., about five minutes for the uniform ground motion and for the differential ground motion with the wave passage effect involved; and about 75 minutes for the differential ground motion with both the wave passage effect and incoherence effect involved.

7 Conclusions

For the earthquake-resistant analyses of long-span bridges, both the seismic spatial effects, and the nonstationary effect are shown to be quite important, and both need be taken into account for more reliable designs. Previously, such analyses, particularly nonstationary, were thought to be very difficult. By using the proposed PEM, these effects can now be included in the computations conveniently and efficiently. From the numerical comparisons, it was also found that for both stationary and nonstationary seismic analyses of long-span bridges, special attention should be paid to the wave passage effect.

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